ECE 637 Lab 6 - Introduction to Colorimetry

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Spring 2018

1: Introduction

In order to understand natural images, one needs to understand how objects reflect light and how humans perceive that reflected light as color. The objective of this laboratory is to introduce the basic ideas behind colorimetry, the quantitative measurement and manipulation of color.

2: Plotting Color Matching Functions and Illuminants

2.1 Plot $x_0(\lambda), y_0(\lambda), z_0(\lambda)$ Color Matching Functions

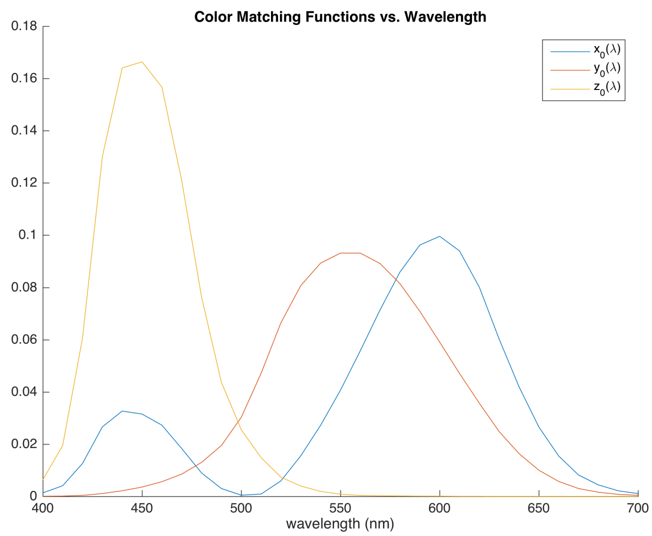

Figure 1: $x_0(\lambda), y_0(\lambda), z_0(\lambda)$ Color Matching Functions

2.2 Plot $l_0(\lambda), m_0(\lambda), s_0(\lambda)$ Color Matching Functions

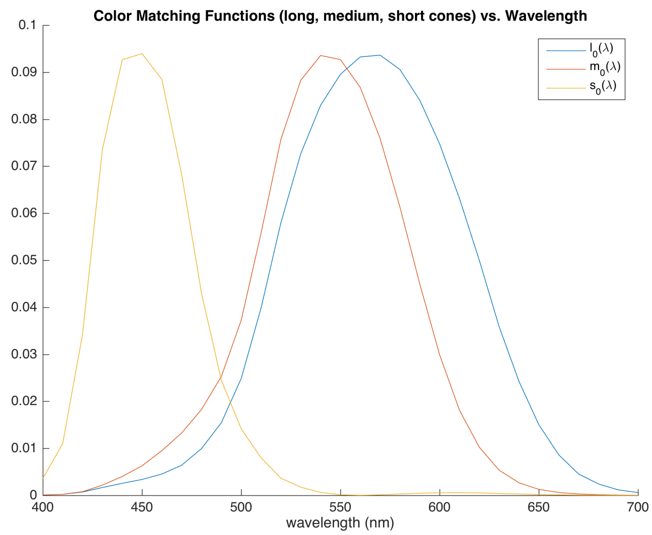

Figure 2: $l_0(\lambda), m_0(\lambda), n_0(\lambda)$ Color Matching Functions

2.3The plot of the D65 and fluorescent illuminants

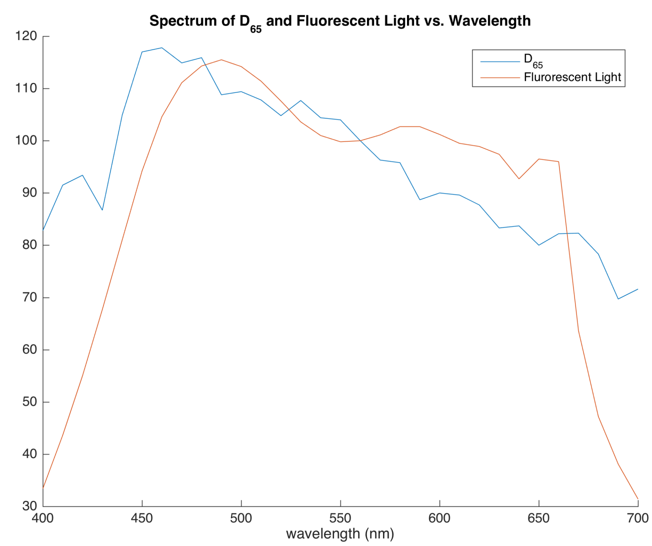

Figure 3: D65 and fluorescent illuminants

3: Chromaticity Diagrams

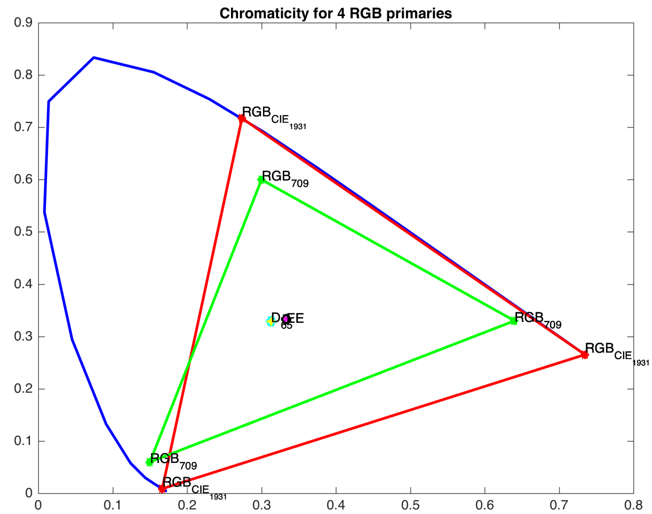

Figure 4: Chromaticity Diagrams

4: Rendering an Image from Illuminant, Reflectance, and Color Matching Functions

4.1 The matrix M709 D65

From equation (15), we have:

$$M = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} \begin{bmatrix} \kappa_r & 0 & 0 \\ 0 & \kappa_g & 0 \\ 0 & 0 & \kappa_b \end{bmatrix} \tag{1}$$

From equation (17), we have:

$$\begin{bmatrix} \kappa_r \\ \kappa_g \\ \kappa_b \end{bmatrix} = \begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix}^{-1} \begin{bmatrix} x_{wp}/y_{wp} \\ 1 \\ z_{wp}/y_{wp} \end{bmatrix}. \tag{2}$$

From equation (19), for 709 RGB we have:

$$\begin{bmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_b \end{bmatrix} = \begin{bmatrix} 0.640 & 0.300 & 0.150 \\ 0.330 & 0.600 & 0.060 \\ 0.030 & 0.100 & 0.790 \end{bmatrix} \tag{3}$$

From equation (9), for D65 illuminant the white point is:

$$\begin{bmatrix} x_{wp} \\ y_{wp} \\ z_{wp} \end{bmatrix} = \begin{bmatrix} 0.3127 \\ 0.3290 \\ 0.3583 \end{bmatrix}. \tag{4}$$

Combining (3) and (4), we have:

$$\begin{bmatrix} \kappa_r \\ \kappa_g \\ \kappa_b \end{bmatrix} = \begin{bmatrix} 0.640 & 0.300 & 0.150 \\ 0.330 & 0.600 & 0.060 \\ 0.030 & 0.100 & 0.790 \end{bmatrix}^{-1} \begin{bmatrix} 0.3127/0.3290 \\ 1 \\ 0.3583/0.3290 \end{bmatrix}$$

yields to:

$$\begin{bmatrix} \kappa_r \\ \kappa_g \\ \kappa_b \end{bmatrix} = \begin{bmatrix} 0.6444 \\ 1.1919 \\ 1.2032 \end{bmatrix}$$

Finally, M_{709_D65} is

$$M = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$$

4.2 Plot Images Obtained from D65 and Fluorescent Light Sources

(a) D65 (b) Fluorescent Light

Figure 5: Images Obtained from D65 and Fluorescent Light Sources

4.3 Qualitative Description of Two Images

The image obtained from the fluorescent light source is a little brighter than than the image obtained from the D_65 light source.

5: Color Chromaticity Diagram

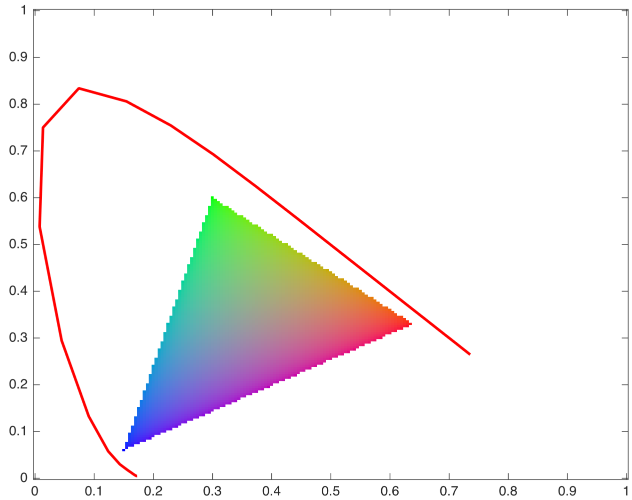

Figure 6: Color Chromaticity Diagrams

The pure spectral source outline is shown in red.

Attachments: code

Q2

```
clear all;
clc;

data = load('data.mat');
lambda = 400:10:700;
x = data.x;
y = data.y;
z = data.z;
illum1 = data.illum1;
illum2 = data.illum2;

A_inv = [0.2430 0.8560 -0.044;
     -0.3910 1.1650 0.0870;
     0.0100 -0.0080 0.5630];

%% Plot
set(0, 'DefaultLineLineWidth', 2);
figure(1);
hold on;
plot(lambda, x);
plot(lambda, y);
plot(lambda, z);
```

```
legend('x_{0}(\lambda)', 'y_{0}(\lambda)', 'z_{0}(\lambda)');
xlabel('wavelength (nm)');
title('Color Matching Functions vs. Wavelength');
print('-dpng', '-r300', 'xyzmatching.png');

lms = A_inv * [x;y;z];
l = lms(1,:);
m = lms(2,:);
s = lms(3,:);

figure(2);
hold on;
plot(lambda, l);
plot(lambda, m);
plot(lambda, s);
legend('l_{0}(\lambda)', 'm_{0}(\lambda)', 's_{0}(\lambda)');
xlabel('wavelength (nm)');
title('Color Matching Functions (long, medium, short cones) vs. Wavelength');
print('-dpng', '-r300', 'lmnmatching.png');

figure(3);
hold on;
plot(lambda, illum1);
plot(lambda, illum2);
legend('D_{65}', 'Flurorescent Light');
xlabel('wavelength (nm)');
title('Spectrum of D_{65} and Fluorescent Light vs. Wavelength');
print('-dpng', '-r300', 'illuminants.png');
```

Q3

```
clear all;
clc;

%% read data
data = load('data.mat');
lambda = 400:10:700;
x0 = data.x;
y0 = data.y;
z0 = data.z;

%% equations (4)
x = x0(:) ./ (x0(:) + y0(:) + z0(:));
y = y0(:) ./ (x0(:) + y0(:) + z0(:));

D_65 = [0.3127 0.3290 0.3583];
EE = [0.3333 0.3333 0.3333];

RGB_CIE_1931 = [0.73467 0.26533 0.0;
    0.27376 0.71741 0.00883;
    0.16658 0.00886 0.82456;
    0.73467 0.26533 0.0];

RGB_709 = [0.64 0.33 0.03;
```

```
    0.3  0.6  0.1;
    0.15  0.06  0.79;
    0.64  0.33  0.03];
%% Plot
figure(5);
set(0, 'DefaultLineLineWidth', 2);
plot(x,y,'b-');
hold on;
plot(RGB_CIE_1931(:,1), RGB_CIE_1931(:,2), 'r*-');
text(RGB_CIE_1931(:,1), RGB_CIE_1931(:,2), 'RGB_{CIE_{1931}}');
plot(RGB_709(:,1), RGB_709(:,2), 'g*-');
text(RGB_709(:,1), RGB_709(:,2), 'RGB_{709}');
plot(D_65(1), D_65(2), 'y+');
text(D_65(1), D_65(2), 'D_{65}');
plot(EE(1), EE(2), 'ko');
text(EE(1), EE(2), 'EE');
title('Chromaticity for 4 RGB primaries');
print('-dpng', '-r300', 'Q3_chromaticity.png');
```

Q4

```
clear all;
clc;

%% read data
data = load('data.mat');
reflect = load('reflect.mat');
lambda = 400:10:700;
R = reflect.R;
x = data.x;
y = data.y;
z = data.z;
illum1 = data.illum1;
illum2 = data.illum2;

%% 170*256*31
[m, n, j] = size(R);
I = zeros(size(R));
for p = 1:m
        for q = 1:n
        for i = 1:j
%                I(p,q,i) = R(p,q,i)* illum1(i);
             I(p,q,i) = R(p,q,i)* illum2(i);
        end
    end
end

%%
[m, n, ~] = size(I);
XYZ = zeros(m,n,3);
for p = 1:m
        for q = 1:n
        XYZ(p,q,:) = permute(I(p,q,:), [1 3 2]) * [x;y;z]';
        end
```

```
end

%%
D_65 = [0.3127 0.3290 0.3583];
RGB_709 = [0.64 0.3 0.15;
     0.33 0.6 0.06;
     0.03 0.1 0.79];
XYZ_wp = [D_65(1)/D_65(2) 1 D_65(3)/D_65(2)];
k_rgb = inv(RGB_709) * XYZ_wp';
M = RGB_709 * diag(k_rgb);

%%
rgb = zeros(m,n,3);
for p = 1:m
        for q = 1:n
                rgb(p,q,:) = inv(M) * permute(XYZ(p,q,:), [1 3 2])';
        end
end

%%
rgb(rgb(:) > 1) = 1;
rgb(rgb(:) < 0) = 0;

figure(1);
image(rgb);

%% Gama-correct
gc_rgb = zeros(m,n,3);
gc_rgb = 255 * ((rgb).^(1/2.2));

figure(2);
image(uint8(gc_rgb));
imwrite(uint8(gc_rgb), 'd65Img2.png');
```

Q5

```
clear all;
clc;

[x, y] = meshgrid(0:0.005:1);
z = 1 - y - x;

RGB_709 = [0.64 0.3 0.15;
     0.33 0.6 0.06;
     0.03 0.1 0.79];
M_709 = RGB_709 * eye(3);

[m, n] = size(x);
XYZ = zeros(m,n,3);

XYZ(:,:,1) = x;
XYZ(:,:,2) = y;
XYZ(:,:,3) = z;
```

```
rgb = zeros(m,n,3);

for p = 1:m
        for q = 1:n
                rgb(p,q,:) = inv(M_709) * permute(XYZ(p,q,:), [1 3 2])';
                if any(rgb(p,q,:) < 0)
                        rgb(p,q,:) = [1, 1, 1];
                end
        end
end

gama_rgb = ((rgb).^(1/2.2));

figure(1);
image([0:0.005:1], [0:0.005:1], gama_rgb);
axis('xy');
hold on;

data = load('data.mat');
x0 = data.x;
y0 = data.y;
z0 = data.z;

x1 = x0(:) ./ (x0(:) + y0(:) + z0(:));
y1 = y0(:) ./ (x0(:) + y0(:) + z0(:));
plot(x1, y1, 'r-');
print('-dpng', '-r300', 'Q5.png');
```