ECE 661 Homework 1

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Problem 1 Solution

All the points:

$$p = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \tag{1}$$

where z is not 0.

Problem 2 Solution

No.

Not all the points at infinity are the same, since those points can reach infinity by different directions in $\mathbb{R}$.

To be more specific, suppose the homogeneous coordinate vector of a point at infinity as $(x, y, 0)$, and the direction of this vector is determined by both x and y. A simple case is that $(1, 0, 0)$ reaches its infinity along x-axis, while $(0, 1, 0)$ reaches its infinity along y-axis.

Problem 3 Solution

The matrix of a degenerate conic is:

$$C = lm^T + ml^T$$

where l and m are the space parameter vectors of two different lines. Assume

$$l = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \tag{2}$$

$$m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \tag{3}$$

Then

$$lm^T = \begin{bmatrix} l_1 m_1 & l_1 m_2 & l_1 m_3 \\ l_2 m_1 & l_2 m_2 & l_2 m_3 \\ l_3 m_1 & l_3 m_2 & l_3 m_3 \end{bmatrix} \tag{4}$$

So $Rank(lm^T) = 1$. Similarly we can get $Rank(l^T m) = 1$ According to the inequality:

$$Rank(A) + Rank(B) \geq Rank(A + B)$$

which means that adding two matrices can not lead to a matrix with bigger ranks. The reason lies in that the output matrix is just linear combinations of the columns the input matrices.

As a result:

$$Rank(C) \leq Rank(lm^T) + Rank(l^T m) = 2$$

which means that the rank of a degenerate conic matrix cannot exceed 2.

Problem 4 Solution

To find the line passing two points, take cross product of the homogeneous coordinate representations:

1. **Step 1:find** l_1

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ 0 \end{bmatrix} \tag{5}$$

2. **Step 2:find** l_2

$$l_2 = \begin{bmatrix} -6 \\ 8 \\ 1 \end{bmatrix} \times \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 12 \end{bmatrix} \tag{6}$$

3. **Step 3:find intersection**

$$p = \begin{bmatrix} -6 \\ 2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 6 \\ 3 \\ 12 \end{bmatrix} = \begin{bmatrix} 24 \\ 72 \\ -30 \end{bmatrix} \tag{7}$$

So the point (x,y) of intersection in the physical plane $\mathbb{R}$ is given by:

$$x = \frac{24}{-30} = -\frac{4}{5}, y = \frac{72}{-30} = -\frac{12}{5}$$

If the second line crosses through (-10,-3) and (10,3), we may take just one step to find the intersection point is actually (0,0). For the first line, it is apparent that it crosses (0,0). For the second line, similarly we have:

$$p = \begin{bmatrix} -10 \\ -3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 20 \\ 0 \end{bmatrix} \tag{8}$$

which means that the third coordinate in HC is zero for the second line, suggesting that this line will have the form $ax + by = 0$. As a result, the second line also passes original point. In conclusion, it will only take us one step to find the intersection point is (0,0).

Problem 5 Solution

To find the line passing two points, take cross product of the homogeneous coordinate representations:

1. **Step 1:find** l_1

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \tag{9}$$

2. **Step 2:find** l_2

$$l_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix} \tag{10}$$

3. **Step 3:find intersection**

$$p = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 18 \\ -18 \\ 0 \end{bmatrix} \tag{11}$$

Comment: For the line representation in HC,

$$l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{12}$$

the slope is given by the ratio a and b. Here $abs(a/b) = 1$, which means these lines are parallel with each other and they will intersect at infinity. From another perspective, the solution shows the intersection point is an ideal point, also proving that the two lines intersect at infinity, along the direction having a slope of -1.

Problem 6 Solution

The equation of a circle with (5,5) as the center and 1 as radius is given by:

$$(x-5)^2 + (y-5)^2 = 1$$

i.e:

$$x^2 + y^2 - 10x - 10y + 49 = 0$$

The homogeneous coordinates of the conic is therefore:

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix} \tag{13}$$

The polar line is then:

$$l = CX = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ 49 \end{bmatrix} \tag{14}$$

The homogeneous coordinate vector of the intersection point p of l and the y-axis(x = 0) is:

$$p = \begin{bmatrix} -5 \\ -5 \\ 49 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 49 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 9.8 \\ 1 \end{bmatrix} \tag{15}$$

As a result, the polar line intersects the y-axis at the physical 2D point (0, 9.8).

Problem 7 Solution

To find the line passing two points, take cross product of the homogeneous coordinate representations:

1. **Step 1:find $x = 1$ by using (1,0) and (1,1)**

$$l_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \tag{16}$$

2. **Step 2:find $y = 1$ by using (0,1) and (1,1)**

$$l_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \tag{17}$$

3. **Step 3:find intersection**

$$p = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \tag{18}$$

So the point (x,y) of intersection in the physical plane $\mathbb{R}$ is given by:

$$x = \frac{-1}{-1} = 1, y = \frac{-1}{-1} = 1$$

i.e, (x,y) = (1,1), which is also true by simply setting the intersection point of x=1 and y=1 as (1,1).