

Birthday paradox

the birthday problem or birthday paradox concerns the probability that, in a set of n randomly chosen people, some pair of them will have the same birthday.

Example: lets assume that we have a group of 23 people.



$$\binom{23}{2} = \frac{23!}{21!2!} = 253 \ pairs$$

We can show that the birthday paradox is larger than 50%!

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Birthday paradox

$$p(n) = 1 - \frac{n! \binom{365}{n}}{365^n}$$

n	p(n)	
10	11.7%	
20	41.1%	
23	50.7%	
30	70.6%	
50	97.0%	
57	99.0%	
100	99.99997%	
200	99.9999999999999 999999999998%	
300	(100 - (6×10 ⁻⁸⁰))%	
350	(100 - (3×10 ⁻¹²⁹))%	
365	(100 - (1.45×10 ⁻¹⁵⁵))%	
367	100%	

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Birthday Attack

- A birthday attack is a name used to refer to a class of brute-force attacks. More precisely,
- "If some function, when supplied with a random input, returns one of $|\mathbf{k}|$ equally-likely values, then by repeatedly evaluating the function for different inputs, we expect to obtain the same output after about $1.2|\mathbf{k}|^{1/2}$."
- $_{\circ}$ Example: for the birthday paradox, we have |k|=365.

Brute force

- ▶ multiple target second preimage (I out of many):
- — if one can attack 2^t simultaneous targets, the effort to find a single preimage is 2^{n-t}
- multiple target second preimage (many out of many):
 - Fig. time-memory trade-off with $\Theta(2^n)$ precomputation and storage $\Theta(2^{2n/3})$ time per (2nd) preimage: $\Theta(2^{2n/3})$ [Hellman'80]
- ► answer: randomize hash function with a parameter S (salt, key, spice,...)

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Brute force attacks in practice

- ▶ (2nd) preimage search
- n = 128:23 B\$ for I year if one can attack 240 targets in parallel
- parallel collision search: small memory using cycle finding algorithms (distinguished points)
 - -n = 128: 1 M\$ for 8 hours (or 1 year on 100K PCs)
- n = 160:90 M\$ for I year
- need 256-bit result for long term security (30 years or more)

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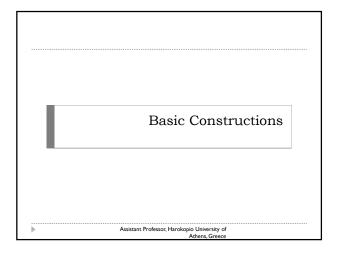
Quantum era

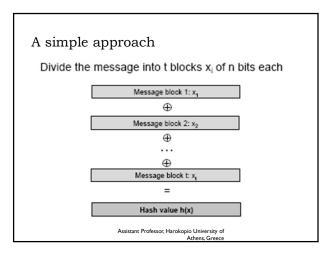
- lacktriangle in principle exponential parallelism
 - $\,\blacktriangleright\,$ inverting a one-way function: 2^n reduced to $2^{n/2}\, [\text{Grover'96}]$
- ▶ collision search:
- $-2^{n/3}$ computation + hardware [Brassard-Hoyer-Tapp'98]
- [Bernstein'09] classical collision search requires $2^{n/4}$ computation and hardware (= standard cost of $2^{n/2}$)

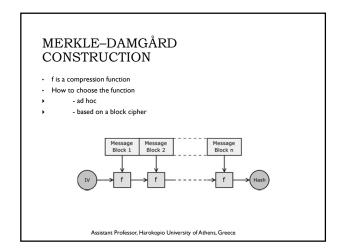
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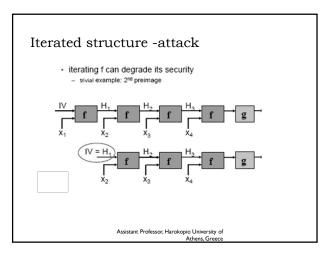
Properties in practice

- ▶ collision resistance is not always necessary
- other properties are needed:
- ▶ PRF: pseudo-randomness if keyed (with secret key)
- PRO: pseudo-random oracle property (formalization of security properties when there is no key)
- ▶ near-collision resistance
- partial preimage resistance (most of input known)
- multiplication freeness
- how to formalize these requirements and the relation between them?









Merkle-Damgard strengthening

Algorithm MD-strengthening

Before hashing a message $x=x_1x_2\dots x_t$ (where x_i is a block of bitlength r appropriate for the relevant compression function) of bitlength b, append a final length-block, x_{t+1} , containing the (say) right-justified binary representation of b. (This presumes $b<2^r$.)

Security relation between f and h

- · solution: Merkle-Damgård (MD) strengthening
 - fix IV, use unambiguous padding and insert length at the end
- f is collision resistant ⇒ h is collision resistant [Merkle'89-Damgård'89]
- f is ideally 2nd preimage resistant ♣ h is ideally 2nd preimage resistant [Lai-Massey'92]
- property preservation has been a heavily studied topic since 2005

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How (NOT) to strengthen a hash function?[Joux'04]

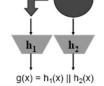
- · answer: concatenation
- h₁ (n1-bit result) and h₂ (n2-bit result)
- intuition: the strength of g against collision/(2nd) preimage attacks is the product of the strength of h₁ and h₂
 - if both are "independent"
- but.... for iterated hash functions only the strongest function matters

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 $g(x) = h_1(x) || h_2(x)$

Multi-collisions [Joux '04]

- finding multi-collisions for an iterated hash function is not much harder than finding a single collision (if the size of the internal memory is n bits)
 - algorithm
 - generate R = 2^{n1/2}-fold multi-collision for h₂
 in R: search by brute force for h₁
- time: n1. 2n2/2 + 2n1/2 << 2^{(n1 + n2)/2}

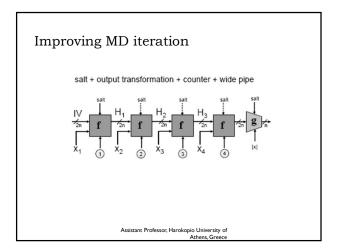


Multi-collisions [Joux '04]

consider h_1 (n1-bit result) and h_2 (n2-bit result), with n1 \geq n2. concatenation of 2 iterated hash functions $(g(x) = h_1(x) \mid\mid h_2(x))$ is as most as strong as the strongest of the two (even if both are independent)

- * cost of collision attack against g at most n1 . $2^{n2/2} + 2^{n1/2} << 2^{(n1+n2)/2}$
- cost of (2nd) preimage attack against g at most
 n1 . 2^{n2/2} + 2ⁿ¹ + 2ⁿ² << 2^{n1 + n2}
- if either of the functions is weak, the attacks may work better

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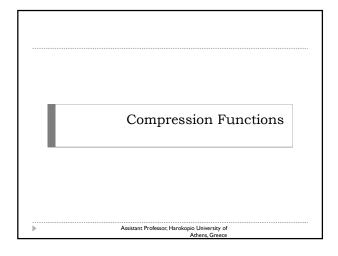


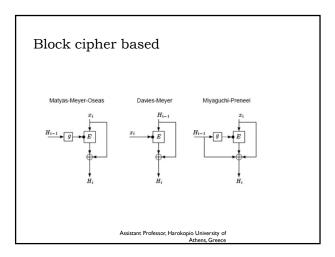
Improving MD iteration

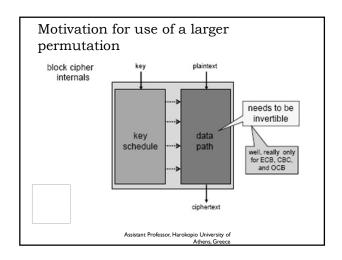
- degradation with use: salting (family of functions, randomization)
- or should a salt be part of the input?
- ▶ PRO: strong output transformation g
- also solves length extension
- ▶ long message 2nd preimage: preclude fix points
- counter $f \rightarrow fi$ [Biham-Dunkelman'07]
- $\,\blacktriangleright\,$ multi-collisions, herding: avoid breakdown at $2^{n/2}$ with larger internal memory: known as wide pipe
- e.g., extended MD4, RIPEMD, [Lucks'05]

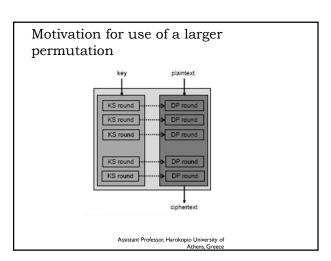
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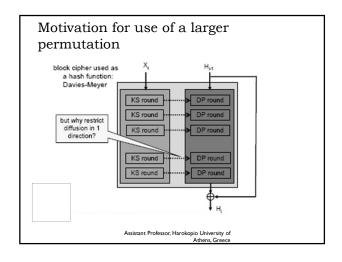
Tree structure: parallelism Assistant Professor, Harokopio University of Athens, Greece

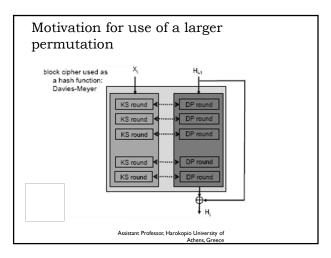


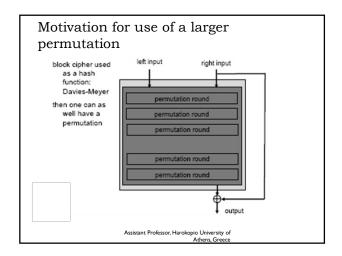


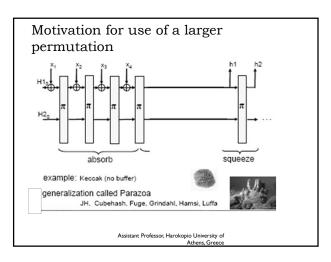


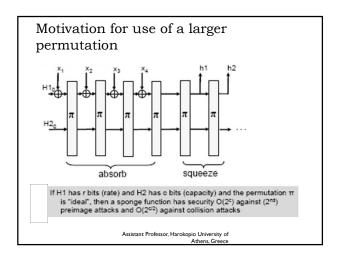






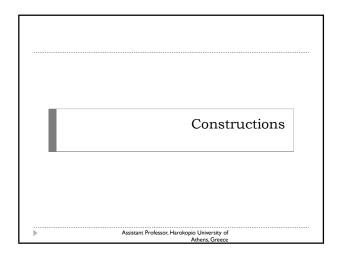


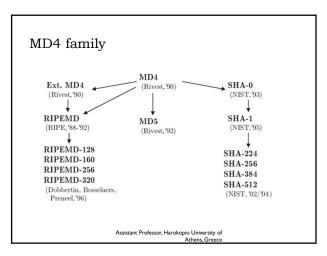




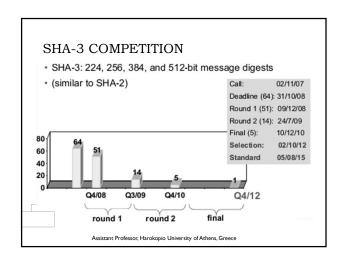
Iteration modes and compression functions

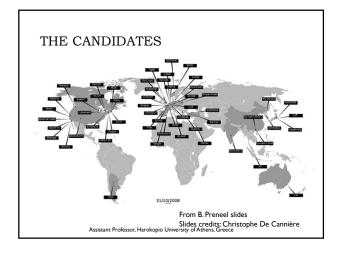
- security of simple modes well understood
 - ▶ powerful tools available
- ▶ analysis of slightly more complex schemes very difficult
 - which properties are meaningful?
 - which properties are preserved?
 - ▶ MD versus sponge is still open debate

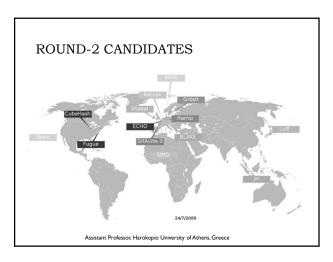




TIMELINE 1990: MD4 by Ron Rivest 1991: MD5 by Ron Rivest (RFC 1321, 1992) 1992: RIPEMD by H. Dobbertin, A. Bosselaers and B. Preneel 1993: SHA-0 by U.S. Government (FIPS PUB 180) 1995: SHA-1 by U.S. Government (FIPS PUB 180-1) 2000: Whirlpool by V. Rijmen and P. Barreto 2001: SHA-2 by U.S. Government (FIPS PUB 180-2) 2005: First attacks against SHA-1 2015: SHA-3 by the Keccak team (FIPS 202) 2017: February 2017, CWI Amsterdam and Google announced they had performed a collision attack against SHA-1 Assistant Professor, Harokopio University of Athens, Greece







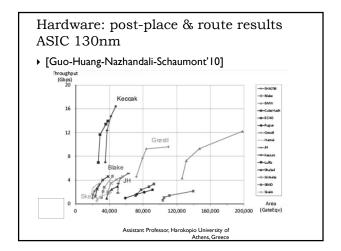
SHA-3 FINALISTS

- ✓ BLAKE (Aumasson et al.)
- ✓ Grøstl (Knudsen et al.)
- √ JH (Hongjun Wu)
- ✓ Keccak (Keccak team, Daemen et al.)
- ✓ Skein (Schneier et al.)

Geography: 3 from Europe, 1 from Asia, 1 from America

Team members also AES finalist: 3

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Keccak: FIPS 202 (published: 5 August 2015)

- append 2 extra bits for domain separation to allow
- flexible output length (XOFs or eXtendable Output Functions)
- tree structure (Sakura) allowed by additional encoding
- ▶ 6 versions
 ▶ SHA3-224: n=224; c = 448; r = 1152 (72%)
 - SHA3-224: n=224; c= 448; r= 1132 (/2%)
 SHA3-256: n=256; c= 512; r= 1088 (68%)
 SHA3-384: n=384; c= 768; r= 832 (52%)
 SHA3-512: n=512; c= 1024; r= 576 (36%)
 SHAKE128: n=x; c= 256; r= 1344 (84%)
 SHAKE256: n=x; c= 512; r= 1088 (68%)
- if result has n bits, H1 has r bits (rate), H2 has c bits (capacity) and the permutation π is "ideal":

 • collisions: min $(2^{\circ/2}, 2^{n/2})$ • 2^{nd} preimage: min $(2^{\circ/2}, 2^n)$

 - ▶ Preimage: min (2^c, 2ⁿ)

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SHA3 WINNER: KECCAK

- ✓ Not an MD construction
- ✓ Based on a new design: sponge



- ✓ Design team: Guido Bertoni, Joan Daemen, Michaël Peeters, Gilles Van Assche
 ✓ FIPS PUB 202:SHA-3 Standard:Permutation-Based Hash and

- Extendable-Output Functions
 https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.202.pdf

STATE OF THE ART

	Output	Classification	
Primitive	Length	Legacy	Future
SHA-2	256, 384, 512	✓	✓
SHA3	256,384,512	✓	✓
Whirlpool	512	✓	✓
SHA3	224	✓	X
SHA-2	224	✓	X
RIPEMD-160	160		X
SHA-1	160	X	X
MD-5	128	X	X
RIPEMD-128	128	X	X

