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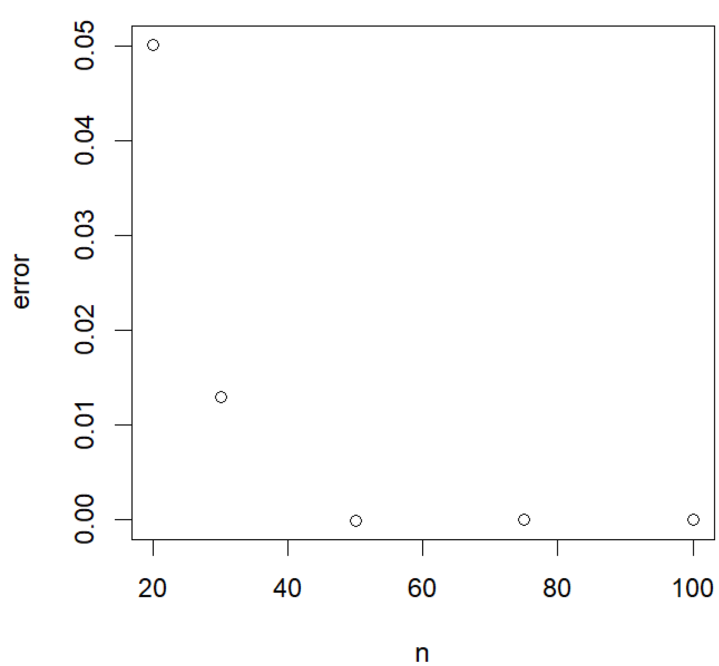
1 Simulation

Problem 1.

i. pbinom(8.25, 20, 0.4)= **0.5955987**
pbinom(8.25, 30, 0.4)= **0.09401122**
pbinom(8.25, 50, 0.4)= **0.0002305229**
pbinom(8.25, 75, 0.4)= **1.826106e-08**
pbinom(8.25, 100, 0.4)= **5.431127e-13**

ii.pnorm((8.25-(0.4*20))/sqrt(0.4*0.6*20)))= **0.5454243**
pnorm((8.25-(0.4*30))/sqrt(0.4*0.6*30)))= **0.08112525**
pnorm((8.25-(0.4*50))/sqrt(0.4*0.6*30)))= **0.0003470073**
pnorm((8.25-(0.4*75))/sqrt(0.4*0.6*30)))= **1.475701e-07**
pnorm((8.25-(0.4*100))/sqrt(0.4*0.6*30)))= **4.557597e-11**

iii. bin=c(0.5955987, 0.09401122, 0.0002305229, 1.826106e-08, 5.431127e-13)
lap= c(0.5454243, 0.08112525,0.0003470073, 1.475701e-07, 4.557597e-11)
error= bin-lap
n=c(20, 30, 50, 75, 100)
plot(n,error)



iv. In this scatter plot we can see that with the more amount of trials, the probability of error goes down, essentially approaching 0 but never actually hitting 0.

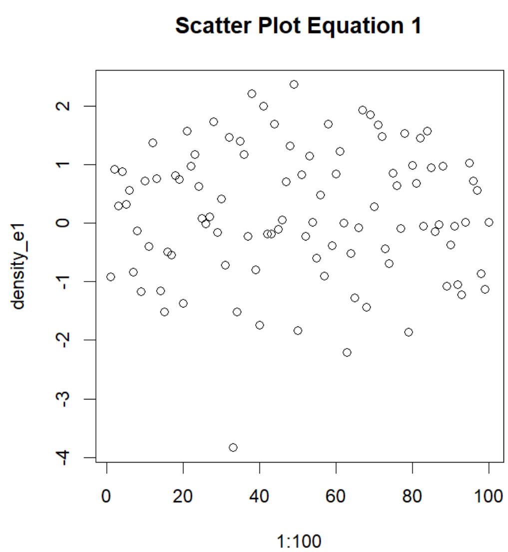
Problem 2.

Code for simulations: Replaced n with size for each simulation

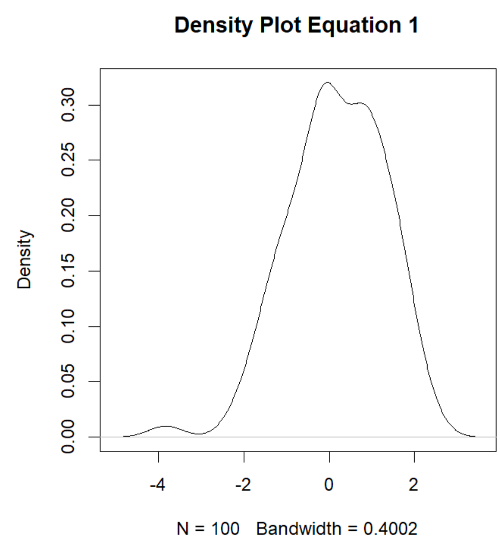
```
density_e1=c(); density_e2=c()
n=20
for (i in 1:100){
  X=rnorm(n,2,3);
  density_e1=c(density_e1,(mean(X)-2)/(sqrt(3^2/n)));
  density_e2=c(density_e2, (((n-1)*var(X))/3^2))
}
plot(1:100,density_e1,main = "Scatter Plot Equation 1")
plot(density(density_e1),main="Density Plot Equation 1")

plot(1:100,density_e2,main = "Scatter Plot Equation 2")
plot(density(density_e2),main="Density Plot Equation 2")
```

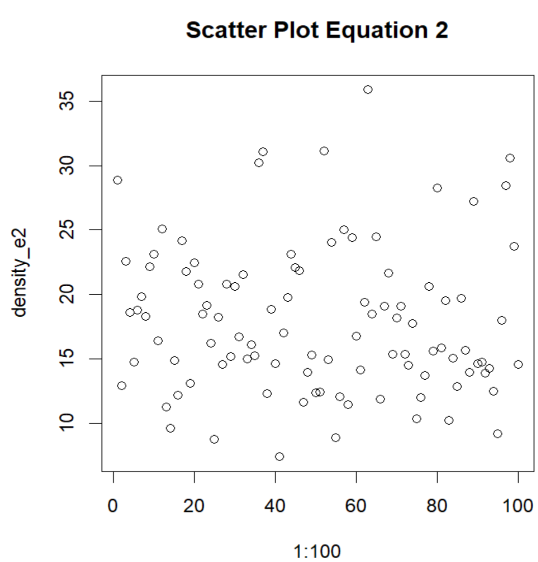
Density_e1 (equation 1) = $\frac{(n-1)S^2}{3^2}$ density_e2 (equation2)= $\frac{(n-1)S^2}{3^2}$

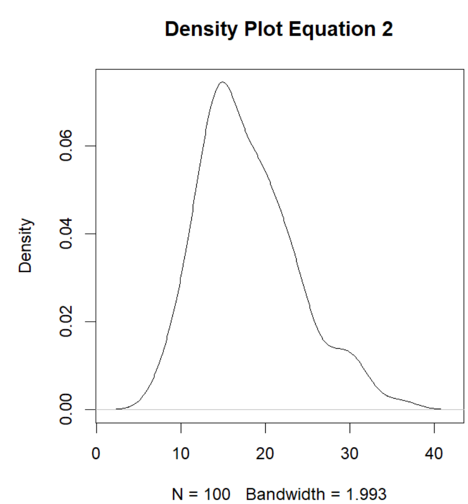
i.n=20 Density

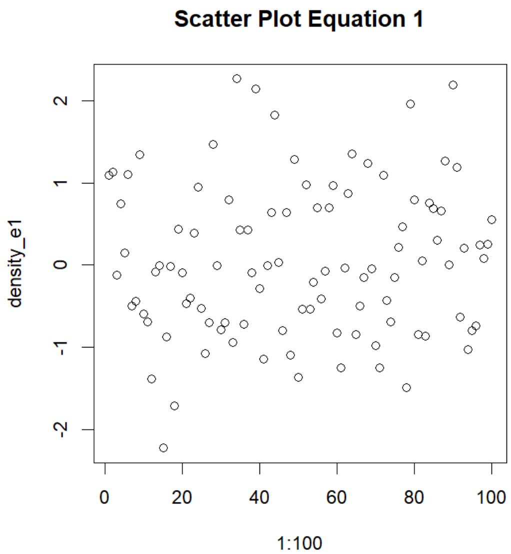


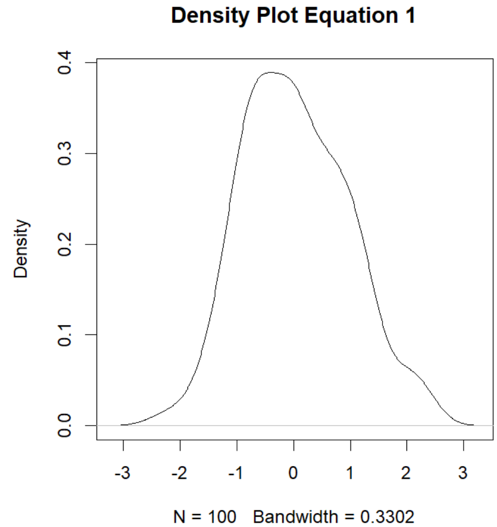
Density





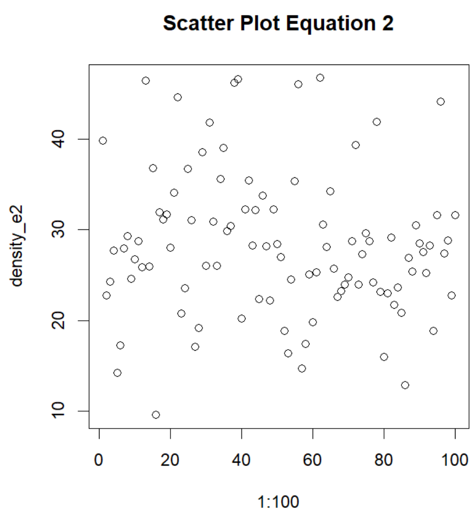




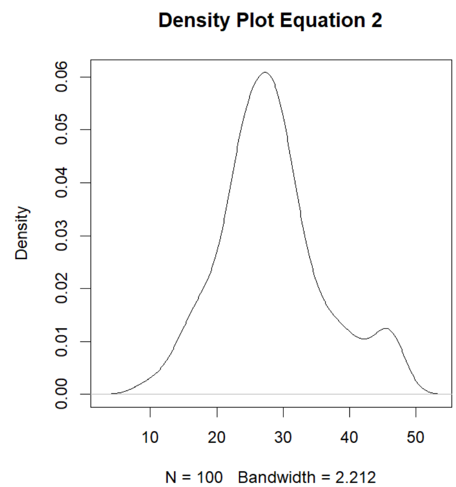


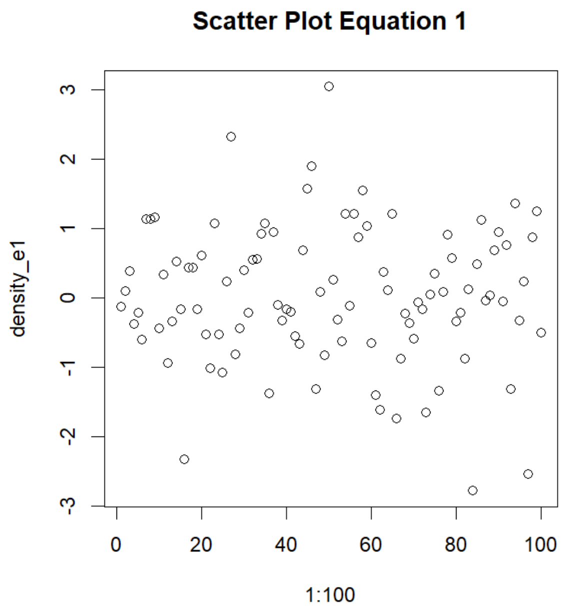
ii.n=30

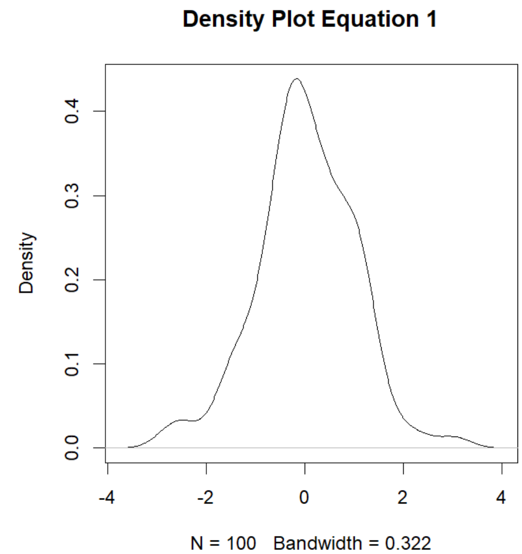
Density



Density

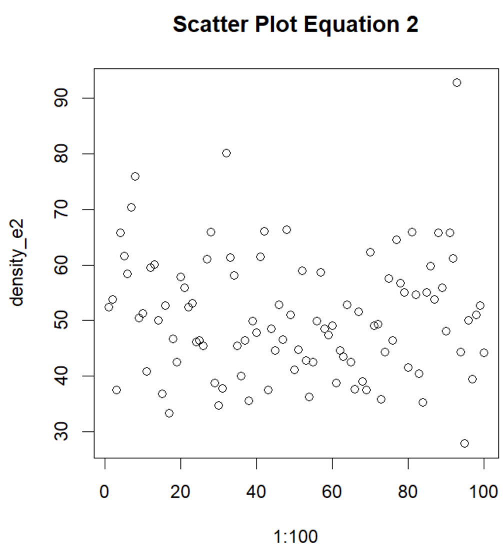




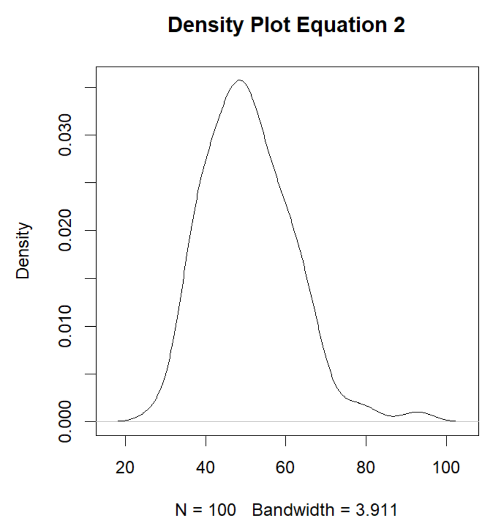


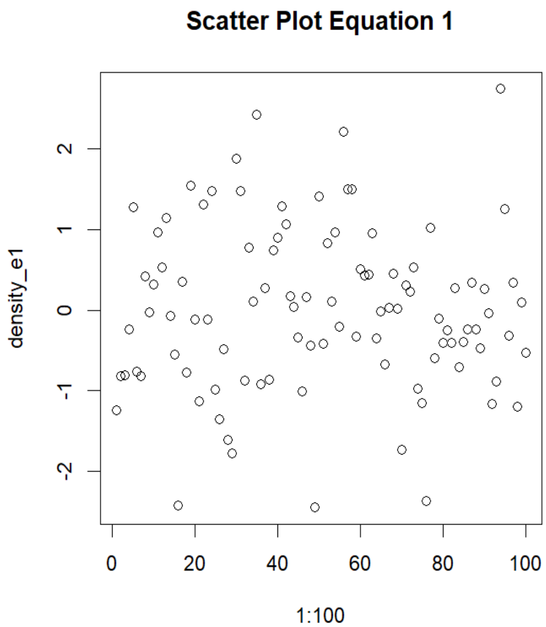
iii.n=50

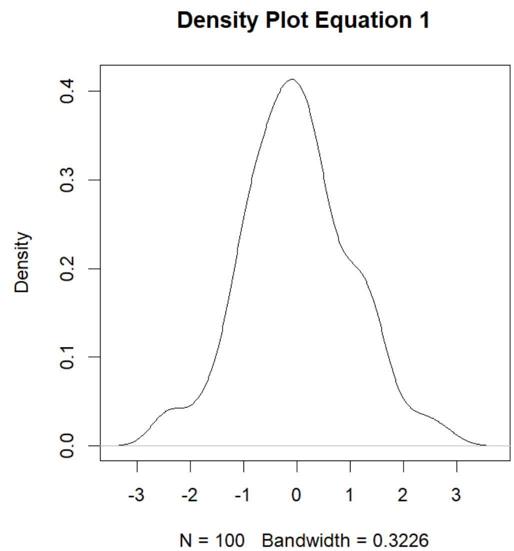
Density



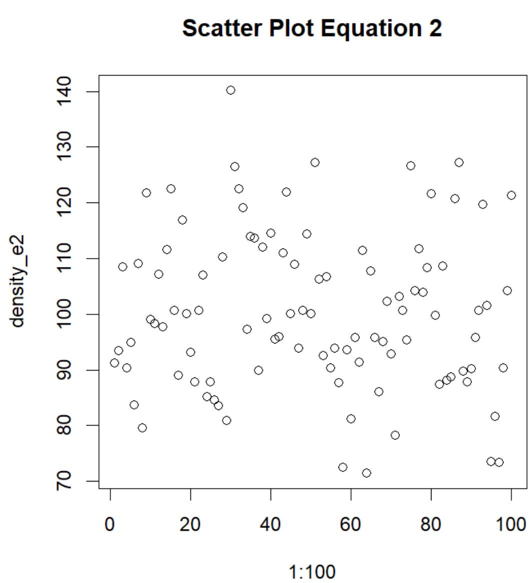
Density



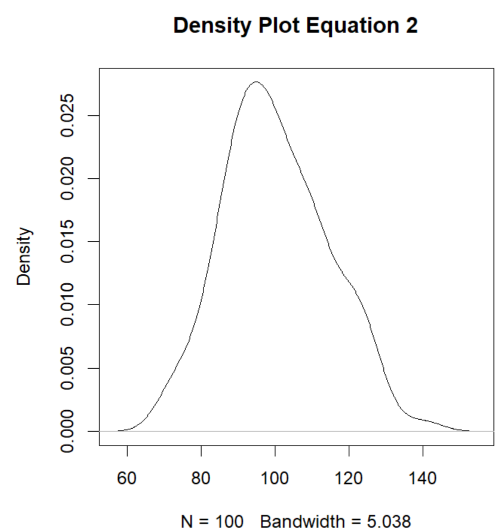




iv.n=100 Density



Density



v. When we look at the density plot (which is density_e1)$\frac{\bar{x}-2}{\sqrt{3^2/n}}$ spans between 0 and the symmetric value of x plane, but these values remain between close to 0. At the same time when we look the the density plot for $\frac{(n-1)S^2}{3^2}$ (which is density_e2) we can notice that as it increases by becoming a smaller value.

Vi. When we look at the scatter plot for $\frac{\bar{x}-2}{\sqrt{3^2/n}}$ we can see that the values are also dispersed between the two symmetric values as density equation 1. When look at the $\frac{(n-1)S^2}{3^2}$ scatter plot we can see that as the n increases, the values would also increase for the density but it remains dispersed. They both present similar findings.

2 Computation

Problem 3.
https://www.investopedia.com/terms/d/degrees-of-freedom.asp#:~:text=Key%20Takeaways-,Degrees%20of%20freedom%20refers%20to%20the%20maximum%20number%20of%20logically,items%20within%20the%20data%20sample. (states “Degrees of freedom is calculated by subtracting one from the number of items within the data sample.” hence why df is set to 9 rather than 10.)

i. $\mu = 1$ and $\sigma^2 = 4$, where $(x_i, ..., x_n) = (0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.75, -1.6, 0.2, 3.15)$

$$a = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2} \rightarrow a = \sum_{i=1}^{n} \frac{(x_i - 1)^2}{4} = 7.30875$$

```
> x = c(0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.75, -1.6, 0.2, 3.15)
> i=x
> sum(((i-1)^2)/4)
[1] 7.30875
```

This statistic is a chi-square distribution.

```
> pchisq(7.30875,df=9)
[1] 0.3949919
```

$$P(\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{\sigma^2} \leq a) =$$

ii. $\sigma^2 = 4$ and $\bar{x} = 0.92$, where $(x_i, ..., x_n) = (0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.75, -1.6, 0.2, 3.15)$

$$b = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(x_i - 0.92)^2}{4} = 7.29275$$

```
> sum(((i-0.92)^2)/4)
[1] 7.29275
```

This statistic is a chi-square distribution.

```
> pchisq(7.29275,df=9)
[1] 0.3933312
```

$$P(\sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2} \leq b) = 0.3933312$$

iii. $\mu = 1$ and $S^2 = 3.241222$, where $(x_i, ..., x_n) = (0.5, 0.9, -0.7, 1.5, -1, 2.5, 3.75, -1.6, 0.2, 3.15)$

$$c = \frac{\bar{x} - \mu}{\sqrt{s^2/n}} = \frac{0.92 - 1}{\sqrt{\frac{3.241222}{n}}} = -0.1405192$$

```
> (0.92-1)/(sqrt(3.241222/10))
[1] -0.1405192
```

This statistic is a T-distribution.

```
> pt(-0.1405192,df=9)
[1] 0.4456722
```

$$P(\frac{\overline{X}-\mu}{\sqrt{S^2/n}} \leq c) = 0.4456722$$

Problem 4.

i. Where $X \sim N(-1, 9)$, $Y \sim X^2_{12}$, $T \sim T_{10}$ and $F \sim F_{8,9}$

a. $P(X \in (0, 1)) =$

```
> x=pnorm(1,-1,3,TRUE)-pnorm(0,-1,3,TRUE)
> x
[1] 0.1169488
```

b. $P(Y \in (3, 14)) =$

```
> y=pchisq(14,12)-pchisq(3,12)
> y
[1] 0.6948357
```

c. $P(T \in (0, 1))$=

```
> z= pt(1,10)-pt(0,10)
> z
[1] 0.3295534
```

d. $P(F \in (0, 1))$=

```
> w=pf(1,8,9)-pt(0,8,9)
> w
[1] 0.5054556
```

ii. For $\alpha = 0.05$

a. Quantiles of X

$\alpha/2\ quantile\ =$

```
> qnorm(0.05/2,-1,9)
[1] -18.63968
```

$(1 - \alpha)/2\ quantile\ =$

```
> qnorm((1-0.05)/2,-1,9)
[1] -1.564361
```

b. Quantiles of Y

$\alpha/2\ quantile\ =$

```
> qchisq(0.05/2,12)
[1] 4.403789
```

$(1 - \alpha)/2\ quantile\ =$

```
> qchisq((1-0.05)/2,12)
[1] 11.04577
```

c. Quantiles of T

$\alpha/2\ quantile\ =$

```
> qt(0.05/2, 10)
[1] -2.228139
```

$(1\ -\ \alpha)/2\ quantile\ =$

```
> qt((1-0.05)/2, 10)
[1] -0.06429815
```

d. Quantiles of F

$\alpha/2\ quantile\ =$

```
> qf(0.05/2,8,9)
[1] 0.2295034
```

$(1\ -\ \alpha)/2\ quantile\ =$

```
> qf((1-0.05)/2,8,9)
[1] 0.9473511
```

3 Verification

Problem 5

1. N ~ B(n,p) → E[N] = np

$$E(X)\ =\ \sum_{x=1}^{n} xp(x\ =\ x)$$

$$=\ \sum_{x=1}^{n} x\binom{n}{x}p^{x}q^{n-x}$$

$$x\binom{n}{x} = x\frac{n!}{x!(n-x)!}$$

$$= x\frac{n(n-1)!}{x(x-1)!(n-x)!}$$

$$= n\binom{n-1}{x-1}$$

After substituting back in…

$$\sum_{x=1}^{n} n \binom{n-1}{x-1} p \, * \, p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{(n-1)-(x-1)}$$

if a = n-1 and b=x-1,

$= np \sum_{b=0}^{a} \binom{a}{b} p^{b} q^{a-b}$ → probability density function!

$\sum_{b=0}^{a} \binom{a}{b} p^{b} q^{a-b}$ = 1, so E(x) = np

2.

$T = \frac{X}{\sqrt{Y/n}}, \rightarrow \frac{1}{\sqrt{2pi}} e^{-x^2/2}$, therefore, E[T] = 0

Problem 6

a. W + 3X - 2Y
   According to the closure of normal distribution:
   ~ N(2 + 3(-1) -2(1), 16 + 9(9) - 4(4))
   ~N(-3, 81)
b. $\frac{(W-2)^2}{16} + \frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4}$
   According to X^2 distribution:
   3 degrees of freedom, therefore, X^2_3
c. $\frac{\frac{W-2}{4}}{\sqrt{(\frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4}) / 2}}$
   According to T distribution:
   $\frac{\frac{W-2}{4}}{\sqrt{(\frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4}) / 2}}$ ~ T_2
d. According to F distribution:
   $\frac{(X+1)^2}{9} / \frac{(Y-1)^2}{4} \sim F(1, 1)$