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I pledge my honor that I have abided by the Stevens Honor System - Kush, Jo-Anne, Simerjeet

### Problem 1.

- The population random variable is represented by  $m$ , which contains both defective and qualified goods.
- The population probability distribution in this case would determine the frequency at which defective goods are drawn ( $r$ ) vs qualified ones ( $m-r$ ) being drawn from ( $m$ ) goods at ( $n$ ) random times. Since we are performing a quality inspection, we are trying to find all the defective products ( $r$ ).
- Assuming 0 for every defect and 1 for every qualified

For the Mean:  $T_1(X_1, \dots, X_n) \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = E[X] \rightarrow E[X] = \left(\frac{m-r}{m}\right)$

For the Variance Using the idea that  $\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$  we can simplify

$$T_1(X_1, \dots, X_n) \quad S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = Var[X] \rightarrow Var[X] = \frac{1}{n-1} \sum_{i=1}^n x_i^2 - \frac{(m-r)^2}{m}$$

### Problem 2.

- Yes, the sample observation is a simple and random sample as each element in the sample is randomly selected and are mutually independent of each other.
- $T(N_1, \dots, N_r)$  This would be Binary Distribution, specifically a binomial distribution. We know this because there are only 2 possible options, either the item is a defect or it is not, and we also know that each good is independent.

This is a binomial distribution hence  $P(N = N_1) = \binom{m}{N_1} \left(\frac{r}{m}\right)^{N_1} * \left(1 - \frac{r}{m}\right)^{m-N_1}$

- For the Mean:  $E[X] = E[n_1] = n_1 * \left(\frac{r}{m}\right)$

For the Variance:  $Var[X] = Var[n_1] = n_1 \left(\frac{r}{m}\right) \left(1 - \left(\frac{r}{m}\right)\right)$

### Problem 3.

- No, the sample is not a simple and random sample since the selection of each element in the sample depends on the other selections since it occurs without replacement.
- $T(N_1, \dots, N_r)$  This would be a normal distribution or hypergeometric distribution; we know this because all the variables depend on one another, as they are drawn without replacement.

$$\mu = n\left(\frac{m-r}{m}\right)$$

$$\sigma = \frac{1}{m-1} \sum_{i=1}^n x_i^2 - \frac{(m-r)^2}{m}$$

$$\text{Normal distribution } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{-x-\mu}{2\sigma^2}\right)}$$

c. For the Mean:  $E[N_2] = N_2 * \left(\frac{m-r}{m}\right)$

