I pledge my Honor that I have abided by the Stevens Honor System

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12.31 Financial incentives for weight loss.

The use of financial incentives has shown promise in promoting weight loss and healthy behaviors. In one study, 104 employees of the Children's Hospital of Philadelphia, with BMIs of 30 to 40 kilograms per square meter (kg/m^2), were each randomly assigned to one of three weight-loss programs.[11] Participants in the control program were provided a link to weight-control information. Participants in the individual-incentive program received this link but were also told that $100 would be given to them each time they met or exceeded their target monthly weight loss. Finally, participants in the group-incentive program received similar information and financial incentives as the individual-incentive program but were also told that they were placed in secret groups of 5 and at the end of each 4-week period, those in their group who met their goals throughout the period would equally split an additional $500. The study ran for 24 weeks and the total change in weight (in pounds) was recorded. LOSS

(a) Make a table giving the sample size, mean, and standard deviation for each group.

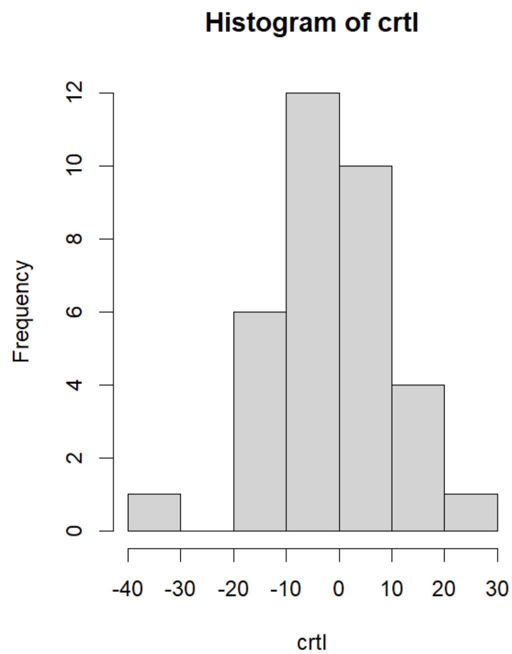
(b) Is it reasonable to pool the variances? Explain your answer.

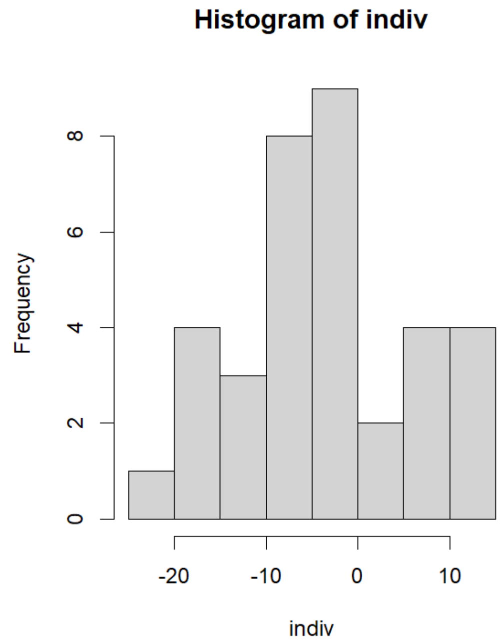
(c) Generate a histogram for each of the programs. Can we feel confident that the sample means are approximately Normal? Defend your answer.

a.

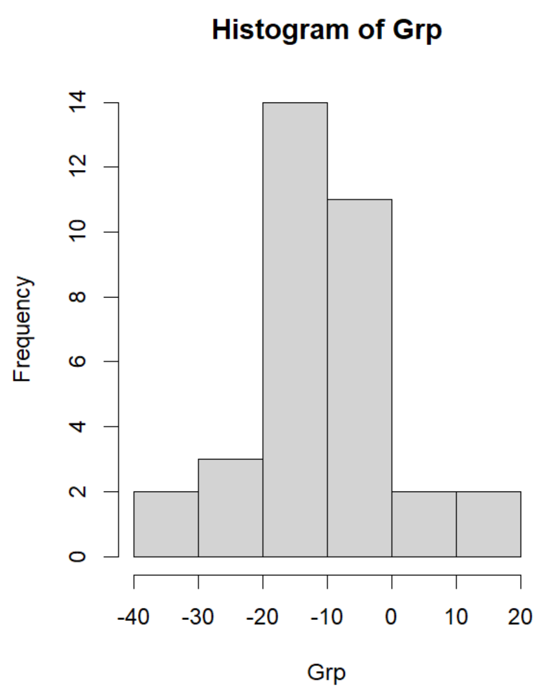
Program	Control	Individual-Incentive	Group-Incentive
Sample Size	35	35	34
Mean	-1.0	-3.7	-10.8
Standard Deviation	11.50073	9.078364	11.13915

b. Yes it is reasonable to pool the variances because twice the smaller standard deviation is still greater than largest standard deviation. In this case (9.078364 *2) > 11.50073 which means even though they are not the same amount, they are still close enough to be pooled.


Histogram of crtl
Frequency
0
2
4
6
8
10
12
-40
-30
-20
-10
0
10
20
30
crtl


Histogram of indiv
Frequency
0
2
4
6
8
-20
-10
0
10
indiv

c.


Histogram of Grp
Frequency
0
2
4
6
8
10
12
14
-40
-30
-20
-10
0
10
20
Grp

12.32 Financial incentives for weight loss, continued.

Refer to the previous exercise. LOSS

(a) Analyze the change in weight using analysis of variance. Report the test statistic, degrees of freedom, *P*-value, and your conclusions.

(b) Even though you assessed the model assumptions in the previous exercise, let's check the assumptions again by examining the residuals. Summarize your findings.

(c) Compare the groups using the least-significant differences method.

(d) Using the results from parts (a), (b), and (c), write a short summary of your conclusions.

(a) H_0: $\mu_1 = \mu_2 = \mu_3$

H_a: $\mu_1 \neq \mu_2 \neq \mu_3$

$$\frac{(35(-1)+35(-3.7)+34(-10.8))}{104} =- 5.1125$$

Ctrl mean = -1.008, Indiv mean = -3.71, Grp mean = -10.8

SSB of crtl = 589.5, SSB of indiv= 69, SSB of Grp =1094.1

Total SSB = 1752.6

SSE=11393.9

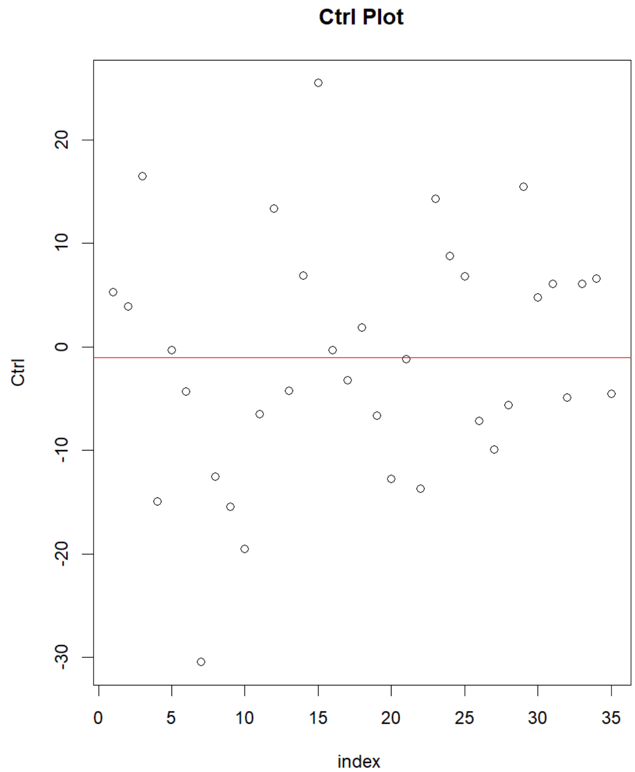
$$\frac{\frac{1752.6}{(k=3)-1}}{\frac{11393.9}{(n=104)-(k=3)}} = f=7.77$$

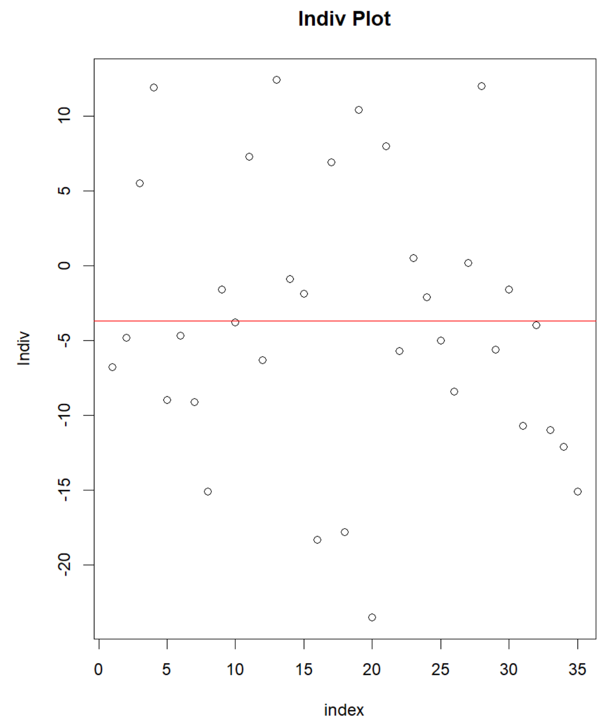
df =2

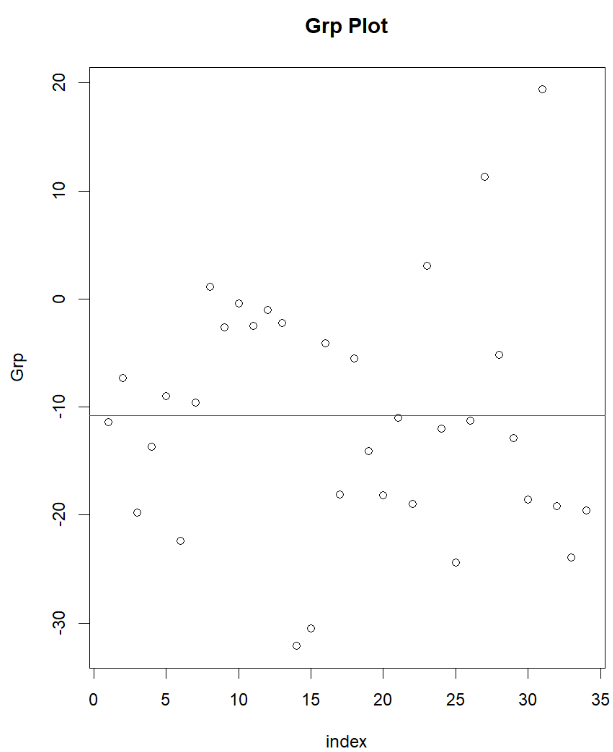
p=P(F>f)=1-pf(f,k-1,n-k) <0.05→0.00073

Source	df	SS	MS	F	p-value
Group	2	1752.6	876.3	7.77	0.00073
Error	101	11393.9	112.81		
Total	103	13146.5			

(b)


Ctrl Plot
Ctrl
index
20
10
0
-10
-20
-30
0
5
10
15
20
25
30
35


Indiv Plot
Indiv
index
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25
30
35


Grp Plot
Grp
index
20
10
0
-10
-20
-30
0
5
10
15
20
25
30
35

When we plot all three graphs, we can see that most of the data points are scattered and don’t focus around their mean lines, showing a lot of variance in each graph.

(c) $Ti, j = \frac{\bar{x}i - \bar{x}j}{\sqrt{(S^2 p(\frac{1}{ni} + \frac{1}{nj}))}}$ $\rightarrow S^2 p = 112.81$

Indiv-ctrl	=	-1.06
Indiv-grp	=	-2.77
ctrl - grp	=	-3.82

n=104, k=3
$p=P(T_{i,j}>t_{i,j})=2(1-pt(|t_{i,j}|, n-k))<\alpha$

Indiv-ctrl = 0.29 → 0.29 > 0.05 hence we fail to reject the H_0, therefore $\mu_{indiv} = \mu_{ctrl}$
Indiv-grp= 0.007 → 0.007< 0.05, hence we fail to accept the H_0, therefore $\mu_{indiv} \neq \mu_{grp}$
ctrl - grp = 0.00023→ 0.00023 <0.05, hence we fail to accept the H_0, therefore $\mu_{ctrl} \neq \mu_{grp}$

(d) When we analyze part A, we can see that this shows that there are differences found within each group since the p-value was less than 0.05. From Part B, we are able to notice diversity in the data when we compare it to the mean of each group. In part C, we can see specifically which group means are not equivalent to one another, which is why we fail to accept the null hypothesis.

12.33 Changing the response variable.

Refer to the previous two exercises, where we compared three weight-loss programs using change in weight measured in pounds. Suppose that you decide to instead make the comparison using change in weight measured in kilograms. LOSS

(a) Convert the weight loss from pounds to kilograms by dividing each response by 2.2.

(b) Analyze these new weight changes using analysis of variance. Compare the test statistic, degrees of freedom, and *P*-value you obtain here with those reported in part (a) of the previous exercise. Summarize what you find.

a)

Program	Control	Individual-Incentive	Group-Incentive
Sample Size	35	35	34

Mean	-0.458	-1.686	-4.902
Standard Deviation	5.228	4.127	5.063

b) H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: $\mu_1 \neq \mu_2 \neq \mu_3$

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Ctrl Group	35	-16.045455	-0.4584416	27.3278283
Indiv Group	35	-59	-1.6857143	17.0282415
Group Group	34	-166.68182	-4.9024064	25.6365054

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	362.108192	2	181.054096	7.76788489	0.00072784	3.08637122
Within Groups	2354.11105	101	23.3080302			
Total	2716.21925	103				

The F-test statistic, degrees of freedom, and the P-value from the previous exercise are the same as the ones obtained from this exercise. The reason why the degrees of freedom are the same can be attributed to the fact that the sample sizes of these three groups did not change upon conversion from lb to kg. With this, the division of the response values by 2.2 (or any constant for that matter) does not change the normality of the data.

12.41 Writing contrasts.

Return to the eye study described in Example 12.25 (page 673). Let μ_1, μ_2, μ_3, and μ_4 represent the mean scores for blue, brown, gaze down, and green eyes.

(a) Because a majority of the population in this study are Hispanic (eye color predominantly brown), we want to compare the average score of the brown eyes with the average of the other two eye colors. Write a contrast that expresses this comparison.

(b) Write a contrast to compare the average score when the model is looking at you versus the score when looking down.

Group	*n*	Mean	Std. dev.
Blue	67	3.19	1.75
Brown	37	3.72	1.72
Down	41	3.11	1.53
Green	77	3.86	1.67

a) $Blue = u_1,\ Brown = u_2,\ Down = u_3,\ Green = u_4$

$\psi_1 = u_2 - \frac{(u_1+u_4)}{2}$

b) $Blue = u_1,\ Brown = u_2,\ Down = u_3,\ Green = u_4$

$a_1 = 1,\ a_2 = 1,\ a_3 =- 3,\ \ a_4 = 1$

$\psi_2 = \frac{(u_1+u_2+u_4)}{3} - u_3$

12.42 Analyzing contrasts.

Answer the following questions for the two contrasts that you defined in Exercise 12.41. EYES

(a) For each contrast give H_0 and an appropriate H_a. In choosing the alternatives you should use information given in the description of the problem, but you may not consider any impressions obtained by inspection of the sample means.

(b) Find the values of the corresponding sample contrasts c_1 and c_2.

(c) Calculate the standard errors SE_{c_1} and SE_{c_2}.

(d) Give the test statistics and approximate P-values for the two significance tests. What do you

(e) Compute 95% confidence intervals for the two contrasts.

a) c_1 Hypotheses:

i) H0: The eyes are brown

ii) Ha: The eyes are not brown.

c_2 Hypotheses:

iii) H0: You are able to see the eye color

iv) Ha: The eye color is not to be seen

b) $\psi_1 = u_2 - \frac{(u_1+u_4)}{2}$

$c_1 = 3.72 - \frac{(3.19+3.86)}{2} = 0.195$

$\psi_2 = \frac{(u_1+u_2+u_4)}{3} - u_3$

$c_2 = \frac{(3.19+3.72+3.86)}{3} - 3.11 = 0.48$

c) $Sp = 1.68$

$SE_{c1} = 1.68 * \sqrt{\frac{1}{37} - \frac{0.25}{67} - \frac{0.25}{77}} = 0.31$

$SE_{c2} = 1.68 * \sqrt{\frac{\frac{1}{9}}{37} + \frac{\frac{1}{9}}{67} + \frac{\frac{1}{9}}{77} + \frac{\frac{1}{9}}{41}} = 0.29$

d) $t1 = \frac{1.68}{0.31} = 0.63$

$df = n - k = 218$

p-value=0.53 hence we fail to reject the H_0, therefore we can see the color

e) c1=0.195± 1.96*(0.31)=(-0.41,0.801)

c2=0.48± 1.96*(0.293)=(-0.095,1.055)