UNIVERSITY OF BATANGAS

GRADUATE SCHOOL

A STUDY ON MATHEMATICAL MODELING IN MEDICINE AND HEALTHCARE

Research Paper

Presented To
Dr. Romell A. Ramos

In Partial Fulfillment
For
Differential Equation

Submitted By

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Contents:

Abstract:

The present study considers mainly a primary idea of mathematical modeling using differential equation. Many issues of mathematical applications in the modern science have been discussed in other different studies. The aim of this paper is to know a little concept on mathematical modeling and modeling in medical science. Its depth is so widespread that the writers know very little about the topic. This literature is based on ideas from online articles. The readers will notice a design of mathematical modeling, its classification, related sectors of modeling, importance and finally an application with an example.

Introduction:

In medical science mathematics application has a long time history. From the beginning of the planet, people tried to find euphony in the shape of the human body. For example, the golden section was used to describe relationship between different parts of the body. Moreover, in the network of ideas generally accepted at that time any disease was considered as lack of balance in the principal body components. Later on this idea became transformed into modern concept of homeostatic equilibrium of human body under health condition and its disturbance during illness.

At present, two vital approaches to mathematical description of medicine phenomena can be distinguished. The first one is to find regularities in quantitative analysis of medicine data. Such investigations are usually attributed to the field of biometrics. Modern statistical methods and computer system are widely used for treatment of huge amount of biometrical data. In turn, the intensive data analysis leads to rapid development of the proper mathematical methods. For instance, the term ‘regression’ was introduced in the 19th century as a result of investigation of inheritance of physiological characteristics of human beings. The mathematical models suggested using this approach have a descriptive design and may be applied to deduce the mechanisms of phenomena under research.

Another approach is to predict the system behavior using the data on mechanisms underlying the described process. This type of mathematical models may have a generalized character and describe the biological processes on any level of complexity. But it can describe adequately the problem only in a short range of conditions. In difference, very complex model can take into account much more real processes, but it is hard to tune and operate.

Definition of mathematical model:

A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Mathematical models are used particularly in the natural sciences and engineering disciplines such as physics, biology and electrical engineering but also used in the field of social sciences such as economics, social science and political science.

Elements of a mathematical model:

Mathematical model can take many forms, including dynamical system, statistical models, differential equations or game theoretic models. These and other types of models can overlap, with a given model involving variety abstract structures.

Classifications:

Mathematical model are usually composed of relationships and variables. Relationship can be described by operators such as algebraic operators, functions, differential operators, etc. Variables are abstractions of systems parameters of interest, than can be quantified. Several classification criteria can be used for mathematical models according to their structures:

01. Linear vs. nonlinear,
02. Static vs. dynamic,
03. Explicit vs. implicit,
04. Discrete vs. continuous,
05. Deterministic vs. probabilistic (stochastic),
06. Deductive, inductive or floating.

Importance of mathematical modeling:

Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application. The necessity of mathematical modeling is as following:

- Mathematical modeling is indispensable in many applications
- It is successful in many further applications
- It gives precision and direction for problem solution
- The modeling enables a thorough understanding of the system modeled.
- It prepares the way for better design or control of a system.
- Mathematical model allows the efficient use of modern computing capabilities.

Respective areas with modeling:

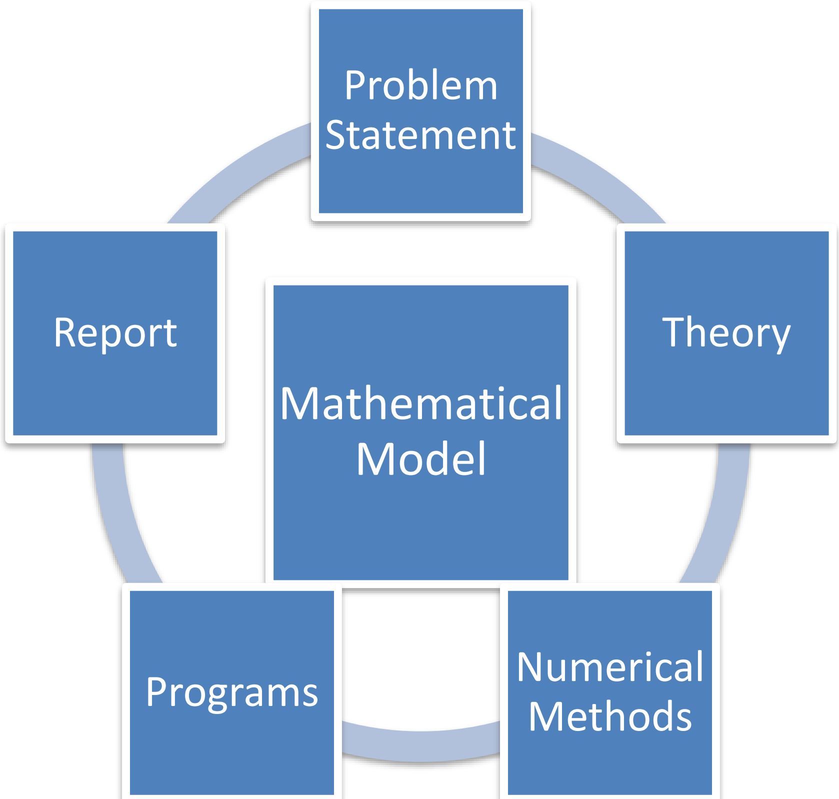
We are going to mention about some fields of applications which are already described by mathematical modeling, although there are an almost endless number of other areas with interesting mathematics problems.

- Astronomy
    - ✓ Detection of planetary system
    - ✓ Correcting the Hubble telescope
    - ✓ Origin of the universe
    - ✓ Evolution of stars
- Biology
    - ✓ Protein folding
    - ✓ Human genome project
    - ✓ Population dynamics
    - ✓ Morphogenesis
    - ✓ Evolutionary pedigrees
    - ✓ Spreading of infectious disease(AIDS)
    - ✓ Animal and plant breeding
- Chemical Engineering
    - ✓ Chemical equilibrium
    - ✓ Planning of production units
- Chemistry
    - ✓ Chemical reaction dynamics
    - ✓ Molecular modeling
    - ✓ Electronic structure calculations
- Artificial Intelligence
    - ✓ Computer vision
    - ✓ Image interpretation
    - ✓ Robotics
    - ✓ Speech recognition
    - ✓ Optical character recognition
    - ✓ Reasoning under uncertainty
- Anthropology
    - ✓ Modeling, classifying and reconstructing skulls
- Archeology
    - ✓ Reconstruction of objects from preserved fragments
    - ✓ Classifying ancient artifices
- Architecture
    - ✓ Virtual reality
- Arts
    - ✓ Computer animation(Jurassic Park)

- Transportation Science:
    - ✓ Air traffic scheduling
    - ✓ Taxi for handicapped people
    - ✓ Automatic pilot for cars and airplanes
- Political Science
    - ✓ Analysis of elections
- Psychology
    - ✓ Formalizing diaries of therapy sessions
- Space Sciences
    - ✓ Trajectory planning
    - ✓ Flight simulation
    - ✓ Shuttle reentry
- Meteorology
    - ✓ Weather prediction
    - ✓ Climate prediction(global warming, ozone hole etcetera)
- Music
    - ✓ Analysis and synthesis of sound
- Neuroscience
    - ✓ Neural networks
    - ✓ Signal transmission in nerves
- Pharmacology
    - ✓ Docking of molecules to proteins
    - ✓ Screening of new compounds
- Physics
    - ✓ Elementary particle tracking
    - ✓ Quantum field theory predictions
    - ✓ Laser dynamics
- Internet
    - ✓ Web search
    - ✓ Optimal routing
- Linguistics
    - ✓ Automatic translations
- Material Science
    - ✓ Microchip production
    - ✓ Microstructures
    - ✓ Semiconductor modeling

- Mechanical Engineering
    - ✓ Stability of structures(high rise buildings, bridges, air planes etcetera)
    - ✓ Structural optimization
- Medical Science
    - ✓ Radiation therapy planning
    - ✓ Computer aided tomography
    - ✓ Blood-circulation models
- Finance and economics
    - ✓ Risk analysis
    - ✓ Value estimation of options
    - ✓ Labor data analysis
- Electrical Engineering
    - ✓ Stability of electric circuits
    - ✓ Microchip analysis
    - ✓ Power supply network optimization
- Fluid mechanics
    - ✓ Wind channel
    - ✓ Turbulence
- Geosciences
    - ✓ Prediction of oil or ore deposits
    - ✓ Map production
    - ✓ Earth quake prediction
- Computer Science
    - ✓ Image processing
    - ✓ Realistic computer graphic (ray tracing)

The modeling diagram:



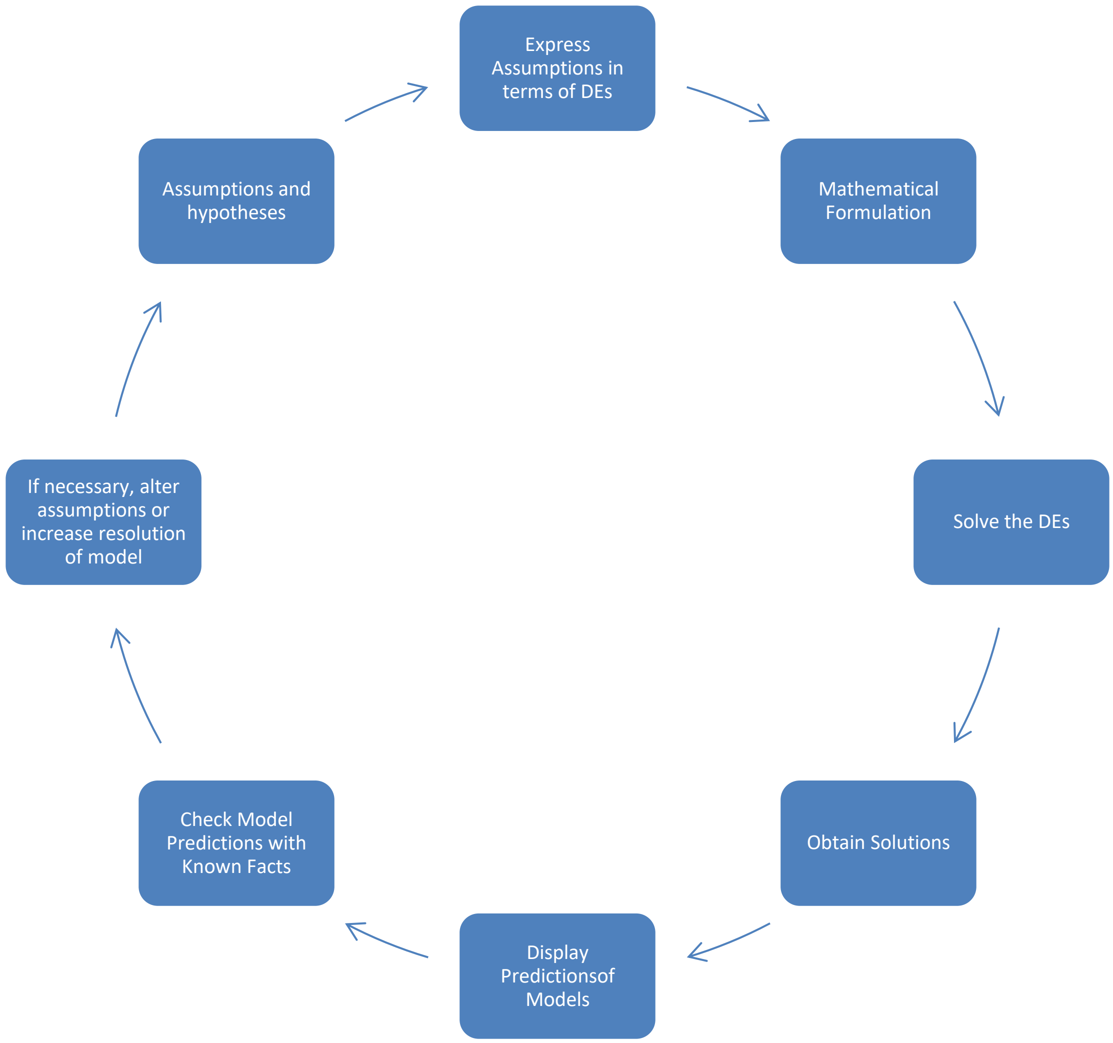
The edges of the diagram represent activities of two way communication (flow of relevant information) between the nodes and the corresponding sources of information.

Differential equation as mathematical model:

The mathematical description of a system of phenomenon is called a mathematical model. Construction of a mathematical model of a system starts with

01. Identification of the variables those are responsible for changing the system. The writer may choose not to incorporate all these variables into the model at first. In this step need to specify the level of resolution of the model.
02. Make a set of reasonable assumptions, or hypotheses, about the system trying to describe. These assumptions will also include any empirical laws that may be applicable to the system.

The steps of the modeling process with differential equation are as follows:



A mathematical model of a physical system will often involve the variable time t. A solution of the model then gives the state of system; in other words, the values of the dependent variable (or variables) of t describe the system in the past, present and future.

An application of modeling:

The model represented by the linear differential equation

$$\frac{dy}{dx} = ky \dots\dots\dots (1)$$

Represent vast variety of situations in medical sciences. Problems in population growth, drug absorption and elimination, alcohol absorption, water cooling carbon dating etcetera can be modeled by equations type (1)

Once, this differential equation has been solved we have effectively solved numerous problems. The solutions to the above problem can be expressed in the form

$$y = y_0 \exp(kt) \dots\dots\dots (2)$$

Where y_0 is initial value. The behavior of the solutions depends on the sign of the constant k.

If k is positive, we have exponential growth; if k is zero, y remains equal to its initial value. And If k is negative we have exponential decay,

$$y \to 0 \; as \; x \to \infty$$

The study of the way in which a drug loses its concentration in the blood of a patient is fundamental to pharmacology. The 'dose-response' relationship plays a vital role in determining the required dosage level and the interval of time between doses for a particular drug.

Suppose, $y = y(t)$ represents the quantity of drug in the blood stream at time t. the simplest way to model such behavior is assume that the rate of change of the concentration is proportional to the concentration of the drug in the blood stream. In mathematical terms

$$\frac{dy}{dx} = -ky \dots\dots\dots (3)$$

Where k is positive constant. Experiments have shown that above equation is a good approximation to reality for many drugs, the most important being penicillin and streptomycin. Having determined the constant k for a particular drug we now use equation (3) as a model.

Suppose, the patient is given an initial dose, y_0, which is assumed instantaneously absorbed by blood at time $t = 0$ resulting in a quantity $y = y_0$ at $t = 0$ in the blood. The actual time of absorption is usually in comparisons small with the time between doses. From the model, the solution is

$$y = y_0 \exp(-kt) \dots \dots \dots (4)$$

Here, $-k$ is showing that the drugs' concentration decays exponentially. After a fixed time T a second dose y_0 is administered. Just before the dose, the amount in the blood is given by,

$$Y(T-) = Y_0 \exp(-kt) \dots \dots \dots (5)$$

Just after the second dose, at time $T = T+$

$$y(T+) = \ y_O + y_0 \exp(-kT)$$

$$= y_0(1 + \exp(-kT)) \dots \dots \dots (6)$$

The new quantity decays according to previous law with initial condition.

$$y = y_0(1 + \exp(-kT)) \text{ , at } t = T.$$

Thus for $t > T$

$$y(t) = y_0(1 + \exp(-kt)) \exp\big(-k(t - T)\big) \dots \dots \dots (7)$$

Again giving the patient a dose yet at $t = 2T$ results in

$$y(2T+) = y_0(1 + \exp(-kT) +) \dots \dots \dots (8)$$

Again,

$$Y = Y_0(1 + \exp(-kT) + \exp(-2kT)) \dots \dots \dots (9)$$

At $t > 2T$

$$y(t) = y_0(1 + \exp(-kT)) + \exp(-2kT) \exp\big(-k(t - 2T)\big) \dots \dots \dots (10)$$

Continuing this way

$$y(nT+= y_0(1 + \exp(-kT) + \cdots + \exp(-nkT)) \dots \dots \dots (11)$$

For, $n = 1,2, \dots$

$$y(nT+) = y_0(1 - \exp(-(n+1)kT))/(1 - \exp(-kT)) \dots \dots \dots (12)$$

As n gets larger, $\exp(-(n+1)kT -) > 0$ so that $y(nT+) = y_0/(1 - \exp(-kT))$

Since, this is independent of $n,$ the model predicts that the quantity of drugs is tending to a saturation level say Y_s where

$$Y_s = y_0/(1 - \exp(-kT))$$

This formula can be used for

(a) the required time interval $T,$ between doses for a given dose y_0 and prescribed final level Y_s.

(b) The dose level $y_0$required to obtain a final dose level Y_s with a prescribed interval between doses, T.

An Example: Spread of a disease

A contagious disease—for example, a flu virus- is spread throughout a community by people coming into contact with other people. Let $x(t)$ denote the number of people who have contracted the disease and $y(t)$ denote the number of people who have not been exposed yet. It seems reasonable to assume that the rate ${}^{dx}/_{dt}$ at which the disease spreads is proportional to the number of encounters or interactions, between these two groups of people. If we assume that the number of interactions is jointly proportional to $x(t)$ and $y(t)$—that is, proportional to the product xy—then

$$\frac{dx}{dt} = kxy \dots \dots \dots (1)$$

Where k is the usual constant of proportionality, Suppose, a small community has a fixed population of n people. If one infected person is introduced into this community, then it could be argued that $x(t)$ and $y(t)$ are related by $x + y = n + 1$. using this last equation to eliminate y in (1) gives us the model

$$\frac{dx}{dt} = kx(n + 1 - x) \dots \dots \dots (2)$$

An obvious initial condition accompanying equation (2) is $x(0) = 1$.

Conclusion:

Mathematical modeling is a great topic, which includes variety of researches. The paper is a summarized basic idea on mathematical modeling in medicine and health and the authors have a great interest to study more and more in future.

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