Estimation of Signal Cable Time Delay (T_0) and T_{max}

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Abstract

In this document the process of estimating the Signal Cable Time Delay (T_0) and T_{max} is described and the results are presented.

Contents

1 Introduction

The recorded TDC values (in nanoseconds) which provide the measurement of drift times for the ions produced by a charged particle passing through the drift chamber are actually a sum of the true drift time and several delays. One major component of this delay is the fixed delay due to the propagation time of the signals through the signal cables that connect the pins in DCRBs (Drift Chamber Readout Boards) and the corresponding pins in the on-board STBs (Signal Translation Boards). If there were no delay of any kind (fixed or event dependent), the signal would be of the shape of a trapezoid with the perfectly vertical leading edge positioned at $t = 0$, and similarly vertical trailing edge positioned at $t = t_{max}$. But, due to the fixed time delays, the whole signal is displayed by a certain amount T0 and the leading and trailing edges smeared out due to event dependent delays - mainly due to the particle flight time (along the volume of the detector) and the signal propagation along the sense wires. In this document, we describe the process of estimating the value of the T0s for all the cables which are 42×18 in number.

2 Fit Equations

There are 9 free parameters that should be deterimined from the fits. four are generic, but the other five are B-field dependent.

$$\begin{aligned}
p_0 &= v_0 \\
p_1 &= deltanm \\
p_2 &= t_{max} \\
p_3 &= distbeta = x_\beta \\
p_4 &= delta_bfield_coefficient = \delta_B \\
p_5 &= b_1 \\
p_6 &= b_2 \\
p_7 &= b_3 \\
p_8 &= b_4
\end{aligned} \tag{1}$$

And, the overall equation is:

$$time = \frac{x}{v_0} + a \cdot \hat{x}^n + b_\alpha \cdot \hat{x}^m + \Delta t_B + \Delta t_\beta \tag{2}$$

with $\hat{x} = x/x_{max}$ and last two terms given by the following expressions:

$$\Delta t_B = \delta_B \cdot B^2 \cdot t_{max} \cdot (b_1 \hat{x_\alpha} + b_2 \hat{x_\alpha}^2 + b_3 \hat{x_\alpha}^3 + b_4 \hat{x_\alpha}^4) \tag{3}$$

$$\Delta t_\beta = \frac{\sqrt{x^2 + (x_\beta \cdot \beta^2)^2} - x}{v_0} \tag{4}$$

3 Procedure for T_0 estimation

To dermine the T_0 for a given cable, a histogram is made of the TDC values recorded for all the hits corresponding to the cable. Then towards the rising edge, the histogram is fit to the following function with five free parameters p_0, .., p_4.

$$f(t) = e^{p_0 + p_1 t} \frac{1}{1 + e^{p_2 + p_3 t}} + p_4 \tag{5}$$

3.1 Constraints on the fit parameters

To get reasonable looking fits, the free parameters had to be constrained to values as listed in the table 1. The resons for those constraints are explained in corresponding sections below.

Parameter	Minimum	Maximum
0	8.6	13.2
1	-1.0	-1.0e-04
2	100.0	4.0e02
3	-1.0	-1.0e-01
4	0.0	1.0e06

Table 1: Limits for fit parameters.

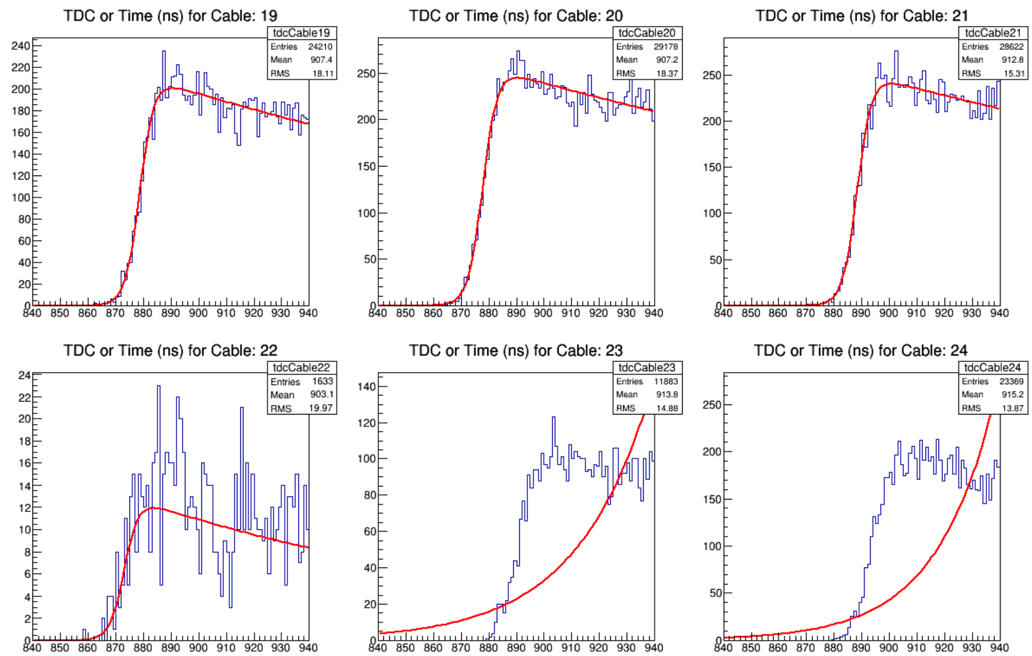

Figure 1: Time distributions showing fits when par[1] wasn't constrained well.

3.1.1 Constraining par[1]

To avoid cases such as in the 5th and 6th plot in figure 1 (where the exp in the numerator rises rather than decays because par[1] is determined to be positive), we ask Minuit to constrain this parameter to be negative (within -1.0 and -4.0e-03), which is decided by looking at par[1] values for cases where it had already worked (such as in the first 4 plots in the same figure above). Initially, a very large range was given and that made it go bersek. Later, the range (-1.0,0.0) was tried but that still left one plot looking unusual such as the fourth plot in figure 2.

3.1.2 Constraining par[4]

And, without a limit on par[4] for the constant noise, sometimes it went negative (as big as 1480 - such as for 81st cable in figure 3. To avoid that situation, the parameter was allowed to vary only above zero and within [0.0,1.0e06].

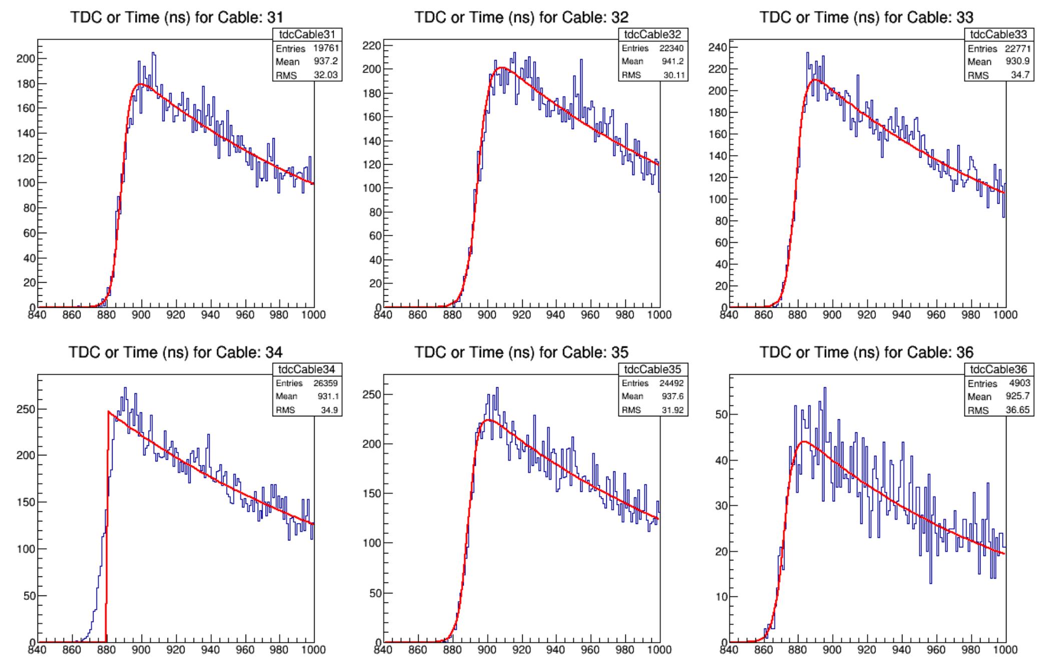

Figure 2: Time distributions showing fits when par[1] wasn't constrained well.

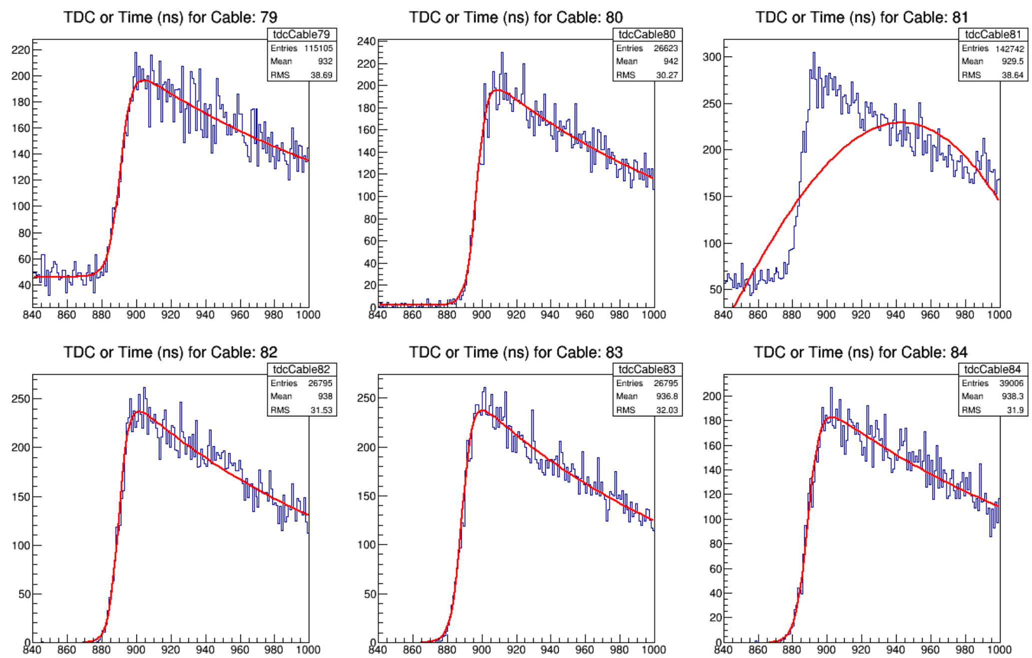

Figure 3: Time distributions showing fits when par[4] wasn't constrained well.

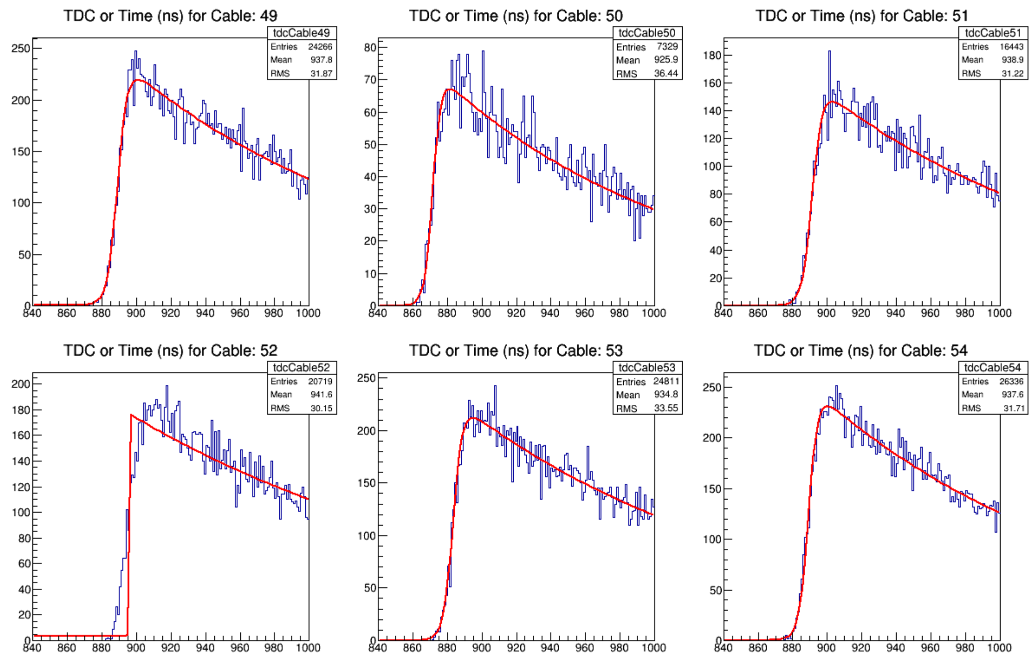

Figure 4: Time distributions showing fits when par[2] wasn't constrained well.

3.1.3 Constraining par[2] and par[3]

Next the cable 52 showed a little abnormal fit as seen in figure 4 When checked the values of the fit parameters, the value of par[2] was seen to be 1.82085e+04, whereas in other cables, it was of the order of 3.0e+2. So, it was imperative to put limits on its value, which was chosen to be [100.0,4.0e02]. With that limit, the issue was resolved, but it created a new problem in Cable 8 as reflected in figure 5 When checked again, it was that it had gotten par[1]=-3.50957e-06 and par[2]=2.18599e+01. In other working cases, these values were of the order of -7.00443e-03 and 3.19779e+02, so the upper limit of par[1] above was changed from 0 to -1.0e-04. This made it better but still not quite as good as we wanted. Upon checking parameters again, it was seen that par[3] was of the order of -4.26634e-02 and par[2] of the order 3.80025e+01, whereas in the working case they were of order 2.83977e+02 and -3.18337e-01. Therefore, par[2], and par[3] were also constrained within [100.0,4.0e02] and [-1.0,-1.0e-01] respectively.

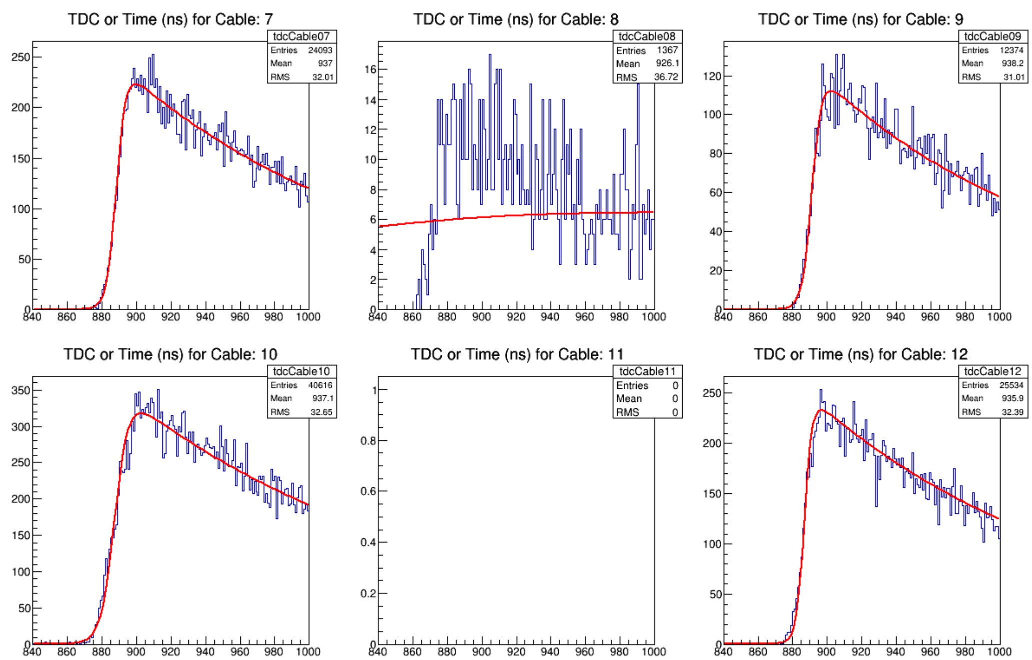

Figure 5: Time distributions showing fits when par[2] wasn't constrained well.

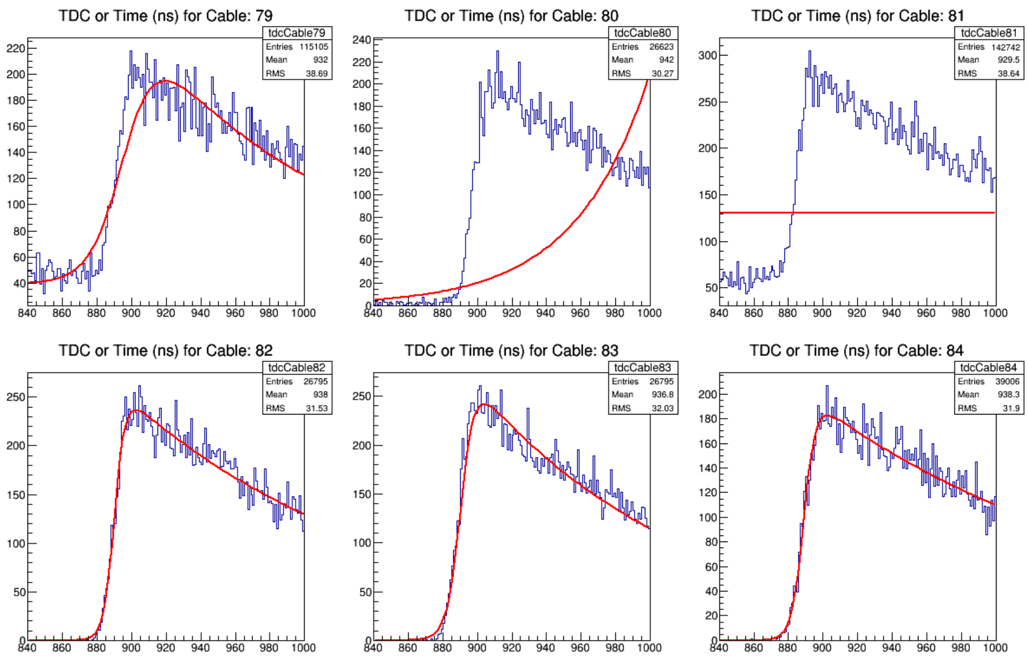

Figure 6: Time distributions showing fits when par[0] wasn't constrained well.

3.1.4 Constraining par[0]

Finally, to avoid getting cases such as that for cable 81 in figure 6, in which case par[0] was negative i.e, -1.09872e+01, the par[0] was also constrained inside [8.6,13.2].

3.2 Estimating T0 values from the fits

Once the overall fit is obtained, corresponding T0 can be extracted from its fit parameters. From the fit fit parameters, we can extract the straight line that represents the leading edge and if this straight line is extrapolated down to the horizontal line that represents the constant noise level (given by the fifth fit parameter p4), time value corresponding to the intersection point gives us the T0 for the given distribution. The straight line representing the rising edge can be defined as the line that passes through the central (half-way) point $P_{1/2}$ of the corresponding sigmoid and having the same slope m as the fit function at that particular reference point.

The reference point $P_{1/2}$ is the one on the fit function where the x-value (or time here) is such that the corresponding sigmoid function i.e. $1.0/(1+e^{p_2+p_3x})$ would have the value of 1/2. This is true when the exponential term in the denominator would be exactly one, which means the x-coordinate of that point is given by:

$$x_{1/2} = xSigm_{1/2} = -\frac{p_2}{p_3} \tag{6}$$

The slope of the fit function at $x = x_{1/2}$, on the other hand, is given by its derivative at that point as follows:

$$slope = m = e^{p_0+p_1x}\Big(\frac{p_1}{1+e^{p_2+p_3x}} - \frac{p_3e^{p_2+p_3x}}{(1+e^{p_2+p_3x})^2}\Big) \tag{7}$$

The value of the function at $x = x_{1/2}$ point is easily evaluated by calling the 'evaluate()' method of the function class that is used for the fitting procedure. Now, using the **point-slope** form for the equation of a straight line, the line representing the leading edge is given by the following equation:

$$y - f(x_{1/2}) = slope(x - x_{1/2}) \tag{8}$$

Likewise, the equation for the noise level is as follows:

$$y = p_4 \tag{9}$$

Now solving these two equations for x gives the T0 defined by the intersection point as follows:

$$T0 = \frac{p_4 - f(x_{1/2})}{slope} + x_{1/2} \tag{10}$$

By using equations 13, 6, and 7 into 10, the expression for T0 can be simplified further, giving the following expression for T0:

$$\begin{aligned} T0 &= -\frac{1+e^{p_2+p_3x_{1/2}}}{p_1+(p_1-p_3)e^{p_2+p_3x_{1/2}}} - \frac{p_2}{p_3} \\ &= -\frac{2}{2p_1-p_3} - \frac{p_2}{p_3} \end{aligned} \tag{11}$$

The second method is extrapolating the same straight line further down such that the corresponding value of time where it meets the time-axis gives us the T0 estimate as follows:

$$\begin{aligned} T0 =& sigmoidHalfwayPoint + \\ & \frac{1}{derivSigHalfPoint}(0.0 - funcValAtSigHalf); \end{aligned} \tag{12}$$

4 Procedure for T_{max} estimation

To dermine the T_{max} for a given cable, a histogram is made of the TDC values recorded for all the hits corresponding to the cable. Then towards the falling edge, the histogram is fit to the following function with five free parameters p_0, .., p_4.

$$f(t) = e^{p_0+p_1t}(1 - \frac{1}{1 + e^{p_2+p_3t}}) + p_4 \tag{13}$$

where $(1 - \frac{1}{1+e^{p_2+p_3t}})$ can be called as the inverse-sigmoid.

4.1 Parameter Constraints

4.2 Estimation of T_{max}

When the values of the parameters are determined by the fits, the corresponding inverse-sigmoids can be extracted, from which a straight line fit to the falling edge at its half-way point ($sigmoidHalfwayPoint$) can be determined. By extrapolating this straigh line, T_{max} can be determined.

Here, $sigmoidHalfwayPoint = |par[2]/par[3]|$ (modulus is for the fact that the ratio is $par[2]/par[3]$ negative due to par[3] being constrained to negative values (see table 1).

5 Conclusion

Appendices

A Propagation Of Uncertainties from fit parameters p_1, p_2, p_3 to T0

From equation 11, we see that T0 depends on three different parameters p_1, p_2, p_3. And, if these parameters have uncertainties Δp_1, Δp_2, and Δp_3 respectively, they will all propagate[1] and contribute to the overall uncertainty in T0 as follows:

$$\Delta T0 = -\sqrt{\left(\frac{\partial T0}{\partial p_1}\right)^2 (\Delta p_1)^2 + \left(\frac{\partial T0}{\partial p_2}\right)^2 (\Delta p_2)^2 + \left(\frac{\partial T0}{\partial p_3}\right)^2 (\Delta p_3)^2} \tag{14}$$

Now, these partial derivatives of T0 w.r.t the three parameters are evaluated as follows:

$$\begin{aligned} \frac{\partial T0}{\partial p_1} &= \frac{4}{(2p_1 - p_3)^2} \\ \frac{\partial T0}{\partial p_2} &= -\frac{1}{p_3} \\ \frac{\partial T0}{\partial p_3} &= -\frac{2}{(2p_1 - p_3)^2} + \frac{p_2}{p_3^2} \end{aligned} \tag{15}$$

References

[1] Wikipedia, https://en.wikipedia.org/wiki/Propagation_of_uncertainty, March, 2017.