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PHS 7010 Fall 2023 - Final Report

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1 Likelihood ratio tests in linear mixed models with one variance component

Consider an LMM with one variance component

$$\mathbf{Y} = X\beta + Z\mathbf{b} + \epsilon,\ E\begin{pmatrix} \mathbf{b} \\ \epsilon \end{pmatrix} = \begin{pmatrix} \mathbf{0}_K \\ \mathbf{0}_n \end{pmatrix},\ \text{cov}\begin{pmatrix} \mathbf{b} \\ \epsilon \end{pmatrix} = \begin{pmatrix} \sigma_b^2\Sigma & \mathbf{0} \\ \mathbf{0} & \sigma_\epsilon^2 I_n \end{pmatrix}$$

where,

- $\mathbf{Y}$ is the $n \times 1$ vector of observations,
- X is the $n \times p$ design matrix for the fixed effects,
- Z is the $n \times K$ design matrix for the random effects,
- β is a p-dimensional vector of fixed effects parameters,
- $\mathbf{b}$ is a K-dimensional vector of random effects,
- $(\mathbf{b}, \epsilon)$ has a normal distribution.

Under these conditions it follows that

$$E(\mathbf{Y}) = X\beta,$$
$$\text{var}(\mathbf{Y}) = \sigma_\epsilon^2 V_\lambda$$

where

- $\lambda = \sigma_b^2/\sigma_\epsilon^2$, which can be considered a signal-to-noise ratio,
- $V_\lambda = I_n + \lambda Z\Sigma Z'$.