Mathematics & Basic Statistics

code → BSCM 103 B

Syllabus : maths4uem.wordpress.com.

Books : Higher engineering mathematics.

B.S. Grewal (44^{th} volume)

K. Das & P. Pal (10^{th} volume)

Syllabus :

- Matrics
- Vector space
- ODE (ordinary Differential equation).
- Basic statistics.

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Matrices

↳ Rectangular arangement of numbers in rows & columns (m x n).

↳ Matrix is represented as $A = [a_{ij}]_{m \times n}$

$i = i^{th}$ row

$j = j^{th}$ column

↳ Types of Matrix :

✓ Row matrix : eg. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

✓ Column matrix : $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

✓ null/void/zero matrix : $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

✓ Square matrix ⇒ when rows = column

$m = n$

eg. 2×2, 3×3

✓ Diagonal matrix = eg. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

only diagonal elements rest are zero.

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Scalar matrix : eg $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
same element diagonal matrix.

Identity matrix : eg $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
or unit matrix

Transpose of a matrix :
interchange of rows & column.

Notes :- $(A^T)^T = A$

$(kA)^T = k(A)^T$

$(AB)^T = B^T A^T$

$(A+B)^T = A^T + B^T$

Symmetric matrix :
if $a_{ij} = a_{ji} \quad \forall\, i, j$

or $\rightarrow$ $\boxed{A^T = A}$

Skew-symmetric matrix $\rightarrow$ $\boxed{A^T = -A}$

Trace of a matrix :
sum of the diagonal elements

- Triangular matrix :

Upper Δ = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

lower Δ = $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 5 & 3 \end{bmatrix}$

- Orthogonal matrix : if $AA^T = I$
- Idempotent matrix : if $A^2 = A$
- Involantry matrix : If $A^2 = I$
- Nilpotent matrix : if $A^k = 0$
- Conjugate of a matrix :

$A = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$; $\bar{A} = \begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix}$

- Conjugate Transpose :

$\bar{A} = \begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix}$

$A^* = (\bar{A})^T = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

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Hermitian matrix :
if $\boxed{A = A^*}$

Skew Hermitian matrix :
if $\boxed{A^* = -A}$

Note : If A is orthogonal matrix
then $AA^T = I$
$\boxed{A^T = A^{-1}}$

Note : for skew symmetric matrix the leading diagonal entries will be 0.

Q. Check weather :

① $\begin{bmatrix} 3 & 2 & 0 \\ 2 & 9 & 1 \\ 0 & 1 & 7 \end{bmatrix}$ is symmetric or not ?
→ it is symmetric bcoz $a_{ij} = a_{ji}$; $A^T = A$

② $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$ ⇒ it is skew symmetric bcoz $\boxed{A^T = -A}$

③ $\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$ is Hermitian ?

$\bar{A} \Rightarrow \begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix} \Rightarrow (\bar{A})^T = \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

yes, it is Hermitian as $\boxed{A^* = A}$

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(4) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ is skew hermitian?

$\bar{A} = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

Yes, it is skew hermitian

as $\boxed{A^* = -A}$

Note → ✓ for skew hermitian matrix diagonal entries will be either zero or purely imaginary.

✓ Diagonal elements for hermitian matrix will always be real.

Q. find trace of $\begin{bmatrix} 1 & 6 & 7 \\ 0 & -5 & 8 \\ 3 & 4 & 2 \end{bmatrix}$

$tr(A) = 1 - 5 + 2$

$= \boxed{-2}$

✓ Algebra of matrix :-

- Addition : order must be same.
  - ✓ Commutative $(a+b = b+a)$
  - ✓ associative $(a + (b+c) = (a+b)+c)$

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- Subtraction : order must be same.
  × Associative
  × Commutative

- Product : ✓ Scalar multiplication
  kA (possible).

✓ $[A]_{m \times n} [B]_{n \times p} = [C]_{m \times p}$

↳ Associative $A(BC) = (AB)C$
↳ Distributive $A(B+C) = AB + AC$.
↳ Commutative $AB \neq BA$ (generally)

✓ Equal matrix :
↳ order must be same.
↳ corresponding elements must be same.

✓ Determinants $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$\Delta \Rightarrow a_{11} a_{22} - a_{21} a_{12}$

✓ Minor, Cofactor, Adjoint.

Note → $|A| = |A^T|$

$(\text{adj } A) A = A(\text{adj } A) = |A| I$

✓ Inverse of matrix : $A^{-1} = \frac{1}{|A|} \text{adj}(A)$.
(If $|A| \neq 0$)

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✓ Singular matrix : if $|A| = 0$

✓ non-singular ⇒ if $|A| \neq 0$

notes
↳ $(AB)^{-1} = B^{-1}A^{-1}$
↳ $(A^T)^{-1} = (A^{-1})^T$

(know about row transformation

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→ Echleon form of a Matrix

Row transformation :

Q.
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$A \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

(echleon form is upper triangular matrix

$|A|$

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→ Rank of a matrix:

↳ No. of non-zero rows in an echleon form of a matrix.

$\rho(A) = \text{Rank}(A) = 2$

Q. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 1 & 6 & 5 \end{bmatrix}$ $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 4 & 2 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

$\Rightarrow 2 - 2(-1)$

$= 2 + 2 = \boxed{4}$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$

$\boxed{\rho(A) = 3}$

Q. $B = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ $R_2 \to R_2 + 2R_1$
$R_3 \to R_3 + R_1$

$B \Rightarrow \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 4 \end{bmatrix}$

$\boxed{\rho(B) = 3}$

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$$C = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

$R_2 \Rightarrow R_2 + 2R_1$
$R_3 \to R_3 + 3R_1$
$R_4 \to R_4 + 5R_1$

$$C \Rightarrow \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix}$$

~~$R_3 \to R_3 + R_1$~~
~~$R_4 \to R_4 + R_1$~~
$R_3 \to R_3 - 2R_2$
$R_4 \to R_4 - 2R_2$

$$C \Rightarrow \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 0 & -0 \\ 0 & 0 & 0 & -0 \end{bmatrix}$$

~~$R_4 \to R_4 - R_3$~~

$\boxed{\rho(C) = 2}$

$$\cancel{C} = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 17 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Q.

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 15 \\ 3 & 7 & 0 & 2 & 16 \\ 5 & 3 & 2 & 3 & 21 \end{bmatrix}_{3\times5}$$

$R_2 \to R_2 - \frac{3}{2}R_1$

$16 - \frac{3}{2} \times 15$

$R_3 \to R_3 - \frac{5}{2}R_1$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 & 1 & 15 \\ 0 & 1 & 4.5 & 1/2 & -6.5 \\ 0 & -7 & 2-\frac{15}{2} & 3-5/2 & \end{bmatrix}$$

$3 - \frac{5}{2} \times 4 = -7$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 & 1 & 15 \\ 0 & 1 & 4.5 & 1/2 & \_\_ \\ 0 & 0 & & & \end{bmatrix}$$

$R_3 \to R_3 + 7R_2$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 & 1 & 15 \\ 0 & 2 & -9 & 1 & -13 \\ 0 & 0 & -74 & 5 & -124 \end{bmatrix}$$

$\boxed{\rho(D) = 3}$

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* Solving linear System of Equations

✓ $AX = B$. (A = matrix of coffiecient of x y z.
B = RHS, X = unknown variable)

eg → $2x + 3y - z = 6$
$x - y + z = 2$
$2x + 3y + 15z = 0$

Matrix form :-

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & 3 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$$

* $AX = B$ (Non - Homogenous)
* $AX = 0$ (Homogenous)

System

$AX = B$

consistent (intersecting or overlapping)

unique

infinite

inconsistent (No sol^n)

$AX = 0$

consistent

unique (Trivial)

infinite (non-Trivial)

$\boxed{\text{Trivial} = (0, 0, 0)}$

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How to solve :-

✓ Matrix inverse method $(X = A^{-1}B)$

✓ Crammer's Rule

✓ Using Rank.

① Using Rank :

Echleon form [A and B]

↓

Rank

n = number of variable

(x, y, z) = n = order

Rank = n

Rank (A) = Rank (A|B)

↓

unique sol^n

Rank < n

↓

Rank (A) = Rank (A|B)

Infinite

Consistent.

Rank ≠ n

inconsistent.

∴ no sol^n

Q.

$$x_1 + 2x_2 - x_3 = 3$$
$$3x_1 - x_2 + 2x_3 = 1$$
$$2x_1 - 2x_2 + 3x_3 = 2$$
$$x_1 - x_2 + x_3 = -1$$

Check for consistency & solve?

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Sol^n ⇒ $AX = B$

step 1

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

step 2 $[A \mid B]$ → Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

step 3 echleon form.

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - \frac{6}{7} R_2 \\ R_4 \to R_4 - \frac{3}{7} R_2 \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5/7 & 20/7 \\ 0 & 0 & -1/7 & -4/7 \end{bmatrix}$$

$R_3 \to 5 - \frac{6}{7} \times 5$
$\to 5 - \frac{30}{7}$
$\frac{35 - 30}{7} = \frac{5}{7}$

$R_3 \to -4 - \frac{6(-8)}{7}$
$-4 + \frac{48}{7}$
$\frac{20}{7} \leftarrow \frac{-28 + 48}{7}$

$R_4 \to -3 - \frac{3}{7} \times -7 = -3 + 3$
$= 0$

$R_4 \to 2 - \frac{3}{7} \times 5 \to 2 - \frac{15}{7}$ $\frac{14 - 15}{7}$ $-1/7$

$R_4 \to$ ~~R_4~~ $-4 - \frac{3(-8)}{7} \Rightarrow -4 + \frac{24}{7} \Rightarrow \frac{-28 + 24}{7}$

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$\rightarrow R_4 \rightarrow R_4 + \frac{1}{5} R_3$

$R_4 \rightarrow -\frac{4}{7} + \frac{1}{5}\cdot\frac{20}{7}$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5/7 & 20/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

step (4) Rank (A) $= 3$
Rank (A|B) $= 3$ $\Big\}$ consistent

Rank (A) $=$ Rank (A/B) $= n$
(unique).

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- Backward substitution.
{Guass - elimination method}

$$\begin{array}{cccc} x_1 & x_2 & x_3 & B \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5/7 & 20/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{5}{7} x_3 = \frac{20}{7}$

$\boxed{x_3 = 4}$

$-7x_2 + 5x_3 = -8$

$-7x_2 + 20 = -8$

$-7x_2 = -8 - 20 = -28$

$\boxed{x_2 = 4}$

$x_1 + 2x_2 - x_3 = 3$

$x_1 + 8 - 4 = 3$

$x_1 + 4 = 3$

$x_1 = 3 - 4$

$= \boxed{-1}$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix}$$

Q4

$$2a + 4b + 3c + d = 15$$
$$3a + 7b + 2d = 16$$
$$5a + 3b + 2c + 3d = 21$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 & 1 \\ 3 & 7 & 0 & 2 \\ 5 & 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 15 \\ 16 \\ 21 \end{bmatrix}$$

$(n =$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & 4 & 3 & 1 & 15 \\ 3 & 7 & 0 & 2 & 16 \\ 5 & 3 & 2 & 3 & 21 \end{array}\right] = [A \,\vdots\, B]$$

$$\Rightarrow R_3 \rightarrow R_3 - \frac{5}{2} R_1$$

$$R_2 \rightarrow R_2 - \frac{3}{2} R_1$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 3 & 1 & 15 \\ 0 & 1 & -9/2 & 1/2 & -13/2 \\ 0 & -7 & -11/2 & 1/2 & -33/2 \end{bmatrix}$$

$$\Rightarrow R_3 \rightarrow R_3 + 7 R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & 4 & 3 & 1 & 15 \\ 0 & 1 & -9/2 & 1/2 & 13/2 \\ 0 & 0 & -37 & 4 & -124/2 \end{array}\right]$$

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$P(A/B) = 3$

$P(A) = 3$

$n = 4$

$3 < 4$

$\therefore$ Infinite

$-37c + 4d = \frac{-124}{2}$

$1b - \frac{9}{2}c + \frac{1}{2}d = \frac{13}{2}$

$2a + 4b + 3c + d = 15$

$$\Rightarrow 4d = \overset{-62}{\frac{-124}{2}} + 37c$$

$4d = -62 + 37c$

$$\boxed{d = \frac{-62 + 37c}{4}}$$

$\cancel{-37c}$

$$\boxed{\frac{-62}{4d} = -37c} \Rightarrow \boxed{c = \frac{+62}{4d \times +37}}$$

$b - \frac{9}{2} \times \frac{62}{4d \times 37} + \frac{1}{2}d = \frac{13}{2}$

$$\boxed{b = \frac{13}{2} - \frac{d}{2} + \frac{9}{2} \times \frac{62}{4d \times 37}}$$

$2a + 4\left(\frac{13}{2} - \frac{d}{2} + \frac{9}{2} \times \frac{62}{4d \times 37}\right) + 3\left(\frac{62}{4d \times 37}\right) + d = 15$

$2a + \frac{52}{2} - \frac{4d}{2} + \left(\frac{36}{2} \times \frac{62}{4d \times 37}\right) + \frac{3 \times 62}{4d \times 37} + d = 15$

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$$2a + 26 - 2d + \left(\frac{18 \times 62}{4d \times 37}\right) + \frac{3 \times 62}{4d \times 37}$$

$$2a - 2d + \frac{18 \times 62}{4d \times 37} + \frac{93}{74d} + d = 15 - 26$$

$$2a - 2d + d + \frac{18 \times 31}{74d} + \frac{93}{74d} = -11$$

$$2a - d + \frac{558}{74d} + \frac{93}{74d} = -11$$

$$2a - d + \frac{651}{74d} = -11$$

~~2a~~

Q_{11}

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \\ 3 & 2 & 3 & 5 \end{array}\right] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -2 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

$\rho(A) = 2 \quad \rho(A|B) = 3$ — inconsistent

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Q. Check the consistency and solve if possible :

(i) $2x - 2y - 4z = 8$
$2x + 3y + 2z = 8$
$-x + y - z = \frac{7}{2}$

(ii) $x + y + z = 6$
$x - y - z = -4$
$x + y - z = 0$
$2x - z = 4$

(step 1 – matrix
2 – ~~echeo~~ Augmented
3 – echeon
4 – Rank
5 – Backward substitution)

(iii) $x + 2y - z = 10$
$x - y - 2z = -2$
$2x + y - 3z = 8$

A (i) $$\begin{bmatrix} 2 & -2 & -4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 7/2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & -4 & 8 \\ 2 & 3 & 2 & 8 \\ -1 & 1 & -1 & 7/2 \end{bmatrix}$$

$\Rightarrow$ now echeon $\rightarrow$

$R_2 \longrightarrow R_2 - R_1$
$R_3 \longrightarrow R_3 + \frac{R_1}{2}$
$$\Rightarrow \begin{bmatrix} 2 & -2 & -4 & 8 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & -3 & 15/2 \end{bmatrix}$$

Rank $(A \vdots B) = 3$; Rank$(A) = 3$

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$n = 3$; unique sol^n.

Backward subs^n →

$$\left[\begin{array}{ccc|c} 2 & -2 & -4 & 8 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & -3 & 15/2 \end{array}\right]$$

$$\Rightarrow 0x + 0y - 3z = 15/2$$

$$z = \frac{\cancel{15}\,5}{2 \times -\cancel{3}}$$

$$\boxed{z = -\frac{5}{2}}$$

$$0x + 5y + 6z = 0$$

$$5y = -6 \times -\frac{5}{2}$$

$$5y = \frac{30}{2}$$

$$y = \frac{6}{2} = 3 \; ; \; \boxed{y = 3}$$

$$2x - 2(3) - \cancel{4}^{2}\left(-\frac{5}{\cancel{2}}\right) = 8$$

$$2x - 6 + 10 = 8$$

$$2x = 8 - 10 + 6$$

$$x = \frac{14 - 10}{2} = \frac{4}{2} =$$

$$\boxed{x = 2}$$

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(ii) $x + y + z = 6$

$x - y - z = -4$

$x + y - z = 0$

$2x - z = 4$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \\ 2 & 0 & -1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & -4 \\ 1 & 1 & -1 & 0 \\ 2 & 0 & -1 & 4 \end{array}\right]$$

$\Rightarrow R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -2 & -6 \\ 0 & -2 & -3 & -8 \end{bmatrix}$$

$R_4 \to \cancel{R_4 - R_3}\ R_4 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -2 & -6 \\ 0 & 0 & -1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -2 & -6 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

$R_4 \to R_4 \cancel{+ R_3} - \frac{R_3}{2}$ $\left(2 - \left(\frac{-6}{2}\right)\right)$

Rank $(A|B) = 4$
$\rho(A) = 3$

$\rho(A|B) \neq \rho(A)$
$\therefore$ $\boxed{\text{no. solution}}$

(iii) $x + 2y - z = 10$
$x - y - 2z = -2$
$2x + y - 3z = 8$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & -2 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 8 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 10 \\ 1 & -1 & -2 & -2 \\ 2 & 1 & -3 & 8 \end{array}\right]$$

$\Rightarrow R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 10 \\ 0 & -3 & -1 & -12 \\ 0 & -3 & -1 & -12 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 10 \\ 0 & -3 & -1 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A|B) = 2$; $\rho(A) = 2$

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$\boxed{x, y, z} = n = 3$

$n > \rho(A) ; \rho(A|B)$

consistent, infinite solution

$0x - 3y - z = -12$

$1x + 2y - z = 10$

$\boxed{-3y + 12 = z}$

$1x + 2y - (-3y + 12) = 10$

$1x + 2y + 3y - 12 = 10$

$x + 5y - 12 = 10$

$\boxed{x + 5y = 22}$ — (1)

$-3y - z = -12$

$(3y + z = 12)$ — (11)

$1x + 2y - z = 10$

$x = 10 + z - 2y$

$x = 10 + z + 2\left(\frac{z - 12}{+3}\right)$

$x = 10 + z + \frac{2z - 24}{3}$

$x = 10 + \frac{3z}{3} + \frac{2z}{3} - \frac{24}{3}$

$x = 10 + \frac{5z}{3} - 8$

$\boxed{x = 2 + \frac{5z}{3}}$ — (111)

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(i) & (ii)

$x + 5y = 22$

$3y + z = 12$

put value of x from (iii)

$x = \frac{2 + 5z}{3}$

$2 + \frac{5z}{3} + 5y = 22$

$3y + z = 12$

$\left[\frac{5z}{3} + 5y = 20\right] \times 3$

$\left[z + 3y = 12\right] \times 5$

$5z + 15y = 60$

$5z + 15y = 60$

(iv) for what values of k, the following equations have solutions (solve all ca

(i)

$x + y + z = 1$

$2x + y + 4z = k$

$4x + y + 10z = k^2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$$

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$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array}\right]$$

$\Rightarrow R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & -3 & 6 & k^2-4 \end{bmatrix}$$

$R_3 \to \cancel{R_3 - 3R_1}\ R_3 - 3R_2$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & (k^2-4)-3k+6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{bmatrix}$$

condition of consistent

$\Rightarrow \rho(A) = \rho(A|B)$

$\cancel{2 \neq 3}$

so, $k^2 - 3k + 2 = 0$

$k^2 - 1k - 2k + 2 = 0$

$k(k-1) - 2(k-1) = 0$

$\boxed{k = 1, 2}$

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$\boxed{\text{If } k=1}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$0x - y + 2z = -1$

$x + y + z = 1$

$2z - y = -1$

$\boxed{z = t}$

$2t - y = -1$

$\boxed{2t + 1 = y}$

$x = 1 - y - z$

$x = 1 - 2t - 1 - t$

$\boxed{x = -3t}$ $\therefore$ Infinite

If $\boxed{k=2}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

$\Rightarrow$ $\rho(A) \neq \rho(A|B)$

$2 \neq 3$

$\therefore$ no soln.

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Q.

$x_1 + 4x_2 + 2x_3 = 1$

$2x_1 + 7x_2 + 5x_3 = 2k$

$4x_1 + mx_2 + 10x_3 = 2k+1$

for what values of m & k, the system will have :

(i) Unique solution ($m \neq 14$, any k)

(ii) No sol^n ($m = 14$, $k \neq 1/2$)

(iii) Many sol^n ($m = 14$, $k = 1/2$)

$$\Rightarrow \begin{bmatrix} 1 & 4 & 2 \\ 2 & 7 & 5 \\ 4 & m & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2k \\ 2k+1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 2 & 7 & 5 & 2k \\ 4 & m & 10 & 2k+1 \end{array}\right]$$

$\Rightarrow R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & 2k-2 \\ 0 & (m-16) & 2 & 2k+1-4 \end{bmatrix}$$

$R_3 \to R_3 + (m-16)R_2$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & 2k-2 \\ 0 & 0 & 2+m-16 & 2k-3+(2k-2)(m-16) \end{bmatrix}$$

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$$\Rightarrow \left[\begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & 2k-2 \\ 0 & 0 & m-14 & 2k-3+(2k-2)(m-16) \end{array}\right]$$

$2k-3+(2k-2)(m-16)$

$2k-3+2km-32k-2m+32$

$2k-32k-3+32-2m+2km$

$-30k-2m+2km+29$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & 2k-2 \\ 0 & 0 & m-14 & -30k-2m+2km+29 \end{array}\right]$$

→ for unique solⁿ

$n = \rho(A) = \rho(A|B)$

So, $m-14 \neq 0$

$\boxed{m \neq 14}$, any k.

$-30k-2m+2km+29 \neq 0$

again → $\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & (2k-2) \\ 0 & (m-16) & 2 & (2k-3) \end{array}\right]$

$R_3 \rightarrow R_3 - 2R_2$

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$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & (2k-2) \\ 0 & m-16+2 & 0 & 2k-3-2(2k-1) \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -1 & 1 & 2k-2 \\ 0 & m-14 & 0 & -2k+1 \end{array}\right]$$

$$x_2(m-14) = -2k+1$$

$$-x_2 = 2k-2$$

$$\boxed{x_2 = 2-2k}$$

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Homogeneous System :-

$$AX = 0$$

trivial (unique) | non-trivial (∞)

Q. $x_1 - x_2 + 2x_3 - 3x_4 = 0$

$3x_1 + 2x_2 - 4x_3 + x_4 = 0$

$5x_1 - 3x_2 + 2x_3 + 6x_4 = 0$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 3 & 2 & -4 & 1 \\ 5 & -3 & 2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 5R_1$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & -3 & 0 \\ 0 & 5 & -10 & 10 & 0 \\ 0 & +2 & -8 & 21 & 0 \end{array}\right]$$

$R_3 \to R_3 - \frac{2}{5} R_2$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & -3 & 0 \\ 0 & 5 & -10 & 10 & 0 \\ 0 & 0 & -4 & 17 & 0 \end{array}\right]$$

$-8 - \frac{2}{\cancel{5}} \times -\cancel{10}^{2}$ $\qquad$ $21 - \frac{2}{\cancel{5}} \times \cancel{10}^{2}$

$-8 + 4 = -4$ $\qquad$ $21 - 4 =$

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⇒ Backward substitution:-

$-4x_3 + 17x_4 = 0$

$5x_2 - 10x_3 + 10x_4 = 0$

$x_1 - x_2 + 2x_3 - 3x_4 = 0$

$x_4 = t$

$-4x_3 = -17t$

$\boxed{x_3 = \frac{17}{4}t}$ $\boxed{x_4 = t}$

$5x_2 - 10\left(\frac{17}{4}\right)t + \frac{40}{4}t = 0$

$5x_2 = -\frac{40t + 170t}{4}$ $\cancel{= 0}$

$5x_2 = \frac{-40t + 170t}{4} = \frac{+130}{4}t$

$x_2 = \frac{+130}{4\times5}t$

$x_2 = \frac{+13\cancel{0}}{2\cancel{0}}t$

$\boxed{x_2 = \frac{+13}{2}t}$

$\cancel{x_1 + \frac{13}{2}t + \frac{2\times17}{4\,2}t - 3t = 0}$

$\cancel{x_1 + \frac{30}{2}t - 3t}$

$x_1 \cancel{+x_2} - \frac{13}{2}t + \frac{2\times17}{4}t - 3t = 0$

$x_1 - \frac{13t}{2} + \frac{17}{2}t - 3t = 0$

$x_1 + 2t - 3t = 0$ $\boxed{x_1 = t}$