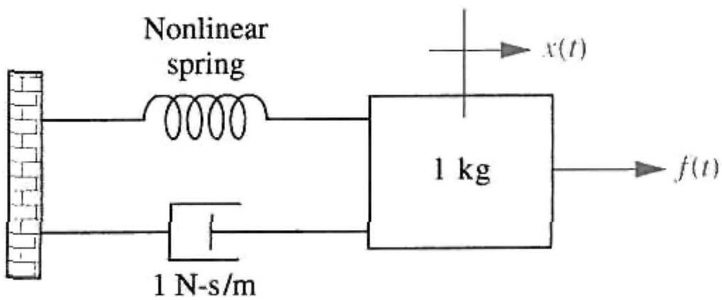
EE2101
Assignment-1

Kota Pranav Kumar
EE19BTECH11051

September 9, 2020

1 Problem

For the translational mechanical system with a nonlinear spring shown in the following figure,find the transfer function, $G(s) = X(s)/F(s)$,for small excursions around $f(t) = 1$.The spring is defined by $x_s(t) = 1 - e^{-f_s(t)}$ $, where$x$_s(t)$ is the spring displacement and $f_s(t)$ is the spring force.



2 Solution

Given: Small excursions around f(t) = 1

Let $x(t) = x_o + \delta x$ and $f(t) = 1 + \delta f$, where x_0 is the point where $f(t) = 1$

We know that $x(t) = x_s(t) = 1 - e^{-f_s(t)}$
$\Longrightarrow$ f$_s(t)$ = -ln(1 - $x(t)$)

As excursions are small,let us linearise the above force expression.
ln(1 - $x(t)$) - ln(1 - x_o) = $-\delta x/1 - x_o$
i.e. ln(1 - $x(t)$) = ln(1 - x_o) - $\delta x/1 - x_o$

At $f(t) = 1$, $x = x_o$
$\Longrightarrow$ x$_o$ =$1 - e^{-1} = 1 - 0.3678 = 0.6321$
$\Longrightarrow$ $ln(1-$x(t)$) = 1 - \delta x/0.3678 = 1 + 2.7188\delta x$

Force due to piston , $f_p(t) = \frac{dx}{dt}$
$\Longrightarrow$ f$_p(t)$ = $\frac{d(x_0+\delta x)}{dt} = \frac{d\delta x}{dt}$

Applying Newtons Second Law to the system, we get

$$f(t) - f_s(t) - f_p(t) = m\frac{d^2x}{dt^2} = m\frac{d^2(x_o + \delta x)}{dt^2} = m\frac{d^2\delta x}{dt^2}$$

$$\implies 1 + \delta f - 1 - 2.7188\delta x - \frac{d\delta x}{dt} = \frac{d^2\delta x}{dt^2}$$

$$\implies \frac{d^2\delta x}{dt^2} + \frac{d\delta x}{dt} + 2.7188\delta x = \delta f$$

$\mathcal{L}\delta x = X(s)$

$\mathcal{L}\frac{d\delta x}{dt} = sX(s) - \delta x(0) = sX(s)$

$\mathcal{L}\frac{d^2\delta x}{dt^2} = s^2X(s) - s\delta x(0) - \delta x'(0) = s^2X(s)$

$\mathcal{L}\delta f = F(s)$

Applying Laplace transform to both the sides,we get

$$s^2X(s) + sX(s) + 2.7188X(s) = F(s)$$

$$\implies X(s)/F(s) = 1/(s^2 + s + 2.7188)$$

$$\therefore G(s) = X(s)/F(s) = 1/(s^2 + s + 2.7188)$$