Part1: Modeling

2. Coordinates and Transformations

Outline

I. Model Transformation
II. 변환
III. 복합변환
IV. 반사
V. 구조왜곡변환
VI. 그래픽 변환
VII. 좌표계

1. Model Transformation

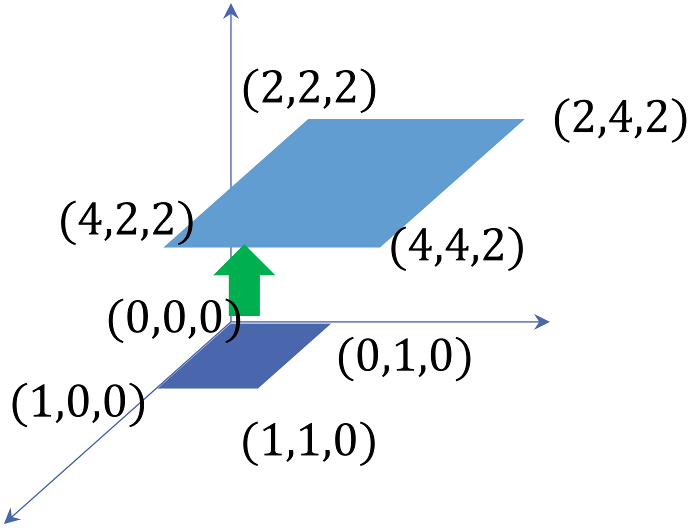
- Vertex로 이루어진 Model을 어떻게 변형 시킬지?

1.1 예) 선형 변환

- 좌표계에서 물체의 확대 와 이동

$$x' = 2 \cdot x + 0 \cdot y + 0 \cdot z + 2$$
$$y' = 0 \cdot x + 2 \cdot y + 0 \cdot z + 2$$
$$z' = 0 \cdot x + 0 \cdot y + 2 \cdot z + 2$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$



1.1 선형 변환

- 좌표계에서 물체의 확대 와 이동 → 확대와 이동

$$x'' = 3 \cdot x' + 3 = 3 \cdot (2 \cdot x + 0 \cdot y + 0 \cdot z + 2\ ) + 3$$
$$y'' = 3 \cdot y' + 3 = 3 \cdot (0 \cdot x + 2 \cdot y + 0 \cdot z + 2) + 3$$
$$z'' = 3 \cdot z' + 3 = 3 \cdot (0 \cdot x + 0 \cdot y + 2 \cdot z + 2\ ) + 3$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

1.1 선형 변환

- 좌표계에서 물체의 확대 와 이동이 100번 반복된다면? (Animation이라고 하면 충분히 가능한 시나리오)
- 모든 Vertex에 100번 연산을 수행해야 함

$$
\begin{aligned}
x'' &= 3 \cdot x' + 3 = 3 \cdot (2 \cdot x + 0 \cdot y + 0 \cdot z + 2\ ) + 3 \\
y'' &= 3 \cdot y' + 3 = 3 \cdot (0 \cdot x + 2 \cdot y + 0 \cdot z + 2) + 3 \\
z'' &= 3 \cdot z' + 3 = 3 \cdot (0 \cdot x + 0 \cdot y + 2 \cdot z + 2\ ) + 3
\end{aligned}
$$

$$
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}
$$

$$
\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}
$$

1.2 Homogeneous Coordinate

- Dimension을 살짝 바꾸면?

$$x' = 2 \cdot x + 0 \cdot y + 0 \cdot z + 2 \qquad x'' = 3 \cdot x' + 0 \cdot y' + 0 \cdot z' + 3$$
$$y' = 0 \cdot x + 2 \cdot y + 0 \cdot z + 2 \qquad y'' = 0 \cdot x' + 3 \cdot y' + 0 \cdot z' + 3$$
$$z' = 0 \cdot x + 0 \cdot y + 2 \cdot z + 2 \qquad z'' = 0 \cdot x' + 0 \cdot y' + 3 \cdot z' + 3$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1' \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 & 9 \\ 0 & 6 & 0 & 9 \\ 0 & 0 & 6 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \text{Homogeneous Coordinate}$$

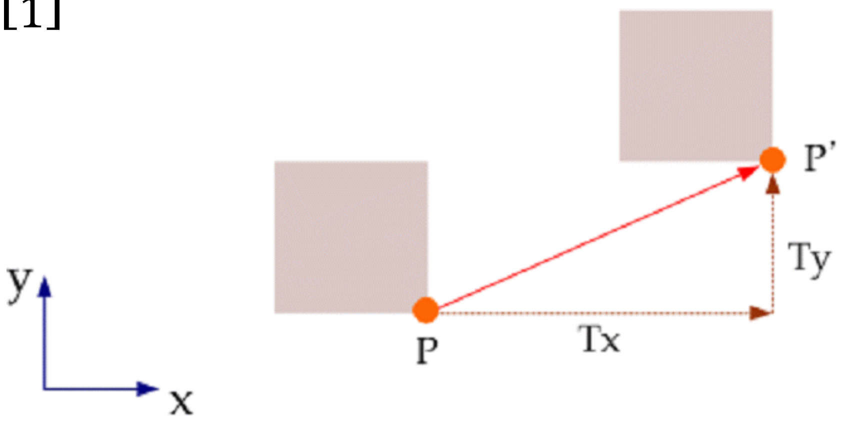
2 변환

- Translation
- Rotation
- Scaling
- Shearing

2.1 Translation (이동)

$$x' = 1 \cdot x + 0 \cdot y + T_x$$
$$y' = 0 \cdot x + 1 \cdot y + T_y$$

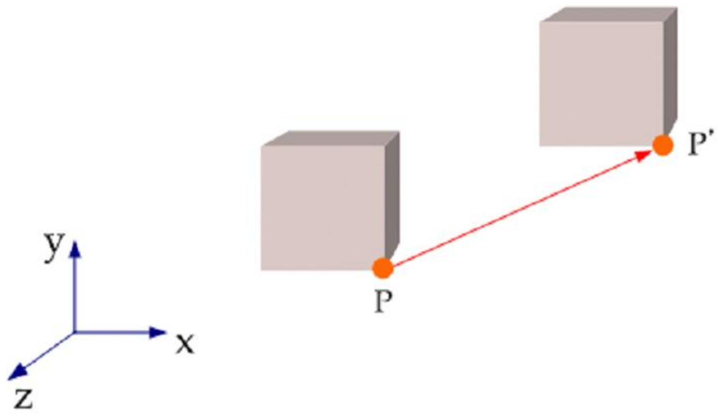
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



2.1 Translation (이동)

$$x' = 1 \cdot x + 0 \cdot y + 0 \cdot z + T_x$$
$$y' = 0 \cdot x + 1 \cdot y + 0 \cdot z + T_y$$
$$z' = 0 \cdot x + 0 \cdot y + 1 \cdot z + T_z$$

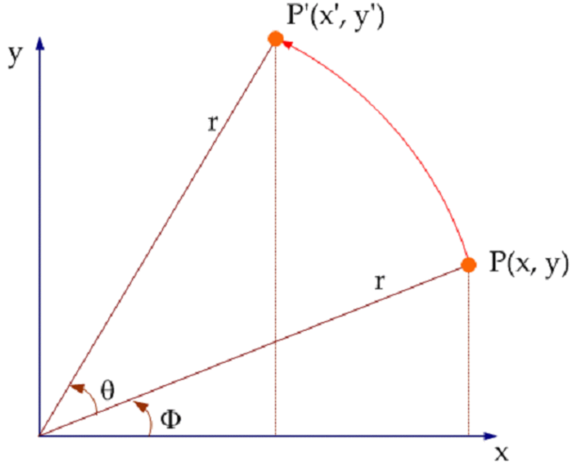
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



2.2 Rotation (회전)

$$x' = r\cos(\varphi+\theta) = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta = \cos\theta\cdot x - \sin\theta\cdot y$$
$$y' = r\sin(\varphi+\theta) = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta = \sin\theta\cdot x + \cos\theta\cdot y$$

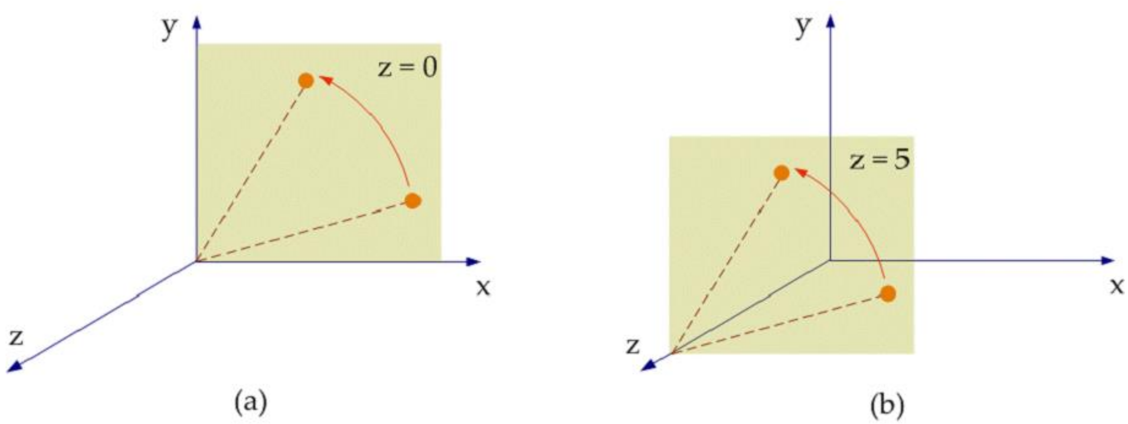
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



2.2 Rotation (회전)

$$x' = r\cos(\varphi+\theta) = r\cos\varphi\cos\theta + r\sin\varphi\sin\theta = \cos\theta\cdot x - \sin\theta\cdot y$$
$$y' = r\sin(\varphi+\theta) = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta = \sin\theta\cdot x + \cos\theta\cdot y$$
$$z' = z$$

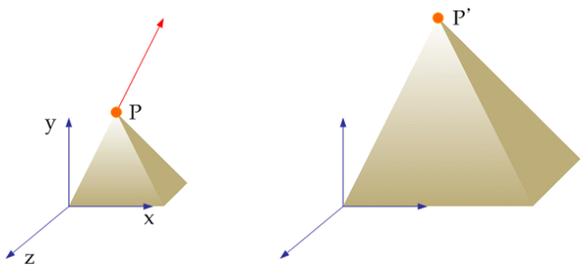
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



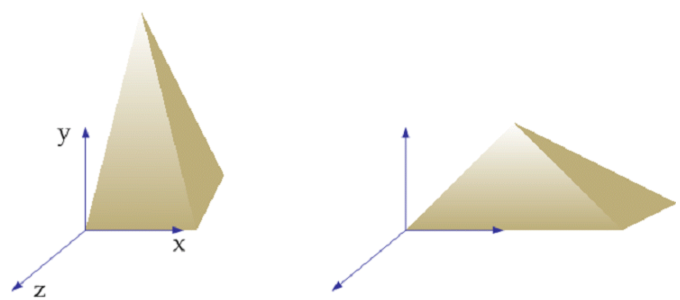
2.3 Scaling (크기조절)

$$
\begin{aligned}
x' &= S_x \cdot x + 0 \cdot y + 0 \cdot z \\
y' &= 0 \cdot x + S_y \cdot y + 0 \cdot z \\
z' &= 0 \cdot x + 0 \cdot y + S_z \cdot z
\end{aligned}
$$

$$
\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$



(a) (b)

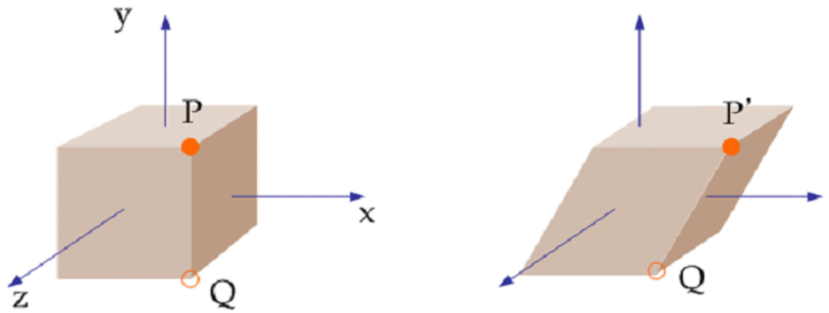


(a) (b)

2.4 Shearing (전단)

$$x' = 1 \cdot x + Sh_y \cdot y + 0 \cdot z$$
$$y' = Sh_x \cdot x + 1 \cdot y + 0 \cdot z$$
$$z' = 1 \cdot z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 0 & 0 \\ Sh_x & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



(a) (b)

3. 복합 변환

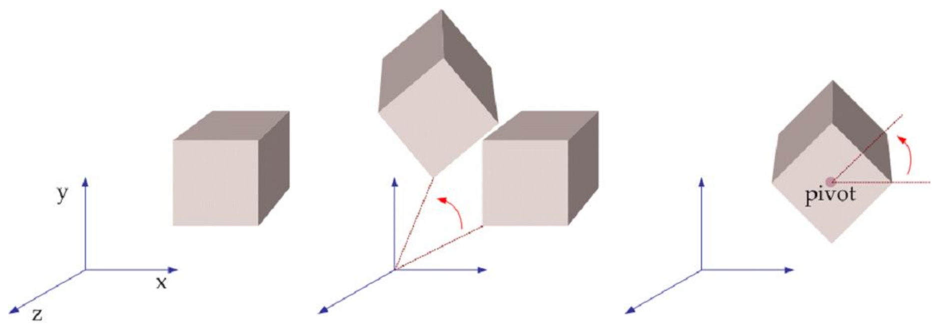
- 변환이 복합적으로 이루어 진 것
- 예) 크기조절 → 회전 → 크기조절

$$P' = S_2 \cdot R_1 \cdot S_2 \cdot P$$

$$(C = S_2 \cdot R_1 \cdot S_2)$$

$$P' = C \cdot P$$

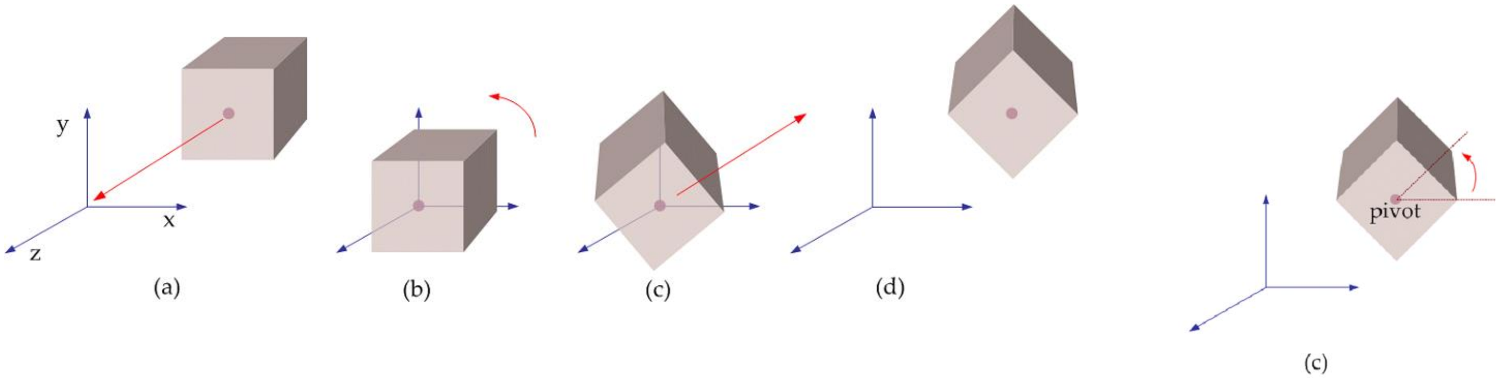
3.1 원점 기준 회전



(a) (b) (c)

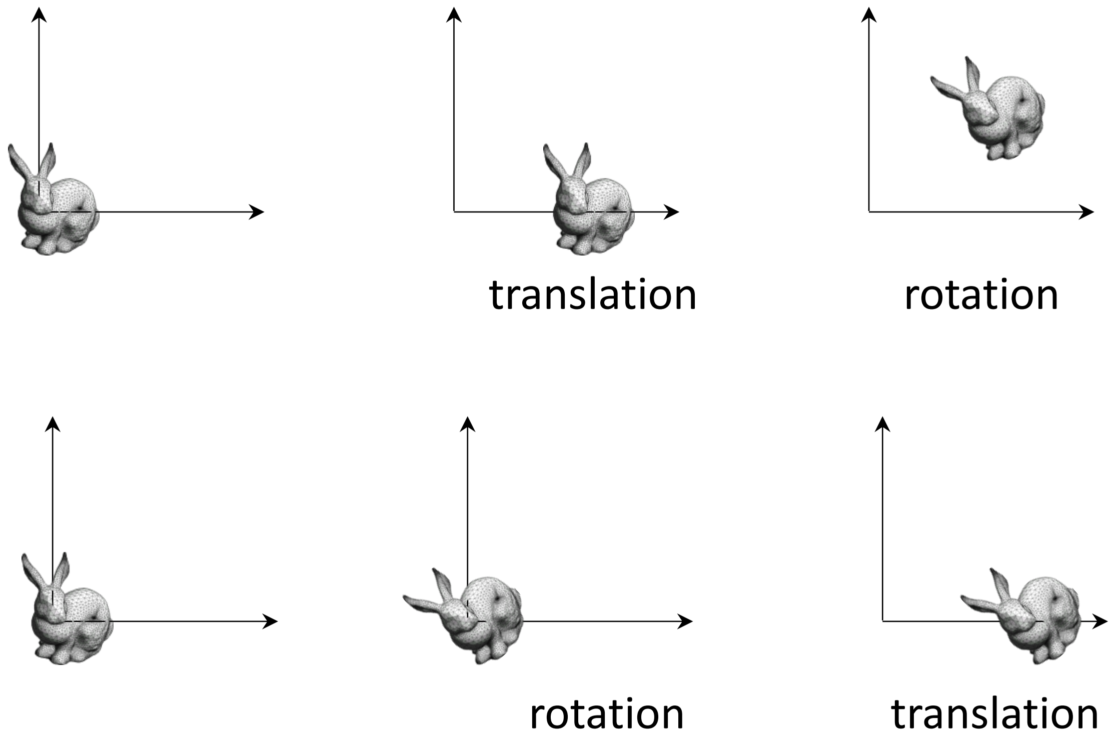
3.1 중심점 기준 회전

$$C = T(Xp, Yp, Zp) \cdot Rz(\theta) \cdot T(-Xp, -Yp, -Zp)$$



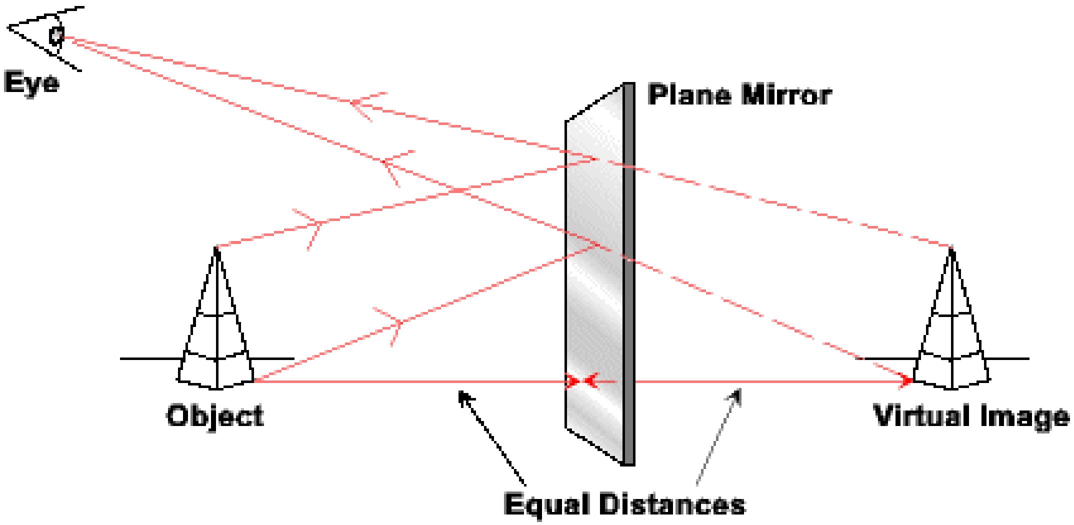
3.2 Translation, Rotation 복합

- 교환 법칙이 성립하지 않음



4 Reflection (반사)

- Ex) 거울에 비추인 object modeling



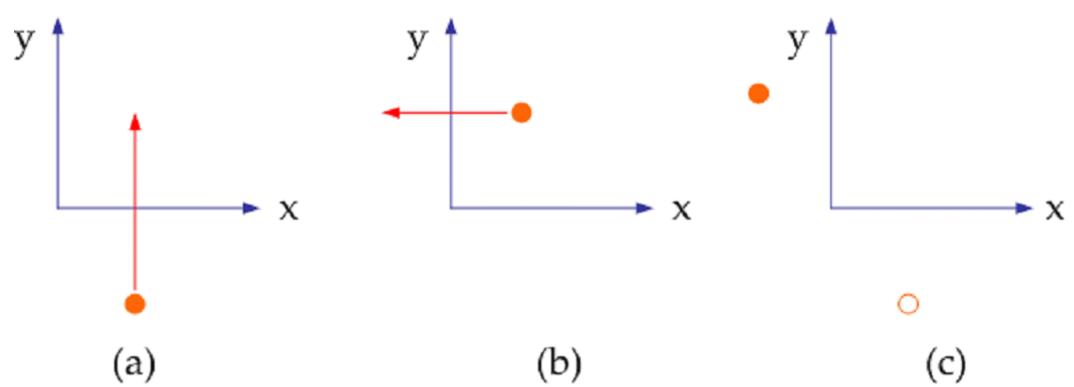
4.1 x=0, y=0 반사

y=0 반사 $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

x=0 반사 $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

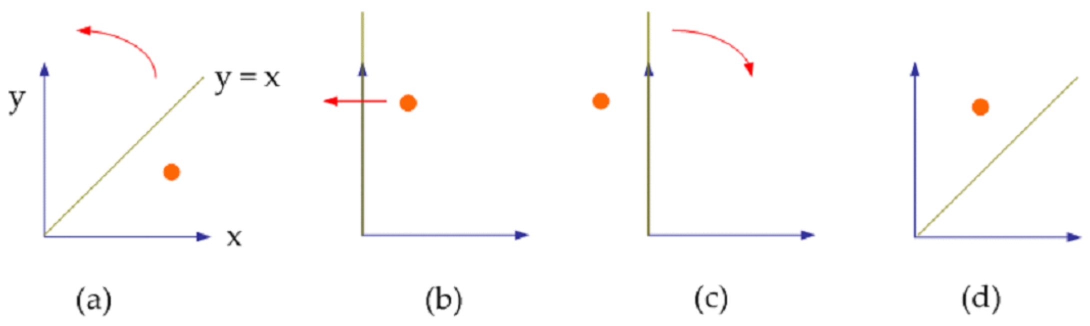
원점 반사 $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$$\mathrm{C} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



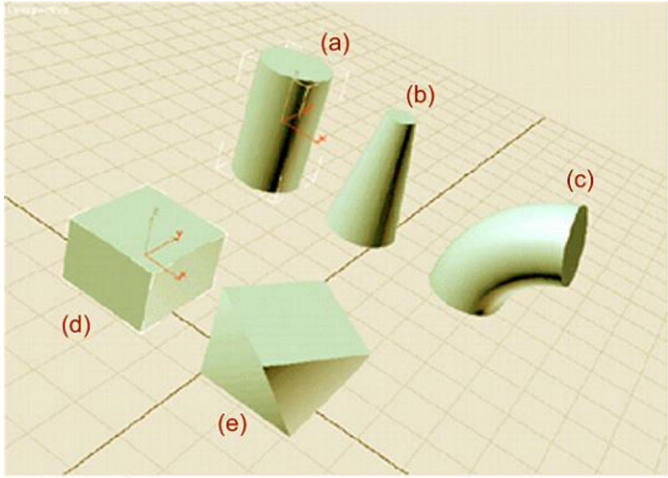
4.2 y=x 기준 반사

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos -45^\circ & -\sin -45^\circ & 0 \\ \sin -45^\circ & \cos -45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



5. 구조 왜곡 변환

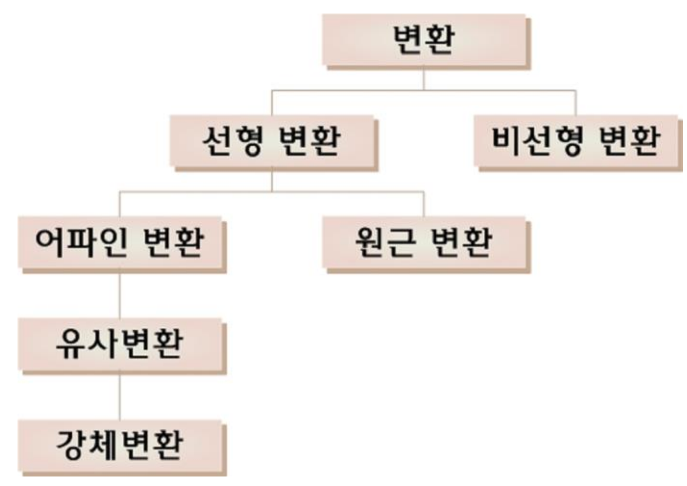
- Tapering : z에 따라 x, y의 크기 조절 (b)
- Bending (휨): 축을 따라 물체가 휨 (c)
- Twisting: z 에 따라 회전각 증가 (e)



6. 그래픽 변환

- Rigid Transformation (강체 변환)
- Translation, Rotation
- 물체 자체의 모습은 불변

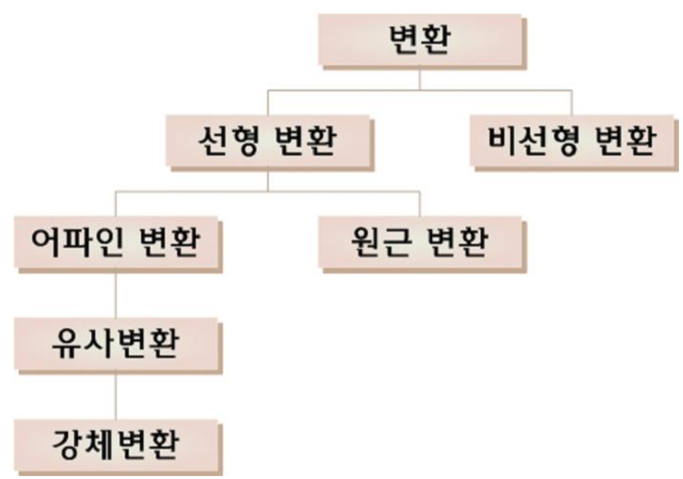
- Similarity Transformation (유사변환)
- 균등 크기 조절 변환, 반사 변환
- 물체면 사이의 각이 유지됨
- 물체내부 정점간의 거리가 일정한 비율로 유지됨



6. 그래픽 변환

- Affine Transformation (어파인 변환)
- 유사변환 + 차등 크기조절 변환, 전단변환
- 물체의 타입이 유지
- 직선은 직선으로, 다각형은 다각형으로, 곡면은 곡면으로
- 평행선이 보존
- 변환행렬의 마지막 행이 항상 (0, 0, 0, 1)

- Perspective Transformation (원근변환)
- 평행선이 만남.
- 직선이 직선으로 유지
- 변환행렬의 마지막 행이 (0, 0, 0, 1) 아님

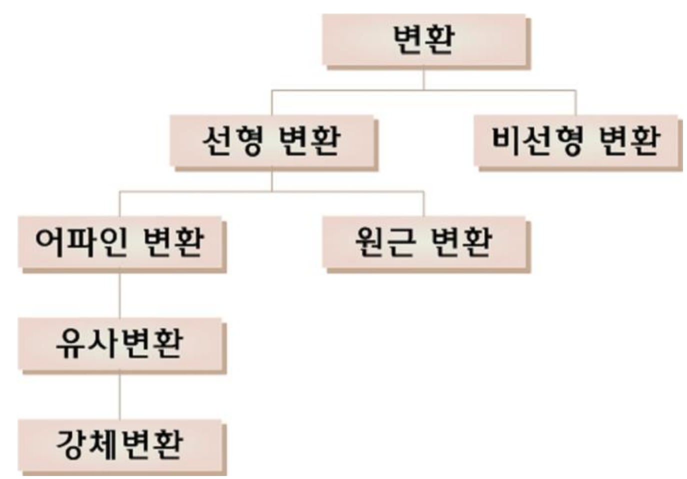


6. 그래픽 변환

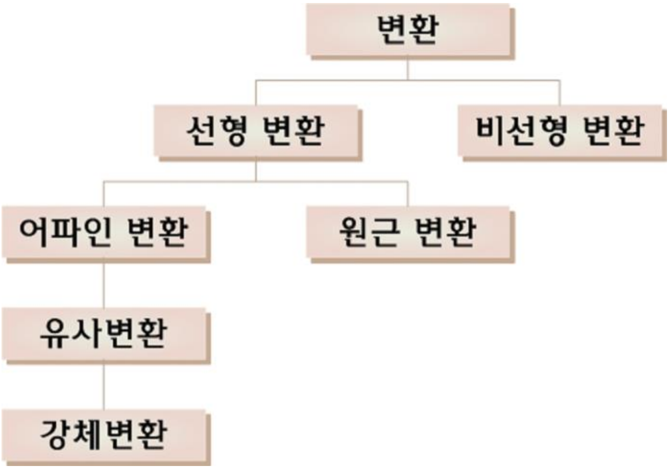
- Linear Transformation (선형 변환)
- 어파인 변환 + 원근 변환
- 선형 조합(Linear Combination)으로 표시되는 변환
- x' = ax + by + cz에서 x'는 x, y, z 라는 변수를 각각 상수 배 한 것을 더한 것이다.
- 예:

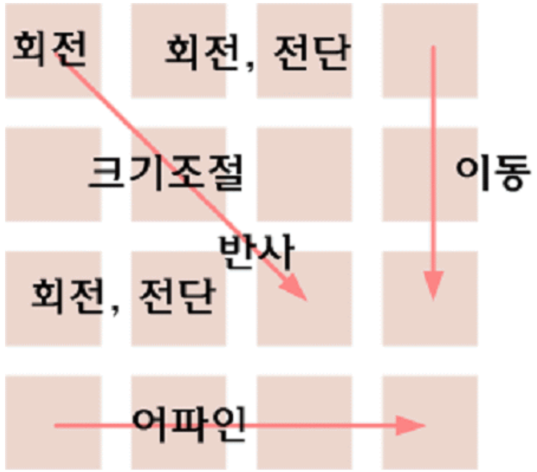
$$x' = r cos(\phi + \theta) = r cos\phi cos\theta - r sin\phi sin\theta = x cos\theta - y sin\theta$$

$$x' = x + Shy \cdot y$$



6. 그래픽 변환



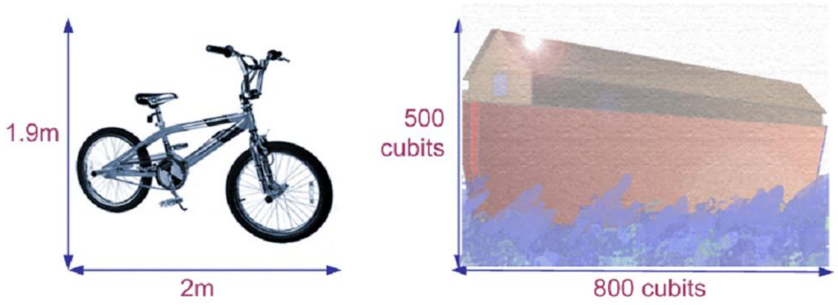


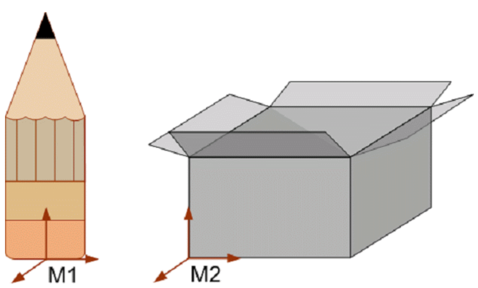
7. 좌표계

- 모델 좌표계
- 전역 좌표계
- 시점 좌표계

7.1 모델 좌표계

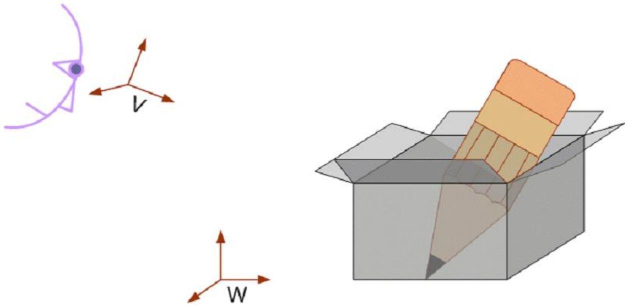
- MCS, Modeling Coordinate System
- LCS, Local Coordinate System
- 모델링
  물체를 설계= 물체 정점을 정의
- 좌표계 단위
  물체 공간
  임의로 설정





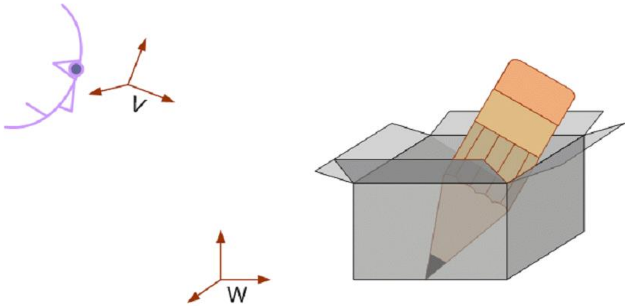
7.2 전역 좌표계

- WCS, World Coordinate System
- 여러 물체가 존재하며 여러 지역 좌표계가 존재하기 때문에 공통된 기준 좌표계가 필요
- 임의 위치에 선정



7.3 시점 좌표계

- VCS, View Coordinate System
- 바라보는 위치에 따라 장면은 달라 보임



7.4 시점좌표, 전역좌표, 모델좌표

- $P_{WCS} = M \bullet P_{MCS}$
- $P_{VCS} = V \bullet P_{WCS} = V \bullet M \bullet P_{MCS}$