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Introduction to Bayesian Statistics

In the last few years the use of Bayesian methods in the practice of applied statistics has greatly increased. In this book we will show how the development of computational Bayesian statistics is the key to this major change in statistics. For most of the twentieth century, frequentist statistical methods dominated the practice of applied statistics. This is despite the fact that statisticians have long known that the Bayesian approach to statistics offered clear cut advantages over the frequentist approach. We will see that Bayesian solutions are easy in theory, but were difficult in practice. It is easy to find a formula giving the shape of the posterior. It is often more difficult to find the formula of the exact posterior density. Computational Bayesian statistics changed all this. These methods use algorithms to draw samples from the incompletely known posterior and use these random samples as the basis for inference. In Section 1.1 we will look briefly at the ideas of the frequentist approach to statistics. In Section 1.2 we will introduce the ideas of Bayesian statistics. In Section 1.3 we show the similarities and differences between the likelihood approach to inference and Bayesian inference. We will see that the different interpretations of the parameters and probabilities lead to the advantages of Bayesian statistics.

1.1 THE FREQUENTIST APPROACH TO STATISTICS

In frequentist statistics, the parameter is considered a fixed but unknown value. The sample space is the set of all possible observation values. Probability is interpreted as long-run relative frequency over all values in the sample space given the unknown parameter. The performance of any statistical procedure is determined by averaging