Assignment 3: Nonlinear Equations and Numerical Optimization

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The goal of this exercise is to introduce basic numerical methods. We use data from a field experiment. Each choice is binary, and the lotteries are denoted ($\mathscr{C}_1$ and $\mathscr{C}_2$). They are composed of a low and high payoffs. Subjects are presented with five gambles entailing binary choices. In each case, the first option has lower expected value and dispersion than the second. At the fifth choice, expected values are similar across lotteries, and the second option still has higher variance. In total, there is 25 decisions, which are stored in dat_choices.

Table 1: Risk Aversion Lotteries: Ordered Lotteries Selection

	$\mathscr{C}_1$				$\mathscr{C}_2$			
	$\mathbb{P}_1$	$\mathbb{P}_2$	EV	SD.	$\mathbb{P}_1$	$\mathbb{P}_2$	EV	SD.
	48	48	48	0	40	64	52	16.97
	40	64	52	16.97	32	80	56	33.94
List 1	32	80	56	33.94	24	96	60	50.91
	24	96	60	50.91	16	112	64	67.88
	16	112	64	67.88	8	120	64	79.2
	48	48	48	0	42	66	54	16.97
	42	66	54	16.97	36	84	60	33.94
List 2	36	84	60	33.94	30	102	66	50.91
	30	102	66	50.91	24	120	72	67.88
	24	120	72	67.88	16	128	72	79.2
	48	48	48	0	38	62	50	16.97
	38	62	50	16.97	28	76	52	33.94
List 3	28	76	52	33.94	18	90	54	50.91
	18	90	54	50.91	8	104	56	67.88
	8	104	56	67.88	0	112	56	79.2
	42	42	42	0	36	60	48	16.97
	36	60	48	16.97	30	78	54	33.94
List 4	30	78	54	33.94	24	96	60	50.91
	24	96	60	50.91	18	114	66	67.88
	18	114	66	67.88	10	122	66	79.2
	54	54	54	0	44	68	56	16.97
	44	68	56	16.97	34	82	58	33.94
List 5	34	82	58	33.94	24	96	60	50.91
	24	96	60	50.91	14	110	62	67.88
	14	110	62	67.88	6	118	62	79.2

Notes: Table lists five lotteries. $\mathscr{C}_1$ and $\mathscr{C}_2$ refer to the choices of the lottery, while $\mathbb{P}_1$ and $\mathbb{P}_2$ refers to the payoffs. $\mathbb{E}V$ is for expected value, while SD is the standard deviation across payoffs. Payoffs are in Canadian $. Source: SRDC-CIRANO Field Experiment on Education Financing.

Exercise 1   Part 1

Let individuals be endowed with a CRRA utility function. The utility of an individual with risk aversion θ over choice c

$$u(c,\theta) = \begin{cases} \dfrac{c^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1 \\ \log(c), & \text{if } \theta = 1 \end{cases} \tag{1}$$

- Write a function crra that as input c and θ and return utility.

- Illustrate the shape of the utility function, for $c = 50$, and risk aversion θ ranging from -5 to 5.

Exercise 2   Part 2

- Create a matrix that records all choices in Table 1.

- Using bisection techniques, find the level of risk aversion that makes an individual indifferent between each of the 25 choices.

- Use turning points to construct identified sets for each set of lotteries.

- Use these identified sets along with dat_choice to construct the distribution of risk aversion in the population for each list of questions.

Exercise 3   Part 3

Consider an individual i endowed with an initial wealth w_i which is set to 20. Consider the choice between $\mathscr{C}_{1l} = (c_l^1, c_l^2)$ and $\mathscr{C}_{2l} = (c_2^1, c_2^2)$. The utility of an agent i for lottery $\mathscr{C}_{1l}$ is:

$$V_i^r(w_i + \mathscr{C}_{1l}) = \overline{V}_{il}^{1r} + \xi_{il}^r = u(w_i + c_{1l}^1, \theta_i) + u(w_i + c_{1l}^2, \theta_i) + \xi_{il}^r \tag{2}$$

The associated probability of choosing $\mathscr{C}_{11}$ over $\mathscr{C}_{2l}$ is:

$$\Pr\left[V_i^r(w_i + \mathscr{C}_1) > V_i^r(w_i + \mathscr{C}_2)\right] = \Pr\left[\xi_{il}^{2r} - \xi_{il}^{1r} < \overline{V}_{il}^{1r} - \overline{V}_{il}^{2r}\right] \tag{3}$$

1. Write the likelihood associated to least risky lottery under the assumption that $\xi_{il}^r \sim EV_1$ (extreme value type 1 = gumbel)

2. One parameter optimization: Grid search: Simulate a grid of potential values for θ, evaluate the likelihood for each of value of the grid and select the maximum. Apply this for individuals 900 and 115.