FREQUENCY MODULATION IN MATLAB

Experiment No: **Date:**

Aim: To implement frequency modulation and demodulation using MATLAB.

Brief Theory/Equations:

When the angle of the carrier wave is varied in some manner with respect to modulating signal $m(t)$, the technique of modulation is known as angle modulation or exponential modulation.

General form, $s(t) = A\cos(\omega_c t + \varphi(t))$, where $s(t)$ is the modulated signal, ω_c is the carrier frequency in radians and $\varphi(t)$ is the phase function which is time varying and captures the information, which you want to convey.

So, modulating signal $m(t)$ somehow modifies this $\varphi(t)$

Let $\theta_i(t) = \omega_c t + \varphi(t)$ be the instantaneous phase of the carrier signal, then instantaneous frequency can be obtained by taking the derivative of instantaneous phase w.r.t. time, t.

$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + \frac{d\varphi(t)}{dt}$ is the instantaneous frequency of the carrier wave or modulated signal

Where, $\varphi(t)$ is the instantaneous phase deviation and $\frac{d\varphi(t)}{dt}$ is the instantaneous frequency deviation.

Phase Modulation: (PM)

The instantaneous phase deviation carries the information or $\varphi(t)$ is varied linearly with $m(t)$

$$\varphi(t) \quad \alpha \quad m(t)$$

$$\varphi(t) = k_p\, m(t)$$

Where, k_p is the phase modulation constant/ phase sensitivity of the modulator measured in radians/volt.

For PM, instantaneous phase $\theta_i(t) = \omega_c t + k_p\, m(t)$

And instantaneous frequency $\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt}$

Modulated signal for PM, $s(t) = A\cos(\omega_c t + k_p\, m(t))$

Frequency Modulation: (FM)

The instantaneous frequency carries the information or message signal.

$$\frac{d\varphi(t)}{dt} \ \alpha \ \ m(t)$$

$$\frac{d\varphi(t)}{dt} = k_f \ m(t)$$

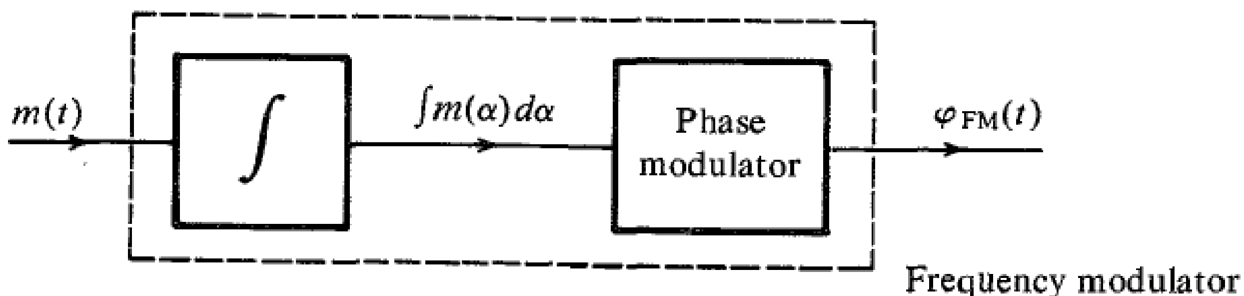
Where, k_f is the frequency modulation constant/ frequency sensitivity of the modulator measured in radians/(seconds*volt) or Hz/volt.

For FM, instantaneous phase deviation, $\varphi(t) = k_f \int_{-\infty}^{t} m(\alpha)d\alpha$

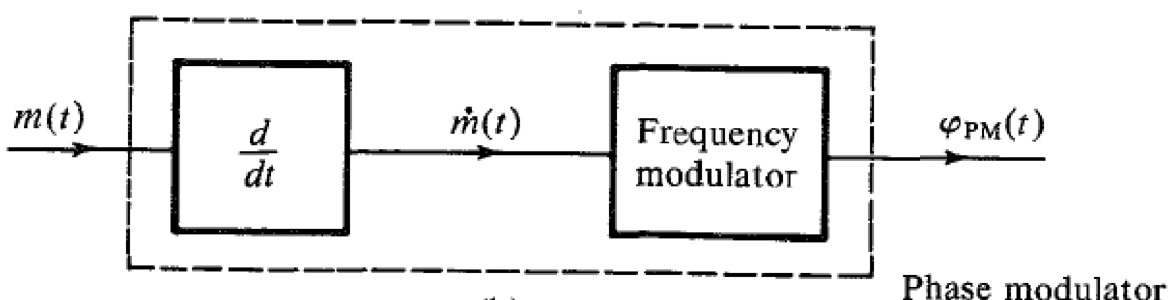
Hence, instantaneous phase, $\theta_i(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha$

And instantaneous frequency, $\omega_i(t) = \omega_c + k_f \ m(t)$

Modulated Signal for FM, $\boldsymbol{s(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha)}$



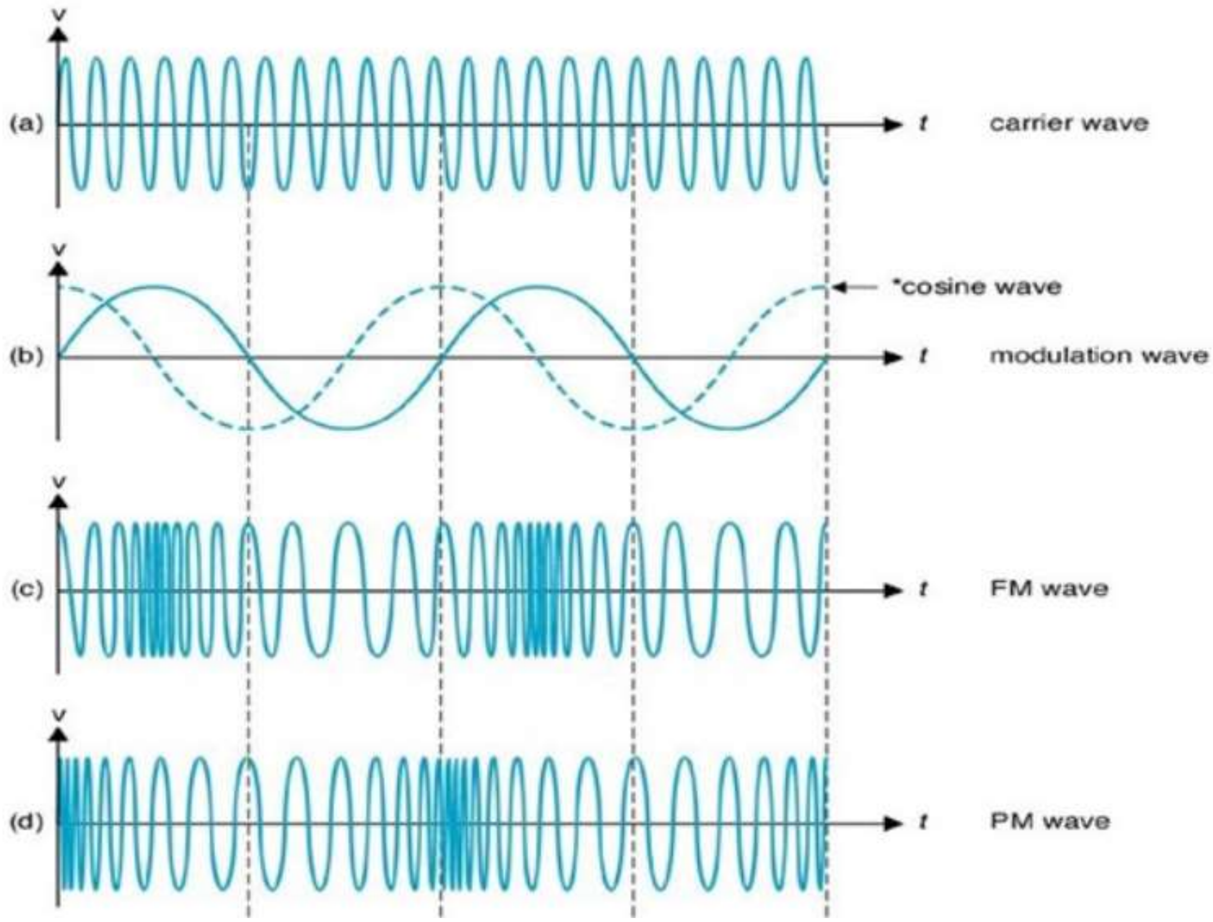
(a)



(b)

- Phase and frequency modulation are inseparable.

Considering, message signal as sine wave, the derivative of it will be cosine and respective FM and PM wave are shown in the next figure.



or else if we consider $m(t) = A_m \cos \omega_m t$, the instantaneous phase deviation for PM and FM will be;

PM,

$$\varphi(t) \quad = \quad k_p\, m(t) = k_p A_m \cos \omega_m t$$

FM,

$$\varphi(t) = k_f \int_{-\infty}^{t} m(\alpha) d\alpha = k_f \int_{-\infty}^{t} A_m \cos \omega_m \alpha \, d\alpha = \frac{k_f A_m \, \sin \omega_m t}{\omega_m}$$

Hence, the modulated signal for PM and FM become

$$\boldsymbol{s(t) = A\cos\left(\omega_c t + k_p\, m(t)\right) = A\cos\left(\omega_c t + k_p\right.} A_m \cos \omega_m t\left.\right) = \boldsymbol{A\cos\left(\omega_c t + \beta_{pm}\right.} \cos \omega_m t\left.\right)$$

Where, $\beta_{pm} = \; k_p \, A_m$ is the modulation index for PM

$$s(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha)$$

$$= A\cos\left(\omega_c t + \frac{k_f A_m \ \sin\omega_m t}{\omega_m}\right) = A\cos\left(\omega_c t + \beta_{fm}\sin\omega_m t\right)$$

Where, $\beta_{fm} = \frac{k_f A_m}{\omega_m} = \frac{\Delta f}{f_m}$ is the modulation index for FM

$\Delta f = k_f A_m$, is the peak frequency deviation

Frequency Deviation:

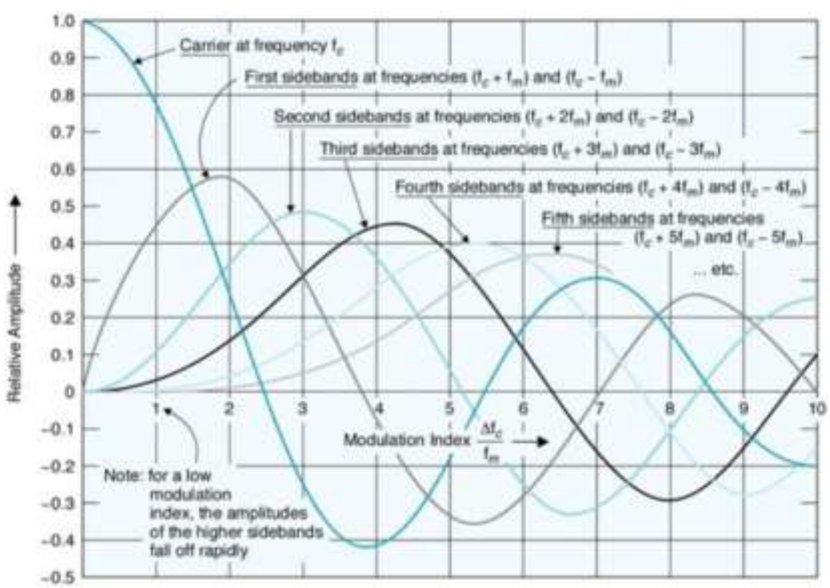
- The amount of change in the carrier frequency produced, by the amplitude of the input modulating signal, is called **frequency deviation**.
- The Carrier frequency swings between fmax and fmin as the input varies in its amplitude.
- The difference between fmax and fc is known as frequency deviation. Δf = fmax – fc
- Similarly, the difference between fc and fmin also is known as frequency deviation. Δf = fc –fmin

The frequency modulated signal can also be expressed in terms of Bessel function by taking the fourier series expansion.

Reference: Modern Digital and Analog Communication by B.P.Lathi

$$s(t)_{FM} = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left((\omega_c + n\omega_m)\right)t$$

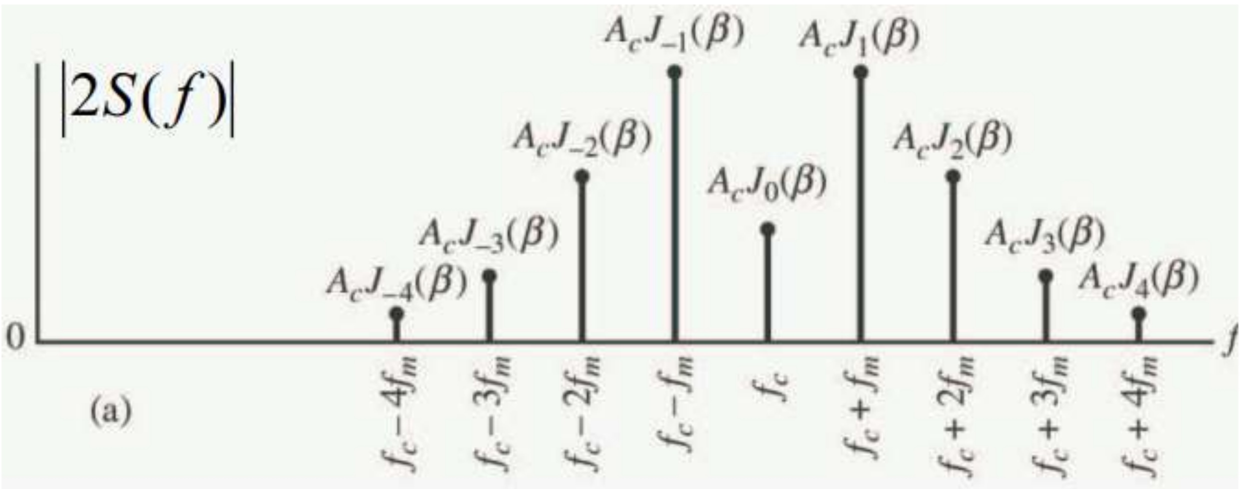
Where, $J_n(\beta)$ is Bessel function of n order with argument β.



By taking the fourier transform of the above equation, we get

$$S(f) = \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta\left(f-(f_c+nf_m)\right)+\delta(f+(f_c+nf_m)\right]$$

Theoretically, single tone FM has carrier component with infinite number of side-bands at $(f_c \pm nf_m)$ and its bandwidth is infinite.



Bandwidth of FM:

As the value of n increase, the significant power level in sideband component decreases

i.e. $lim_{n\to\infty} J_n(\beta) = 0$.

So we consider only k sidebands on either sides of f_c and look at these 2k+1 components i.e. from $(f_c + kf_m)$ to $(f_c - kf_m)$ and if we consider the power ratio,

Power ratio = $\frac{\frac{1}{2}A_c^2\sum_{k=-\infty}^{\infty} J_n(\beta)^2}{\frac{1}{2}A_c^2} = \frac{sideband\ power}{carrier\ power} = J_0(\beta)^2 + 2\sum_{n=1}^{k} J_n(\beta)^2$

($since, J_0(\beta)$ is the carrier related component and $J_n(\beta)$ is an even function)

The bandwidth is defined as band of frequencies over which signal contains 98% of its power that means the power ratio is 0.98.

In general, $BW = 2kf_m$

In empirical sense, if we choose $k = \beta + 1$, then power ratio turns to 0.98. (depends on the Bessel function values)

Hence, $BW = 2(\beta + 1)f_m = 2(\beta f_m + f_m) = 2(\Delta f + f_m)$ $[since\ \beta = \frac{\Delta f}{f_m}]$

Frequency Demodulation:

We know that the equation of FM wave is

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

Differentiate the above equation with respect to '**t**'.

$$\frac{ds(t)}{dt} = -A_c\left(2\pi f_c + 2\pi k_f m(t)\right)\sin\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$

We can write,

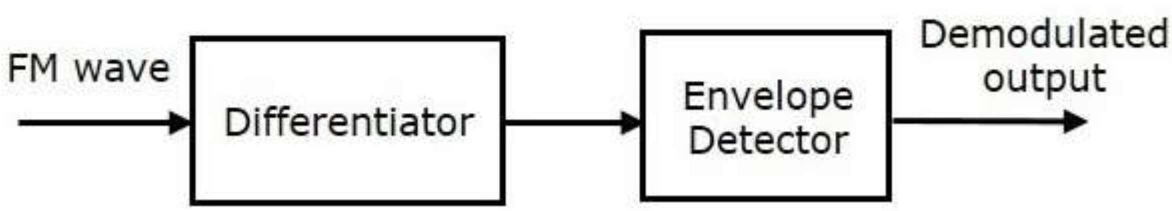
$-\sin\theta$ as $\sin(\theta - 180^0)$.

$$\Rightarrow \frac{ds(t)}{dt} = A_c\left(2\pi f_c + 2\pi k_f m(t)\right)\sin\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt - 180^0\right)$$

$$\Rightarrow \frac{ds(t)}{dt} = A_c(2\pi f_c)\left[1 + \left(\frac{k_f}{k_c}\right)m(t)\right]\sin\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt - 180^0\right)$$

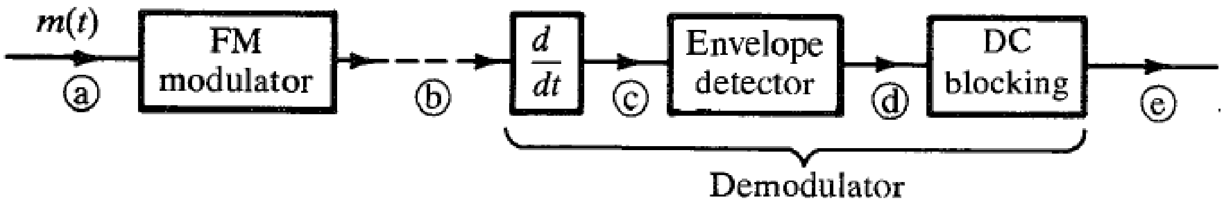
In the above equation, the amplitude term resembles the envelope of AM wave and the angle term resembles the angle of FM wave. Here, our requirement is the modulating signal m(t)m(t). Hence, we can recover it from the envelope of AM wave.

The following figure shows the block diagram of FM demodulator using frequency discrimination method.



This block diagram consists of the differentiator and the envelope detector. Differentiator is used to convert the FM wave into a combination of AM wave and FM wave. This means, it converts the frequency variations of FM wave into the corresponding voltage (amplitude) variations of AM wave. We know the operation of the envelope detector. It produces the demodulated output of AM wave, which is nothing but the modulating signal.

The FM communication link is shown below:



(b)

Algorithm:

- Define the sampling frequency $f_s > 2(f_c + k_{max} f_m)$ say, $f_s = 100Hz$;
- Define the time range using the sampling frequency t= -10 : $1/f_s$: 10
- Consider, message signal, $m = A_m \cos \omega_m t$, where $f_m = 1Hz$ and carrier signal $c = A_c \cos \omega_c t$ where $f_c = 10Hz$. Keep the amplitude of message and carrier signal same.
- Assume $k_p = 1$ and $k_f = 2\pi f_m$
- For PM, $\boldsymbol{s = A\cos\left(\omega_c t + k_p\, m(t)\right) = A\cos\left(\omega_c t + k_p\right.} A_m \cos \omega_m t)$
- For FM, $\boldsymbol{s(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha) = A\cos\left(\omega_c t + \frac{k_f A_m \sin \omega_m t}{\omega_m}\right)}$

  Use integral function in MATLAB in general so it can be done for any input signal.
- Figure1: Plot input signal, carrier signal, FM signal, derivative of input signal and PM signal using subplot(511) to subplot(515);

  Use diff function in matlab for the derivative of the signal

  For example, y=diff(x), pad the last value of the signal so as to ensure the same size because difference values will be one less than the given values.

  y=[diff(x) diff(end)]
- Figure 2: plot the frequency spectrum of FM and PM signals using the commands fft and fftshift.
  - Define n=length(t) %gives no. of columns in t
  - Define the step size for frequency axis fp which should be of same size as that of t. (matrix dimensions must match for plotting).
    - df=fs/n; where fs is the sampling frequency
  - Define frequency axis fp = -fs/2:df:fs/2-df ($\pm$df for getting same size matrices)
  - Take the fourier transform of FM and PM using fft command
  - Y = fft(x) returns the discrete Fourier transform (DFT) of vector x, computed with a fast Fourier transform (FFT) algorithm.

- Y = fftshift(X) rearranges the outputs of fft by moving the zero-frequency component to the center of the array. It is useful for visualizing a Fourier transform with the zero-frequency component in the middle of the spectrum.
- Can write in a single syntax as y=fftshift(fft(x));
- Plot the frequency spectrum of FM and PM using the command plot(fp,y).Use subplot(211) to (212).

- **Demodulate the FM signal**
  - Take the derivative of FM signal using diff function of MATLAB as mentioned in one of the previous step.
  - Multiply the above signal with carrier signal, this will look like AM wave.
    - Do .* element wise multiplication
- Do the low pass filtering of the above signal using butter and filter command.
    - [b,a] = butter(n,Wn,'ftype'). **This creates a filter**
    - [b,a] = butter(n,Wn) designs an order n lowpass digital Butterworth filter with normalized cutoff frequency Wn. It returns the filter coefficients in length n+1 row vectors b and a, with coefficients in descending powers of z.

$$H(z) = \frac{b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \ldots + a(n+1)z^{-n}}$$

    - where the string 'ftype' is one of the following:
      - 'high' for a highpass digital filter with normalized cutoff frequency Wn
      - 'low' for a lowpass digital filter with normalized cutoff frequency Wn
      - 'stop' for an order 2*n bandstop digital filter if Wn is a two-element vector, Wn = [w1 w2]. The stopband is w1 < ω < w2.
      - 'bandpass' for an order 2*n bandpass filter if Wn is a two-element vector, Wn = [w1 w2]. The passband is w1 < ω < w2. Specifying a two-element vector, Wn, without an explicit 'ftype' defaults to a bandpass filter.
      - Cutoff frequency, Wn is that frequency where the magnitude response of the filter is . For butter, the normalized cutoff frequency Wn must be a number between 0 and 1, where 1 corresponds to the Nyquist frequency, π radians per sample.
      - Choose order, n=4 ; Wn=fm/fs; ftype='low'
- Use the filter command to apply the filter
  - y = filter(b,a,X), where b and a are the coefficients obtained in the previous step and X will the multiplied signal output (derivative of FM signal*Carrier Signal). y is the filtered output. Adjust order so that you get approximate input signal.
- Figure 3: Plots for FM demodulation
  - Include three figures using subplot(311) to subplot(313)
    - 1^{st} – derivative of FM signal
    - 2^{nd} - multiplied signal output (derivative of FM signal*Carrier Signal)
    - 3^{rd} – filtered output y. Include message signal as well in this

- Example: plot(t,m,t,y)

Expected Output waveforms:

Figure 1:

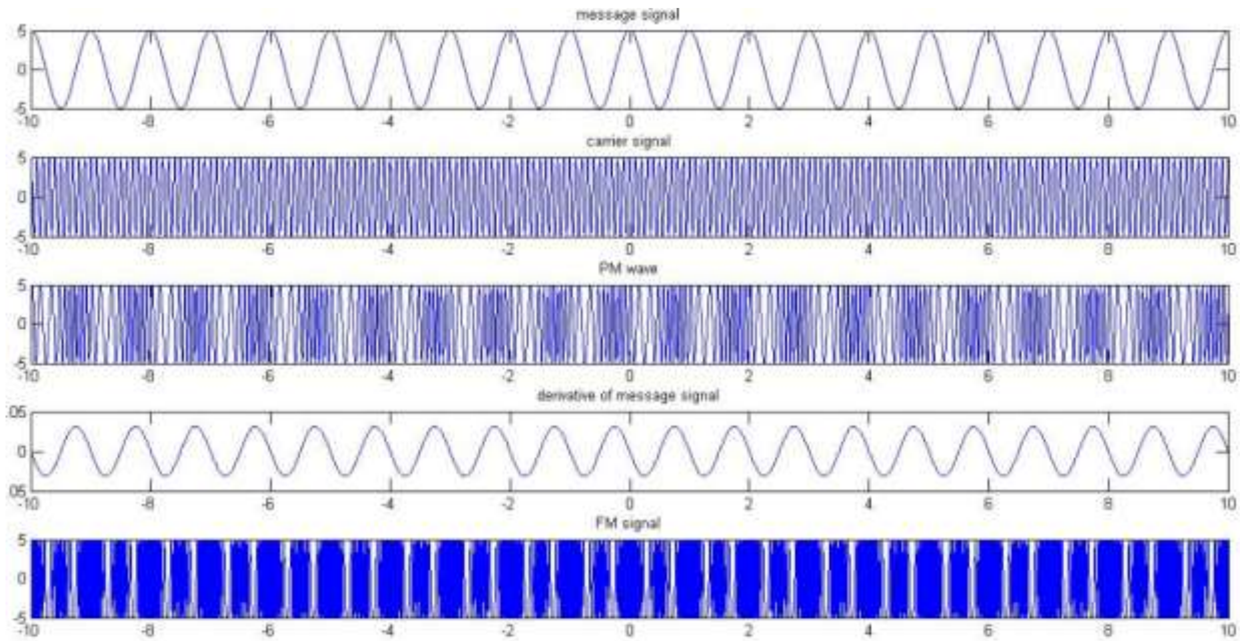


Figure 2:

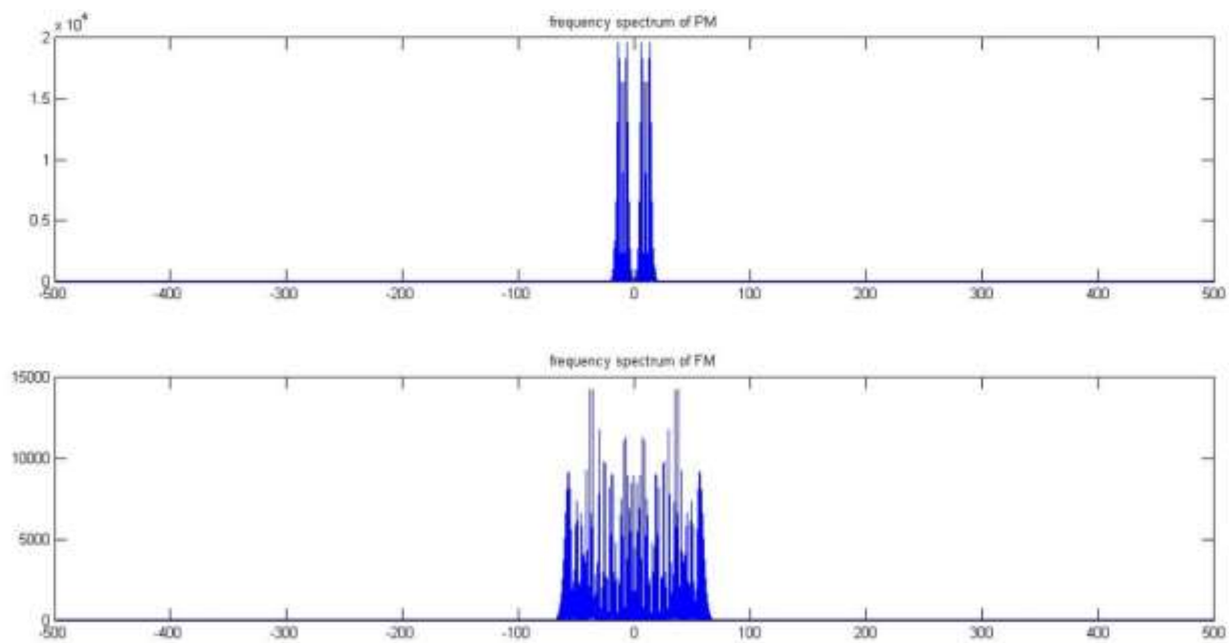
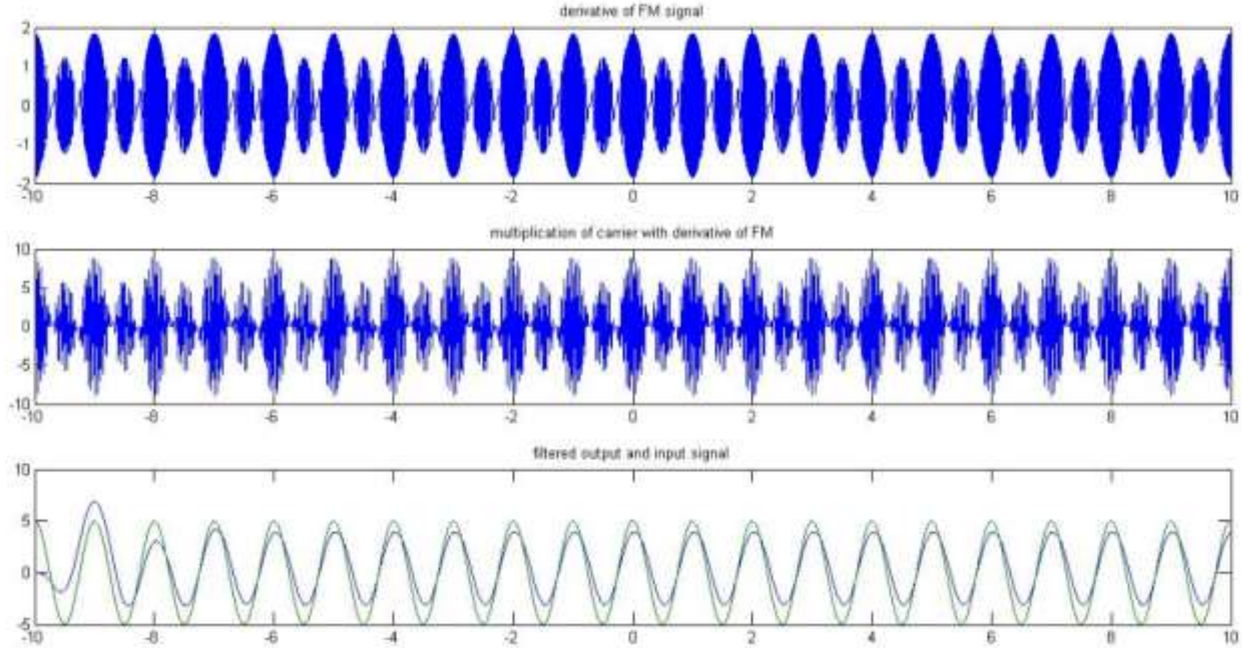


Figure 3:



Code:

Output Waveforms:

Conclusion:

Remarks: **Signature:**

References:

- NPTEL communication systems lectures https://www.youtube.com/watch?v=gsUaHawPy-w&list=PL7748E9BEC4ED83CA&index=15
- Modern Analog and Digital Communication by B.P. Lathi (3rd or 4th edition)
- Communication Systems by Simon Haykin (4th edition)