NAME:- KRUNAL RANK

ADM. NO:- U18CO081

B TECH 3RD YEAR

Scanned with CamScanner

U18CO081

27/08/20

DAA Tutorial 2

1. Given $f(n) = 3n^2 + 4n - 2$

let

$$3n^2 + 4n - 2 \le c_1 n^2$$

Here for $c_1 = 5$, ~~LHS =~~ & $n_0 = 1$,

LHS = 5 RHS = 5

for $n_0 = 2$,

LHS = 30 RHS = 20

Thus, for $c_1 = 6$ it will satisfy.

For $n_0 = 3$ & $c_1 = 6$,

LHS = 27 + 12 − 2 = 37

RHS = 9 × 6 = 54

Hence, $\forall$ $c_1 = 6$ & $n \geq n_0 = 2$

$$3n^2 + 4n - 2 \le 6n^2$$

Thus, it is $O(n^2)$.

2. Given $f(n) = n^3$

let $n^3 \le c_1 n^2$.

For $n_0 = 1$, $c_1 = 1$ satisfies it.

For $n_0 = 2$, $c_1 = 2$ satisfies it.

For $n_0 = 3$, $c_1 = 3$ satisfies it.

As $c_1 \geq n$.

Hence, c_1 is growing indefinitely.

Hence, $n^3 != O(n^2)$

Scanned with CamScanner

Ans 3. Given $f(n) = 3n^2+5n+6$

$g(n) = n^2$

let $f(n) \le c_1 g(n)$.

$3n^2+5n+6 \le c_1 n^2$

~~Hence~~

Here for $n_0=1$,

$LHS = 3+5+6 = 14$

$c_1 = 14$ satisfies it.

for $n_0=2$,

$LHS = 12+10+6 = 28$

$c_1 = 6$ satisfies it.

for $n_0=3$,

$LHS = 27+15+6 = 48$.

$c_1=6$ satisfies it.

Hence,

for $\forall\, n_0 \ge 2$,

$3n^2+5n+6 \le 6n^2$

Hence, $f(n) = O(g(n))$

Ans 4. Given $f(n) = 3n^2+5n+6$, $g(n) = n^2$

let

$c_1 n^2 \le f(n) \le c_2 n^2$

For first inequality,

$c_1 n^2 \le 3n^2+5n+6$

For $c_1 = 3$,

the above inequality satisfies.

Now, $3n^2+5n+6 \le c_2 n^2$

For $c_2=6$ $n_0 \ge 2$

$3n^2+5n+6 \le 6n^2$

Scanned with CamScanner

Date / /
Page

Hence, $3n^2 \le 3n^2+5n+6 \le 6n^2$.

Hence, $f(n) = \Theta(n^2)$.

Ans 5:

a) $f(n) = 3n^2 + 10n\log n$

$f(n) = O(n^2)$.

Hence, it is not $O(n\log n)$. Hence, false.

b) $3n^2+10n\log n$

$f(n) = \Omega(n^2)$.

Hence, it is correct.

$c_1 n^2 \le 3n^2+10n\log n$

~~let~~ for $c_1 = 3$, it satisfies the inequality.

Hence, true.

c) $3n^2+10n\log n$

From above 2 cases,

$3n^2+10n\log n = \Theta(n^2)$.

Hence, true.

d) $n\log n + n/2$.

~~fe~~ $n\log n + n/2 \le cn$.

$c \ge \log n + \frac{1}{2}$

c here grows indefinitely as n increases.

Hence, $n\log n + n/2 \ne O(n)$.

Hence, false.

Scanned with CamScanner

e) Given,

$f(n) = \Theta(g(n))$

$g(n) = \Theta(h(n))$

Now,

$c_1 g(n) \le f(n) \le c_2 g(n)$

$c_1' h(n) \le g(n) \le c_2' h(n)$

Now,

$c_1' h(n) c_1 g(n) \le f(n) . g(n) \le c_2 g(n) . c_2' h(n)$

Since all values are positives & ≥ 1

Hence,

$c_1 c_1' h(n) \le f(n) \le c_2 c_2' h(n)$

Hence,

$\therefore \frac{c_1'}{c_1} h(n) \le f(n) \le \frac{c_2'}{c_2} h(n)$

$\frac{c_2}{c_2'} f(n) \le h(n) \le \frac{c_1}{c_1'} f(n)$

Hence, $h(n) = \Theta(f(n))$

It is true.

f) Given,

$f(n) = O(g(n))$

$g(n) = O(h(n))$

Now,

$f(n) \le c_1 g(n)$

$g(n) \le c_1 h(n)$

Hence, $f(n) \le c_1' h(n)$.

Hence, $h(n) = \Omega(f(n))$. Thus true

g) $f(n) = O(g(n))$
$g(n) = O(f(n))$

Now, $f(n) \le c_1 g(n)$
$g(n) \le c_2 f(n)$

Then either $c_1 c_2 \geq 1$ or $f(n) = g(n)$.
Thus, it is not always that $f(n) = g(n)$.
Hence, false.

h) Now,
$c_1 n \le n/100$ ~~$\le c_2 n$~~

For $c_1 = \frac{1}{100}$.

It is satisfies the inequality.
Hence, it is $\Omega(n)$. It is true.

i) Now,
$2^{n+1} \le c_1 2^n$
For ~~$c_1 \geq 2$~~ $c_1 = 2$.
The inequality satisfies.
Hence, it is ~~$\Theta(n)$~~ $O(2^n)$. It is true.

j) $2^{2n} \le c_1 2^n$
For $n_0 = 1$, LHS $= 4$ $c_1 = 2$ satisfies.
For $n_0 = 2$, LHS $= 16$ $c_1 = 4$ satisfies.
For $n_0 = 3$, LHS $= 64$ $c_1 = 8$ satisfies
For $n_0 = 4$, LHS $= 256$ $c_1 = 16$ satisfies.
Hence, c_1 grows indefinitely with n.
Hence, $2^{2n} \, != O(2^n)$. It is actually $O(4^n)$.
Thus, it is false.

Scanned with CamScanner

Ans 6:

a) Given $f(n) = 3n+5$

$c_1 n \leq f(n) \leq c_2 n$

for $c_1 = 3$, $n_0 \geq 1$

$3n \leq 3n+5$

It is satisfied.

for $c_2 = 8$ $n_0 \geq 1$

$3n+5 \leq 8n$.

It is satisfied.

Hence, $f(n) = O(n)$

$f(n) = \Omega(n)$.

b) Given $f(n) = 100n+7$

$c_1 n \leq f(n) \leq c_2 n$

for $c_1 = 100$, $n_0 \geq 1$.

$100n \leq 100n+7$.

It is satisfied.

for $c_2 = 107$, $n_0 \geq 1$

$100n+7 \leq 107n$

It is satisfied.

$f(n) = O(n) = \Omega(n)$.

c) Given $f(n) = 27n^2+16n$

$c_1 n^2 \leq 27n^2+16n \leq c_2 n^2$.

for $c_1 = 27$, $n_0 \geq 1$

$27n^2 \leq 27n^2+16n$

It is satisfied.

Scanned with CamScanner

Date / /
Page

~~For~~ $c_2 n^2 \geq 27n^2 + 16n$

$c_2 \geq 27 + \frac{16}{n}$

Hence, for $c_2 = 43$, $n_0 \geq 1$

$27n^2 + 16n \leq 43n^2$

It is satisfied.

Hence, $f(n) = O(n^2) = \Omega(n^2)$

d) $f(n) = 45n^2 + 23n + 36$,

let $c_1 n^2 \leq 45n^2 + 23n + 36 \leq c_2 n^2$

for $c_1 = 45$ $n_0 \geq 1$

& $c_2 = 104$, $n_0 \geq 1$

~~Her~~ Both inequalities satisfy.

Hence, $O(n^2)$ & $\Omega(n^2)$ is the correct answer.

e) $f(n) = 3n^3 + 4n$

let $c_1 n^3 \leq 3n^3 + 4n \leq c_2 n^3$

for $c_1 = 3$ & $n_0 \geq 1$ and $c_2 = 7$ & $n_0 \geq 1$.

Both inequalities satisfy.

Hence, $f(n) = O(n^3)$

$f(n) = \Omega(n^3)$

Scanned with CamScanner

Ans 7. Given,

$f(n) = n$

$g(n) = n^{(1+\sin n)}$

Here,

the power of $g(n)$ varies between -1 and 1.

Hence, it is not always possible

that $f(n) \le c_1 g(n)$

and $c_1 g(n) \le c_1 f(n)$.

Hence, both are wrong.

Ans 8. As seen from the given 'C' function,

the value of p is always $\lfloor \log_2 n \rfloor$ for all i

from 1 to $n-1$.

Also, the value of q is incremented by

$\lfloor \log_2(\log_2 n) \rfloor$ for all i from 1 to $n-1$.

Hence,

q is approximately $n \log_2(\log_2 n)$.

Scanned with CamScanner