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SEMESTER : 7

DIVISION : B

Ans 1:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

TP = True Positive
TN = True Negative

$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

FP = False Positive
FN = False Negative

$$\text{Specificity} = \frac{TN}{TN + FP}$$

$$P = TP + FN$$

$$N = TN + FP$$

$$\text{RHS} = \text{Sensitivity} \times \frac{P}{P+N} + \text{Specificity} \times \frac{N}{P+N}$$

$$\frac{TP}{TP+FN} \times \frac{(TP+FN)}{(TP+TN+FP+FN)} + \frac{TN}{TN+FP} \times \frac{(TN+FP)}{(TP+TN+FP+FN)}$$

$$= \frac{TP + TN}{TP + TN + FP + FN}$$

\therefore Accuracy

$\therefore P \text{ LHS} \quad [\text{Hence, proved}].$

a) $P(X = \{MP=Y, WP=Y, CC1=N, G=F\})$

$$\text{Let } P(UP=Y|X) = \frac{P(X|LIP=Y) \cdot P(G=F|Y)}{P(X)}$$

$$P(X|LIP=Y) = P(MP=Y|LIP=Y) \times P(WP=Y|LIP=Y) \times P(CC1=N|LIP=Y) \\ \times P(G=F|LIP=Y) \\ = \frac{4}{5} \times \frac{4}{5} \times \frac{2}{5} \times \frac{3}{5}$$

$$= \frac{96}{625}$$

$$P(UP=Y|X) = \frac{\frac{96}{625} \times \frac{1}{2}}{P(X)} = \frac{48}{625}$$

$$P(LIP=N|X) = \frac{P(X|LIP=N) \cdot P(LIP=N)}{P(X)}$$

$$P(X|LIP=N) = P(MP=Y|LIP=N) \times P(WP=Y|LIP=N) \\ \times P(CC1=N|LIP=N) \times P(Gender=F|LIP=N) \\ = \frac{1}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{4}{5} \\ = \frac{8}{625}$$

$$P(LIP=N|X) = \frac{\frac{8}{625} \times \frac{1}{2}}{P(X)} = \frac{4}{625}$$

Since $P(LIP=Y|X) > P(LIP=N|X)$
LIP=Y

b)

If gender is unknown, $\Rightarrow UP=Y$

Ans 3: $H(S) = - \left(\frac{4}{10} \log_2 \left(\frac{4}{10} \right) + \frac{3}{10} \log_2 \left(\frac{3}{10} \right) + \frac{3}{10} \log_2 \left(\frac{3}{10} \right) \right)$

$= 1.5709$ (Ans) $\therefore H(S) = 1.5709$ (Ans)

For attribute Gender,

	Male	Female
Bus	1	1
Car	1	1
Train	1	1

$H(\text{Gender} = \text{Male}) = - \left(\frac{3}{5} \log_2 \left(\frac{3}{5} \right) + \frac{1}{5} \log_2 \left(\frac{1}{5} \right) + \frac{1}{5} \log_2 \left(\frac{1}{5} \right) \right)$

$= 1.3709$ (Ans)

$H(\text{Gender} = \text{Female}) = - \left(\frac{1}{5} \log_2 \left(\frac{1}{5} \right) + \frac{2}{5} \log_2 \left(\frac{2}{5} \right) + \frac{2}{5} \log_2 \left(\frac{2}{5} \right) \right)$

$= 1.5219$ (Ans)

$H(\text{Gender}) = \frac{1}{2} \times 1.3709 + \frac{1}{2} \times 1.5219$ (Ans)

$= 1.4464$ (Ans)

$I_G = 1.5709 - 1.4464 = \underline{\underline{0.1245}}$ (Ans)

For attribute Car Ownership,

	0	1	2
Bus	1	1	1
Car	1	1	1
Train	1	1	1

$H(\text{Car Ownership} = \text{No}) = - \left(\frac{2}{3} \log_2 \left(\frac{2}{3} \right) + \frac{1}{3} \log_2 \left(\frac{1}{3} \right) \right)$

$= 0.9234$ (Ans)

$H(\text{Car Ownership} = \text{Yes}) = - \left(\frac{2}{3} \log_2 \left(\frac{2}{3} \right) + \frac{1}{3} \log_2 \left(\frac{1}{3} \right) + \frac{2}{3} \log_2 \left(\frac{2}{3} \right) \right)$

$= 1.5219$ (Ans)

$$H(\text{Car Ownership} = 2) = - \left(1 \log_2(1) \right) = 0$$

$$H(\text{Car Ownership}) = \frac{3}{10} \times 0.9234 + \frac{57}{10} \times 1.5219 + 0$$

~~1.0379~~

$$IG = 1.5709 - 1.0379 = 0.5320$$

For Attribute Travel Cost

	Cheap	Standard	Expensive
Bus	1111	1111	1111
Car	1111	1111	1111
Train	1111	1111	1111

$$H(\text{Travel Cost} = \text{Cheap}) = -\left(\frac{4}{5} \log_2\left(\frac{4}{5}\right) + \frac{1}{5} \log_2\left(\frac{1}{5}\right)\right)$$

P = 0.17219

$$H(\text{Travel Cost} = \text{Standard}) = 0$$

$H(\text{Travel Cost} = \text{Expensive}) > 0$ (abnormal) H

$$H(\text{Travel Cost}) = \underline{0.3609}$$

$$IG = 1.5709 - 0.3609 = \underline{\underline{1.21}}$$

For attribute Income level, $\chi^2_{(1)} = 10.19$ and $p < 0.05$.

		Low	Medium	High
Bus	II	II	II	II
Car	II	II	II	II
Train	I	II	III	II

$$H(\text{Income level} = \text{Basic}) = -\left(\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) = 1 = 0$$

$$H(\text{Income Level}=\text{Medium}) = -\left(\frac{1}{3}\log_2\left(\frac{1}{3}\right) + \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right) - \frac{1}{6}\log_2\left(\frac{1}{6}\right) = 1.4586$$

$$H(\text{Income level} = \text{High}) = 0$$

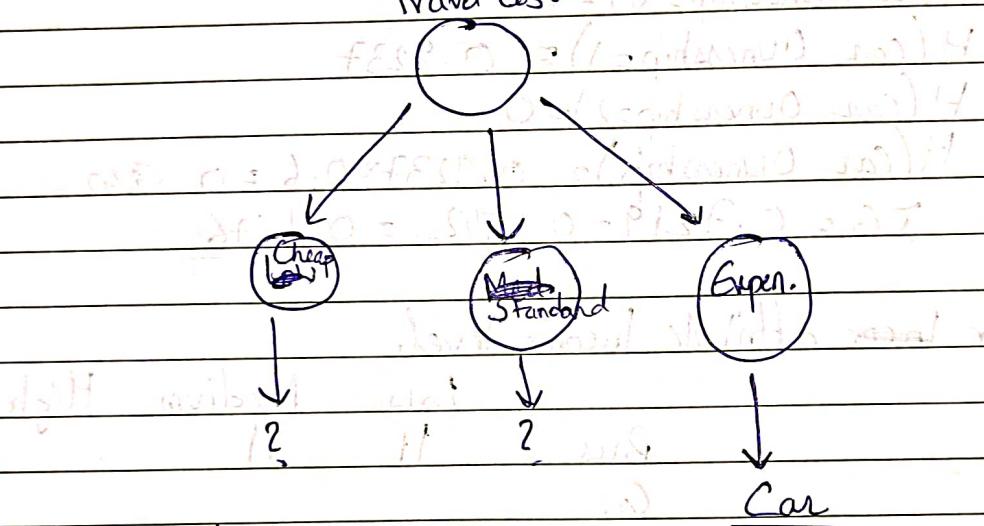
$$H(\text{Income level}) = 1.4586 \times 0.6 \\ = 0.8751$$

$$IG = 1.5709 - 0.8751 = \underline{\underline{0.6958}}$$

Hence, ~~IG~~ Highest IG is for ~~Attribute~~ Travel Cost.
So root node is Travel Cost.

$$\text{Travel Cost} = (0.2919)(0.6) +$$

$$0.4586(0.4) + 0.7219(0.4)$$



lets solve for ~~Travel Cost~~ ~~IG~~ Cheap

$$H(\text{Travel Cost} = \text{IG}(\text{Cheap})) = 0.7219$$

Now, for attribute Gender, then ~~Attribute~~ Male & Female

Bus

Car

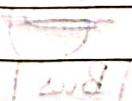
Train

$$H(\text{Gender} = \text{Male}) = 0.3219$$

$$H(\text{Gender} = \text{Female}) = 0.6781$$

$$H(\text{Gender}) = 0.4$$

$$IG = \underline{\underline{0.3219}}$$



For attribute Car Ownership,

	0	1	2
Bus	11	11	11
Car	11	11	11
Train	11	11	11

$$H(\text{Car Ownership} = 0) = 0$$

$$H(\text{Car Ownership} = 1) = 0.9237$$

$$H(\text{Car Ownership} = 2) = 0$$

$$H(\text{Car Ownership}) = 0.9237 \times 0.6 = 0.5542$$

$$IG = 0.7219 - 0.5542 = 0.1676$$

For ~~linear~~ attribute Income level,

	Low	Medium	High
Bus	11	11	11
Car	11	11	11
Train	11	11	11

$$H(\text{Income level} = \text{low}) = 0$$

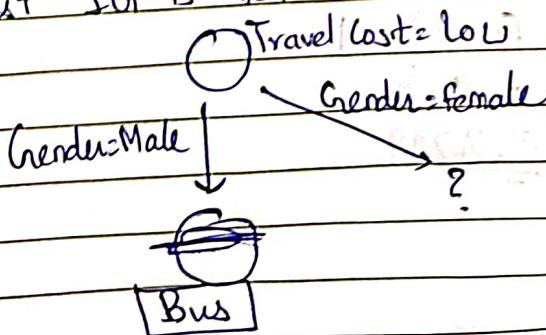
$$H(\text{Income level} = \text{medium}) = 0.9237$$

$$H(\text{Income level} = \text{high}) = 0$$

$$H(\text{Income level}) = 0.5542$$

$$IG = 0.7219 - 0.5542 = 0.1676$$

Hence, Highest IG is for gender attribute.
So,

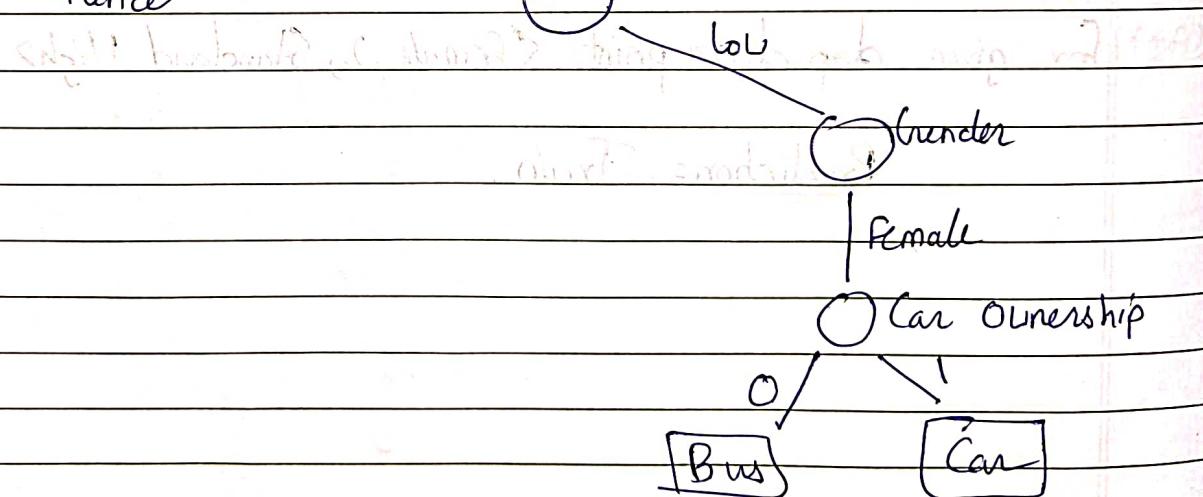


Now, $H(\text{Travel Cost} = \text{low}) | \text{Gender} = \text{Female}) = 1$

for attribute Car Ownership

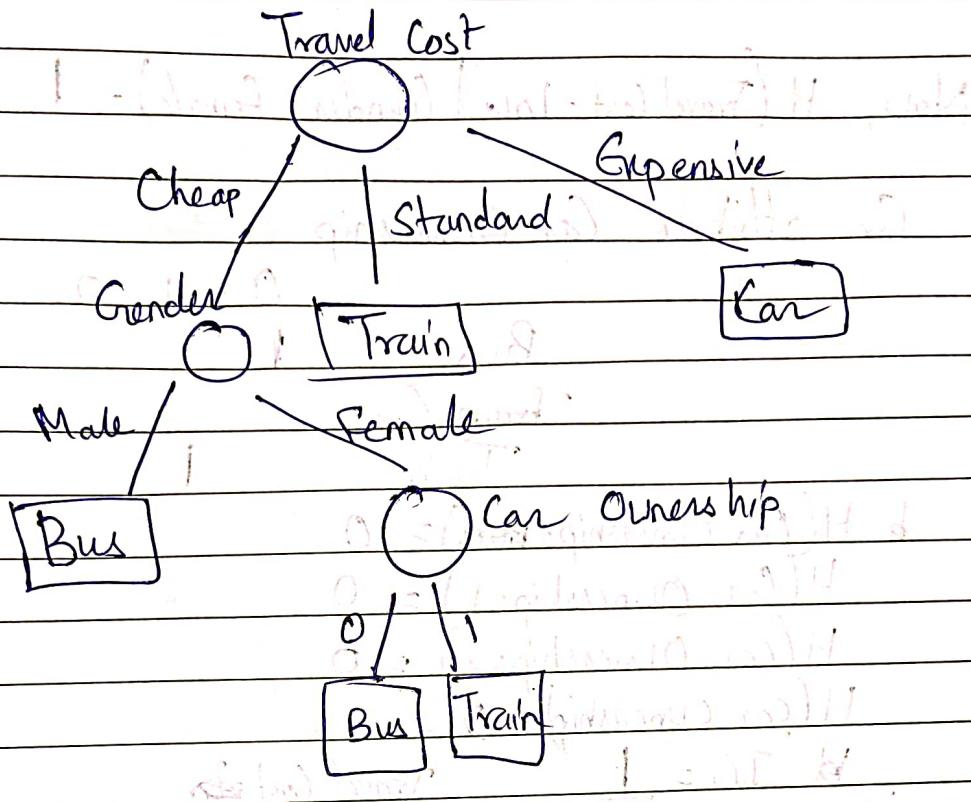
E	O	I	S
Bus	1	0	2
Train	Car	1	
Train		1	
$H(\text{Car Ownership} = \text{None}) = 0$			
$H(\text{Car Ownership} = 1) = 0$			
$H(\text{Car Ownership} = 2) = 0$			
$H(\text{Car Ownership} = 0) = 1$			
$H(G) = 1$	Travel Cost		

Hence



Now, let us calculate sub tree for Travel Cost = Standard

For attribute Travel Cost = Standard, all outputs are True. Hence, the constructed Decision Tree is as follows:-



for given data point {female, 2, Standard, High}

Prediction = Train.