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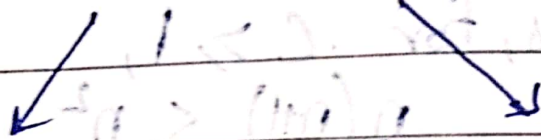
BTECH 3RD YEAR

10/9/20 U18C0081

Ans. Given the array $A = \{34, 56, 21, 22, 54, 32, 1, 12, 3, 4, 5\}$

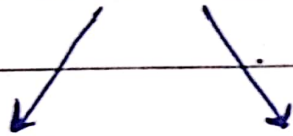
(Divide)

34 56 21 22 54 32 1 12 3 4 5

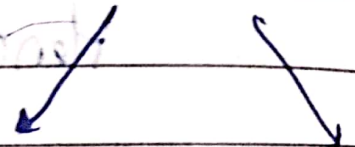


34 56 21 22 54 32 1 12 3 4 5

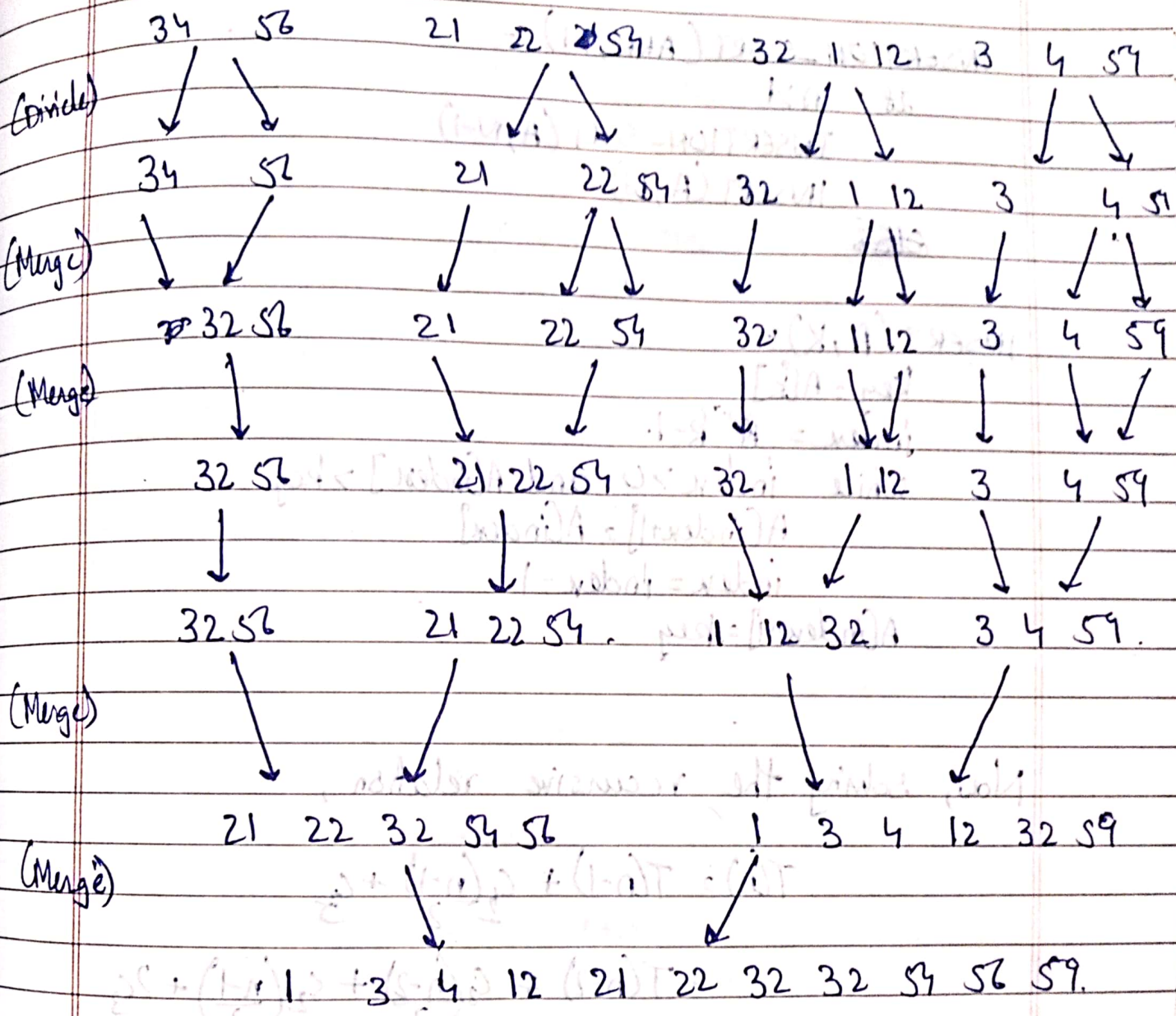
(Divide)



34 56 21 22 54 32 1 12 3 4 5



34 56 21 22 54 32 1 12 3 4 5



Ans: For the given problem,

$$T(n) = \begin{cases} T(n-1) + O(n) & ; n > 1 \\ O(1) & ; n = 1 \end{cases}$$

Thus, we get,

$$T(n) = \begin{cases} c & ; n = 1 \\ T(n-1) + c(n-1)/2 + c & ; n > 1 \end{cases}$$

This is because of the following recursive procedure:-

INSERTION_SORT (ARR, N) :-

if $N > 1$

INSERTION_SORT (A, N-1)

INSERT (A, N)

~~else~~

INSERT (A, K) :-

key = A[K]

index = K-1

while index > 0 and A[index] > key

A[index+1] = A[index]

index = index - 1

A[index+1] = key

Now, solving the recursive relation,

$$T(n) = T(n-1) + C_2 \left(\frac{n-1}{2} \right) + C_3$$

$$= T(n-2) + C_2 \left(\frac{n-2}{2} \right) + C_2 \left(\frac{n-1}{2} \right) + 2C_3$$

$$= C_1 + \frac{C_2}{2} (1 + 2 + 3 + \dots + n-1) + 4C_3$$

$$\approx \underline{\underline{O(n^2)}}$$

MERGESORT (ARR, LOW, HIGH) :-

MID $\leftarrow (LOW + HIGH) / 2$

MERGESORT (ARR, LOW, MID)

MERGESORT (ARR, MID, HIGH)

MERGE (ARR, LOW, HIGH, MID)

MERGE (ARR, LOW, HIGH, MID) :-

K \leftarrow LOW I \leftarrow 0 ARR1 \leftarrow {}

L \leftarrow MID

While K \leq MID and L \leq HIGH;

If ARR[K] \leq ARR[L],

ARR1[I++] = ARR[K++]

Else

ARR1[I++] = ARR[L++]

While K \leq MID,

ARR1[I++] = ARR[K++]

While L \leq HIGH,

ARR1[I++] = ARR[L++]

I \leftarrow 0

While I \leq I

ARR[J++] = ARR1[I++]

Ans 3:

Binary search is a divide-and-conquer based search technique which requires sorted array.

BINARY_SEARCH (ARR, LOW, HIGH, KEY) :-

IF LOW = HIGH AND ARR[LOW] = KEY,

RETURN LOW

ELSE IF LOW > HIGH:

RETURN -1

MID ← (LOW + HIGH) / 2

IF ARR[MID] > KEY,

RETURN BINARY_SEARCH (ARR, LOW, MID - 1, KEY)

ELSE IF ARR[MID] < KEY

RETURN BINARY_SEARCH (ARR, MID + 1, HIGH, KEY)

ELSE

RETURN MID

The recurrence for the above function is:

$$T(N) = T(N/2) + O(1)$$

$$T(N) = O(1) + O(1) + T(N/4)$$

$$T(N) = O(1) + O(1) + \dots + O(1)$$

$\log_2 N$ times

Hence,

~~$O(N)$~~

$$O(T(N)) = O(\log N)$$

Ans 4

Example:- Consider the array:-
 $\{8, 9, 13, 17, 21, 25, 28\}$

Dry run for finding 25 is as follows:-

Binary Search (0, 7, 25) has $mid = 3$ and $arr[mid] = 17$

$17 < 25$, Hence,

Binary Search (4, 7, 25) has $mid = 5$ and $arr[mid] = 25$

Which is equal to 25. Hence, returns 5.

Ans 4: Given a sorted ^{binary} array we need to find number of
in it.

Let the sorted array have size N .

SEARCH (ARR, LOW, HIGH, N)

If $LOW = HIGH$, Then,

RETURN $N - LOW$

$MID \leftarrow (LOW + HIGH) / 2$

If $ARR[MID] = 1$, Then,

~~HIGH = MID~~ RETURN SEARCH (ARR, LOW, ^{MID-1} ~~HIGH-1~~, N)

ELSE,

RETURN SEARCH (ARR, MID, HIGH, N)

$T(N) = T(N/2) + O(1)$ Hence, $O(T(N)) = O(\log N)$

Example:- Given an array:-

$\{0, 0, 0, 0, 0, 0, 1, 1, 1, 1\}$ $N = 10$

Call SEARCH (ARR, 0, 10) $\rightarrow MID = 5 \rightarrow ARR[MID] = 0$

Hence, Call SEARCH (ARR, 6, 10) $\rightarrow MID = 8 \rightarrow ARR[MID] = 1$

Hence, Call SEARCH (ARR, 6, 7) $\rightarrow MID = 6 \rightarrow ARR[MID] = 1$

Hence, Call SEARCH (ARR, 6, 6) $\rightarrow LOW = HIGH \rightarrow$ Return $N - LOW$

Ans: MAXSUBARRAY.SUM(ARR, N):

If $N = 0$ and $ARR[N] > 0$

Return $ARR[N]$

Else if $N = 1$

Return 0

SUM \leftarrow MAXSUBARRAY.SUM(ARR, N-1)

If $SUM + ARR[N] > \text{SUM}$

Return $SUM + ARR[N]$

Else

Return $\text{MAX}(SUM, ARR[N])$

$$T(N) = T(N-1) + O(1)$$

$$= \underbrace{O(1) + O(1) + \dots + O(1)}_{N\text{-times}}$$

$$\text{Hence, } O(T(N)) = O(N)$$

Here, Example = $\{-1, 3, 7, 2, -2, -9\}$

For $N=1$, It returns 0

For $N=2$, $SUM = 0$, $SUM + ARR[2] > \text{SUM}$, Hence, Returns
 $SUM + ARR[2] = 3$

For $N=3$, $SUM = 7$, $SUM + ARR[3] > \text{SUM}$, Hence, Returns
 $SUM + ARR[3] = 10$

For $N=4$, $SUM = 10$, $SUM + ARR[4] > \text{SUM}$, Hence, Returns
 $SUM + ARR[4] = 12$

For $N=5$, $SUM = 12$, $SUM + ARR[5] < \text{SUM}$. Hence, Returns
 $SUM = 12$

For $N=6$, $SUM = 12$, $SUM + ARR[6] < \text{SUM}$, Hence, Returns
 $SUM = 12$.

Ans 6: SECOND-MAXIMUM(ARR, N):
 If $N=2$ Then,
 Return $\{ \text{MAX}(\text{ARR}[1], \text{ARR}[2]), \text{MIN}(\text{ARR}[1], \text{ARR}[2]) \}$

$(\text{FIRSTMAX}, \text{SECONDMAX}) \leftarrow \text{SECOND-MAXIMUM}(\text{ARR}, N-1)$
 If $\text{ARR}[N] > \text{FIRSTMAX}$ Then,
 Return $\text{FIRST} \{ \text{ARR}[N], \text{FIRSTMAX} \}$
 If $\text{ARR}[N] > \text{SECONDMAX}$; Then,
 Return $\{ \text{FIRSTMAX}, \text{ARR}[N] \}$
 Return $\{ \text{FIRSTMAX}, \text{SECONDMAX} \}$

$$\begin{aligned} \text{Here, } T(N) &= T(N-1) + O(1) \\ &= O(1) + O(1) + \dots + O(1) \end{aligned}$$

Hence, $O(T(N)) = O(N)$ N times

The method returns minimum and second maximum values.