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BTECH 3RD YEAR

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DAA Tutorial 3
(Divide and Conquer)

Ans 1;
i) Given $T(n) = 2T(n/2) + 2$

$= 2(2T(n/4) + 2) + 2$

$\ldots = 2(2(2T(n/8) + 2) + 2) + 2$

$= \ldots$

For this we guess the solution as $T(n) \le \cancel{c\log n}\ cn$

Now, $T(n) \le 2(c\,\cancel{\log_2(n/2)}) + 2$

$T(n) \le 2c\log_2 n - 2c\log_2 2 + 2$

$T(n) \le 2c\log_2 n - 2(c-1)$

When we put $c = 1$,

$T(n) \le \cancel{2\log_2} n$

Hence, it is correct. $T(n) = \cancel{c\log n}\ O(n)$.

$T(n) = 2^{\log n - 1} + 2^{\log n - 1} + \ldots + 1$

$= \sum_{k=0}^{k=\log n - 1} 2^k$

$= \underline{\underline{O(n)}}$

2. Given $T(n) = T(n/2) + n^2$

Let us guess its solution as $T(n) \le cn^2\log_2 n$

Now,

$T(n) \le cn^2\log_2(n/2) + n^2$

$T(n) \le cn^2(\log_2 n - \log_2 2) + n^2$

$T(n) \le cn^2\log_2 n - cn^2 + n^2$

$T(n) \le cn^2\log_2 n$

Hence, $T(n) = cn^2\log_2 n$

3. Given $T(n) = 2T(n-1)+1$

Using iteration we get,

$$T(n) = 1 + 2(T(n-1))$$
$$= 1 + 2(1 + 2T(n-2))$$
$$= 1 + 2(1 + 2(1 + T(n-3)))$$
$$\cancel{= 1 + 2}$$

For base cases,

$$T(0) = 0$$
$$T(1) = 1$$
$$T(2) = 3$$
$$T(3) = 7$$

Let us assume that $T(k) = 2^k - 1$

Then,

$$T(k+1) = 2(T(k-1)) + 1$$
$$= 2(2^k - 1) + 1$$
$$= 2^{k+1} - 1$$

Hence, our assumption is correct.

$T(n) = 2^n - 1 \quad \forall n \in N$

4. $$T(n) = \begin{cases} C_1 & \text{if } n = 0 \\ C_2 + T(n-1) & \text{otherwise} \end{cases}$$

$$T(0) = C_1$$
$$T(1) = C_1 + C_2$$
$$T(2) = C_2 + C_1 + C_2 = 2C_2 + C_1$$
$$T(3) = 3C_2 + C_1$$

Hence, let us assume that $T(k) = kC_2 + C_1$

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Now, let us prove it true for $T(k+1)$.

$$T(k+1) = T(k) + c_2$$
$$= kc_2 + c_1 + c_2$$
$$= (k+1)c_2 + c_1$$

Hence, it is correct.

Hence, $T(n) = nc_2 + c_1 \mid \forall n$

Ans 2: Given $T(n) = T(n-1) + n$

Using recurrence relation, we get,

$$T(n) = n + n-1 + T(n-2)$$
$$= n + n-1 + n-2 + T(n-3)$$
$$= n + n-1 + n-2 + \dots + 3 + 2 + 1$$
$$= \frac{n(n+1)}{2}$$

Now, $\frac{n(n+1)}{2} \leq cn^2$

$$\frac{n^2}{2} + \frac{n}{2} \leq cn^2$$

$$c \geq \frac{1}{2} + \frac{1}{2n}$$

Thus, for $c \geq 1$,

$$\frac{n(n+1)}{2} \leq n^2$$

Hence, $T(n) = O(n^2)$

Ans 3: The recurrence relation is given by,
$T(n) = T(n-1) + 1$

For $T(0) = 0$
$T(1) = 1$
$T(2) = 2$
|

Let us assume that $T(k) = k$
Here, let us try to prove it for $k+1$.

$T(k+1) = T(k) + 1$
$= k+1$

Hence, it is true.
~~For~~ Hence, $T(n) = n$

Similarly let $n \leq c_1 n$
$c_1 = 1$
Hence, $T(n) = n$

Ans 4: Given the array $A = \{34, 56, 21, 22, 54, 32, 1, 12, 3$

34 56 21 22 54 32

(Divide)

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Ans 4. Given

$$T(n) = T(n/2) + T(n/2) + n$$
$$= 2T(n/2) + n$$
$$= 2T(n/4) + n + n/2$$
$$+ 2T(n/8) + n + 2n/2 + 4n/4$$
$$= \underbrace{n + n + n + \dots + n}_{\log_2 N \text{ times}}$$

$$= cn\log n$$

Hence: $T(n) = cn\log n$

Now, let

$$T(n) \le c'n^2$$
$$n\log n \le c'n^2$$
$$c' \ge \frac{\log n}{n}$$

$$c' = 1,$$
$$n\log n \le c'n^2$$

Hence, $T(n) = O(n\log n) = O(n^2)$

[Hence, shown]

Ans 5. Given $T(n) = \begin{cases} 8T(n/2) + n^2 & ; n > 1 \\ 1 & ; n = 1 \end{cases}$

Given $T(n) = 8T(n/2) + n^2$

$$= 8\left(8T(n/4) + \frac{n^2}{4}\right) + n^2$$

$$= 64T(n/4) + 3n^2$$

$$= 64\left(8T(n/8) + \frac{n^2}{16}\right) + 3n^2$$

$$= 8^3 T(n/8) + 7n^2$$

Let us assume the solution as $(2^n - 1)n^2$.

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Let us observe $T(n)$ for n as any power of 2,

$$T(1) = 1$$
$$T(2) = 8 \times 1 + 4$$
$$= 12$$
$$T(4) = 8 \times 12 + 16$$
$$= 112$$

$$T(n) = n^2 + 2n^2 + 2^2 n^2 + 2^3 n^3 + \dots + 2^{\log_2 n - 1} n^2 + 8^{\log_2 n}$$
$$= n^2 \sum_{k=0}^{\log n - 1} 2^k + 8^{\log_2 n}$$
$$= n^2 \sum_{k=0}^{\log n - 1} 2^k + (2^3)^{\log n}$$

Now,
$$\sum_{k=0}^{\log n - 1} 2^k = \Theta(2^{\log n - 1}) = \Theta(n)$$

Also, $(2^{\log n})^3 = \underline{\underline{n^3}}$

Hence, $T(n) = n^2 \Theta(n) + \Theta(n^3)$
$$= \underline{\underline{\Theta(n^3)}}$$