NAME:- KRUNAL RANK

ADM. NO:- U18CO081

B TECH 3RD YEAR

U18CO081

20/8/20

DAA Tutorial 1 Asymptotic Analysis

Big-Oh Notation

1. $6n+2$

let $6n+2 \le c_1 n$

let $c_1 = 8$ Then,

$6n+2 \le 8n \quad n \geq 1$.

Hence, it is $O(n)$.

2. $150n+8$

let $150n+8 \le c_1 n$

let $c_1 = 158$. Then,

$150n+8 \le 158n \quad n \geq 1$.

Hence, it is $O(n)$

3. $10n^2+3n+2$

let $10n^2+3n+2 \le c_2 n^2$

$c \geq 10 + \frac{3}{n} + \frac{2}{n^2}$

Now for $n \geq n_0 = 1$, $c \geq 15$

Thus, $10n^2+3n+2 \le 15n^2 \quad \forall n \geq n_0$

Hence, it is $O(n^2)$

4. Given $f(n) = 6 \cdot 2^n + n^2$

let $f(n) \leq c\,2^n$

$$c \geq 6 + \frac{n^2}{2^n}$$

for $n=1$, $c \geq 6.5$

for $n=2$ $c \geq 6 + 1 = 7$

for $n=3$ $c \geq 6 + \frac{9}{8}$

for $n=4$ $c \geq 6 + 1 = 7$

Thus, $6 \cdot 2^n + n^2 \leq 7\,2^n$ $n \geq 4$

$\Rightarrow O(2^n)$

5. Given $3n+5 = O(2^n)$

let $f(n) \leq c \cdot 2^n$

$3n+5 \leq c \cdot 2^n$

$$c \geq \frac{3n}{2^n} + \frac{5}{2^n}$$

For $n=1$, $c \geq 4$

For $n=2$, $c \geq 1.5 + 1.25 = 2.75$

Hence, it is correct that

$3n+5 = O(2^n)$ $n \geq 1$ with $c = 4$.

6. Given $f(n) = n^4 + n + 6$

let $n^4 + n + 6 \leq c_1 n^4$

$c_1 \geq 1 + \frac{1}{n^3} + \frac{6}{n^4}$

for $n_0 = 1$, $c_1 \geq 8$

for $n_0 = 2$, $c_1 \geq 1 + 1/8 + 6/16$

Hence, $n^4 + n + 6 \leq 8n^4$ ~~for~~ $n \geq 1$

Hence, it is $O(n^4)$

Omega Notation

1. Given $f(n) = 6n + 2$

let $f(n) \geq c\,g(n)$

$6n + 2 \geq c_1$

Then, for $n_0 = 1$ $c_1 = 8$.

Hence, it is $\Omega(1)$

2. Given $f(n) = 10n^2 + 3n + 2$

let $f(n) \geq c_1 n$

$10n^2 + 3n + 2 \geq c_1 n$

$c_1 \leq 10n + 3 + \frac{2}{n}$

for $n_0 = 1$, $c_1 \leq 15$

for $n_0 = 2$, $c_1 \leq 20 + 3 + 1 = 24$.

Hence, it is $\Omega(n)$

3: Given $f(n) = n^3 + n + 5$

let $f(n) \geq c_1 n^2$

$c_1 \leq n + \frac{1}{n} + \frac{5}{n^2}$

for $n=1$, $c_1 \leq 1 + 1 + 5 = 7$

for $n=2$, $c_1 \leq 2 + 0.5 + 1.25 = 3.75$

for $n=3$, $c_1 \leq 3 + 0.33 + 0.55 = 3.9$

Hence, $n^3 + n + 5 \geq 3.75 n^2 \ \forall n \geq 2$

Hence, it is $\Omega(n^2)$

4: Given $2n^2 + n\log n + 1$

let $f(n) \geq c_1 n\log n$

$2n^2 + n\log n + 1 \geq c_1 n\log n$

$c_1 \leq \frac{2n}{\log n} + 1 + \frac{1}{n\log n}$

for $n_0 = 2$ $c_1 \leq 5.5$

Hence, $f(n) \geq 5.5\, n\log n$

it is $\Omega(n\log n)$

Theta Notation

1. let

~~$\leq 7n \leq$~~

$c_1 n \leq 7n + 15^{10} \leq c_2 n$

$c_2 \geq 7 + \frac{15^{10}}{n}$

~~For $n_0 = 15^{10}$, $c_2 = 8$~~ for $n=1$, $c_2 \geq 7+15^{10}$

$c_1 \leq 7 + \frac{15^{10}}{n}$

~~For $n_0 = 15^{-10}$, $c_1 = 8$~~ for $n_0 = 1$, $c_1 \leq 7+15^{10}$

$(7+15^{10})n \leq 7n + 15^{10} \leq (7+15^{10})n$

Hence it is $\Theta(n)$

2. let

$c_1 n^2 \leq 8n^2 + 4n + 2 \leq c_2 n^2$

$c_2 \geq 8 + \frac{4}{n} + \frac{2}{n^2}$

For $n_0 = 1$, $c_2 \geq 14$

Also, $c_1 n^2 \leq 8n^2 + 4n + 2$

$c_1 \leq 8 + \frac{4}{n} + \frac{2}{n^2}$

For $n_0 = 1$, $c_1 \leq 8$

Hence, $8n^2 \leq 8n^2 + 4n + 2 \leq 14n^2$

Hence it is $\Theta(n^2)$

3/2 let

$$c_1 n^2 \le 2n^2 + n\log n + 1 \le c_2 n^2.$$

$$c_1 n^2 \le 2n^2 + n\log n + 1$$

c_1 for $n = n_0 \; \forall n_0 \ge 1$,

$$c_1 = 2.$$

$$2n^2 \le 2n^2 + n\log n + 1$$

Now,

$$c_2 n^2 \ge 2n^2 + n\log n + 1$$

$$c_2 \ge 2 + \frac{\log n}{n} + \frac{1}{n^2}$$

$n=1$, $c_2 \ge 2+1 = 3$

$n=2$, $c_2 \ge 2 + 0.5 + 0.25 = 2.75$

Hence,

$$2n^2 \le 2n^2 + n\log n + 1 \le 3n^2$$

Thus, it is $\Theta(n^2)$.

4/2 let

$$c_1(1) \le 3n+6 \le c_2(1)$$

Here, the right inequality,

$$c_2 \ge 3n+6$$

The right side is growing infinitely and c_2 cannot contain it.

Hence, it is not $\Theta(1)$.

Date / /
Page

5. let

$$C_1 n^2 \leq 4n+2 \leq C_2 n^2$$

Examining the left side inequality,

$$C_1 \leq \frac{4}{n} + \frac{2}{n^2}$$

for $n_0 = 1$ $C_1 \leq 6$
for $n_0 = 2$ $C_1 \leq 2 + 0.5 = 3$
for $n_0 = 3$ $C_1 \leq 1.33 + 0.22 = 1.55$
Hence, the left side is shrinking infinitely.

Thus, it is not $\Theta(n^2)$

6. let

$$C_1 n^{1.00001} \leq 7n + 8 \leq C_2 n^{1.00001}$$

Here, for $C_2 \geq 15$,

$$7n + 8 \leq C_2 n^{1.00001} \quad n \geq n_0 = 1$$

for $C_1 \leq \frac{7}{n^{0.00001}} + 8$

No matter how small, right side is shrinking indefinitely.
Hence,
it is not $\Theta(n^{1.00001})$.

Scanned with CamScanner