

AI1103

Assignment 5

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Download Python codes from below link :

<https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/code/Assignment%205.py>

Download LaTex file from below link :

<https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/LaTex/Assignment%205.tex>

Problem number GATE EC 2019 Q.20

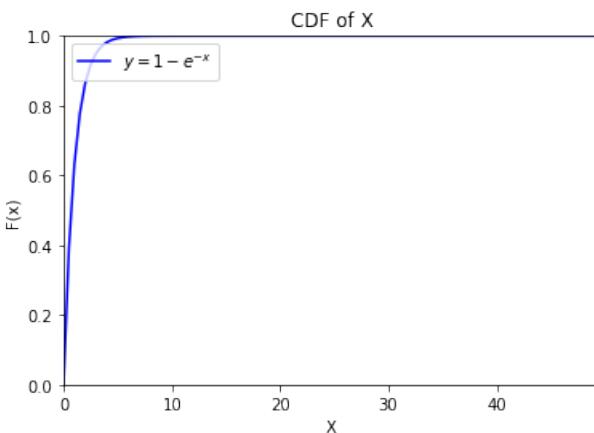
Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_Z(x) = \begin{cases} 1 - e^{-x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$

Then $\Pr(Z>2 | Z>1)$, rounded off to two decimal places, is equal to

Solution

Given that Z is an exponential distribution with cumulative function $F_Z(x)$



We know that probability density function

$$f_Z(x) = F'_Z(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The PDF of X is,

$$\begin{aligned} f_Z(x) &= \int_{-\infty}^{\infty} f(x)dx \\ \Pr(Z > 2 | Z > 1) &= \frac{\Pr((Z > 2) \cap (Z > 1))}{\Pr(Z > 1)} \\ &= \frac{\Pr(Z > 2)}{\Pr(Z > 1)} \\ &= \frac{\int_{-\infty}^{\infty} e^{-x} dx}{\int_{-\infty}^{\infty} e^{-x} dx} \\ &= \frac{2}{\int_1^{\infty} e^{-x} dx} \\ &= \frac{2}{\int_1^{\infty} e^{-x} dx} \\ &= \frac{-(e^{-\infty} - e^{-2})}{-(e^{-\infty} - e^{-1})} \\ &= \frac{e^{-2}}{e^{-1}} \\ &= e^{-1} \\ &= 0.3679 \end{aligned}$$

