

AI1103

Assignment 5

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Download Python codes from below link :

<https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/code/Assignment%205.py>

Download LaTeX file from below link :

<https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/LaTex/Assignment%205.tex>

Problem number GATE EC 2019 Q.20

Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_Z(x) = \begin{cases} 1 - e^{-x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$

Then $\Pr(Z > 2 \mid Z > 1)$, rounded off to two decimal places, is equal to

Solution

Given that Z is an exponential distribution with cumulative function $F_Z(x)$.

We know that probability density function

$$f_Z(x) = F'_Z(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The CDF of X is,

$$F_Z(x) = \Pr(Z \leq x), \text{ for all } x \in R$$

$$\Pr(Z \leq 2) = F_Z(2)$$

$$= 1 - e^{-2}$$

$$\Pr(Z \leq 1) = F_Z(1)$$

$$= 1 - e^{-1}$$

$$\Pr(Z > 2) = 1 - \Pr(Z \leq 2)$$

$$= e^{-2}$$

$$\Pr(Z > 1) = 1 - \Pr(Z \leq 1)$$

$$= e^{-1}$$

$$\Pr(Z > 2 \mid Z > 1) = \frac{\Pr((Z > 2) \cap (Z > 1))}{\Pr(Z > 1)}$$

$$= \frac{\Pr(Z > 2)}{\Pr(Z > 1)}$$

$$= \frac{e^{-2}}{e^{-1}}$$

$$= e^{-1}$$

$$= 0.3679$$

