

AI1103

Assignment 6

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https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_6/LaTex/Assignment%206.tex

Problem number CSIR UGC NET Dec 2014 Q.104

Suppose X_1, X_2, X_3 and X_4 are independent and identically distributed random variables, having density function f . Then,

- 1) $\Pr(X_4 > \max(X_1, X_2) > X_3) = \frac{1}{6}$
- 2) $\Pr(X_4 > \max(X_1, X_2) > X_3) = \frac{1}{6}$
- 3) $\Pr(X_4 > X_3 > \max(X_1, X_2)) = \frac{1}{12}$
- 4) $\Pr(X_4 > X_3 > \max(X_1, X_2)) = \frac{1}{6}$

Solution

The probability density function (pdf) $f(x)$ of a random variable X is defined as the derivative of the cdf $F(x)$:

$$f(x) = \frac{d}{dx} F(x).$$

It is sometimes useful to consider the cdf $F(x)$ in terms of the pdf $f(x)$:

$$F(x) = \int_{-\infty}^x f(t) dt$$

The PDF of X is,

$$F_X(x) = \int_{-\infty}^{\infty} f(x) dx \quad (0.0.1)$$

$$(0.0.2)$$

$$\Pr(X_2 > X_1) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) dt dx \quad (0.0.3)$$

$$= \int_{-\infty}^{\infty} f_X(x) F_X(x) dx \quad (0.0.4)$$

$$= \frac{F_X^2(x)}{2} \Big|_{-\infty}^{\infty} \quad (0.0.5)$$

$$= \frac{1}{2}. \quad (0.0.6)$$

$$\Pr(X_4 > \max(X_1, X_2) > X_3) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) dt dx$$

$$= {}^2C_1 \left[\int_{-\infty}^t f_X(w) dw \right] \int_{-\infty}^t f_X(z) dz dt dx \quad (0.0.7)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x 2f_X(t) F_X^2(t) dt dx \quad (0.0.8)$$

$$= \int_{-\infty}^{\infty} f_X(x) \frac{2}{3} F_X^3(x) dx \quad (0.0.9)$$

$$= \frac{2}{3} \frac{F^4 X(x)}{4} \Big|_{-\infty}^{\infty} \quad (0.0.10)$$

$$= \frac{1}{6} \quad (0.0.11)$$

$$\Pr(X_4 > X_3 > \text{Max}(X_1, X_2)) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) \int_{-\infty}^t f_X(z) \cdot {}^2\text{C}_1 \left[\int_{-\infty}^t f_X(w) dw \right] dz dt dx \quad (0.0.12)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) \int_{-\infty}^t 2f_X(z) F_X(t) dz dt dx \quad (0.0.13)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) F_X^2(t) dt dx \quad (0.0.14)$$

$$= \int_{-\infty}^{\infty} f_X(x) \cdot \frac{1}{3} F_X^3(x) dx \quad (0.0.15)$$

$$= \frac{1}{3} \frac{F_X^4(x)}{4} \Big|_{-\infty}^{\infty} \quad (0.0.16)$$

$$= \frac{1}{12}. \quad (0.0.17)$$

\therefore **Option 1,3** are **correct** answers.