# AI1103
# Challenging Problem 7

Nagubandi Krishna Sai
MS20BTECH11014

Download LaTex file from below link :

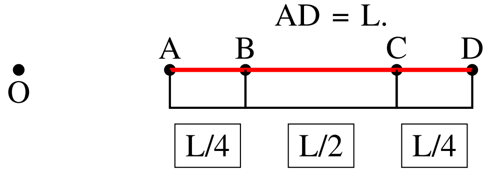
https://github.com/KRISHNASAI1105/demo/blob/main/Challenging%20problem%207/LaTex/Challenging%20problem%207.tex

***IES/ISS STATISTICS* 2015 *Q*.3(*c*)**

Two points are chosen on a line of unit length. Find the probability that each of the 3 line segments will have length greater than $\frac{1}{4}$?

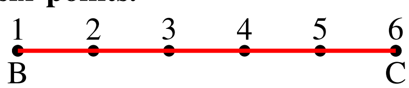
***Solution***

AD = L.




∴ You cannot take a point from AB and CD region. So, this means we have to choose the two points apart from $\frac{L}{4} distance$.

Suppose, the line of length $\frac{L}{2}$ consists of **six points.**




$$\begin{aligned}\text{Probability} = {} & \text{(If you pick point 1, then} \\ & \text{we can pick a point from 4,5,6)} \\ & + \text{(If you pick point 2, then} \\ & \text{we can pick a point from 5,6)} \\ & + \text{(If you pick point 3, then} \\ & \text{we can pick a point from 6)}\end{aligned} \tag{0.0.1}$$

$$\text{Probability} = \frac{1}{2}\left(\left(\frac{1}{6}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{5}\right)\right) \tag{0.0.2}$$

Consider that the line of length $\frac{L}{2}$ has n points and n$\to \infty$.

$$\begin{aligned}\text{Probability} &= \lim_{n\to\infty}\left(\frac{1}{2n}\right)\left(\frac{\frac{n}{2}}{n-1}\right) + \left(\frac{1}{2n}\right)\left(\frac{\frac{n}{2}-1}{n-1}\right) \\ &\quad ..... + \frac{1}{2n}\left(\frac{1}{n-1}\right) \\ &= \lim_{n\to\infty}\sum_{r=0}^{\left(\frac{n}{2}-1\right)}\frac{\left(\frac{n}{2}-r\right)}{2n(n-1)}\end{aligned}$$

As $n \to \infty, n-1 \to n$.
$$\tag{0.0.3}$$

$$\begin{aligned}\text{Probability} &= \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{\left(\frac{n}{2}-1\right)}\frac{n-2r}{4n} \\ &= \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{\left(\frac{n}{2}-1\right)}\frac{1}{4}-\frac{r}{2n} \\ &= \int_0^{\frac{1}{2}}\left(\frac{1}{4}-\frac{x}{2}\right)dx \\ &= \left(\frac{x}{4}-\frac{x^2}{4}\right)\Bigg|_0^{\frac{1}{2}} \\ &= \frac{1}{8}-\frac{1}{16} \\ &= \frac{1}{16}.\end{aligned} \tag{0.0.4}$$