AI1103
Assignment 6

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https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_6/LaTex/Assignment%206.tex

***Problem number CSIR UGC NET Dec* 2014 *Q*.104**

Suppose X_1,X_2,X_3 and X_4 are independent and identically distributed random variables, having density function f. Then,

1) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{6}$
2) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{8}$
3) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{12}$
4) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{6}$

Solution

$$\begin{aligned} Pr(X2 > X1) &= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)\ dtdx \\ &= \int_{-\infty}^{\infty} f_X(x) F_X(x) dx \\ &= \frac{F_X^2(x)}{2}\Big|_{-\infty}^{\infty} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Pr(X_4 > Max(X_1, X_2) > X_3) &= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t).{}^2\mathrm{C}_1. \\ &\quad \left[\int_{-\infty}^{t} f_X(w)dw\right] \int_{-\infty}^{t} f_X(z)\ dzdtdx \\ &= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} 2 f_X(t) F_X^2(t)\ dtdx \\ &= \int_{-\infty}^{\infty} f_X(x).\frac{2}{3} F_X^3(x) dx \\ &= \frac{2}{3}\frac{F_X^4(x)}{4}\Big|_{-\infty}^{\infty} \\ &= \frac{1}{6} \end{aligned}$$

$$Pr(X_4 > X_3 > Max(X_1, X_2)) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t) \int_{-\infty}^{t} f_X(z).$$

$$^2C_1 \left[\int_{-\infty}^{t} f_X(w)dw \right] dzdtdx$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)$$

$$\int_{-\infty}^{t} 2f_X(z)F_X(t)dzdtdx$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)F_X^2(t)\ dtdx$$

$$= \int_{-\infty}^{\infty} f_X(x).\frac{1}{3}F_X^3(x)dx$$

$$= \frac{1}{3}\frac{F_X^4(x)}{4}\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{12}$$

$\therefore$ **Option 1,3** are **correct** answers.