

AI1103

Assignment 7

Nagubandi Krishna Sai
MS20BTECH11014

Download LaTex file from below link :

https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_7/LaTex/Assignment_7.tex

Problem number CSIR UGC NET 2014 Q.106

Consider a Markov chain with state space $1, 2, \dots, 100$. Suppose states $2i$ and $2j$ communicate with each other and states $2i-1$ and $2j-1$ communicate with each other for every $i, j = 1, 2, \dots, 50$. Further suppose that $p_{3,3}^{(2)} > 0, p_{4,4}^{(3)} > 0$ and $p_{2,5}^{(7)} > 0$. Then

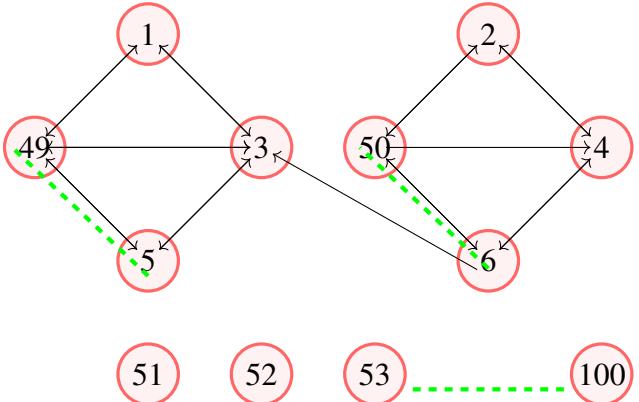
- 1) The Markov chain is irreducible.
- 2) The Markov chain is aperiodic.
- 3) State 8 is recurrent.
- 4) State 9 is recurrent.

Solution

Definition 1. We say that Markov chain is irreducible if and only if all states belong to one communication class and all states communicate with each other.

Definition 2. In an **irreducible chain** all states belong to a single communicating class. This means that, if one of the states in an irreducible Markov chain is **aperiodic**. Then, all the remaining states are also aperiodic.

$$S = \{1, 2, \dots, 100\}.$$



Consider, the communication classes of the given Markov chain as follows :

$$C_1(1) = \{1, 3, 5, 7, \dots, 49\}.$$

$$C_1(2) = \{2, 4, 6, 8, \dots, 50\}.$$

$$C_1(51) = \{51\}, C_1(52) = \{52\}, \dots, C_1(100) = \{100\}.$$

∴ As there are 52 communication classes, the given Markov chain is reducible.

$\Rightarrow [i \in C_{(i)}] (\because i \text{ communicate with } i \text{ in zero steps})$

Given, even states communicate with each other. Similarly odd states communicate with each other. Regarding periodicity,

$$d(K) = \gcd(m \geq 1 : P_{k,k}^m > 0).$$

For all odd states in $\{1, 3, 5, 7, \dots, 49\}$, Periodicity = $d(1)$.

Similarly, For all even states in $\{2, 4, 6, 8, \dots, 50\}$, Periodicity = $d(2)$.

$$d(1) = d(2) = \gcd\{2, 3, \dots\} = 1. \therefore \text{Aperiodic.}$$

$$d(51) = d(52) = \dots = d(100) = 0. \therefore \text{periodic.}$$

Hence, The given **Markov chain** is reducible and not a aperiodic chain.

$\{1, 3, 5, 7, \dots, 49, 51, 52, 53, \dots, 100\}$ are recurrent states.
 $\{2, 4, 6, 8, \dots, 50\}$ are transient states.

Option 4 is a correct answer