AI1103
Assignment 7

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https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_7/LaTex/Assignment_7.tex

Problem number CSIR UGC NET 2014 Q.106

Consider a Markov chain with state space 1,2,....,100. Suppose states 2i and 2j communicate with each other and states 2i-1 and 2j-1 communicate with each other for every i,j = 1,2,...,50. Further suppose that $p^{(2)}_{3,3} > 0, p^{(3)}_{4,4} > 0$ and $p^{(7)}_{2,5} > 0$. Then

1) The Markov chain is irreducible.
2) The Markov chain is aperiodic.
3) State 8 is recurrent.
4) State 9 is recurrent.

Solution

Definition 1. *We say that Markov chain is* ***irreducible*** *if and only if all states belong to one communication class and all states communicate with each other.*

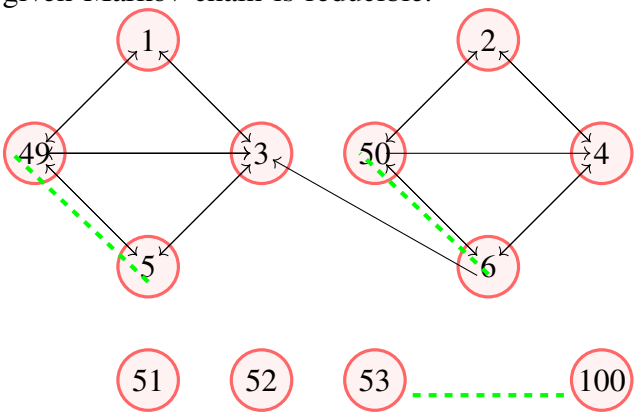
$$S = \{1,2,....,100\}.$$

Consider, the communication classes of the given Markov chain as follows :

Communication class	set of elements
$C_1(1)$	{1,3,5,7,....,49}
$C_1(2)$	{2,4,6,8,....,50}
$C_1(51)$	{51}
$C_1(52)$	{52}
⋮	⋮
$C_1(100)$	{100}

TABLE 4: Communication class

∴ As there are 52 communication classes, the given Markov chain is reducible.



Regarding periodicity,

$$d(K) = gcd(m \geq 1 : P^m_{k,k} > 0).$$

Periodicity of elements	set of elements
d(1)	{1,3,5,7,....,49}
d(2)	{2,4,6,8,....,50}

TABLE 4: Periodicity of some of elements of set S

d(1) = d(2) = gcd{2,3,....} = 1. ∴ Aperiodic.
d(51) = d(52) = ... = d(100) = 0. ∴ periodic.
Hence, The given **Markov chain** is **reducible** and not a **aperiodic** chain.
{1,3,5,7,...,49,51,52,53,....,100} are recurrent states.
{2,4,6,8,....,50} are transient states.

∴ **Option 4** is a **correct** answer