AI1103 Assignment 5

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Download Python codes from below link :

https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/code/Assignment%205.py

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https://github.com/KRISHNASAI1105/demo/blob/main/Assignment%205/LaTex/Assignment%205.tex

Problem number GATE EC 2019 *Q*.20

Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_Z(x) = \begin{cases} 1 - e^{-x}, & if\ x \geq 0 \\ 0, & if\ x < 0. \end{cases}$$

Then Pr(Z>2 | Z>1), rounded off to two decimal places, is equal to

Solution

Given that Z is an exponential distribution with cumulative function $F_Z(x)$

We know that probability density function

$$f_Z(x) = F'_Z(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The PDF of X is,

$$f_Z(x) = \int_{-\infty}^{\infty} f(x)dx$$

$$\begin{aligned} Pr(Z > 2|Z > 1) &= \frac{Pr((Z > 2) \cap (Z > 1))}{Pr(Z > 1)} \\ &= \frac{Pr(Z > 2)}{Pr(Z > 1)} \\ &= \frac{\int_2^{\infty} e^{-x}dx}{\int_1^{\infty} e^{-x}dx} \\ &= \frac{-(e^{-\infty} - e^{-2})}{-(e^{-\infty} - e^{-1})} \\ &= \frac{e^{-2}}{e^{-1}} \\ &= e^{-1} \\ &= 0.3679 \end{aligned}$$