

# AI1103

## Assignment 6

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### **Problem number CSIR UGC NET Dec 2014 Q.104**

Suppose  $X_1, X_2, X_3$  and  $X_4$  are independent and identically distributed random variables, having density function  $f$ . Then,

- 1)  $\Pr(X_4 > \max(X_1, X_2) > X_3) = \frac{1}{6}$
- 2)  $\Pr(X_4 > \max(X_1, X_2) > X_3) = \frac{1}{8}$
- 3)  $\Pr(X_4 > X_3 > \max(X_1, X_2)) = \frac{1}{12}$
- 4)  $\Pr(X_4 > X_3 > \max(X_1, X_2)) = \frac{1}{6}$

### **Solution**

The probability density function (pdf)  $f(x)$  of a random variable  $X$  is defined as the derivative of the cdf  $F(x)$ :

$$f(x) = \frac{d}{dx} F(x).$$

It is sometimes useful to consider the cdf  $F(x)$  in terms of the pdf  $f(x)$ :

$$F(x) = \int_{-\infty}^x f(t) dt$$

The PDF of  $X$  is,

$$F_X(x) = \int_{-\infty}^{\infty} f(x) dx \quad (0.0.1)$$

$$(0.0.2)$$

$$\Pr(X_2 > X_1) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) dt dx \quad (0.0.3)$$

$$= \int_{-\infty}^{\infty} f_X(x) F_X(x) dx \quad (0.0.4)$$

$$= \frac{F_X^2(x)|_{-\infty}^{\infty}}{2} \quad (0.0.5)$$

$$= \frac{1}{2}. \quad (0.0.6)$$

$$\Pr(X_4 > \max(X_1, X_2) > X_3) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t).^2 C_1. \left[ \int_{-\infty}^t f_X(w) dw \right] \int_{-\infty}^t f_X(z) dz dt dx \quad (0.0.7)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x 2f_X(t) F_X^2(t) dt dx \quad (0.0.8)$$

$$= \int_{-\infty}^{\infty} f_X(x) \cdot \frac{2}{3} F_X^3(x) dx \quad (0.0.9)$$

$$= \frac{2}{3} \frac{F_X^4(x)|_{-\infty}^{\infty}}{4} \quad (0.0.10)$$

$$= \frac{1}{6}. \quad (0.0.11)$$

$$\Pr(X_4 > X_3 > \text{Max}(X_1, X_2)) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) \int_{-\infty}^t f_X(z) \cdot {}^2\text{C}_1 \left[ \int_{-\infty}^t f_X(w) dw \right] dz dt dx \quad (0.0.12)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) \int_{-\infty}^t 2f_X(z) F_X(t) dz dt dx \quad (0.0.13)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^x f_X(t) F_X^2(t) dt dx \quad (0.0.14)$$

$$= \int_{-\infty}^{\infty} f_X(x) \cdot \frac{1}{3} F_X^3(x) dx \quad (0.0.15)$$

$$= \frac{1}{3} \left. \frac{F_X^4(x)}{4} \right|_{-\infty}^{\infty} \quad (0.0.16)$$

$$= \frac{1}{12}. \quad (0.0.17)$$

$\therefore$  **Option 1,3** are **correct** answers.