AI1103
Assignment 6

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https://github.com/KRISHNASAI1105/demo/blob/main/Assignment_6/LaTex/Assignment%206.tex

Problem number CSIR UGC NET Dec 2014 *Q*.104

Suppose X_1,X_2,X_3 and X_4 are independent and identically distributed random variables, having density function f. Then,

1) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{6}$
2) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{8}$
3) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{12}$
4) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{6}$

Solution

The probability density function (pdf) f(x) of a random variable X is defined as the derivative of the cdf F(x):

$$f(x) = \frac{d}{dx}F(x).$$

It is sometimes useful to consider the cdf F(x) in terms of the pdf f(x):

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

The PDF of X is,

$$F_X(x) = \int_{-\infty}^{\infty} f(x)dx \tag{0.0.1}$$

1) $\Pr(X_2 > X_1)$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)\ dtdx \tag{0.0.2}$$

$$= \int_{-\infty}^{\infty} f_X(x)F_X(x)dx \tag{0.0.3}$$

$$= \left.\frac{F_X^2(x)}{2}\right|_{-\infty}^{\infty} \tag{0.0.4}$$

$$= \frac{1}{2}. \tag{0.0.5}$$

2) $\Pr(X_4 > Max(X_1, X_2) > X_3)$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t).{}^2\mathrm{C}_1.$$

$$\left[\int_{-\infty}^{t} f_X(w)dw\right]\int_{-\infty}^{t} f_X(z)dzdtdx \tag{0.0.6}$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} 2f_X(t)F_X^2(t)dtdx \tag{0.0.7}$$

$$= \int_{-\infty}^{\infty} f_X(x).\frac{2}{3}F_X^3(x)dx \tag{0.0.8}$$

$$= \left.\frac{2}{3}\frac{F_X^4(x)}{4}\right|_{-\infty}^{\infty} \tag{0.0.9}$$

$$= \frac{1}{6}. \tag{0.0.10}$$

3) $\Pr\left(X_4 > X_3 > Max(X_1, X_2)\right)$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t) \int_{-\infty}^{t} f_X(z).{}^2\mathrm{C}_1. \left[\int_{-\infty}^{t} f_X(w)dw\right] dzdtdx \tag{0.0.11}$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t) \int_{-\infty}^{t} 2f_X(z)F_X(t)\ dzdtdx \tag{0.0.12}$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)F_X^2(t)dtdx \tag{0.0.13}$$

$$= \int_{-\infty}^{\infty} f_X(x).\frac{1}{3}F_X^3(x)dx \tag{0.0.14}$$

$$= \frac{1}{3}\frac{F_X^4(x)}{4}\bigg|_{-\infty}^{\infty} \tag{0.0.15}$$

$$= \frac{1}{12}. \tag{0.0.16}$$

$\therefore$ $\boxed{\textbf{Option 1,3} \text{ are } \textbf{correct} \text{ answers.}}$