AI1103
Assignment 6

Nagubandi Krishna Sai
MS20BTECH11014

Download LaTex file from below link :

Problem number CSIR UGC NET Dec 2014 Q.104

Suppose X_1, X_2, X_3 and X_4 are independent and identically distributed random variables, having density function f. Then,

1) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{6}$
2) $\Pr(X_4 > Max(X_1, X_2) > X_3) = \frac{1}{8}$
3) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{12}$
4) $\Pr(X_4 > X_3 > Max(X_1, X_2)) = \frac{1}{6}$

Solution

The probability density function (pdf) f(x) of a random variable X is defined as the derivative of the cdf F(x):

$$f(x) = \frac{d}{dx}F(x).$$

It is sometimes useful to consider the cdf F(x) in terms of the pdf f(x):

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

The PDF of X is,

$$F_X(x) = \int_{-\infty}^{\infty} f(x)dx \qquad (0.0.1)$$

$$(0.0.2)$$

$$\Pr(X_2 > X_1) = \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)\ dt dx \qquad (0.0.3)$$

$$= \int_{-\infty}^{\infty} f_X(x)F_X(x)dx \qquad (0.0.4)$$

$$= \frac{F_X^2(x)}{2}\bigg|_{-\infty}^{\infty} \qquad (0.0.5)$$

$$= \frac{1}{2}. \qquad (0.0.6)$$

$$\Pr(X_4 > Max(X_1, X_2) > X_3)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t).^2C_1.\left[\int_{-\infty}^{t} f_X(w)dw\right]\left|\int_{-\infty}^{t} f_X(z)dz dt dx\right.$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} 2f_X(t)F_X^2(t)dt dx \qquad (0.0.7)$$

$$= \int_{-\infty}^{\infty} f_X(x).\frac{2}{3}F_X^3(x)dx \qquad (0.0.8)$$

$$= \frac{2}{3}\frac{F_X^4(x)}{4}\bigg|_{-\infty}^{\infty} \qquad (0.0.9)$$

$$= \frac{1}{6}. \qquad (0.0.10)$$

$\Pr(X_4 > X_3 > Max(X_1, X_2)$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t) \int_{-\infty}^{t} f_X(z).^2C_1 \left[\int_{-\infty}^{t} f_X(w)dw \right] dzdtdx$$

$$(0.0.11)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t) \int_{-\infty}^{t} 2f_X(z)F_X(t) \; dzdtdx$$

$$(0.0.12)$$

$$= \int_{-\infty}^{\infty} f_X(x) \int_{-\infty}^{x} f_X(t)F_X^2(t)dtdx \qquad (0.0.13)$$

$$= \int_{-\infty}^{\infty} f_X(x).\frac{1}{3}F_X^3(x)dx \qquad (0.0.14)$$

$$= \frac{1}{3}\frac{F_X^4(x)}{4}\Big|_{-\infty}^{\infty} \qquad (0.0.15)$$

$$= \frac{1}{12}. \qquad (0.0.16)$$

$\therefore$ Option 1,3 are **correct** answers.