Assignment (Recursion)

Date
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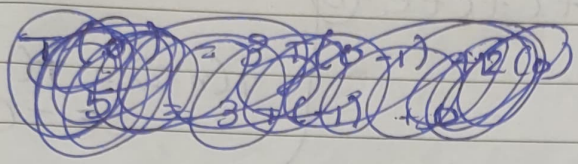
(Ex 1) $T(n) = 3T(n-1) + 12n$, $T(0) = 5$, $T(2) = ?$

$T(n) = 3T(n-1) + 12n$



$T(1) = 3T(1-1) + 12(1)$
$= 3T(0) + 12$
$= 3(5) + 12$
$= 15 + 12$
$= 27$

$T(2) = 3T(2-1) + 12(2)$
$= 3T(1) + 24$
$= 3(27) + 24$
$= 81 + 24$
$T(2) = 105$

(Q2) Using Substitution method.

(a) $T(n) = T(n-1) + c$

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + c & n > 1 \end{cases}$$

$T(n) = T(n-1) + c$ — (1)

$T(n) = T(n-2) + c + c$ — (2)

$T(n) = T(n-3) + c + c + c$ — (3)

⋮

K times

⋮

$T(n-1) = T(n-1-1) + c$
$= T(n-2) + c$

$T(n-2) = T(n-2-1) + c$
$= T(n-3) + c$

$n - K = 1$
$K = n - 1$

$T(n) = T(n-K) + K \cdot c$

$= T(n-n+1) + (n-1) \cdot c$
$= T(1) + c(n-1)$
$= 1 + c(n-1)$
$= cn$

∴

∴ Time complexity = $O(n)$

(b) $T(n) = 2T(n/2) + n$ — (1)

$T(n) = 2 \cdot 2T(n/2^2) + n + n$ — (2)

$T(n) = 2 \cdot 2 \cdot 2\, T\left(\frac{n}{2^3}\right) + n + n + n$ — (3)

$= 2^3 \cdot T\left(\frac{n}{2^3}\right) + 3n$ — (3)

⋮

K times

$T(n/2) = 2T(n/2^2) + n$

$T(n/2^2) = 2T(n/2^3) + n$

$\log_2 n = K \log_2 2$

$K = \log_2 n$

$\frac{n}{2^K} = 1$

$n = 2^K$

$= 2^K T\left(\frac{n}{2^K}\right) + K.n$

$= 2^K T(1) + K.n$

$= n\,T(1) + \log n . n$

$= n + n \log n$

Time complexity	=	$O(n \log n)$

(c) $T(n) = 2T(n/2) + C$ — ①

$= 2.2\,T\left(\frac{n}{2^2}\right) + C + C$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2C$ — ②

$T(n/2) = 2T\left(n/2^2\right) + C$

$T\left(n/2^2\right) = 2T\left(n/2^3\right) + C$

$= 2^2 . 2\,T\left(\frac{n}{2^3}\right) + C + 2C$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3C$

$\vdots$ K times

$T(n) = 2^K T\left(\frac{n}{2^K}\right) + K.C$

$\frac{n}{2^K} = 1$

$n = 2^K$

$\log n = K \log 2$

$\log n = K$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k.c$$

$$= n\,T(1) + \log n . c$$

$$= n + \log n$$

Time complexity $= O(\log n)$

(d) $T(n) = T(n/2) + c$ — ①

$T(n/2) = T(n/2^2) + c$

$T(n/2^2) = T(n/2^3) + c$

$$T(n) = T\left(\frac{n}{2^2}\right) + c + c$$

$$= T\left(n/2^2\right) + 2c \quad -②$$

$$= T\left(n/2^3\right) + c + 2c$$

$$T(n) = T\left(n/2^3\right) + 3c \quad -③$$

} k times

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log n = k \log 2$

$k = \log n$

$$T(n) = T\left(n/2^k\right) + k.c$$

$$T(n) = T(1) + k.c$$

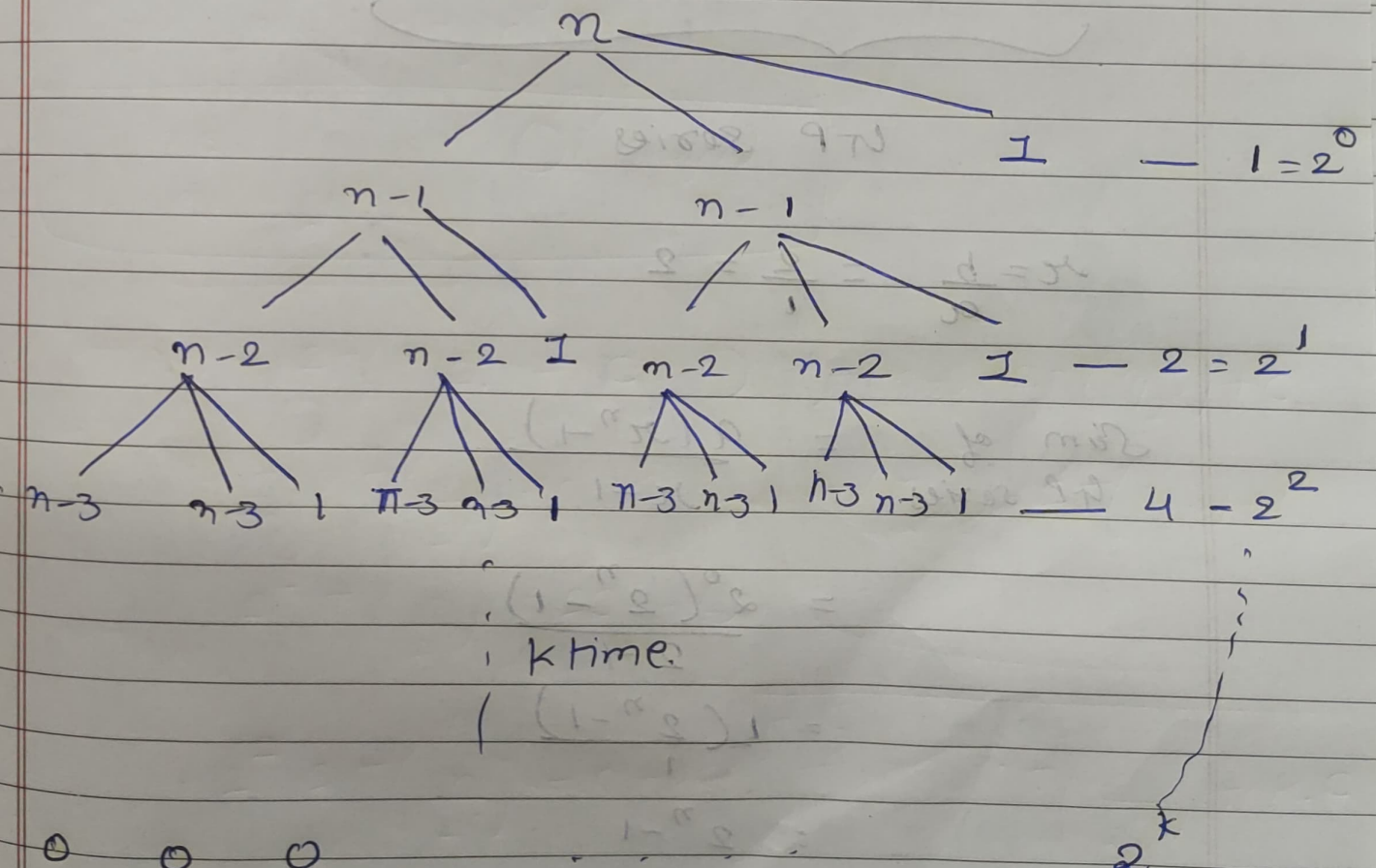
$$= 1 + \log n . c$$

$T(n) = c \log n$

Time complexity $= O(\log n)$

(Ex 3)

(a) $$T(n) = \begin{cases} 2T(n-1) + 1 & n > 1 \\ c & n = 1 \end{cases}$$



Left side	Right side
$n-k=0$	$n-k=0$
$n=k$	$n=k$
$k=n$	$k=n$

$$= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^k$$

$$= \underbrace{2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n}_{} \quad (k=n)$$

GP series

$$r = \frac{b}{a} = \frac{2}{1} = 2$$

Sum of GP series $= \frac{a(r^n-1)}{r-1}$

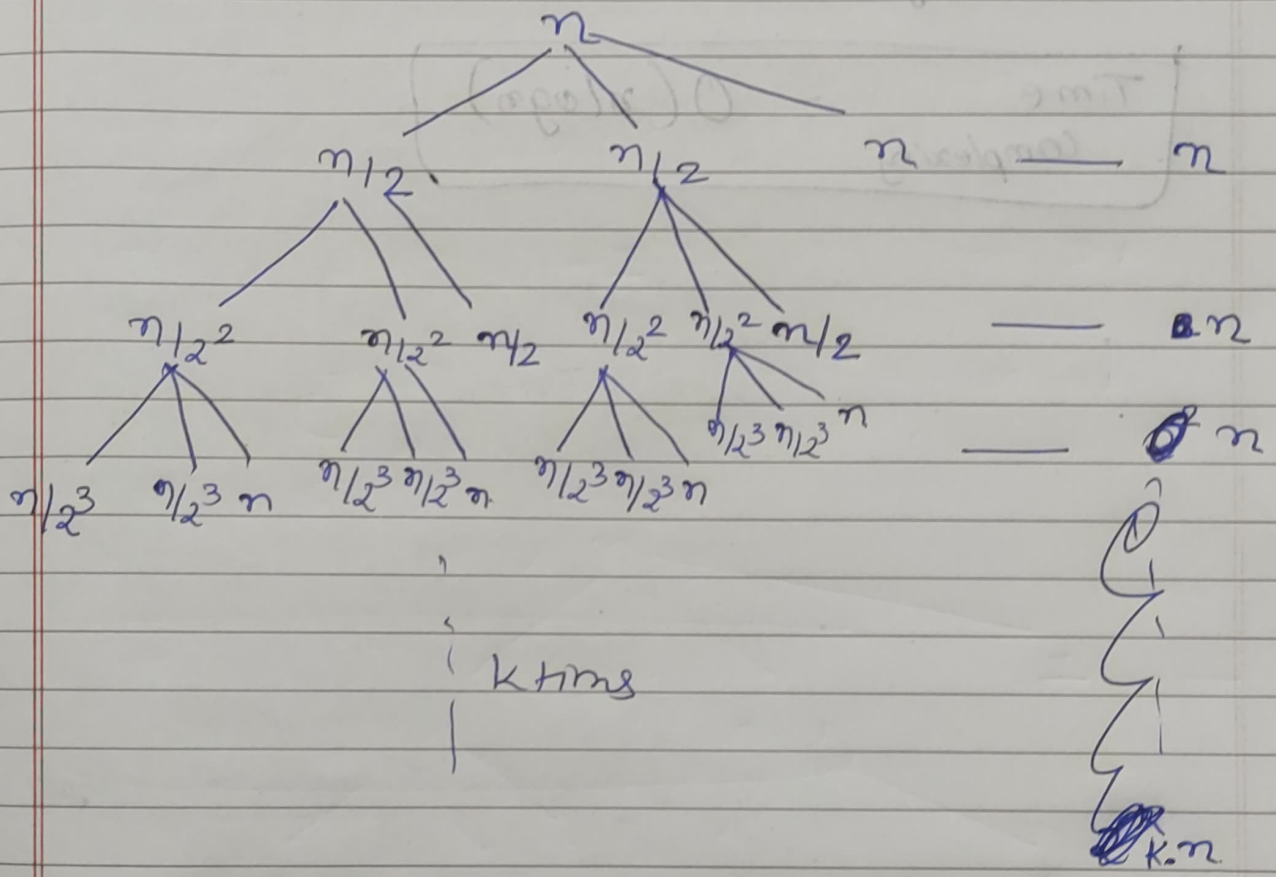
$$= \frac{2^0(2^n-1)}{2-1}$$

$$= \frac{1(2^n-1)}{1}$$

$$= 2^n - 1$$

Time complexity $= O(2^n)$

(b) $T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$



Left side	Right side
$\frac{n}{2^k} = 1$	$\frac{n}{2^k} = 1$
$n = 2^k$	$n = 2^k$
$\log n = k \log 2$	$\log n = k \log 2$
$k = \log n$	$k = \log n$

$= n + n + n + \ldots \quad n \cdot K$

$= n \cdot K$

$= n \log n$

Time complexity $= O(n \log n)$