

# Assignment (Recursion)

Date \_\_\_\_\_

Page \_\_\_\_\_

(Ex 1)

$$T(n) = 3T(n-1) + 12n, \quad T(0) = 5, \quad T(2) = ?$$

$$T(n) = 3T(n-1) + 12n$$

~~$$T(0) = 5$$
  
$$5 = 3T(-1) + 12(0)$$~~

$$T(1) = 3T(1-1) + 12(1)$$

$$= 3T(0) + 12$$

$$= 3(5) + 12$$

$$= 15 + 12$$

$$= 27$$

$$T(2) = 3T(2-1) + 12(2)$$

$$= 3T(1) + 24$$

$$= 3(27) + 24$$

$$= 81 + 24$$

$$T(2) = 105$$

(Ex 2)

Using Substitution method:

(a)

$$T(n) = T(n-1) + C$$

$$T(n) = \begin{cases} T(1) + C(n-1) & n > 1 \\ T(1) & n = 1 \end{cases}$$

$$T(n) = T(n-1) + c \quad (1)$$

$$T(n) = T(n-2) + c + c \quad (2)$$

$$T(n) = T(n-3) + c + c + c \quad (3)$$

K times

$$n-k=1 \\ k=n-1$$

$$T(n) = T(n-k) + k \cdot c$$

$$= T(n-n+1) + (n-1) \cdot c$$

$$= T(1) + c(n-1)$$

$$= 1 + c(n-1)$$

$$= cn$$

$\therefore$

$$\therefore \text{Time complexity} = O(n)$$

$$(b) \quad T(n) = 2T(n/2) + n \quad (1)$$

$$T(n) = 2 \cdot 2T(n/2^2) + n + n \quad (2)$$

$$T(n) = 2 \cdot 2 \cdot 2T(n/2^3) + n + n + n \quad (3)$$

$$T(n) = 2^3 \cdot T(n/2^3) + 3n \quad (3)$$

K times

$$T(n-1) = T(n-1-1) + c \\ = T(n-2) + c$$

$$T(n-2) = T(n-2-1) + c \\ = T(n-3) + c$$

$$T(n/2) = 2T(n/2^2) + n$$

$$T(n/2^2) = 2T(n/2^3) + n$$



$$\log_2 n = K \log_2 2$$

$$K = \log_2 n$$

$$n = 1$$

$$\frac{n}{2^K} = 2^K$$

$$= 2^K T\left(\frac{n}{2^K}\right) + K \cdot n$$

~~$$T(n) = 2 T\left(\frac{n}{2}\right) + \log_2 n$$~~

~~$$T(n) = 2 T\left(\frac{n}{2}\right) + \log_2 n$$~~

~~$$T(n) = 2 T\left(\frac{n}{2}\right) + \log_2 n$$~~

~~$$T(n) = 2 T\left(\frac{n}{2}\right) + \log_2 n$$~~

$$= 2^K T(1) + K \cdot n$$

$$= n \cdot 1 + \log n \cdot n$$

$$= n + n \log n$$

Time complexity =  $O(n \log n)$

③  $T(n) = 2T(n/2) + C$  - ①

$= 2 \cdot 2 T\left(\frac{n}{2^2}\right) + C + C$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2C$  - ②

$T(n/2) = 2T(n/2^2) + C$

$T(n/2^2) = 2T(n/2^3) + C$

$$= 2^2 \cdot 2 T\left(\frac{n}{2^3}\right) + C + 2C$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3C$$

K times

$$T(n) = 2^K T\left(\frac{n}{2^K}\right) + K \cdot C$$

$$\frac{n}{2^K} = 1$$

$$\log n = K \log 2$$

$$\log n = K$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k \cdot C$$

$$= n T(1) + \log n \cdot C$$

$$= n + \log n$$

Time complexity =  $O(\log n)$

(d)  $T(n) = T(n/2) + C$  — (1)

$$T(n) = T\left(\frac{n}{2}\right) + C + C$$

$$= T\left(\frac{n}{2^2}\right) + 2C$$
 — (2)

$$= T\left(\frac{n}{2^3}\right) + C + 2C$$

$$T(n) = T\left(\frac{n}{2^3}\right) + 3C$$
 — (3)

k times

$$\log n = k \log 2$$

$$k = \log n$$

$$\frac{n}{2^k} = 1$$

$$T(n) = T\left(\frac{n}{2^k}\right) + k \cdot C$$

$$T(n) = T(1) + k \cdot C$$

$$= 1 + \log n \cdot C$$

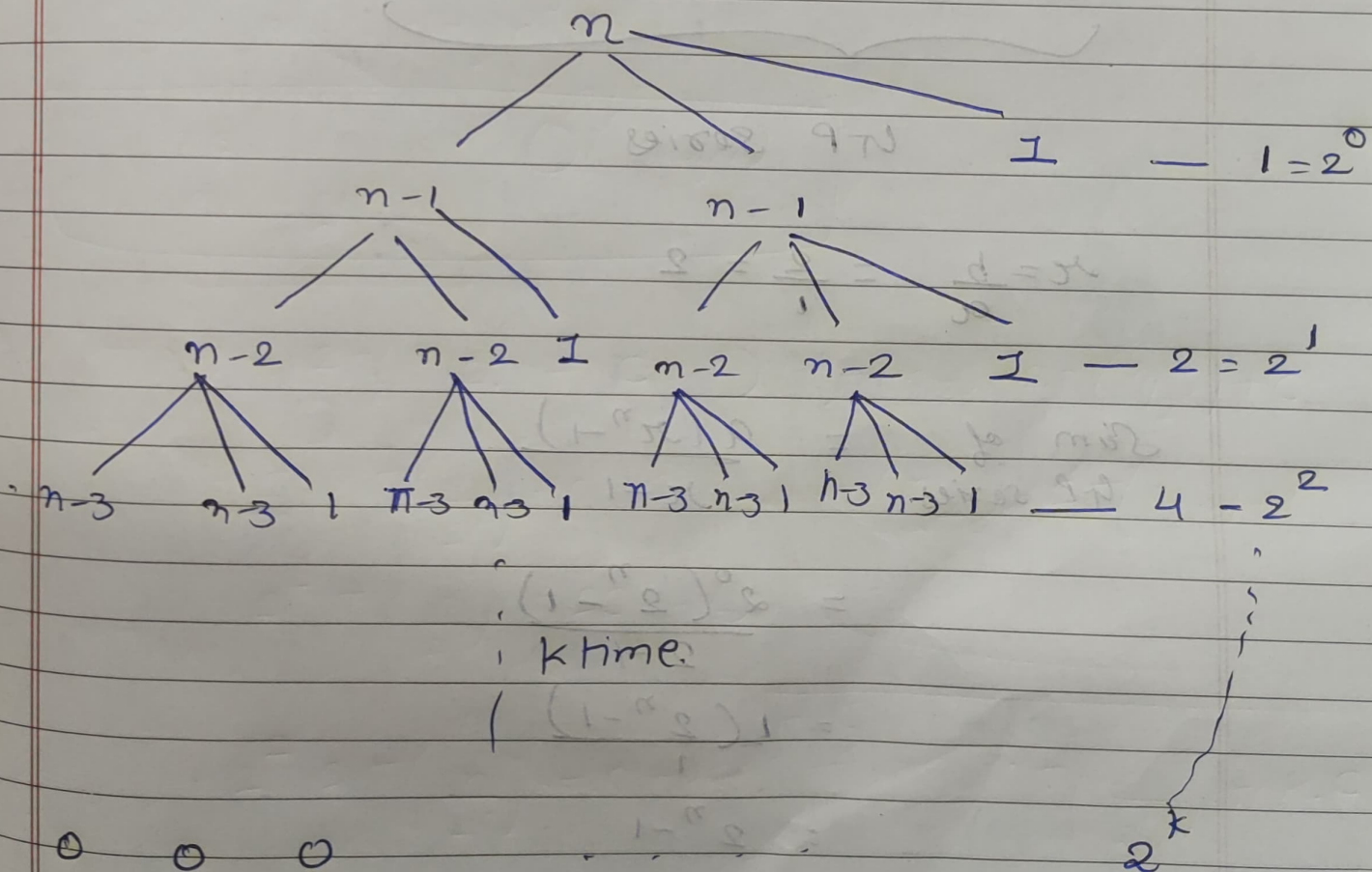


$$T(n) = c \cdot \log n$$

$$\text{Time complexity} = O(\log n)$$

(Ex 3)

$$(a) \quad T(n) = \begin{cases} 2T(n-1) + 1 & n > 1 \\ c & n = 1 \end{cases}$$



left side

$$n - k = 0$$

$$n = k$$

$$k = n$$

Right side

$$n - k = 0$$

$$n = k$$

$$k = n$$

$$= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^k$$

$$= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n \quad (k = n)$$

G.P series

$$r = \frac{b}{a} = \frac{2}{1} = 2$$

$$\text{Sum of G.P series} = \frac{a(r^n - 1)}{r - 1}$$

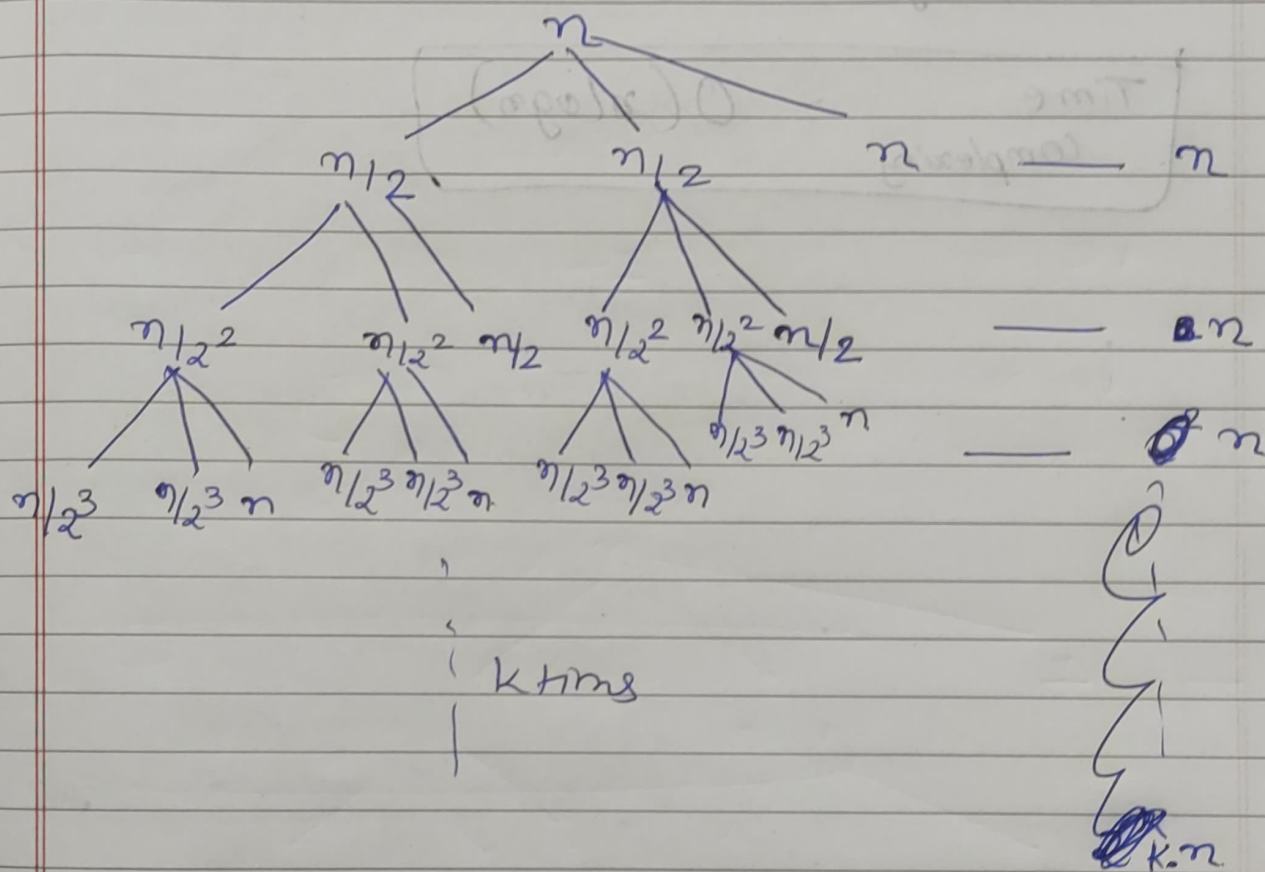
$$= \frac{2^0(2^n - 1)}{2 - 1}$$

$$= 1(2^n - 1)$$

$$= 2^n - 1$$

$$\boxed{\text{Time complexity} = O(2^n)}$$

$$(b) \quad T(n) = \begin{cases} c \cdot I & n = 1 \\ 2T(n/2) + n & n \neq 1 \end{cases}$$



Left side

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k \log_2 2$$

$$k = \log n$$

Right side

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k \log_2 2$$

$$k = \log n$$

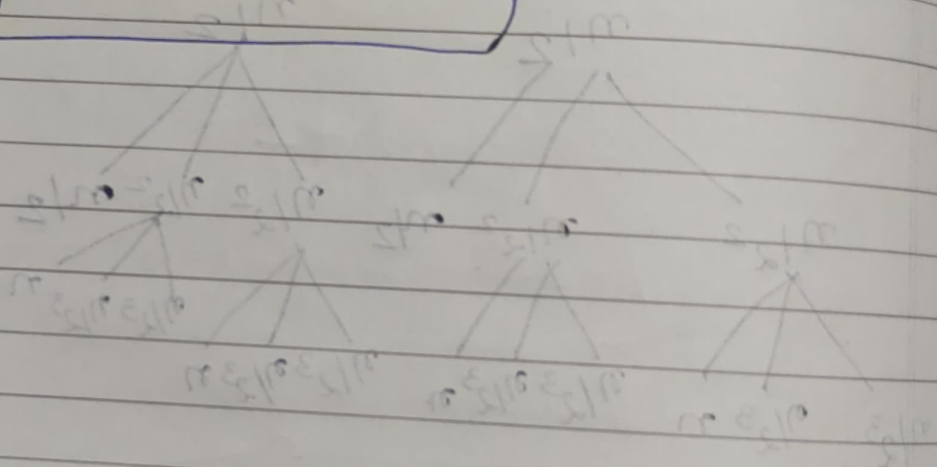


$$= n + n + n + \dots + n \cdot K$$

$$= n \cdot K$$

$$= n \log n$$

Time Complexity =  $O(n \log n)$



Right side

$$n = 1$$

$$n = 2$$

$$n = 4$$

$$n = 8$$

$$n = 16$$

$$n = 32$$

$$n = 64$$

$$n = 128$$

$$n = 256$$

$$n = 512$$

$$n = 1024$$

$$n = 2048$$

$$n = 4096$$

$$n = 8192$$

$$n = 16384$$

$$n = 32768$$

$$n = 65536$$

$$n = 131072$$

$$n = 262144$$

$$n = 524288$$

$$n = 1048576$$

$$n = 2097152$$

$$n = 4194304$$

$$n = 8388608$$

$$n = 16777216$$

$$n = 33554432$$

$$n = 67108864$$

$$n = 134217728$$

$$n = 268435456$$

$$n = 536870912$$

$$n = 1073741824$$

$$n = 2147483648$$

$$n = 4294967296$$

$$n = 8589934592$$

$$n = 17179869184$$

$$n = 34359738368$$

$$n = 68719476736$$

$$n = 137438953472$$

$$n = 274877906944$$

$$n = 549755813888$$

$$n = 1099511627776$$

$$n = 2199023255552$$

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