**Algorithm 1:** Guarded Saturation

**Function** $\mathsf{GSat}$**:**
  **Input :** Guarded TGDs $\Sigma$
  **Output :** Atomic rewriting of $\Sigma$
  $\mathcal{W} = \mathsf{VNF}\big(\mathsf{HNF}(\Sigma)\big)$ `// working set`
  $\mathcal{F} = \mathcal{N} = \emptyset$ `// full and non-full set`
  **while** $\mathcal{W} \neq \emptyset$ **do**
    Let $\sigma$ be a rule in $\mathcal{W}$
    **if** *$\sigma$ is full* **then**
      $\mathcal{F} = \mathcal{F} \cup \{\sigma\}$
      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\tau, \sigma) \mid \tau \in \mathcal{N}\big\}$
    **else**
      $\mathcal{N} = \mathcal{N} \cup \{\sigma\}$
      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\sigma, \tau) \mid \tau \in \mathcal{F}\big\}$
    **end**
    $\mathcal{W} = \mathcal{W} \cup \mathcal{E} \setminus \big(\mathcal{F} \cup \mathcal{N} \cup \{\sigma\}\big)$
  **end**
  **return** $\mathcal{F}$

**Function** $\mathsf{EVOLVE}$**:**
  **Input :** Non-full $\sigma = \beta(\vec{x}) \to \exists \vec{y}\, \eta$ and full $\sigma' = \beta' \to \eta'$
  **Output :** Derived rules of $\sigma$, $\sigma'$
  Rename $\sigma'$ s.t. $\mathsf{Vars}(\sigma) \cap \mathsf{Vars}(\sigma') = \emptyset$
  Let $G'$ be a guard of $\sigma'$
  $\mathcal{E} = \emptyset$
  **foreach** $H \in \eta$ **do**
    **if** *there is an mgu $\theta$ of $G'$ and $H$ with $\theta\big|_{\vec{y}} = id_{\vec{y}}$ and $\vec{x}\theta \cap \vec{y} = \emptyset$* **then**
      $\sigma = \sigma\theta$
      $\sigma' = \sigma'\theta$
      $S' = (B'_1, \ldots, B'_n)$ s.t. $B'_i \in \beta'$ and $B'_i \cap \vec{y} \neq \emptyset$ and $G' = B'_1$
      **for** $2 \leq i \leq n$ **do**
        `// get matches for` $B'_i = R(w_1, \ldots, w_m)$ `by selecting atoms with matching`
        `// predicates and positions of existentials`
        $S_i = \{R(v_1, \ldots, v_m) \in \eta \mid \forall 1 \leq j \leq m.\ v_j \in \vec{y} \vee w_j \in \vec{y} \implies v_j = w_j\}$
      **end**
      **foreach** $S \in \big(\{H\} \times S_2 \times \cdots \times S_n\big)$ **do**
        Let $\theta^*$ be the mgu of $S$ and $S'$
        `// Note:  evc is satisfied by construction`
        $\beta'' = \big(\beta \cup \big(\beta' \setminus \{B_1, \ldots, B_n\}\big)\big)\theta^*$
        $\eta'' = \big(\eta \cup \eta'\big)\theta^*$
        $\mathcal{E} = \mathcal{E} \cup \mathsf{VNF}\big(\mathsf{HNF}(\beta'' \to \exists \vec{y}\, \eta'')\big)$
      **end**
    **end**
  **end**
  **return** $\mathcal{E}$