Algorithm 1: Guarded Saturation

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1 Function GSat:
          Input : Guarded TGDs \Sigma
          Output: Atomic rewriting of \Sigma
          W = \mathsf{VNF}(\mathsf{HNF}(\Sigma))
 2
                                                                                           // working set
          \mathcal{F} = \mathcal{N} = \dot{\emptyset}
                                                                          // full and non-full set
 3
          while W \neq \emptyset do
 4
                Let \sigma be a rule in W
 5
                if \sigma is full then
 6
                     \mathcal{F} = \mathcal{F} \cup \{\sigma\}
 7
                     \mathcal{E} = \{ \mathsf{EVOLVE}(\tau, \sigma) \mid \tau \in \mathcal{N} \}
 8
                else
                     \mathcal{N} = \mathcal{N} \cup \{\sigma\}
10
                   \mathcal{E} = \{ \mathsf{EVOLVE}(\sigma, \tau) \mid \tau \in \mathcal{F} \}
11
12
               \mathcal{W} = \mathcal{W} \cup \mathcal{E} \setminus (\mathcal{F} \cup \mathcal{N} \cup \{\sigma\})
13
          end
14
          return \mathcal{F}
15
16
17 Function EVOLVE:
          Input: Non-full \sigma = \beta(\vec{x}) \to \exists \vec{y} \, \eta and full \sigma' = \beta' \to \eta'
          Output: Derived rules of \sigma, \sigma'
          Rename \sigma' s.t. Vars(\sigma) \cap Vars(\sigma') = \emptyset
18
          Let G' be a guard of \sigma'
19
          \mathcal{E}=\emptyset
20
          foreach H \in \eta do
\mathbf{21}
                if there is an mgu \theta of G' and H with \theta|_{\vec{y}} = id_{\vec{y}} and \vec{x}\theta \cap \vec{y} = \emptyset then
22
                     \sigma = \sigma \theta
23
                      \sigma' = \sigma'\theta
24
                      S' = (B'_1, \ldots, B'_n) s.t. B'_i \in \beta' and B'_i \cap \vec{y} \neq \emptyset and G' = B'_1
25
26
                           // get matches for B_i' = R(w_1, \dots, w_m) by selecting atoms with matching
                           // predicates and positions of existentials
                           S_i = \{ R(v_1, \dots, v_m) \in \eta \mid \forall 1 \le j \le m. \ v_j \in \vec{y} \lor w_j \in \vec{y} \implies v_j = w_j \}
27
                      end
28
                      foreach S \in (\{H\} \times S_2 \times \cdots \times S_n) do
29
                           Let \theta^* be the mgu of S and S'
                           // Note: evc is satisfied by construction
                           \beta'' = (\beta \cup (\beta' \setminus \{B'_1, \dots, B'_n\}))\theta^*
31
                           \eta'' = (\eta \cup \eta')\theta^*
32
                           \mathcal{E} = \mathcal{E} \cup \mathsf{VNF} \big( \mathsf{HNF} (\beta'' \to \exists \vec{y} \, \eta'') \big)
33
                     end
34
               end
35
          end
36
          return \mathcal{E}
37
```