Algorithm 1: Guarded Saturation

Function GSat**:**

  Input : Guarded TGDs Σ

  Output : Atomic rewriting of Σ

  $\mathcal{W} = \mathsf{VNF}\big(\mathsf{HNF}(\Sigma)\big)$ `// working set`

  $\mathcal{F} = \mathcal{N} = \emptyset$ `// full and non-full set`

  while $\mathcal{W} \neq \emptyset$ **do**

    Let σ be a rule in $\mathcal{W}$

    if σ *is full* **then**

      $\mathcal{F} = \mathcal{F} \cup \{\sigma\}$

      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\tau, \sigma) \mid \tau \in \mathcal{N}\big\}$

    else

      $\mathcal{N} = \mathcal{N} \cup \{\sigma\}$

      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\sigma, \tau) \mid \tau \in \mathcal{F}\big\}$

    end

    $\mathcal{W} = \mathcal{W} \cup \mathcal{E} \setminus \{\sigma\}$

  end

  return $\mathcal{F}$

Function EVOLVE**:**

  Input : Non-full $\sigma = \beta(\vec{x}) \to \exists \vec{y}\, \eta$ and full $\sigma' = \beta' \to \eta'$

  Output : Derived rules of σ, σ'

  Rename σ' s.t. $\mathsf{Vars}(\sigma) \cap \mathsf{Vars}(\sigma') = \emptyset$

  Let G' be a guard of σ'

  $\mathcal{E} = \emptyset$

  foreach $H \in \eta$ **do**

    if *there is an mgu* θ *of* G' *and* H *with* $\theta\big|_{\vec{y}} = id_{\vec{y}}$ *and* $\vec{x}\theta \cap \vec{y} = \emptyset$ **then**

      $\sigma = \sigma\theta$

      $\sigma' = \sigma'\theta$

      $S' = (B'_1, \ldots, B'_n)$ s.t. $B'_i \in \beta'$ and $B'_i \cap \vec{y} \neq \emptyset$ and $G' = B'_1$

      for $2 \leq i \leq n$ **do**

        `// get matches for` $B_i = R(w_1, \ldots, w_m)$ `by selecting atoms with matching`

        `// predicates and positions of existentials`

        $S_i = \{R(v_1, \ldots, v_m) \in \eta \mid \forall 1 \leq j \leq m.\ v_j \in \vec{y} \vee w_j \in \vec{y} \implies v_j = w_j\}$

      end

      foreach $S \in \big(\{G'\} \times S_2 \times \cdots \times S_n\big)$ **do**

        Let θ^* be the mgu of S and S'

        `// Note: evc is satisfied by construction`

        $\beta'' = \big(\beta \cup \big(\beta' \setminus \{B_1, \ldots, B_n\}\big)\big)\theta^*$

        $\eta'' = \big(\eta \cup \eta'\big)\theta^*$

        $\mathcal{E} = \mathcal{E} \cup \mathsf{VNF}\big(\mathsf{HNF}(\beta'' \to \exists \vec{y}\, \eta'')\big)$

      end

    end

  end

  return $\mathcal{E}$