Algorithm 1: Guarded Saturation

Function GSat:
  Input : Guarded TGDs Σ
  Output : Atomic rewriting of Σ
  $\mathcal{W} = \mathsf{VNF}\big(\mathsf{HNF}(\Sigma)\big)$ `// working set`
  $\mathcal{F} = \mathcal{N} = \emptyset$ `// full and non-full set`
  while $\mathcal{W} \neq \emptyset$ **do**
    Let σ be a rule in $\mathcal{W}$
    if σ *is full* **then**
      $\mathcal{F} = \mathcal{F} \cup \{\sigma\}$
      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\tau, \sigma) \mid \tau \in \mathcal{N}\big\}$
    else
      $\mathcal{N} = \mathcal{N} \cup \{\sigma\}$
      $\mathcal{E} = \big\{\mathsf{EVOLVE}(\sigma, \tau) \mid \tau \in \mathcal{F}\big\}$
    end
    $\mathcal{W} = \mathcal{W} \cup \mathcal{E} \setminus \big(\mathcal{F} \cup \mathcal{N}\big)$
  end
  return $\mathcal{F}$

Function EVOLVE:
  Input : Non-full $\sigma = \beta(\vec{x}) \to \exists \vec{y}\, \eta$ and full $\sigma' = \beta' \to \eta'$
  Output : Derived rules of σ, σ'
  Rename σ' s.t. $\mathsf{Vars}(\sigma) \cap \mathsf{Vars}(\sigma') = \emptyset$
  Let G' be a guard of σ'
  $\mathcal{E} = \emptyset$
  foreach $H \in \eta$ **do**
    if *there is an mgu* θ *of* G' *and* H *with* $\theta\big|_{\vec{y}} = id_{\vec{y}}$ *and* $\vec{x}\theta \cap \vec{y} = \emptyset$ **then**
      $\sigma = \sigma\theta$
      $\sigma' = \sigma'\theta$
      $S' = (B'_1, \ldots, B'_n)$ s.t. $B'_i \in \beta'$ and $B'_i \cap \vec{y} \neq \emptyset$ and $G' = B'_1$
      for $2 \leq i \leq n$ **do**
        `// get matches for` $B'_i = R(w_1, \ldots, w_m)$ `by selecting atoms with matching`
        `// predicates and positions of existentials`
        $S_i = \{R(v_1, \ldots, v_m) \in \eta \mid \forall 1 \leq j \leq m.\ v_j \in \vec{y} \vee w_j \in \vec{y} \implies v_j = w_j\}$
      end
      foreach $S \in \big(\{H\} \times S_2 \times \cdots \times S_n\big)$ **do**
        Let θ^* be the mgu of S and S'
        `// Note: evc is satisfied by construction (update: if we use constants, this unification can fail)`
        $\beta'' = \big(\beta \cup \big(\beta' \setminus \{B'_1, \ldots, B'_n\}\big)\big)\theta^*$
        $\eta'' = \big(\eta \cup \eta'\big)\theta^*$
        $\mathcal{E} = \mathcal{E} \cup \mathsf{VNF}\big(\mathsf{HNF}(\beta'' \to \exists \vec{y}\, \eta'')\big)$
      end
    end
  end
  return $\mathcal{E}$