Assignment 7

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March 17, 2020

Abstract

This week's Python assignment focuses on the following topics.

- Analysis of filters using laplace transforms.
- Learn to use python tools such as Symbolic python known as *Sympy* and Signal toolbox.

1 Introduction

The goal of this assignment is to look at how to analyze second-order Lowpass and Highpass filters realised using single opamp using Python's symbolic solving library, Sympy and the Signal-Processing toolbox.

2 Assignment Tasks

2.1 Low-Pass Filter System

The nodal equation for the lowpass filter circuit given is given below.

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G} \\ -\frac{1}{1+sR2C2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -\frac{1}{R_1}-\frac{1}{R_2}-sC_1 & \frac{1}{R2} & 0 & sC_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)/R_1 \end{pmatrix}$$

The Magnitude response of the system is obtained using Sympy toolbox along with Signal-Processing toolbox as follows

```
def lowpass(R1,R2,C1,C2,G,Vi):

    s = symbols('s')
    A = Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],
                  [0,-G,G,1],[-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
```

```
        b = Matrix([0,0,0,-Vi/R1])
        V = A.inv()*b
        return (A,b,V)

def lti_conversion(xpr, s=symbols('s')):

        num, denom = simplify(xpr).as_numer_denom()
        H = Poly(num, s).all_coeffs() ,Poly(denom, s).all_coeffs()
        l_num, l_den = [lambdify((), c)() for c in H]
        return sp.lti(l_num, l_den)

s = symbols('s')
A,b,V = lowpass(10000,10000,1e-9,1e-9,1.586,1)
Vo = V[3]
H = lti_conversion(Vo)
w = p.logspace(0,8,801)
ss = 1j*w
hf = lambdify(s,Vo,'numpy')
v = hf(ss)

p.loglog(w,abs(v),lw=2)
p.xlabel(r'$w\rightarrow$')
p.ylabel(r'$|H(jw)|\rightarrow$')
p.title('Magnitude response for Circuit 1')
p.grid(True)
p.show()
```

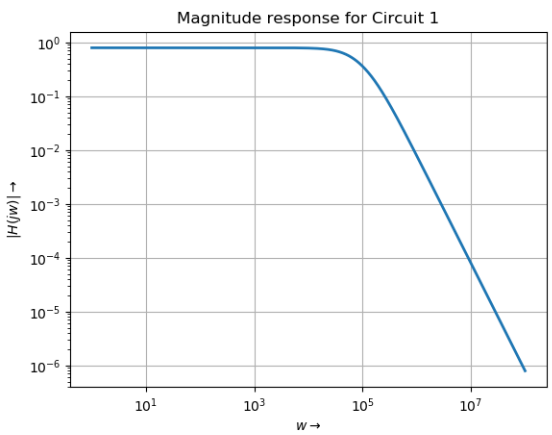

Figure 1: 'Magnitude response for Circuit 1'

As we observe the Bode plot and the circuit that we know it is a low pass filter with bandwidth $0<\omega<10^4$. So the circuit will only pass input with frequencies which are in range of bandwidth only and attenuates other frequencies largely since its second order filter with -40dB/dec drop in gain.

The unit step response corresponding to the system is obtained as follows.

```
t = np.linspace(0,0.001,1000)
Vo = sp.step(H,T=t)
```

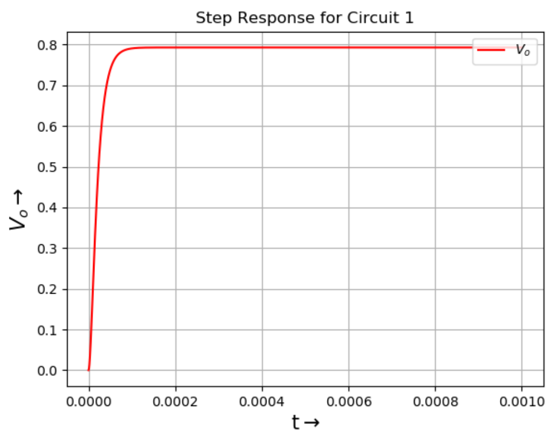

Figure 2: Step Response of Circuit-1

we observe the step response plot that $V_o(t)$ increases quickly from 0 to 0.8 and settles at 0.8 for after some time and remains constant. Because since the network is lowpass filter, the output must be dominated by DC gain at steady state.

Obtain and analyse the response for sinusoid with a low frequency and high frequency component of $\omega_1 = 2000\pi\ rads^{-1}\ and\omega_2 = 2*10^6\pi\ rads^{-1}$.

$$V_i(t) = (\ \sin(2000\pi t) + \cos(2*10^6\pi t)\ )u_o(t)\ V \tag{1}$$

```
t = np.linspace(0,0.01,100000)
Vi = np.multiply((np.sin(2000*np.pi*t)+np.cos(2000000*np.pi*t))
                ,np.heaviside(t,0.5))
Vo = sp.lsim(H,Vi,T=t)
p.figure(1)
p.plot(Vo[0],Vi,label=r'$V_{in}$')
```

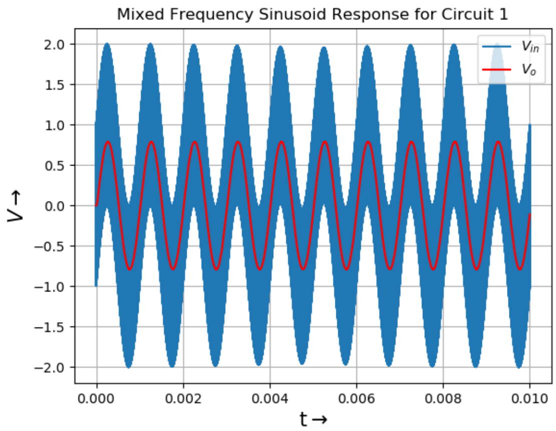

Figure 3: Response of Sinusoid Input

From the plot, we can see that the circuit will only pass input with frequencies which are in range of its bandwidth only. But since its not a ideal low pass filter as its gain doesn't drop abruptly at 10^4 rather gradual decrease which is observed from magnitude response plot.So the output $V_o(t)$ will be mainly of $\sin(2000\pi t)$ with higher frequencies attenuated largely since its second order filter so gain drops 40dB/dec.

2.2 High-Pass Filter System

The nodal equation for the highpass filter circuit given is given below.

$$\begin{pmatrix} 0 & 0 & 1 & -\frac{1}{G} \\ -\frac{-sR_3C_2}{1+sR_3C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -1-(sR_1C_1)-(sR_3C_2)) & sC_2R_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)sR_1C_1 \end{pmatrix}$$

The Magnitude Response of the system is obtained just as Low-pass system except the modification of Matrix as mentioned above.The corresponding plot is given below.

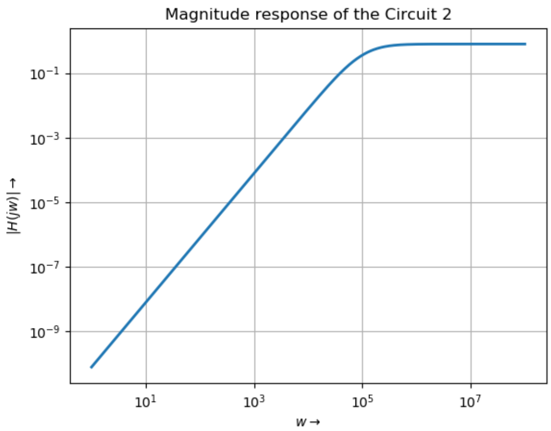

Figure 4: Magnitude response for Circuit 2

We observe the plot and the circuit that we know it is a high pass filter with bandwidth $\omega > 10^5$.So the circuit will only pass input with frequencies which are in range of bandwidth only.

The corresponding step-response of the system is given as below

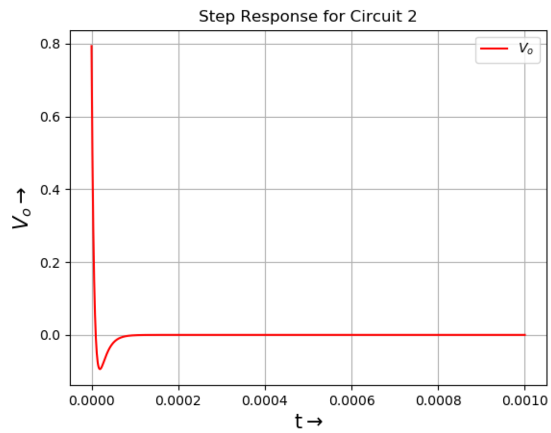

Figure 5: Step Response of Circuit-2

The Transient Response of the system for the damped sinusoid input.

$$V_i(t) = e^{-0.5t} \sin(2\pi t) u_o(t) \ V \tag{2}$$

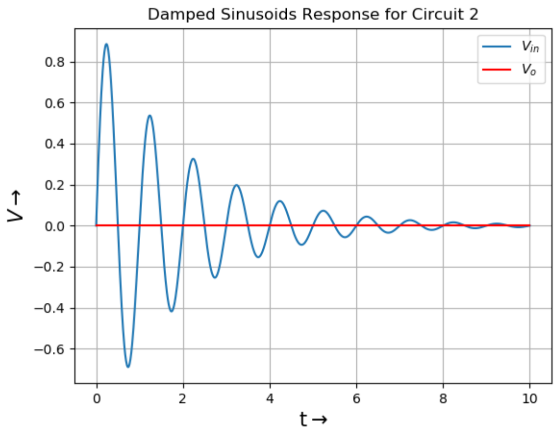

Figure 6: Transient Response of Damped Sinusoid

Since the Frequency of the input is low, the system suppress the input.The change in the exponential would only affect the rate at which the sinusoid amplitude decays to zero.

3 Conclusion

Sympy provides a convenient way to analyse LTI systems using their Laplace transforms. The toolbox was used to study the behaviour of a low pass filter,implemented using an opamp of gain G. For a mixed frequency sinusoid as input, it was found that the filter suppressed the high frequencies while allowing the low frequency components. Similarly, a high pass filter was implemented using an op-amp with the same gain. The magnitude response of the filter was plotted and its output was analysed for damped sinusoids. The step response of the filter was found to have a non-zero peak at t = 0, due to the sudden change in the input voltage.