EE2703 : Applied Programming Lab
Final Exam 2020

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System Description

A rectangular tank (Ly = 20cm and Lx = 10cm) is partially filled with a fluid of dielectric constant $\epsilon_r = 2$. (the dielectric constant of air may be taken to be $\epsilon_r = 1$.The height of the filled portion is h. The walls of the tank are ideal conductors.The sides and bottom of the tank are grounded.

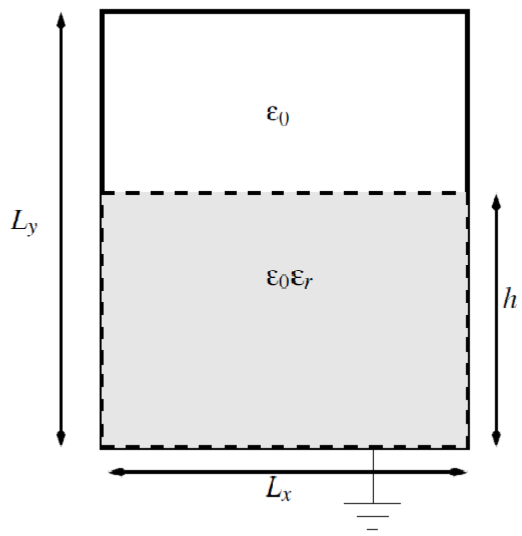

Figure 1: Capacitor Diagram

Answer(b)

The top of the tank is connected to the sides via a series RLC circuit (the tank is the C) and an AC source. As h changes, the capacitance seen by the circuit changes and the resonant frequency shifts.The aim of the question is to determine the height of dielectric medium from the resonant frequency noted.I refer to the plot obtained in question (e) to develop my algorithm.

For the convenience of designing algorithm I have interpolated between the data points obtained in (e)

Assuming that the observed resonant frequency (f) of the RLC circuit, area of cross-section of the tank and inductance (L) value are obtained from the user as input, we can calculate the capacitance of the tank as follows.

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

$$C = \frac{1}{4\pi^2 f^2 L} \tag{2}$$

It is interesting to note that the plot of Qtop (charge per meter) vs h/Ly ratio is obtained at constant voltage of 1V. Therefore this literally translates into Capcitance vs h/Ly ratio, if Qtop (charge per meter) is multiplied by the cross-sectional area (A) of the capacitor tank. Once the capacitance value is computed, with the help of plot in (e) we can backtrack (infer) the value of h.Here we present the pseudo algorithm for the same.

```
dielectric_height(f,L,A,Ly):
    -> Calculate C given by (2)
    -> Q_top = C*A
    -> Evaluate h from the Q_top vs h/Ly plot
```

Answer(c)

It is important to note that we deal with manipulation large arrays of potential (40 X 20) and electric field (38 X 18) matrix.Implementing these computation across arrays with loops is extremely time consuming.With the availability of multiple processors in the system and flexibility of parallelizing these computation with the equivalent vector implementation of these loops, makes computation fast.It is known and easily verifiable that vectorized implementation of these computation are faster than loop implementation.

Here we show the effect of vectorized implementation via a small example of copying scaled values of one array to other, to prove the effectiveness of implementing parallel computing.

Example:

```
A = np.linspace( 100, -100, num = 100000, dtype=float)
B = np.zeros(100000)

t1 = time.time()
for i in range(100000):
  B[i] = 0.5*A[i]
t2 = time.time()

print("Time taken by loop implementation:",1e6*(t2-t1))
```

```
C = np.linspace( 100, -100, num = 100000, dtype=float)
D = np.zeros(100000)

t3 = time.time()
D[:] = (0.5)*C[:]
t4 = time.time()

print("Time taken by vectorised implementation:",1e6*(t4-t3))
```

Output: the time taken by loop[implementation and vector implementation is obtained and we clearly see the effectiveness of vector implementation by the order of time consumed less than loop implementation of an array of 100000 elements.

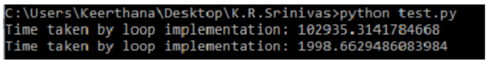

Figure 2: Vector Implementation vs Loop Implementation

Time taken in Loop implementation = 102935.3ms
Time taken in Vector implementation = 1998.6ms

The Vector implementation is almost 50 times faster than iterative implementation.

Below we see the vector handling of update of potential update at the mesh nodes, taking care of the interface of dielectric and air, following the direction given in question paper.

```
def phi_new(phi,phiold,t):
    phi[1:t,1:-1] = 0.25*(phiold[1:t,0:-2]+ phiold[1:t,2:]+
                       phiold[0:t-1,1:-1] + phiold[2:t+1,1:-1])
    phi[t,1:-1] = (epsilon_a*phiold[t-1,1:-1] +
                  epsilon_l*phiold[t+1,1:-1])/(epsilon_a+epsilon_l)
    phi[t+1:-1,1:-1] = 0.25*(phiold[t+1:-1,0:-2]+ phiold[t+1:-1,2:]
                        + phiold[t:-2,1:-1] + phiold[t+2:,1:-1])
    return phi
```

We perform the update of potential in the air region, liquid region and at the interface separately by vector implementation. While, phi[1:t,1:-1] correpond to region in air, phi[t+1:-1,1:-1] correspond to liquid region and phi[t,1:-1] correspond to the interface layer.

Answer(d)

In this section we look at how we solve the laplace equation to find the potential at each node in the mesh of capacitor tank.We approximate the the poisson's equation by difference equations. Since, we know the voltage on the top metal plate is constant 1 V, it directly translates the poisson's equation as

$$\nabla^2 \phi = 0 \tag{3}$$

As we use a 2D plate, the numerical solutions in 2D can be easily transformed into difference equation. The equation can be written out as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial \phi}{\partial x}_{(x_i,y_j)} = \frac{\phi(x_{i+1/2}, y_j) - \phi(x_{i-1/2}, y_j)}{\Delta x}$$

$$\frac{\partial^2 \phi}{\partial x^2}_{(x_i,y_j)} = \frac{\phi(x_{i+1}, y_j) - 2\phi(x_i, y_j) + \phi(x_{i-1}, y_j)}{(\Delta x)^2}$$

Using above equations we get

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{4}$$

While updating the interior points for potential we should also take care of the boundary conditions.It is nice that the boundary conditions are already mentioned in the question. so we accordingly, initialise the top row to 1V and the sides of the rectangular grid to 0 (since it is grounded).

It should be kindly noted that in order to handle continuity of Displacement vector at the interface (condition demanded under electrostatic situation), we adhere to the potential function at interface given in the question paper.Also the results derived above are implemented in vectorized form for faster computation.

Starting with the ϕ matrix's interior points initialised to 0 and boundary conditions set, we iterate through the process of updation of potential until we achieve an accuracy of $1X10^{-7}$ and also extrapolate the error for infinite iterations.Below, we show a semi-log plot of the error over iteration and show how it reduces exponentially

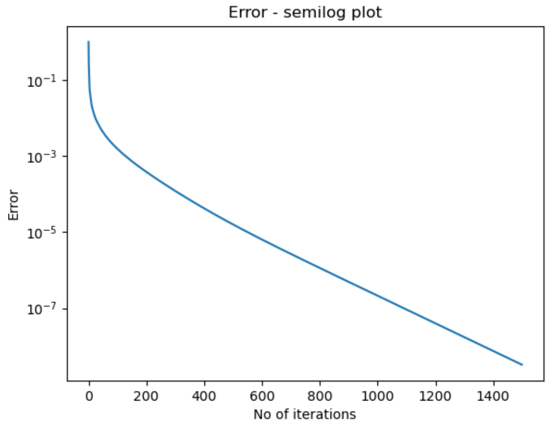

Figure 3: Semi-log Plot of Error over iteration

In order to compute the electric field at any point in the mesh, we take the gradient of the potential in that region. This gradient can similarly be translated into difference equation.

$$\vec{E} = \vec{E_x} + \vec{E_y} \tag{5}$$

$$\vec{E}_{(i,j)} = -\frac{\partial \phi}{\partial x}_{(i,j)} - \frac{\partial \phi}{\partial y}_{(i,j)} \tag{6}$$

$$\vec{E}_{x(i,j)} = \frac{1}{2\Delta}(\phi_{i,j-1} - \phi_{i,j+1}) \tag{7}$$

$$\vec{E}_{y(i,j)} = \frac{1}{2\Delta}(\phi_{i-1,j} - \phi_{i+1,j}) \tag{8}$$

where Δ is the distance between successive nodes and for convenience we assume this to be the same for rows and coloumns.

The quiver plot of the electric field vectors show that the field emanates from positive plate and bend towards the grounded wall.The strength of the electric field vector is high in the air medium compared to that in the dielectric liquid.

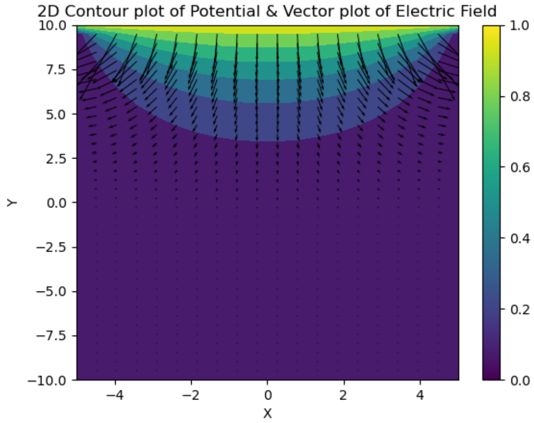

Figure 4: Contour Plot of Potential and Quiver Plot of Electric Field

Answer(e)

In this question we have to plot Qtop (charge on the top metal plate) and Qliq (charge on the wall surface in contact with liquid) vs the ratio h/Ly.It is quite direct to obtain the charge per length on these surfaces with the help of Gauss law.

$$\iint_A \vec{D}.\, dA = Q_{free} \tag{9}$$

$$\vec{D} = \epsilon_o \epsilon_r \vec{E} \tag{10}$$

Since we have a 2D mesh to deal with, we can compute the **charge per length** on the desired sides by summing up the displacement vector's normal component along the length.This can be given as

$$\Delta * \sum_{n=0}^{N-3} \epsilon_o \epsilon_r \vec{E_\perp} = Q_{free} \tag{11}$$

It should be clear why the summation runs from 0 to N-3, as the size of E vector is smaller than potential ϕ.With this logic we compute Qtop and Qliq for different values of h/Ly ratio. the plots are given below.

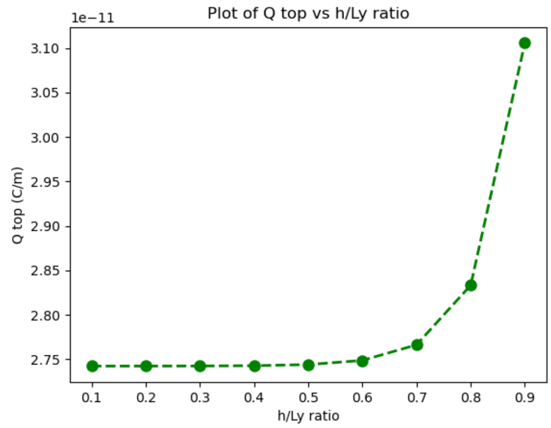

Figure 5: Q top vs h/Ly ratio

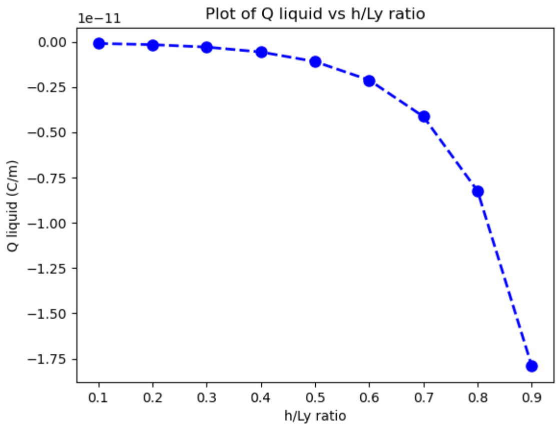

Figure 6: Q liquid vs h/Ly ratio

Answer(f)

In this question we are asked to compute the electric field at the center of each mesh cells given the potential at the corners of the cell and hence show if Displacement vector $\vec{D}$ is continuous at the interface of air and liquid.We proceed computing the electric field at mesh cell center as follows.

$$\vec{E}_{x(i,j)} = \frac{1}{\Delta}(\phi_{i,j} - \phi_{i,j+1}) \tag{12}$$

$$\vec{E}_{y(i,j)} = \frac{1}{\Delta}(\phi_{i,j} - \phi_{i+1,j}) \tag{13}$$

$$\vec{E'_x} = \frac{1}{2}[E_x(i,j) + E_x(i+1,j)] \tag{14}$$

$$\vec{E'_y} = \frac{1}{2}[E_x(i,j) + E_x(i,j+1)] \tag{15}$$

Equation (14) and (15) basically averages out the electric field along the corners of the mesh, thus giving an approximate value to the electric field at the center of the mesh cell. hereby I present the quiver plot of these electric field obtained.

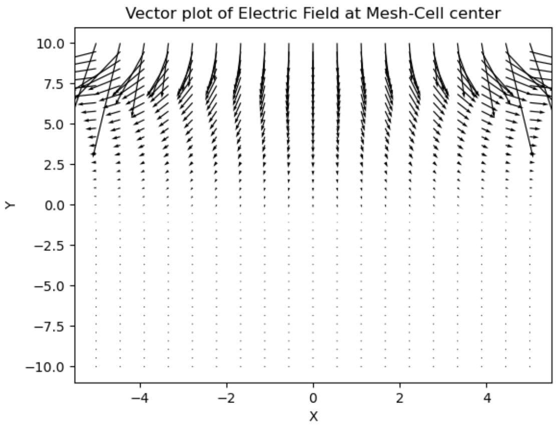

Figure 7: Quiver Plot of Electric field obtained at center of Mesh-cells

The continuity of Displacement Vector at the interface demands the value of normal component of $\vec{D}$ to be same on either side of the interface.With the electric field computed at center of mesh cells, we verify the same on the either side immediate to level k (index corresponding to interface of dielectrics).

```
D_air = epsilon_a*E_y1[k-1,:]
D_liq = epsilon_l*E_y1[k,:]

D_diff = (np.abs(D_air-D_liq)/D_air)*100
```

```
The percentage change in value of Displacement vector (normal component) across the interface (m-k) is:
[9.40117900e-05 9.40436246e-05 9.41038094e-05 9.41857695e-05
 9.42805739e-05 9.43779247e-05 9.44672824e-05 9.45390038e-05
 9.45853693e-05 9.46013977e-05 9.45853693e-05 9.45390038e-05
 9.44672824e-05 9.43779247e-05 9.42805739e-05 9.41857695e-05
 9.41038094e-05 9.40436246e-05 9.40117900e-05]
```

Figure 8: Percentage Difference in the Displacement vector across interface

Clearly the difference in normal component of $\vec{D}$ is very very small in percentage, hence showing that the Displacement vector is continuous across the interface.

Answer(g)

In this section, we shall see if in this scenario, the electric fields obey Snell's Law at the dielectric interface. The Snell's Law states

$$\epsilon_a.sin(\theta_i) = \epsilon_l.sin(\theta_r) \tag{16}$$

where,

$$\theta_i = \arctan \frac{E_{y(a)}}{E_{x(a)}} \tag{17}$$

$$\theta_r = \arctan \frac{E_{y(l)}}{E_{x(l)}} \tag{18}$$

Code:

```
incidence = np.zeros(N-2)
refraction = np.zeros(N-2)
angle_change = np.zeros(N-2)

for i in range(N-2):
    incidence[i]= epsilon_a*np.sin(math.atan2(Ey[k-2,i],Ex[k-2,i]))
    refraction[i] = epsilon_l*np.sin(math.atan2(Ey[k,i],Ex[k,i]))
    angle_change[i] = (math.atan2(Ey[k-2,i],Ex[k-2,i])
                       -math.atan2(Ey[k,i],Ex[k,i]))

snell_difference_percentage = ((np.abs(incidence-refraction))
                               /incidence)*100
```

Output:

```
The percentage difference in the value of incidence and refraction is:
[36.7508925  43.02406441 52.16618762 62.75665268 73.46392008 83.20136728
 91.15352258 96.75078989 99.63545165 99.63545165 96.75078989 91.15352258
 83.20136728 73.46392008 62.75665268 52.16618762 43.02406441 36.7508925 ]
```

Figure 9: Validity of Snell's law

We clearly see that $\epsilon_a sin(\theta_i)$ is not equal to $\epsilon_l sin(\theta_r)$ and hence Snell's law doesn't hold in this scenario.Snell's law is valid only in the scenario of electromagnetic waves and not fair to expect the same in electrostatic conditions.in this situation the change in direction of electric field clearly depends on the boundary conditions. This is clearly shown by the change in angle of electric field on crossing the dielectric.

```
The change in direction of electric field in terms of angle deviation (radians):
[-0.0789552  -0.14285694 -0.18285374 -0.19739784 -0.18954228 -0.16402823
 -0.12564492 -0.07865234 -0.02674855  0.02674855  0.07865234  0.12564492
  0.16402823  0.18954228  0.19739784  0.18285374  0.14285694  0.0789552 ]
```

Figure 10: Change in direction of electric field (in Radians)

The data clearly shows that change in angle of electric field close to right and left boundary are of opposite sign.This is evident due to the fact that the walls are grounded, hence under static condition electric field will converge towards the lower potential region.

Note:

The system descriptions and question details are taken from the question paper itself. Also methods of updating of ϕ matrix and evaluating electric field are borrowed from Assignment-5 of the course.