# An Optimized Proportionate LMS Algorithm for Sparse System Identification

K.R.Srinivas

EE18B136
Indian Institute of Technology - Madras

EE6110 - Adaptive Signal Processing, Course Project

# Table of Contents

## Table of Contents

# System Identification

## Goal :

Identify an unknown system with its output corrupted by an apparently “undesired” signal. Find the FIR filter that best approximates the unknown system.

We consider the application of Acoustic Echo Cancellation to study Sparse System Identification.

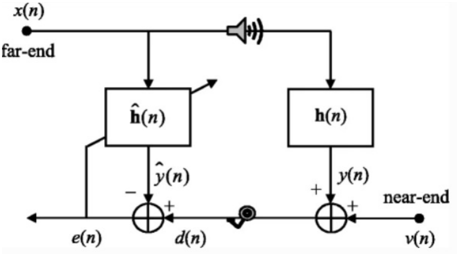


Figure: System Identification (Acoustic echo cancellation) configuration

# System Characteristics & Requirements for the Algorithm

## System Characteristics

- System paths can have excessive lengths in time. (AEP:50-400ms)
- Time variant nature of system response.
- The additive noise can be non-stationary and highly correlated.

# System Characteristics & Requirements for the Algorithm

## System Characteristics

- System paths can have excessive lengths in time. (AEP:50-400ms)
- Time variant nature of system response.
- The additive noise can be non-stationary and highly correlated.

## Requirements for the Adaptive Algorithm

- Long length adaptive filter.
- High convergence rate and good tracking capabilities (in order to deal with the high length and time varying nature of the system impulse responses).
- Lower misadjustment (to be able to attenuate the far-end speaker echoes) .

# Table of Contents

# Sparse Adaptive Filters

A small fraction of system impulse response have significant amplitude while the rest has small or zero magnitude.

**Example:**

- Network echo - bulk delay: 32-128 ms (depending on the network conditions) "active" region: 8-12 ms [1].
- Sparseness of Acoustic echo depends on reverberation time, the distance between loudspeaker and microphone, different changes in the environment (e.g., temperature or pressure).

# Sparse Adaptive Filters

A small fraction of system impulse response have significant amplitude while the rest has small or zero magnitude.

**Example:**

- Network echo - bulk delay: 32-128 ms (depending on the network conditions) "active" region: 8-12 ms [1].
- Sparseness of Acoustic echo depends on reverberation time, the distance between loudspeaker and microphone, different changes in the environment (e.g., temperature or pressure).

**Idea!**

. *Proportionate* the Algorithm .Update each coefficient of the filter independent of each other. Redistribute adaptation gain, emphasizing larger coefficients (speed-up convergence) [2][3].

# Table of Contents

# System Model

An adaptive filter is used to model an unknown system, both driven by the same zero-mean input signal $x(n)$. The reference signal at the discrete-time index $n$ is,

$$d(n) = \mathrm{h}^T(n)\mathrm{x}(n) + v(n), \tag{1}$$

where,

$$\mathrm{x}(n) = [x(n) \quad x(n-1) \quad \cdots \quad x(n-M+1)]^T$$

$$\mathrm{h}(n) = [h_0(n) \quad h_1(n) \quad \cdots \quad h_{M-1}(n)]^T$$

where $M$ is the length of system impulse response. $v(n)$ is the system noise, usually considered as a zero-mean white Gaussian noise signal of variance $\sigma_v^2 = E[v(i)]^2$.

# System Model

The system impulse response follows first-order Markov Model. It's evolves as follows,

$$\mathrm{h}(n) = \mathrm{h}(n-1) + \mathrm{w}(n), \tag{2}$$

where $\mathrm{w}(n)$ is a zero-mean white Gaussian noise signal vector, uncorrelated with $\mathrm{h}(n-1)$ .

$$\mathrm{R}_w = \sigma_w^2 \mathrm{I}_M,$$

where $\mathrm{I}_M$ is the $M \times M$ identity matrix. The variance, $\sigma_w^2$ ,captures the uncertainties in $\mathrm{h}(n)$. Notice, equations (1) and (2) define a state variable model

# Proportionate LMS Algorithm

The general update rule for Proportionate-LMS is,

$$\hat{\mathrm{h}}(n) = \hat{\mathrm{h}}(n-1) + \mu \mathrm{G}(n-1)\mathrm{x}(n)e(n), \tag{3}$$

where,

$$e(n) = d(n) - \mathrm{x}^T(n)\hat{\mathrm{h}}(n-1) \tag{4}$$

If the a-posteriori misalignment is defined as $\mathrm{c}(n) = \mathrm{h}(n) - \hat{\mathrm{h}}(n)$, (3) and (4) can be rewritten as,

$$\mathrm{c}(n) = \mathrm{c}(n-1) + \mathrm{w}(n) - \mu \mathrm{K}(n-1)\mathrm{x}(n)e(n), \tag{5}$$

$$e(n) = \mathrm{x}^T(n)\mathrm{c}(n-1) + \mathrm{x}^T(n)\mathrm{w}(n) + v(n), \tag{6}$$

# Table of Contents

# Minimizing System Misalignment

**Objective:** $min\ E\left[\|\mathrm{c}(n)\|_2^2\right]$ : Take $l_2$ on both sides of eqn(5), The following assumptions and notations are taken to simplify the objective.

- The input signal is white and stationary to some degree.
- The filter length M is large.
- Orthogonality Principle holds good (algorithm has converged fairly).
- $\mathrm{R_c}(n) = E\left[\mathrm{c}(n)\mathrm{c}^T(n)\right]$ and $m(n) = E\left[\|\mathrm{c}(n)\|_2^2\right]$.

$$m(n) = m(n-1) + M\sigma_w^2 - 2\mu\sigma_x^2 \mathrm{tr}\left\{\left[\mathrm{R_c}(n-1) + \sigma_w^2 \mathrm{I}_M\right]\mathrm{K}(n-1)\right\} + \mu^2\sigma_x^2 \mathrm{tr}\left[\mathrm{K}^2(n-1)\right]\left[\sigma_v^2 + M\sigma_w^2\sigma_x^2 + \sigma_x^2 m(n-1)\right]. \tag{7}$$

# Minimizing System Misalignment

To minimize $m(n)$, impose $\frac{\partial m(n)}{\partial \mu(n)} = 0$. We get step-size as,

$$\mu(n) = \frac{\mathrm{tr}\left\{\left[\mathrm{R_c}(n-1) + \sigma_w^2 \mathrm{I}_M\right] \mathrm{K}(n-1)\right\}}{\mathrm{tr}\left[\mathrm{K}^2(n-1)\right]\left\{\sigma_v^2 + \sigma_x^2[m(n-1) + M\sigma_w^2]\right\}}. \quad (8)$$

With $\Gamma(n) = \mathrm{R_c}(n) + \sigma_w^2 \mathrm{I}_M$ , apply Cauchy-Schwartz Inequality to Nr term

$$\{\mathrm{tr}\ [\Gamma(n-1)\mathrm{K}(n-1)]\}^2 \leq \|\boldsymbol{\gamma}(n-1)\|_2^2 \|\mathrm{k}(n-1)\|_2^2$$

Equality achieved for $\mathrm{k}(n-1) = q\gamma(n-1)$ with $p > 0$.

# Minimizing System Misalignment

Assume

$$\mathrm{tr}\left[\mathrm{K}(n-1)\right] = M$$

$$\Rightarrow \mathrm{tr}\ \left[\mathrm{K}(n-1)\right] = q\mathrm{tr}\left[\Gamma(n-1)\right] = q\left[m(n-1) + M\sigma_w^2\right]$$

$$q = \frac{M}{m(n-1) + M\sigma_w^2} \tag{9}$$

Substituting the value of $q$ back in eq (8) and (7) we get the iteration expression for $\mu(n)$ and $m(n)$. To determine $\boldsymbol{\gamma}(n-1)$, compute $\mathrm{R_c}(n)$ similar to (7).Thus,

$$\gamma(n) = \gamma(n-1) + \sigma_w^2 1_{M\times 1} + \sigma_x^2\left\{1 - 2q^2\mu^2(n)\right\} \times \gamma(n-1) \odot \gamma(n-1), \tag{10}$$

To ensure stability, normalize $\gamma(n)$. Set $\gamma(n) = \gamma(n)/max(q\gamma(n))$.

# Algorithm

**Optimized Proportionate LMS Algorithm**

**Initialization:** $\hat{\mathrm{h}}(0) = 0 \quad \hat{m}(0) = \epsilon > 0 \quad \hat{\sigma}_w^2 = 0 \quad \gamma(0) = 1_{M\times 1}$

**Iteration :**

. $e(n) = d(n) - \mathrm{x}^T(n)\hat{\mathrm{h}}(n-1),$

. $\hat{\sigma}_x^2 = \frac{1}{M}\mathrm{x}^T(n)\mathrm{x}(n),$

. $q = M/(m(n-1) + M\hat{\sigma}_w^2)$

. $\mu(n) = \frac{1}{q\sigma_v^2 + M\sigma_x^2},$

. $m(n) = m(n-1) + M\sigma_w^2 - q\mu(n)\sigma_x^2\|\gamma(n-1)\|_2^2,$

. $\widehat{\mathrm{h}}(n) = \widehat{\mathrm{h}}(n-1) + q\mu(n)\gamma(n-1) \odot \mathrm{x}(n)\mathrm{e}(n),$

. $\gamma(n) = \gamma(n-1) + \sigma_w^2 1_{M\times 1} + \sigma_x^2\left\{1 - 2q^2\mu^2(n)\right\} \times \gamma(n-1) \odot \gamma(n-1),$

. $\gamma(n) = \gamma(n)/max(q\gamma(n))$

. $\hat{\sigma}_w^2 = \frac{1}{M}||\widehat{\mathrm{h}}(n) - \widehat{\mathrm{h}}(n-1)||^2$

# Table of Contents

# Simulation

The adaptive filter is used to identify the acoustic echo path between the loudspeaker and the microphone,i.e., the room impulse response. We consider two impulse responses, one of them being quite sparse and the other being dense. Each of them have 512 coefficients with 8kHz sampling rate [4]. The length of the adaptive filter is usually less than impulse response length (in practical AEC scenarios), however for convenience we set it to $M = 512$ as well.

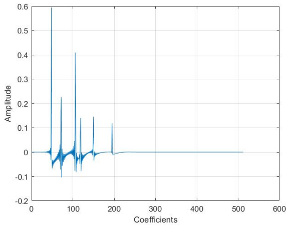


Figure: Sparse Impulse Response used in simulation

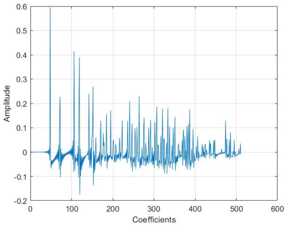


Figure: Dense Impulse Response used in simulation

# Simulation

The far end signal is either a unit variance AR(1) process or a speech sequence. The echo signal is corrupted with white gaussian noise signal (near end) with 20dB SNR. In order to evaluate the tracking capabilities of the algorithms, an echo path change scenario is simulated, by shifting the impulse response to the right by 50 samples. The measure of performance is the normalized misalignment (dB) $= 20log_{10}||\mathrm{h}(n) - \widehat{\mathrm{h}}(n)||_2/||\mathrm{h}(n)||_2$.

# Simulation Results

Compare with IPNLMS [2] at two different $\mu$ values. For $\mu = 1$ (fastest convergence mode), both the algorithms have almost same initial convergence. However, the IPNLMS achieves lower (poor) steady-state misalignment. On the other hand, for $\mu = 0.1$, in steady-state (provided echo path doesn't change), both algorithm tend to achieve almost same misalignment but IPNLMS has poor convergence rate. These conclusions are valid irrespective of sparseness of the impulse response.

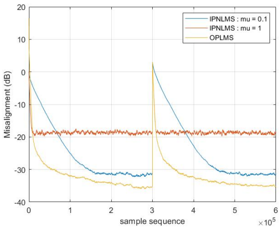


Figure: Misalignment under Sparse IR

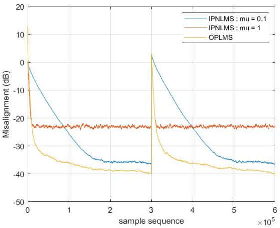


Figure: Misalignment under Dense IR

# Simulation Results

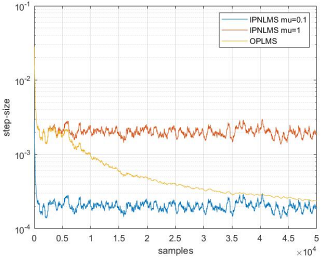


Figure: Comparison of step size evolution in IPNLMS and OPLMS filters.

Initially, the stepsize of OPLMS is quite high and comparable to IPNLMS ($\mu = 1$) aiding faster convergence. It decreases gradually on achieving steady-state, comparable to IPNLMS ($\mu = 0.1$) where their misalignments are also similar. This feature of OPLMS filter helps it achieve faster convergence rate.

# Simulation Results

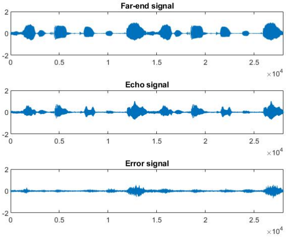


Figure: IPNLMS Response

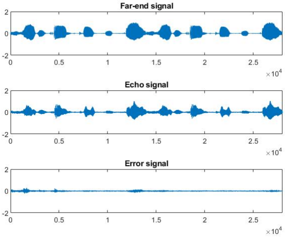


Figure: OPLMS Response

We now use real speech sequence (of length $N = 28017$) to visualise the attenuation of echo on using the adaptive filter. It is quite evident from the simulation results that the strength of error signal resulting in OPLMS filter weaker than that of IPNLMS filter.

# Table of Contents

# References


- C Paleologu, J Benesty, S Ciochina,"Sparse Adaptive Filters for Echo Cancellation",Synthesis Lectures on Speech and Audio Processing, 2010.
- D. L. Duttweiler, "Proportionate normalized least-mean-squares adaptation in echo cancelers," in IEEE Transactions on Speech and Audio Processing, vol. 8, no. 5, pp. 508-518, Sept. 2000, doi: 10.1109/89.861368.
- J. Benesty and S. L. Gay, "An improved PNLMS algorithm," in Proc. IEEE ICASSP, 2002, pp. II-1881–II-1884.
- J.B. Allen and D.A. Berkley, "Image method for efficiently simulating small-room acoustics," Journal Acoustic Society of America, 65(4), April 1979, p 943. www.audiolabs-erlangen.de/fau/professor/habets/software/rir-generator

Thank You!