Summary | Dynamics

Introduction

A branch of mechanics, which deals with motion of bodies.

2 parts:

- **Kinematics**: the study of geometric aspects of motion (not referencing the forces)
- **Kinetics**: the analysis of the forces that cause the motion

Kinematics of a particle

A particle has a mass and negligible size.

> **Note**
>
> When bodies of finite size is of interest, the body might be considered as particles **provided** motion of the body is characterized by motion of its center of mass and any rotation of the body is neglected.

Rectilinear motion

When the motion of a particle is along a straight line.

Suppose x is the distance to the particle from a fixed point on its motion path.

- $\dot{x}$ is its instantaneous velocity.
- $\ddot{x}$ is its instantaneous acceleration.

Curvilinear motion

When the motion of a particle is along a curve.

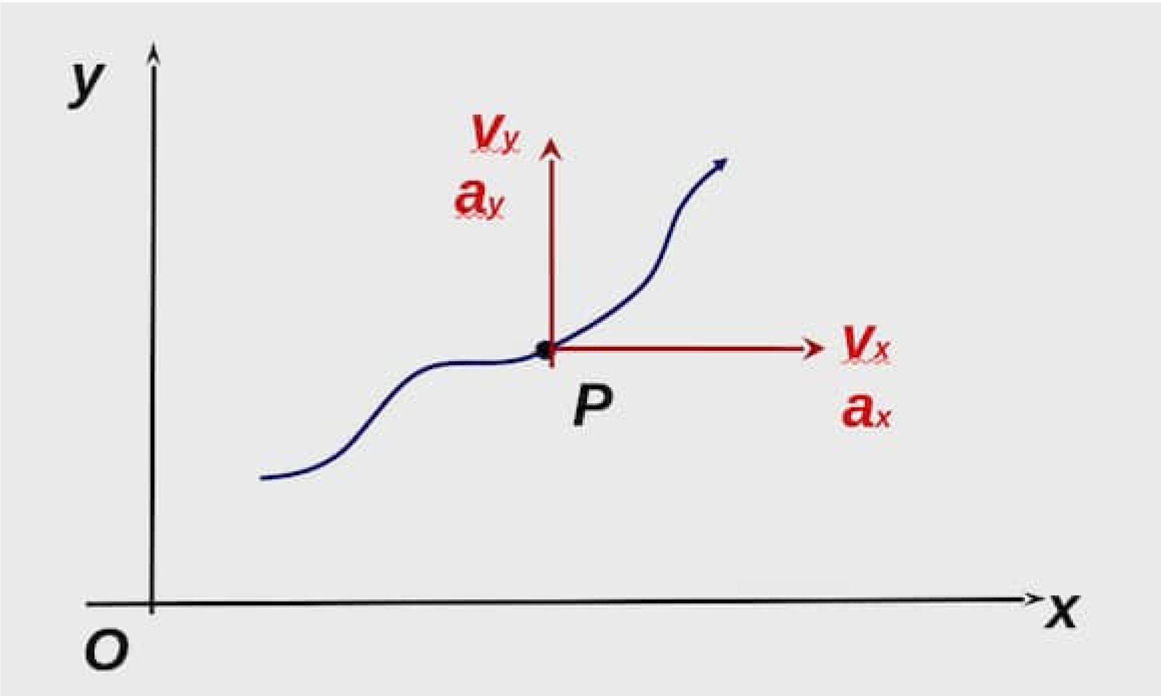
Suppose $\overline{r}$ is the position vector of the particle.

- Instantaneous velocity $\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}$
- Instantaneous speed $|\boldsymbol{v}| = \frac{\mathrm{d}s}{\mathrm{d}t}$
- Instantaneous acceleration $\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}$

> **Note**
>
> Right hand rule is used here to denote the direction of any rotary motions.

2D motion of a particle

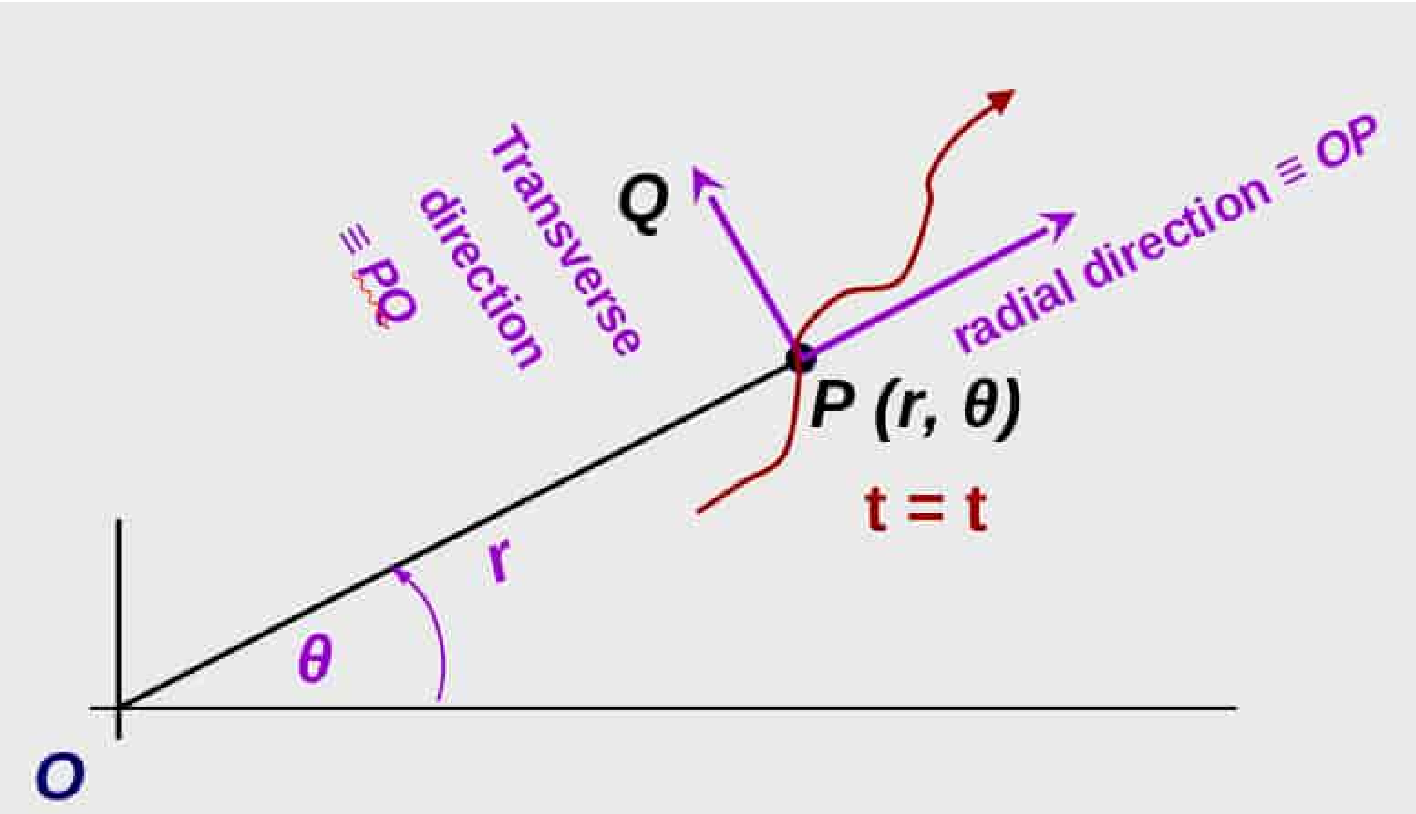
Rectangular form



$$v_y = \frac{\mathrm{d}y}{\mathrm{d}t} = \dot{y} \quad \wedge \quad v_x = \frac{\mathrm{d}x}{\mathrm{d}t} = \dot{x}$$

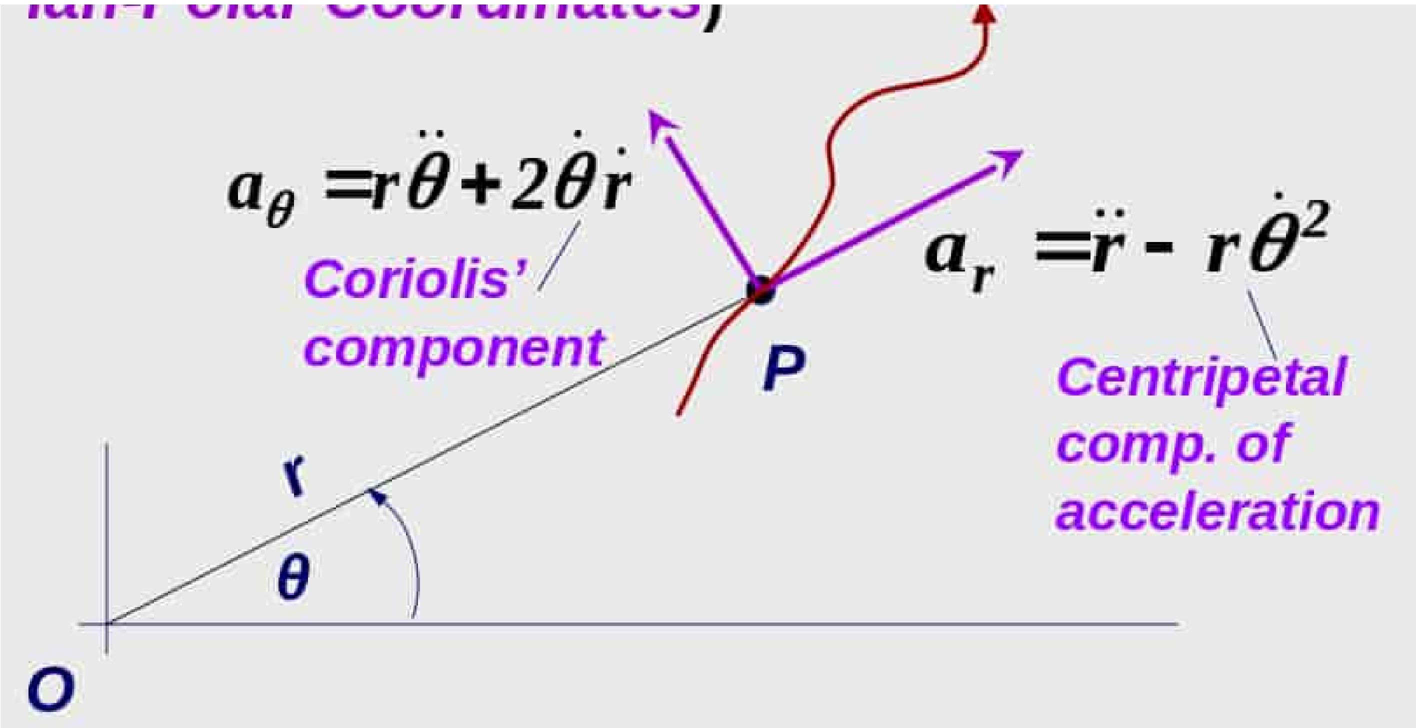
$$a_y = \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \ddot{y} \quad \wedge \quad a_x = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \ddot{x}$$

Polar form



Velocity have a transverse and radial components.

- Transverse component
  $v_\theta = \underline{\dot{\theta}} \times \underline{r}$
- Radial component
  $v_r = \underline{\dot{r}}$



Acceleration also have a transverse and radial components.

- Transverse component
    - $a_\theta = r\ddot{\theta} + 2\dot{\theta}\dot{r}$
    - In vector equation: $\underline{a_\theta} = \underline{\ddot{\theta}} \times \underline{r} + 2(\underline{\dot{\theta}} \times \underline{\dot{r}})$
- Radial component
    - $a_r = \ddot{r} - r\dot{\theta}^2$
    - $\underline{a_r} = \underline{\ddot{r}} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

In the acceleration:

- Coriolis' component of acceleration: $2\dot{\theta}\dot{r}$
- Centripetal component of acceleration: $-r\dot{\theta}^2 = \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

Effects of Coriolis' component

- Objects reflect to the right in the northern hemisphere
- Objects reflect to the left in the southern hemisphere
- Maximum deflections occur at the poles. No deflection at the equator.

Unit vectors

Unit vectors in transverse and radial directions are denoted by e_θ and e_r respectively.

$$\dot{e}_r = \dot{\theta} e_\theta \quad \wedge \quad \dot{e}_\theta = -\dot{\theta} e_r$$

Velocity

$$v = \frac{\mathrm{d}}{\mathrm{d}t}(r e_r) = \dot{r} e_r + r \dot{e}_r = \dot{r} e_r + r \dot{\theta} e_\theta$$

Acceleration

$$a = \frac{\mathrm{d}}{\mathrm{d}t}(r \dot{\theta} e_\theta) = (\ddot{r} - r \dot{\theta}^2) e_r + (r \ddot{\theta} + 2 \dot{\theta} \dot{r}) e_\theta$$

2D kinematics of a rigid body

Rigid body

A solid body that doesn't deform.

Degrees of freedom

In the motion of a rigid body in 2D kinematics, there are 3 degrees of freedom.

- Movement along x direction
- Movement along y direction
- Rotation about z direction

In 3D, there are 6 degrees of freedom: movement and rotation along each direction.

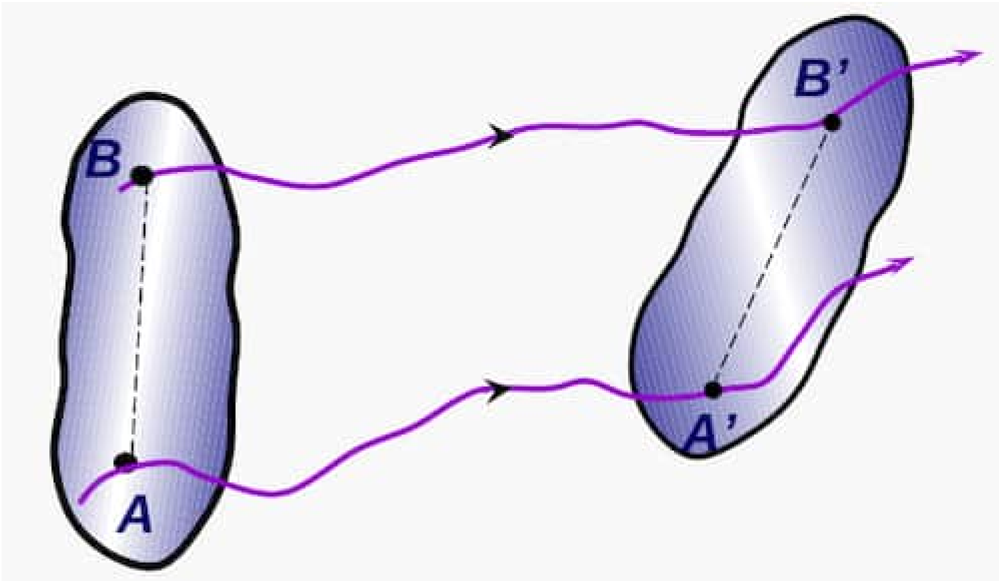
Translation

Movement that changes the position of an object. Translation can be done through a rectilinear or curvilinear path. Axes of the body always stays parallel.

Rotation

Circular movement of an object about a fixed axis that is perpendicular to the plane.

General 2D motion



Mixture of translation and rotation.

$$v_{\mathrm{B}} = v_{\mathrm{A}} + v_{\mathrm{B/A}} = v_{\mathrm{A}} + \underline{\dot{\theta}} \times r_{\mathrm{B/A}}$$

$$a_{\mathrm{B}} = a_{\mathrm{A}} + a_{\mathrm{B/A}} = a_{\mathrm{A}} + \underline{\ddot{\theta}} \times r_{\mathrm{B/A}} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times r_{\mathrm{B/A}})$$

Here:

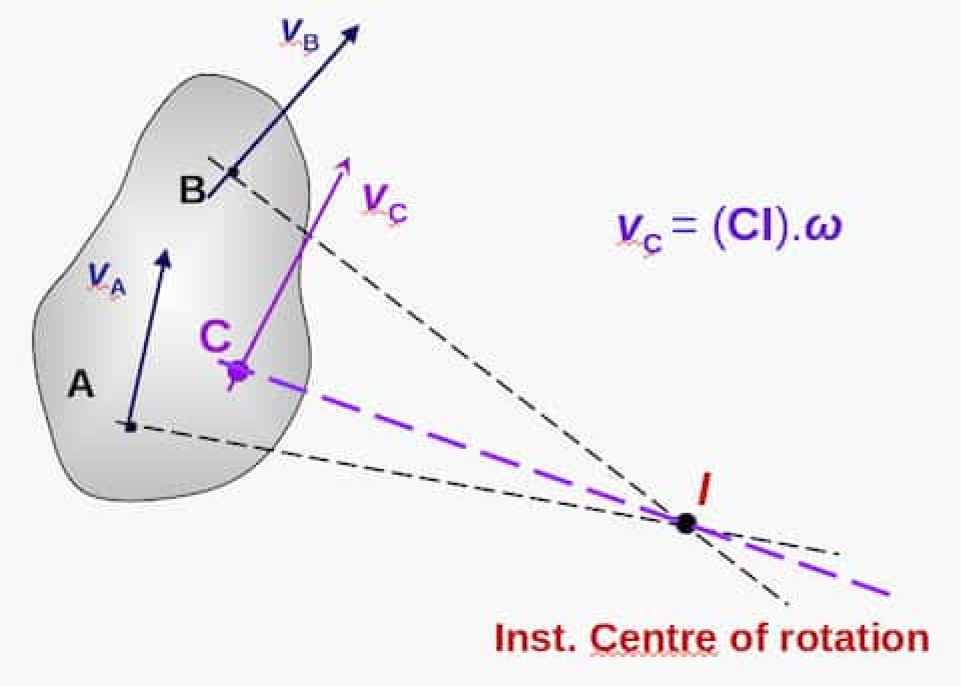
- $\underline{\dot{\theta}}$ - Angular velocity of B relative to A
- $v_{\mathrm{B/A}}$ - Velocity of B relative to A
- $a_{\mathrm{B/A}}$ - Acceleration of B relative to A
- $r_{\mathrm{B/A}}$ - Position vector of B relative to A. It’s constant.

In general motion, each particle of the body has a different velocity at every instance.

Instantaneous centre of rotation

The point that has 0 velocity at a particular instant of time. This point might be changing throughout the motion. Denoted by I.

It can be imagined that the object is momentarily having a pure rotation about this centre I.



I can be found by drawing a line perpendicular at the velocity vectors at 2 different points and finding their intersection point.

Centrode

The locus of instantaneous centres during the motion.

Mechanisms

Mechanism

An assembly of rigid bodies or links designed to obtain a desired motion from an available motion while transmitting appropriate forces and moments. Motion of the links have definite relative motion with other links.

Simple mechanisms

- Lever
- Pulley
- Gear trains
- Belt and chain drive
- Four bar linkage

Other complex mechanisms

- Lock stitch mechanism (used in sewing machine)
- Geneva mechanism

  Constant rotational motion to intermittent rotational motion. mostly used in watches.
- Scotch yoke mechanism

  Constant rotational motion to linear motion (vice versa.). Mainly used as valve actuators in high pressure gas pipelines.
- Slider crank mechanism

  Used in internal combustion engines

2D link mechanisms

Method of instantaneous centre of rotation

- Find the instantaneous centre of the rotation from known velocities at known points
- Use the instantaneous centre to find velocities at other points

Kinematic chain

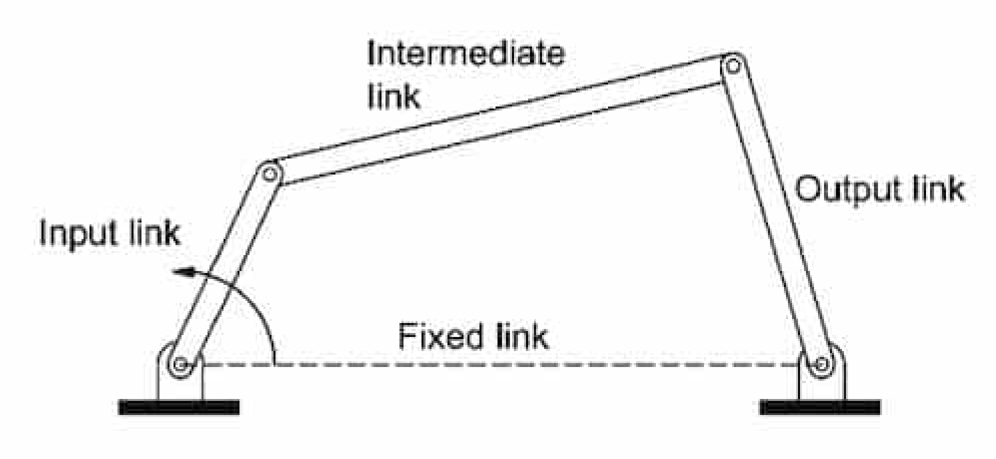
An arbitrary collection of links (forming a closed link) that is capable of relative motion and that can be made into a rigid structure by an additional single link.

Four-bar Mechanism

Four bar-shaped members connected to each other in one plane.

Usually:

- 1 fixed link + 3 moving links
- 4 pin joints
- 2 moving pivots + 2 fixed pivots
- 4 turning pairs



- **input link** - usually denoted in the left.
- **output link** - usually denoted in the right.
- **coupler** - intermediate link
- **frame** - fixed link

Grashof's law

A four bar mechanism has at least one revolving link **if** $l_0 + l_3 \leq l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

Modes of motions

Mechanism	Shortest link	Criteria
Crank rocker	Input link	$s + l < p + q$
Double crank	Fixed link	$s + l < p + q$
Double rocker	Coupler link	$s + l < p + q$
Change point	Any	$s + l = p + q$
Triple rocker	Any	$s + l > p + q$

crank means a link that makes a full revolution. **rocker** means a link that doesn't make a full revolution.

Crank rocker mechanism

Shortest link rotates a full revolution. Output link oscillates.

Double crank mechanism

Shortest link is fixed. Both input and output links rotates a full revolution.

Double rocker mechanism

Shortest link make full resolution. Input and output links makes a full revolution.

Special cases

$l_0 + l_3 = l_1 + l_2$.

Mechanism	Orientation
Parallelogram linkage or anti-parallelogram linkage	Equal links are opposite to each other
Deltoid linkage	Equal links are adjacent to each other

Parallelogram linkage

Double crank mechanism. Opposite links are equal and parallel. Angular velocity of input crank & output crank is same. Orientation of the coupler doesn't change during the motion.

Anti-parallelogram linkage

Double crank mechanism. Angular velocity of input crank is different to output crank.

Deltoid linkage

- Longest link is fixed: crank rocker mechanism
- Shortest link is fixed: double crank mechanism

Non-Grashof's condition

A four bar mechanism with the property **if** $l_0 + l_3 > l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

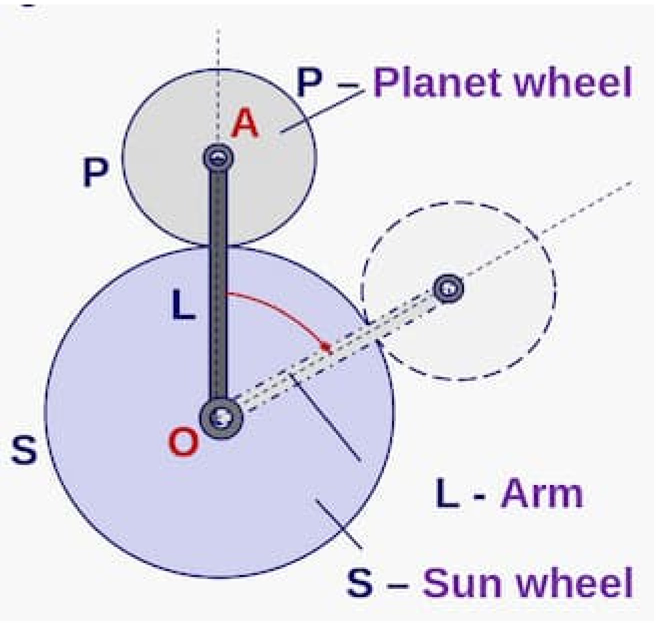
Three links are in oscillation.

Epicyclic Gears

In below equations:

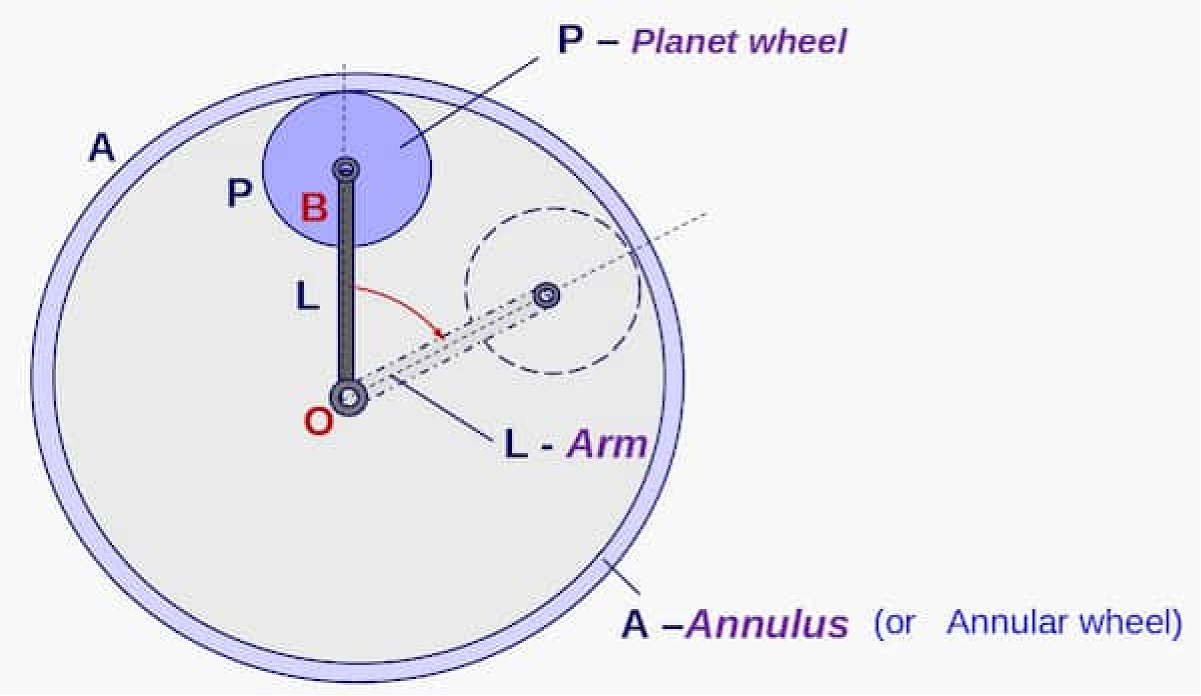
- ω_p - Absolute angular speed of planet wheel P

External



$$\omega_p = \left(1 + \frac{r_S}{r_P}\right)\omega_L - \left(\frac{r_S}{r_P}\right)\omega_S$$

Internal



$$\omega_p = \left(1 - \frac{r_A}{r_P}\right)\omega_L + \left(\frac{r_A}{r_P}\right)\omega_A$$

Mobility of Mechanisms

Independent object

Has 3 degrees of freedom.

Lower Pair

A pair of kinematic elements which share a surface of contact.

When a rigid body is constrained by a lower pair, which allows only rotational or sliding movement. It has 1 degree of freedom, and the 2 degrees of freedom are lost.

Some examples:

- Turning pair
- Sliding pair
- Helical thread

Higher Pair

A pair of kinematic elements which share only a line or a point of contact.

When a rigid body is constrained by a higher pair, it has 2 degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

Gear is an example.

When 2 independent objects are brought together to create a link, some degree of freedom will be lost.

> “You lose some freedom when you become a couple.” — Our Dynamics Lecturer

Grubler’s Equation

Suppose N kinematic elements are brought together. 1 of them is fixed. The remaining elements have $3(N-1)$ degrees of freedom. Each lower pairs loses 2 degrees of freedom. Each higher pairs loses 1 degree of freedom. For a workable mechanism, resultant degrees of freedom must be 1.

$$F = 3(N-1) - 2L - H = 1 \implies 3N - 2L + H = 4$$

Here:

- F - degree of freedoms
- N - number of kinematic elements
- L - number of lower pairs
- H - number of higher pairs

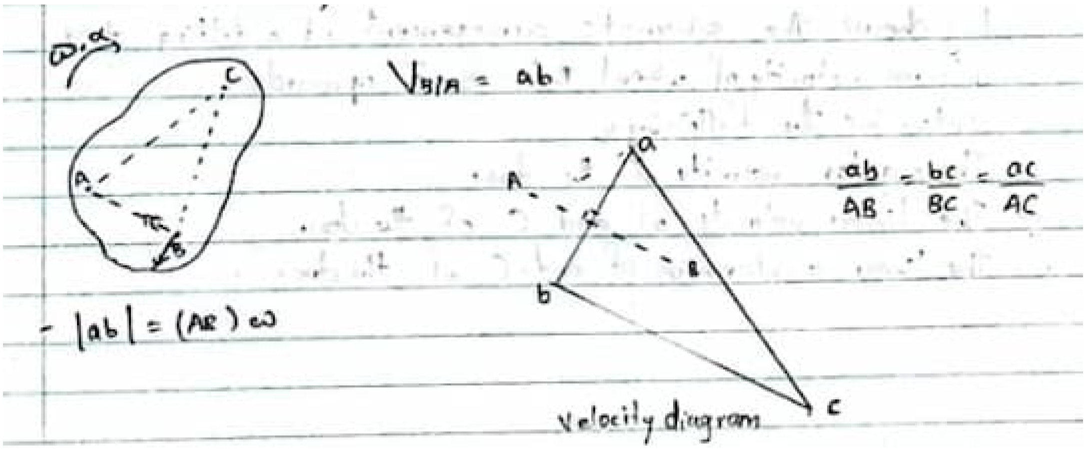
Velocity & Acceleration Diagram

O is a fixed point.

Velocity diagram

Notation

- oa - Absolute velocity of point A
- ab - Velocity of point B relative to point A



The above illustration is from Ruththiragayan, one of my friends.

Acceleration diagram

Notation

- o_1a_1 - Absolute acceleration of point A
- a_1b_1 - Velocity of point B relative to point A

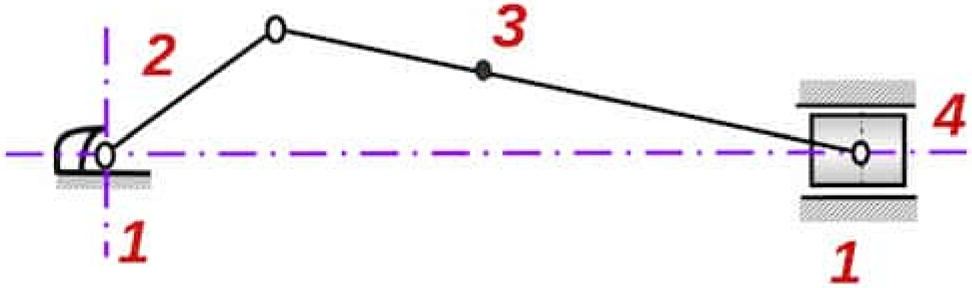
$$a_1b_1 = a_1x_1 + x_1b_1 = (\mathrm{AB})\,\omega_{AB}^2 + (\mathrm{AB})\,\alpha_{AB}$$

Here:

- a_1x_1 - the radial component of the relative acceleration between A and B
- x_1b_1 - the transverse component of the relative acceleration between A and B

Inversions of a mechanism

The inversions are obtained by making different kinematic element stationary (one at a time) while keeping the same set of kinematic pairs.



For example, in slider crank mechanism:

- When link 2 is fixed: Whitworth quick-return mechanism
- When link 3 is fixed: The oscillating cylinder engine
- When link 4 is fixed: Hand pump