# Introduction to Statics

## Centroid / Centre of area

The point where the area of a figure is assumed to be concentrated.

Located on the axes of symmetry.

## First moment of area

$$\text{About x-axis} = \int_A y \mathrm{d}A$$

$$\text{About y-axis} = \int_A x \mathrm{d}A$$

## Second moment of area

$$\text{About x-axis} = I_{xx} = I_x = \int_A y^2 \mathrm{d}A$$

$$\text{About y-axis} = I_{yy} = I_y = \int_A x^2 \mathrm{d}A$$

## The product of moment of area about x,y axes

$$I_{xy} = \int_A xy \mathrm{d}A$$

## The polar moment of area about z axis

$$I_{zz} = J_0 = \int_A r^2 \mathrm{d}A = I_{xx} + I_{yy}$$

## Radius of gyration

$$\text{About x-axis} = r_x^2 = \frac{I_{xx}}{A}$$

$$\text{About y-axis} = r_y^2 = \frac{I_{yy}}{A}$$

$$\text{About z-axis} = r_z^2 = \frac{I_{zz}}{A}$$

# Derived Formulas for Common Shapes

| Shape | Description | $I_{xx}$ |
|---|---|---|
| Rectangle | Base $b$. Height $h$. About centroidal axis parallel to base. | $\frac{bh^3}{12}$ |
| Triangle | Base $b$. Height $h$. About base. | $\frac{bh^3}{12}$ |
| Triangle | Base $b$. Height $h$. About centroidal axis parallel to base. | $\frac{bh^3}{36}$ |
| Circle | Diameter $d$. About centroidal axis. | $\frac{\pi d^4}{64}$ |
| Parallelogram | Base $b$. Height $h$. About centroidal axis parallel to base. | $\frac{bh^3}{12}$ |

# Parallel Axis Theorem

$$I_x = I_{x_1} + A\bar{y}^2$$

$$I_y = I_{y_1} + A\bar{x}^2$$

$$I_{xy} = I_{x_1y_1} + A\bar{x}\bar{y}$$

Here

- On LHS, the moments of area are about some $x$, $y$ axes.
- On RHS, the moments of area are about centroidal axes $x_1$, $y_1$ parallel to x, y.
- $\bar{x}$ is the distance between $x$ and $x_2$ axes.
- $\bar{y}$ is the distance between $y$ and $y_1$ axes.

# Perpendicular Axis Theorem

$$I_{zz} = I_{xx} + I_{yy} = I_{mm} + I_{nn}$$

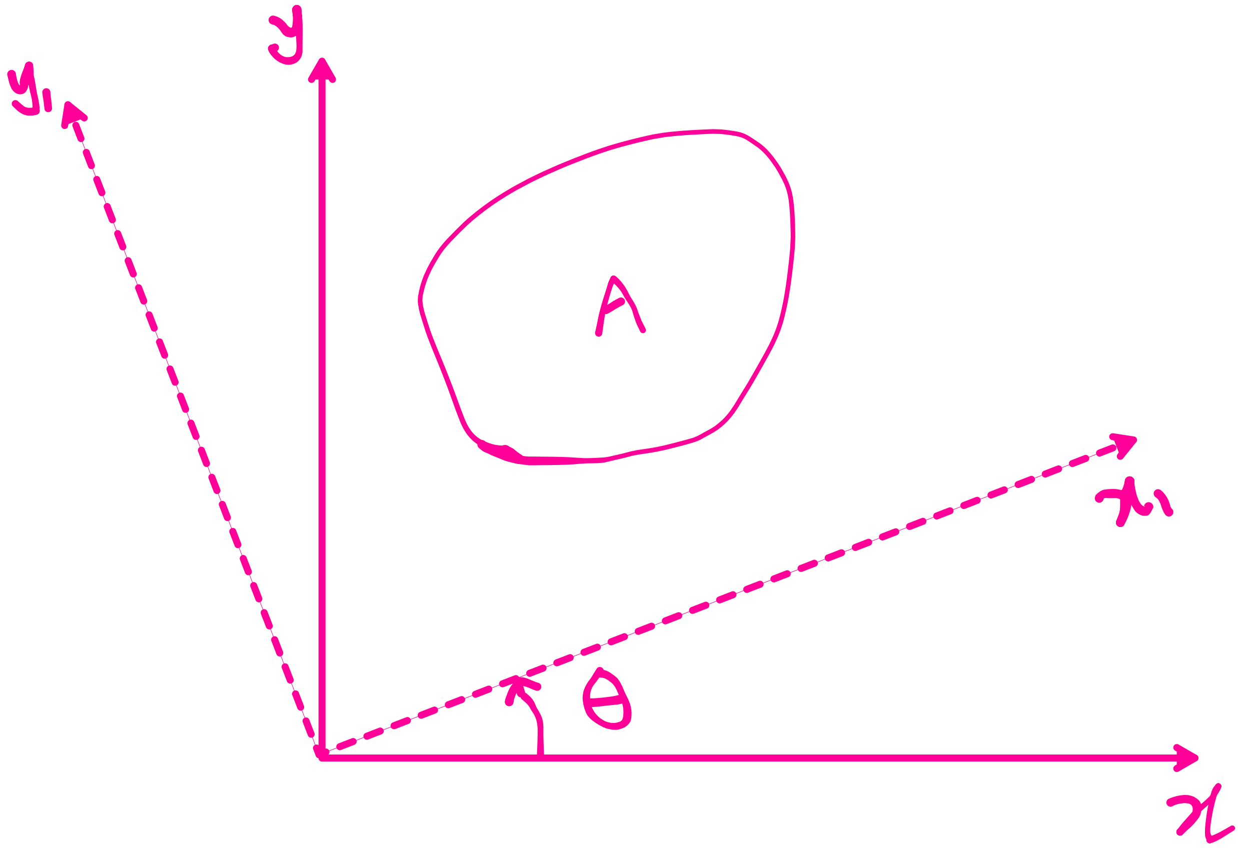
$x$, $y$, $z$ are a set of axes. $m$, $n$, $z$ are another set of axes.

If $I_{xx}$ is at maximum, $I_{yy}$ will be at minimum.

# Transformation Law

The 2 sets of axes must share the origin.

> **Note**
> Don't have to memorize this. Will be given on exams, if required.




$$I_{x_1x_1} = \frac{I_{xx}+I_{yy}}{2} + \left(\frac{I_{xx}-I_{yy}}{2}\right)cos2\theta - I_{xy}sin2\theta$$

$$I_{y_1y_1} = \frac{I_{xx}+I_{yy}}{2} - \left(\frac{I_{xx}-I_{yy}}{2}\right)cos2\theta + I_{xy}sin2\theta$$

$$I_{x_1y_1} = \left(\frac{I_{xx}-I_{yy}}{2}\right)sin2\theta + I_{xy}cos2\theta$$

# Principal Axes

The product of moment of area is zero about principal axes.

$$I_{xy} = 0$$

There will be 2 directions of principal axes which are perpendicular to each other.

Any axis of symmetry is a principal axis. Any axis through centroid is a principal axis.

## Principal second moments of area

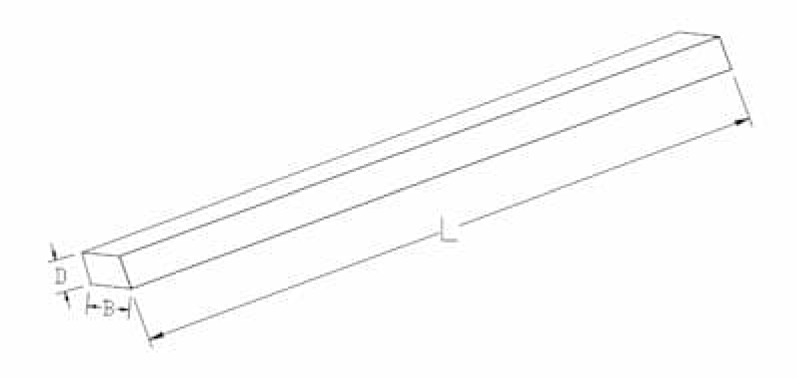
Second moments about the principal axes.

About principal axes second moments of area will be at minimum and maximum.

## Centroidal principal axes

Principal axes through the centroid.

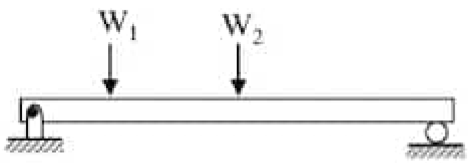
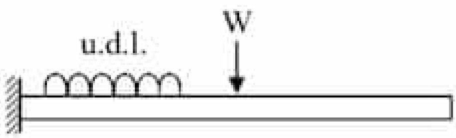
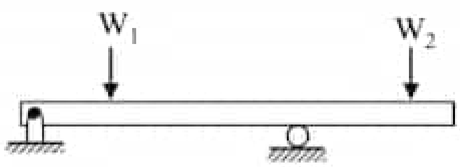
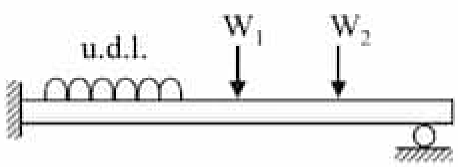
# Beams

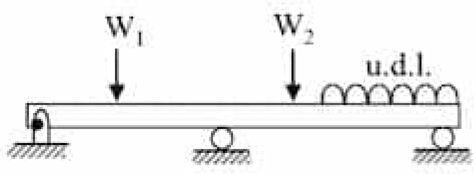
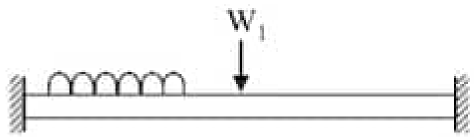



- long ( $L >> B, D$ )
- axis of the beam is straight
- constant cross-section throughout its length

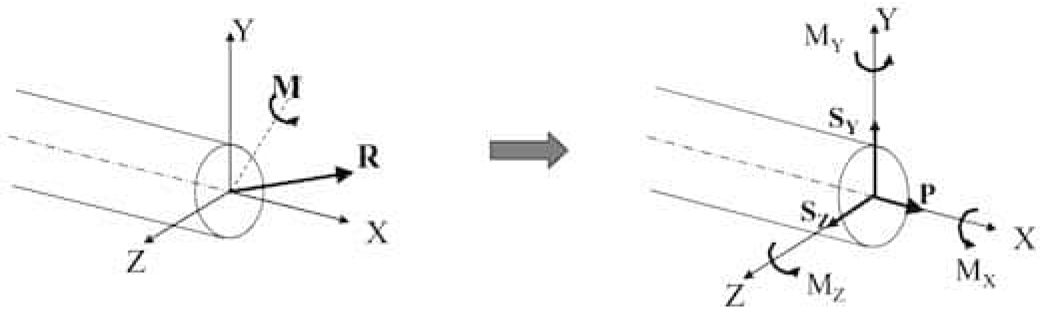
## Classified by supporting conditions

First 3 are the mandatory ones.

| Type | Image |
| --- | --- |
| Simply supported beam |  |
| Cantilevered beam |  |
| Overhanging beam |  |
| Propped cantilevered beam |  |

| Type | Image |
|---|---|
| Continuous beam |   |
| Fixed beam |   |

# At a section




- $P$ - Normal force / Axial force
- $S_y, S_y$ - Shear forces along $y$ and $z$ axis
- $M_x$ - Twisting moment / Torque
- $M_y, M_z$ - Bending moments about $y$ and $z$ axis

# Degress of freedom

A plane member have 3 degress of freedom. Any of the 3 can be restrained.

- Displacement in x-direction
- Displacement in y-direction
- Rotation about z-direction

# SFD & BMD

## Sign convention

- Bending moment
  - Hogging (curves upwards) is **(+)ve**
  - Sagging (curves downwards) is **(-)ve**
- Shear force
  - Clockwise shear is **(+)ve**.
  - Counterclockwise shear is **(-)ve**.

> **Note**
>
> A member is in pure bending when shear force is 0 and bending moment is a constant in a part of a beam.

# Distributed load, shear force & bending moment

When a beam is under a distributed load of $w = f(x)$ per unit length.

$$\frac{\mathrm{d}S}{\mathrm{d}x} = -w$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = -S \; ; \; \frac{\mathrm{d}^2M}{\mathrm{d}x^2} = w$$

# Principle of Superposition

A beam with multiple loads can be split into multiple systems each with a single load. Reason for doing so is the ease of calculations.

Values will be the sum of each system’s corresponding value.

# Structural Elements

3 types

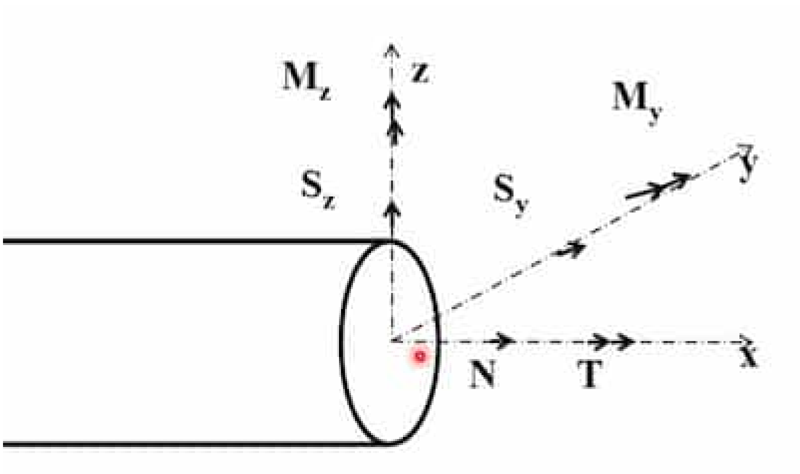
- Bars (1D)
- Plates and Shells (2D)
- Blocks (3D)

Bar elements are only focused for 1st semester.

## Pin Joint

Doesn't exert a moment. Free rotations are allowed. When only pin joints are used, bars will have only axial forces.

## Bars




Here

- $N$
  - Axial force
- $S_x, S_y$
  - Shear force
- $M_x$

### Types of bars

#### Axially loaded

Generally in trusses, **pin joints** are considered.

- Predominant tension - Ties
- Predominant compression - Struts

#### Flexural

- Predominant bending - beams

## Torsional

- Predominant torque - shafts

# Trusses

Also known as Ties-Struts model.

## Definition

An assembly of members used to span long distances. Idealized as

- Connected by **frictionless** pin joints at their ends
- Developing axial forces

## Types

2 types

- Plane truss (2D)
- Space truss (3D)

A truss requires 3 external reactions for equilibrium.

Predominant force is axial force.

## Advantages of truss

- High/length span
- Material efficiency

## Triangulation

- Start with a triangle (3 bars and 3 joints)
- Add 2 more bars and a joint repeatedly to create a truss

This type of truss is a **simple truss**.

## Simple (Closed) Truss

When a truss is pinned only made of bars and joints

## Open Truss

When a truss is pinned directly to a foundation. It has 1 member & 2 free joints less than a closed truss.

# Stability of trusses

When a truss is:

- unstable: it’s called a mechanism
- stable: it’s called a structure

## Stable truss

When the shape cannot be altered, the structure is **internally stable**.

### Stable & determinate (simply stiff)

**Determinate** means internal forces can be determined by laws of statics alone.

### Stable & indeterminate

**Indeterminate** means laws of statics alone are not sufficient to determine forces; relative stiffness of members will influence the solution (Indeterminate trusses are safer)

## Unstable truss

When the shape can be altered, the truss is called a mechanism.

## Necessary condition for a 2D simple (closed) truss

$m = 2j - 3$ is a necessary but not sufficient condition being simply stiff.

- $m < 2j - 3$
  - truss is unstable
- $m = 2j - 3$
  - truss is determinate if stable
- $m > 2j - 3$
  - truss is indeterminate if stable

## Necessary condition for a 2D open truss

$m = 2j$ is a necessary but not sufficient condition being simply stiff.

- $m < 2j$
  - truss is unstable
- $m = 2j$
  - truss is determinate if stable
- $m > 2j$
  - truss is indeterminate if stable

## Necessary condition for a 3D simple (closed) truss

$m = 3j - 6$ is a necessary but not sufficient condition for being simply stiff.

## Necessary condition for a 3D open truss

$m = 3j$ is a necessary but not sufficient condition for being simply stiff.

# Analysis of Trusses

Deviations from the ideal in real trusses

- Loads are not applied only at joints; hence there is bending in members
- Joints are not perfectly pinned, so moments can be developed at joints

# Method of Joints

## Principle

Since the truss is in equilibrium, each pin joint must be in equilibrium.

> **Note**
>
> 2 equilibrium equations can be written at each joint – vertical & horizontal

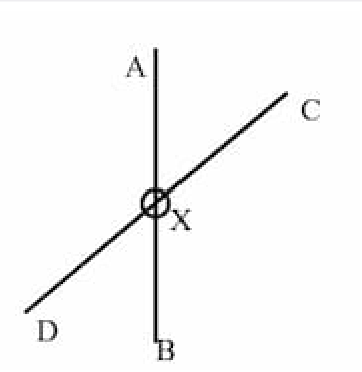
## Sign convention

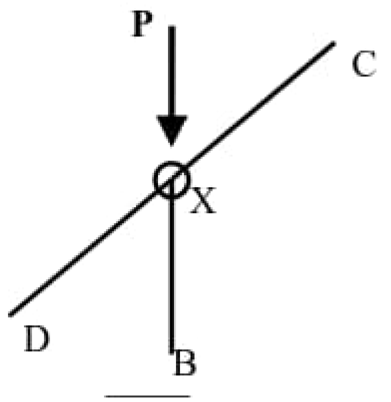
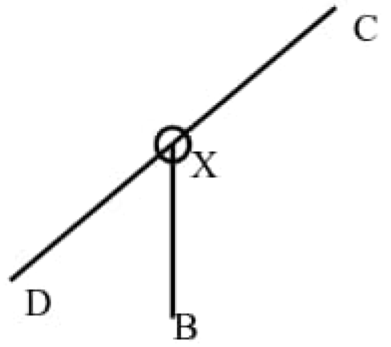
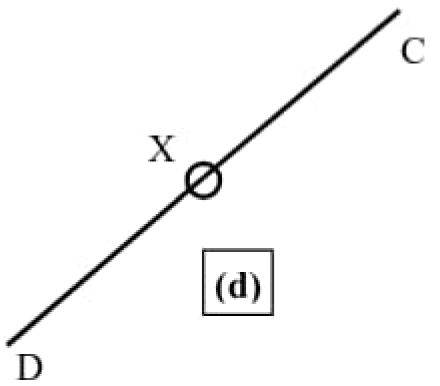
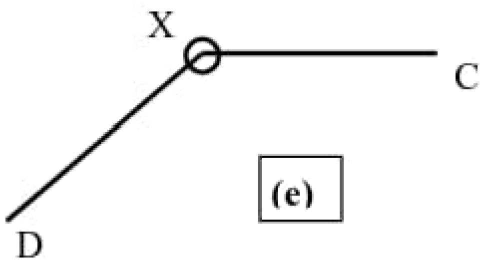
Tensile forces are positive. Compressive forces are negative.

## Method

- Find external reactions using equilibrium using equilibrium equations for the entire truss.
- Start with a joint with only 2 unknown joint forces.
- Mark the tensile forces (consider all forces are tensile) acting on the join.
- Find the unknown forces at the selected joint, using 2 equilibrium equations for the joint.
- Go to all other joints in turn and find forces in all the members.

## Special cases

| Case | Description |
|---|---|
|  | $F_{\mathrm{AX}} = F_{\mathrm{XB}} \wedge F_{\mathrm{DX}} = F_{\mathrm{XC}}$ |

| Case | Description |
| --- | --- |
|  | $F_{\mathrm{AX}} = F_{\mathrm{XB}} \wedge F_{\mathrm{DX}} = F_{\mathrm{XC}}$ |
|  | $F_{\mathrm{XB}} = 0 \wedge F_{\mathrm{DX}} = F_{XC}$ |
|  | $F_{\mathrm{DX}} = F_{\mathrm{XC}}$ |
|  | $F_{\mathrm{DX}} = F_{\mathrm{XC}} = 0$ |

# Method of Sections

## Principle

Since the truss is in equilibrium, each part of it must be in equilibrium in stable equilibrium.

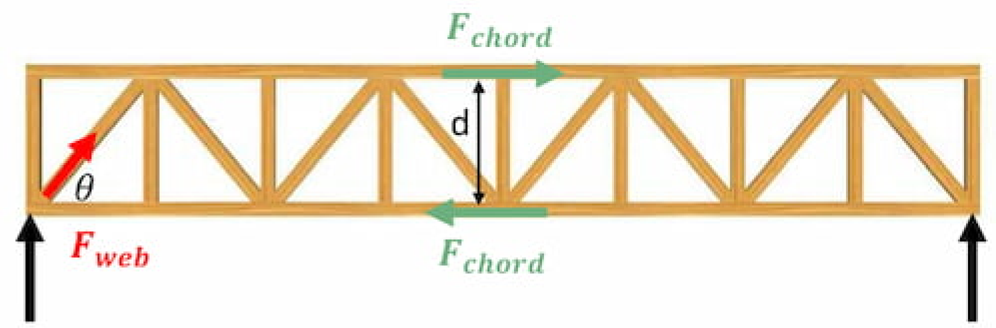
## Method

- Decide on which member’s internal force must be calculated.
- Cut the truss **3 or less** members including the target member.
- Internal forces in cut members become external forces. Can be represented as tensile forces.
- Use equilibrium equations for RHS or LHS section to find the internal forces.

# Beam Analogy (Approximate) method

We find the internal forces assuming the elongated truss is a beam.

> **For a simply supported beam**
>
> - Maximum bending moment is at mid-span: $M_{\max} = \frac{wL^2}{8}$
> - Maximum shear force is at the supports: $\frac{wL}{2}$

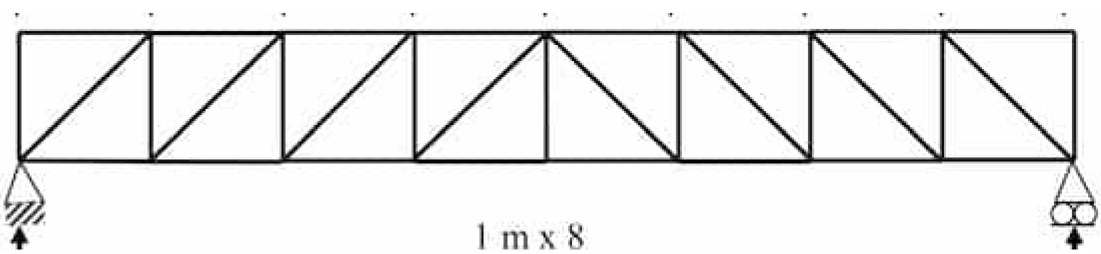



Here:

- Chord members - horizontal members
- Web members - diagonal members
- $d$ - truss depth

In the truss,

- Bending moment is carried by chord members.
  $\text{Bending moment} = F_{\text{chord}} \times d$
- Shear force is carried by vertical component of web member force

> **Pratt & Howe type trusses**
>
> Above-mentioned truss is **Pratt type**.
>
> **Howe type truss** is a similar structure.
>
> 
> 
>
> In pratt type truss, internal force in web members are tensile. In howe type trusses, internal force in web members are compressive. Usually **Pratt type** is cost-efficient. To

> make sure a howe type truss is strong enough like pratt type, web members must be shorter and thicker.

# Indeterminate Trusses

When a truss is indeterminate, one or more compatibility equations (related to truss deformation) must be used.