Summary | Electrical Fundamentals

Introduction

An electric power system consists of 3 principle sections

- power stations (generation)
- transmission
- distribution

Balanced Three-Phase System

In power stations, AC current is generated in 3 different phases. All have the same max. voltage, but 120 degrees phase difference. At any given time, the resultant of all 3 phase currents is 0. Hence **balanced**.

Single-line diagram

A balanced 3-phase circuit can be represented by a single-phase equivalent circuit. The diagram showing the single-phase equivalent of the power system using standard symbols.

Variable load

Load on a power station changes with to uncertain demands of consumers. This is called the **variable load**.

Load vs time curve is called the **load curve**. Area under this curve is the **total energy requirement**.

Basics

Be sure to revise the Electricity unit of G.C.E. (A/L) Physics.

Charge

- measured in Coulomb (C) = 6.25×10^{18} number of electrons
- quantized
- conserved

Time invariant charge is denoted as Q. And time varying charge is denoted as q.

Current

Amount of charges (in C) flowing through a point in unit time. Conventional current (opposite to electron flow) flows from positive to negative potentials.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Time invariant current (DC) is denoted as I. And time varying current (AC) is denoted as i.

Voltage

Voltage at a point is the work that must be done against the electric field to move a unit positive charge from infinity to that point.

One volt is the potential difference between two points when one joule of energy is used to move one coulomb of charge from one point to the other.

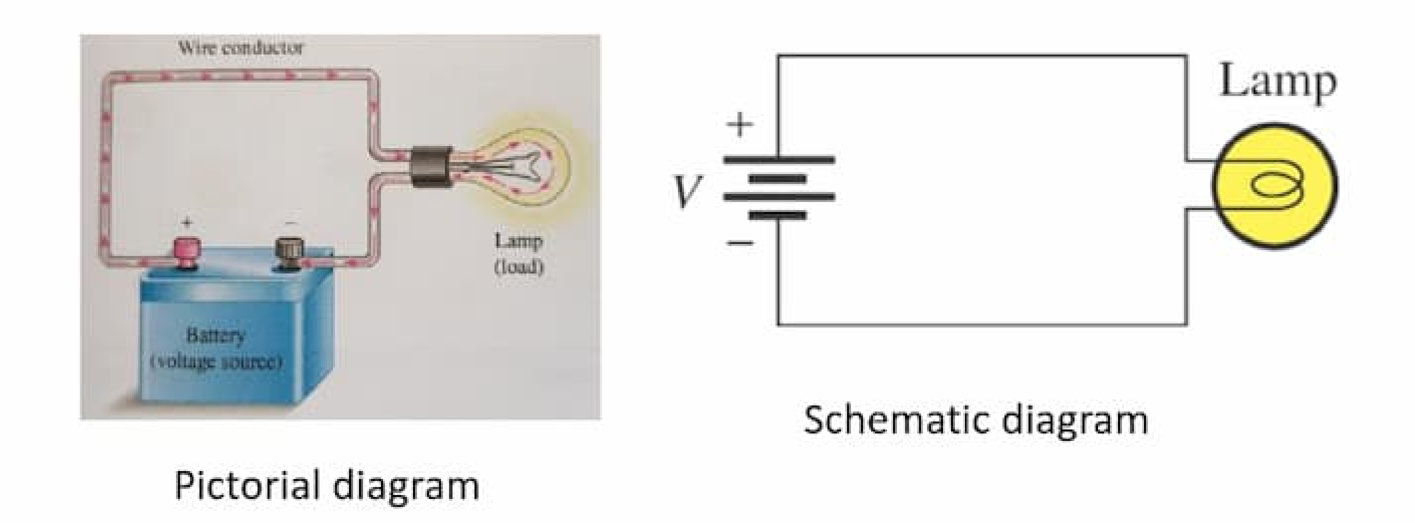
$$V = \frac{E}{Q}$$

Time invariant voltage is denoted as V. And time varying voltage is denoted as v.

Voltage difference is the work that must be done against the electric field to move a unit positive charge from one point to another.

$$V_{AB} = V_A - V_B$$

Electric Circuit



Types of circuits

- Closed circuit - the electricity flows
- Open circuit - the electricity doesn’t flow. current = 0. infinite resistance.
- Short circuit - very large current. 0 resistance.

Power

$$p = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}w}{\mathrm{d}q}\frac{\mathrm{d}q}{\mathrm{d}t} = vi$$

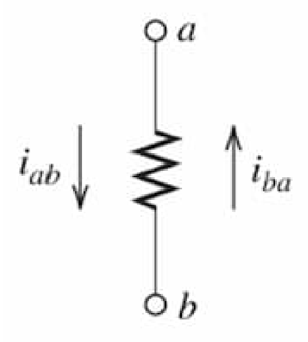
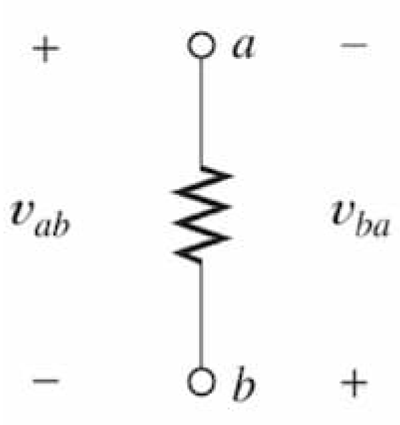
Total Work

$$w = \int_{t_0}^{t} p \mathrm{d}t = \int_{t_0}^{t} vi \mathrm{d}t$$

####### When v and i is constant

$$w = vi \int_{t_0}^{t} \mathrm{d}t = vi(t - t_0)$$

Double subscript notation

-	Current	Voltage
Double subscript		
Equation	$i_{ab} = -i_{ba}$	$v_{ab} = -v_{ba} = v_a - v_b$
Description	Current is flowing **from point a to point b**	Voltage is **higher at point a and lower at point b**

Common Terms

Two terminal element

An element connected to two nodes.

Branch

A branch represents a single element, such as a resistor or a battery. A branch is a two terminal element.

Node

A node is the point connecting more than 1 branches. Denoted by a dot.

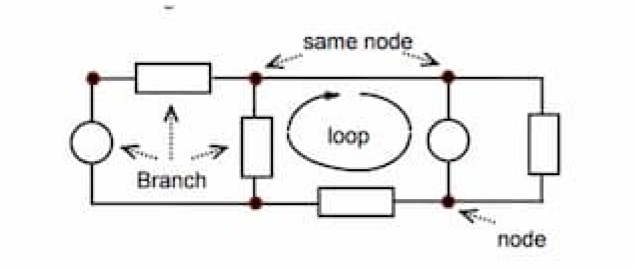
All points in a circuit that are connected directly by ideal conductors can be considered to be a single node.

Loop

A loop is a closed path through a circuit in which no node is encountered more than once except for the same start/finish node.

Mesh

A mesh is a loop without having other loops inside it.



Circuit elements

Two types of circuit elements.

- Active
- Passive

Active

Capable of generating electrical energy.

- Voltage sources
- Current sources

These can interchangeably be used.

Passive

Either consumes or stores electrical energy.

- Resistors
- Inductors
- Capacitors
- Any other elements

Voltage sources

- Batteries - electrochemical
- Solar cells - photo voltaic
- Generators - electromagnetic

Ideal voltage source

Constant voltage for any required currents. Does not exist.

Resistors

Resistance, in terms of physical dimensions:

$$R = \frac{\rho l}{A}$$

Here the l is the length, and A is the cross-sectional area, ρ is the resistivity.

If a voltage V is applied across a conductor, then a given current I will flow through the conductor $V \propto I$. The proportionality constant is called resistance R.

$$R = \frac{V}{I}$$

Capacitors

Made of two conductive plates separated by an insulating (dielectric) layer.

Capacitance (C), in terms of physical dimensions:

$$C = \frac{\epsilon A}{d}$$

Here the d is the distance between the plates, and A is the area of a plate.

In an ideal capacitor, the charge imbalance Q is proportional to the voltage V across the plates.

$$Q = CV$$

v and i

As C is constant, current i passing through the capacitor and the voltage v across the capacitor are related by:

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$$

Energy stored

Assume voltage across an initially uncharged capacitor rises to V during a time period of t seconds.

$$e = \int_0^t pdt = \int_0^t vidt = C\int_0^v vdv$$

$$E = \frac{1}{2}CV^2$$

Inductors

When there is a current in the inductor, a magnetic field is created. Any change in current causes the magnetic field to change, this in turn induces a voltage across the inductor that opposes the original change in current.

A length of wire turned into a coil works as a inductor.

Inductance (L)

For an ideal inductor:

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Here the v is the voltage difference between the inductor, and i is the current through the inductor.

The polarity is such as to oppose the change in current.

Energy stored

Assume voltage across an inductor rises from 0 to i during a time period of t seconds.

$$e = \int_0^t pdt = \int_0^t vidt = L\int_0^i idi$$

$$E = \frac{1}{2}Li^2$$

Kirchhoff Laws

Kirchhoff Current Law

The algebraic sum of all the currents entering and leaving a node is zero.

$$\sum_{\text{node}} I = 0 \implies \sum_{\text{in}} I = \sum_{\text{out}} I$$

Based on principle of conversation of charge.

Kirchhoff Voltage Law

The algebraic sum of voltages around a loop is zero.

$$\sum_{\text{node}} V = 0$$

Based on principle of conversation of energy.

Voltage division

Series connection is used to divide voltage. Potentiometeres are commonly used to create voltage divider circuits.

Current division

Parallel connection is used to divide current.

Introduction to Waves

Waveform

Obtained by plotting instantaneous values of a time-varying quantity against time.

Periodic Waveform

A pattern repeats after T time. Periodic time is T and frequency f is $\frac{1}{T}$.

Alternating Waveform

A waveform that changes in magnitude and direction with time. Is also a periodic waveform.

Sinusoidal Waves

Same as $sin\theta$ vs θ (in rad). Also called sine waves, and sinusoid.

$$y = Asin(\omega t + \phi)$$

When ϕ is:

- > 0 - the wave is said to be **leading** by ϕ
- $= 0$ - the wave is the **reference**
- < 0 - the wave is said to be **lagging** by ϕ

Sinusoidal voltages are be easily generated using rotating machines.

Complex Waveforms

Periodic non-sinusoidal waveforms can be split into its fundamental and harmonics.

Fundamental Waveform

$$f_0 = f_{\text{complex}}$$

Harmonics

Sine waves with higher frequencies which is a multiple of f_0.

$$f_{\text{harmonic}} = n \cdot f_0 \; ; \, n \in \mathbb{Z}$$

Harmonics are grouped into

- **odd harmonic** when n is odd.
- **even harmonic** when n is even.

Definitions in AC Theory

Say v is alternating as in $v = v_m sin(\omega t + \phi)$.

Peak value

Maximum instantaneous value. v_m in the example.

Peak-to-peak value

Maximum variation between maximum positive and negative instantaneous values. $2v_m$ in the example.

For a sinusoidal waveform, this is twice the peak value.

Mean value

$$v_{\text{mean}} = \frac{1}{T} \int_{T_0}^{T_0+T} v(t) \mathrm{d}t$$

Here:

- T_0 is the starting time of a cycle
- T is the periodic time

For any symmetric waveform, mean value is zero.

Average value

Mean value of the rectified version of a waveform.

For symmetric waveforms, half-cycle mean value is taken as the average value.

$$v_{\text{average}} = \frac{1}{\frac{T}{2}} \int_{T_0}^{T_0+\frac{T}{2}} v(t)\,\mathrm{d}t$$

For sinusoidal waveforms,

$$v_{\text{average}} = \frac{1}{\frac{T}{2}} \int_{T_0}^{T_0+\frac{T}{2}} v_m sin(\omega t + \phi)\,\mathrm{d}t = \frac{2}{\pi} v_m = 0.637 v_m$$

Effective value or rms (root mean square) value

$$v_{\text{rms}} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} v(t)^2\,\mathrm{d}t}$$

For sinusoidal waveforms:

$$v_{\text{rms}} = v_m \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} sin^2(\omega t + \phi)\,\mathrm{d}t} = \frac{v_m}{\sqrt{2}}$$

> **Note**
>
> i_{rms} is the equivalent current that dissipates same amount of power across a resistor R in time T as $i(t)$. Similar for voltage.

> **Note**
>
> rms value is always used to express the magnitude of a time varying quantity.

Instantaneous power

$$P = vi = iRi = i^2 R$$

Form factor

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}} = \frac{v_m}{\sqrt{2}} \times \frac{2}{\pi v_m} = 1.111$$

Peak factor

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}} \$ = v_m \times \frac{\sqrt{2}}{v_m} = 1.412$$

Phasor Representation

Phasor (phase vector) is a vector representing a sinusoidal function.

- Magnitude of the phasor: rms value of the wave
- Angle of the phasor: The angular position ϕ, with respect to a reference direction

Can also be represented by a complex number.

Representation

- Polar form: $A = |A| \angle \phi$
- Cartesian or rectangular form: $A = A_x + jA_y$

Here:

- $|A| = A_{\text{rms}} = \sqrt{A_x^2 + A_y^2}$
- $A_x = |A| \cos \phi$
- $A_y = |A| \sin \phi$
- $j = \sqrt{-1}$
- $\tan \phi = \frac{A_y}{A_x}$

Impedance & Admittance

$$Z = \frac{V}{I} = R + jX$$

Here:

- R: Resistance
- X: Reactance

Admittance (Y)

Inverse of impedance.

$$Y = \frac{1}{Z} = \frac{I}{V} = G + jB$$

Here:

- G: Conductance
- B: Susceptance

From the definitions:

$$G = \frac{R}{R^2 + X^2} \ \wedge B = -\frac{X}{R^2 + X^2}$$

For simple circuit elements

Resistor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across a resistor with resistance R. From Ohm's law:

$$v = RI_m \sin(\omega t + \phi_0)$$

> **Note**
>
> No changes in frequency, phase angle. v is in phase with i. R doesn't have reactance.

Inductor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across an inductor with inductance L.

$$v = L\omega I_m \sin\left(\omega t + \left(\phi_0 + \frac{\pi}{2}\right)\right)$$

Reactance of the inductor is $X_L = L\omega$.

> **Note**
>
> v leads i by $\frac{\pi}{2}$. No changes in frequency.

Capacitor

Let $i = I_m \sin\left(\omega t + \phi_0\right)$ is applied across an capacitor with capacitance c.

$$v = \frac{I_m}{c\omega} \sin\left(\omega t + \left(\phi_0 - \frac{\pi}{2}\right)\right)$$

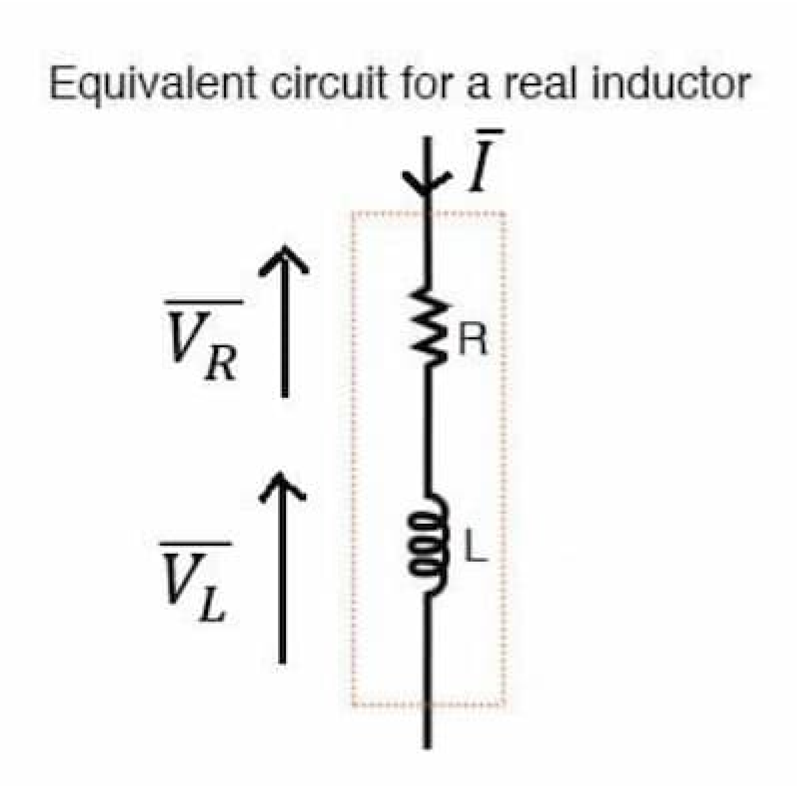
Reactance of the capacitor (capacitive reactance) is $X_c = -\frac{1}{c\omega}$.

> **Note**
>
> v lags i by $\frac{\pi}{2}$. No changes in frequency.

> **Note**
>
> If v:
>
> - lags i - circuit is capacitive
> - leads i - circuit is inductive

For complex circuit elements

Real Inductor



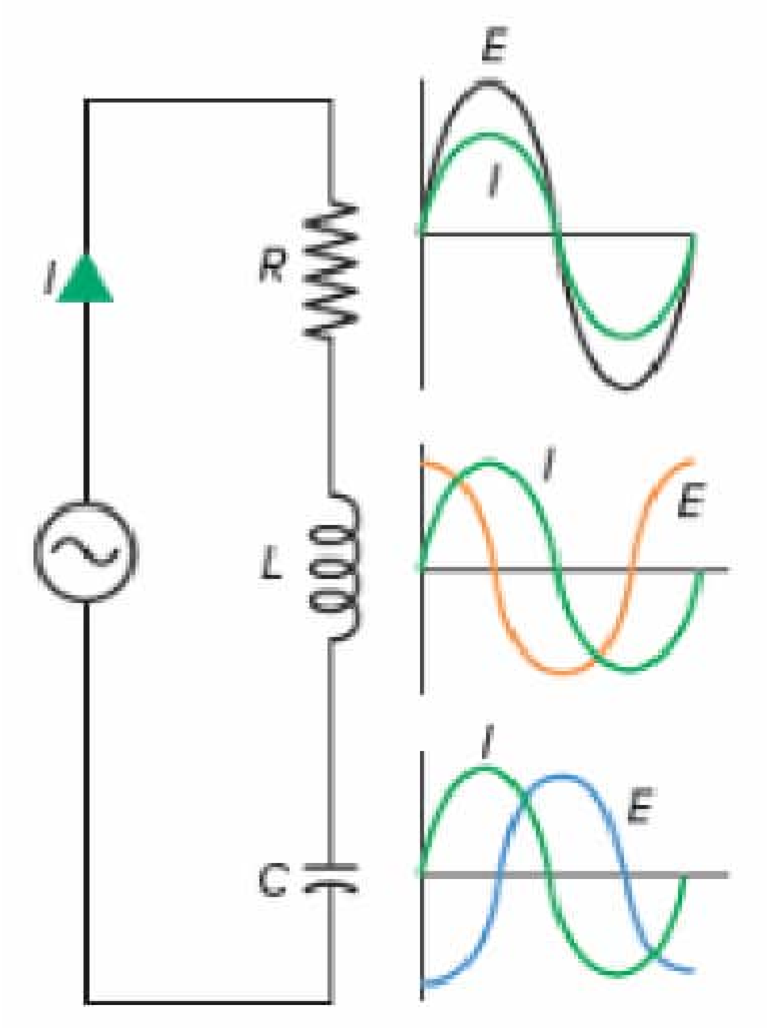
Take $\overline{I}$ as the reference. We get:

$$\overline{V} = \overline{I}(R + j\omega L)$$

From here $\overline{Z}$ can be written (in cartesian or polar form):

$$\overline{Z} = R + j\omega L = |\overline{Z}|\angle\phi$$

RLC series circuit



Complex impedances are added up to find the total impedance of a series circuit.

$$\overline{Z} = R + j(\omega L - \frac{1}{\omega C})$$

For a series circuit

Total impedance is the sum of each component's impedance.

For a parallel circuit

Total admittance is the sum of each component's admittance.

Power and Power factor

- In a purely resistive AC circuit, the energy delivered by the source will be dissipated in the form of the heat by the resistance.
- In a purely capacitive or purely inductive circuit, all of the energy will be stored during one half of each cycle, and then returned to the source during the other half cycle – there will be no net conversion to heat.
- When there is both a resistive component and a reactive component, some energy will be stored, and some will be converted to heat during each cycle.

Power of a purely resistive circuit

Suppose a circuit with load R resistance is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{R} \cos^2 (\omega t)$$

Here

- $p(t)$ is always positive
- $\text{Average power} = \frac{1}{2} \text{Peak power}$

Power of a purely inductive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{2\omega L} \sin (2\omega t)$$

Power of a purely capacitive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = -\frac{V_m^2 \omega C}{2} \sin(2\omega t)$$

Reactive Power

Power delivered to/from a pure energy storage element is known as reactive power.

- Average power consumed by a pure energy storage element is zero.
- Current associated with it is **not** 0. Transmission lines, transformers, fuses, etc. must all be designed to be capable of withstanding this current.
- Loads with energy storage elements will draw large currents and require heavy duty wiring even though little average power is consumed.
- In all electrical and electronic systems, it is the true power (the resistive power) that does the work, the reactive power simply shuttles back and forth between the source and the load.
- This means that the apparent power supplied is a combination of the true and the reactive power.

Power of a general load

Consider a general load with both resistive and reactive components. Depending on how inductive or capacitive the reactive component, the phase shift between voltage and current phasor lies between 90° and -90°.

Suppose the circuit is supplied a voltage of $v(t) = V_m \cos(\omega t)$. And the current phasor shifts in θ phase angle.

$$i(t) = I_m \cos(\omega t) - \theta$$

This ends up with:

$$p(t) = \frac{1}{2} V_m I_m \left[\cos\theta + \cos\left(\omega t - \frac{\theta}{2}\right) \right]$$

Average of over 1 cycle

$$P_{\text{avg}} = \frac{1}{T} \int_{t_0}^{t_0+T} p(t) \mathrm{d}t = \frac{1}{2} V_m I_m \cos\theta$$

Power factor

In the above equation of P_{avg}, the $\cos\theta$ is called the power factor.

Reactive power

$$Q_{\text{reactive}} = V_{\text{rms}} I_{\text{rms}} \sin\theta$$

Apparent power

$$S = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

The apparent power is essentially the effective power that the source “sees”

Power triangle

TODO