

Summary | Dynamics

Introduction

⚠️ Todo

This page is not very well organized yet. Let me know how it can be improved.

A branch of mechanics, which deals with motion of bodies.

2 parts:

- **Kinematics**: the study of geometric aspects of motion (not referencing the forces)
- **Kinetics**: the analysis of the forces that cause the motion

Kinematics of a particle

A particle has a mass and negligible size.

📌 Note

When bodies of finite size is of interest, the body might be considered as particles **provided** motion of the body is characterized by motion of its center of mass and any rotation of the body is neglected.

Rectilinear motion

When the motion of a particle is along a straight line.

Suppose x is the distance to the particle from a fixed point on its motion path.

- \dot{x} is its instantaneous velocity.
- \ddot{x} is its instantaneous acceleration.

Curvilinear motion

When the motion of a particle is along a curve (and not a straight line).

Suppose \vec{r} is the position vector of the particle from a fixed point.

- Instantaneous velocity $\mathbf{v} = \frac{d\mathbf{r}}{dt}$
- Instantaneous speed $|\mathbf{v}| = \frac{ds}{dt}$
- Instantaneous acceleration $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

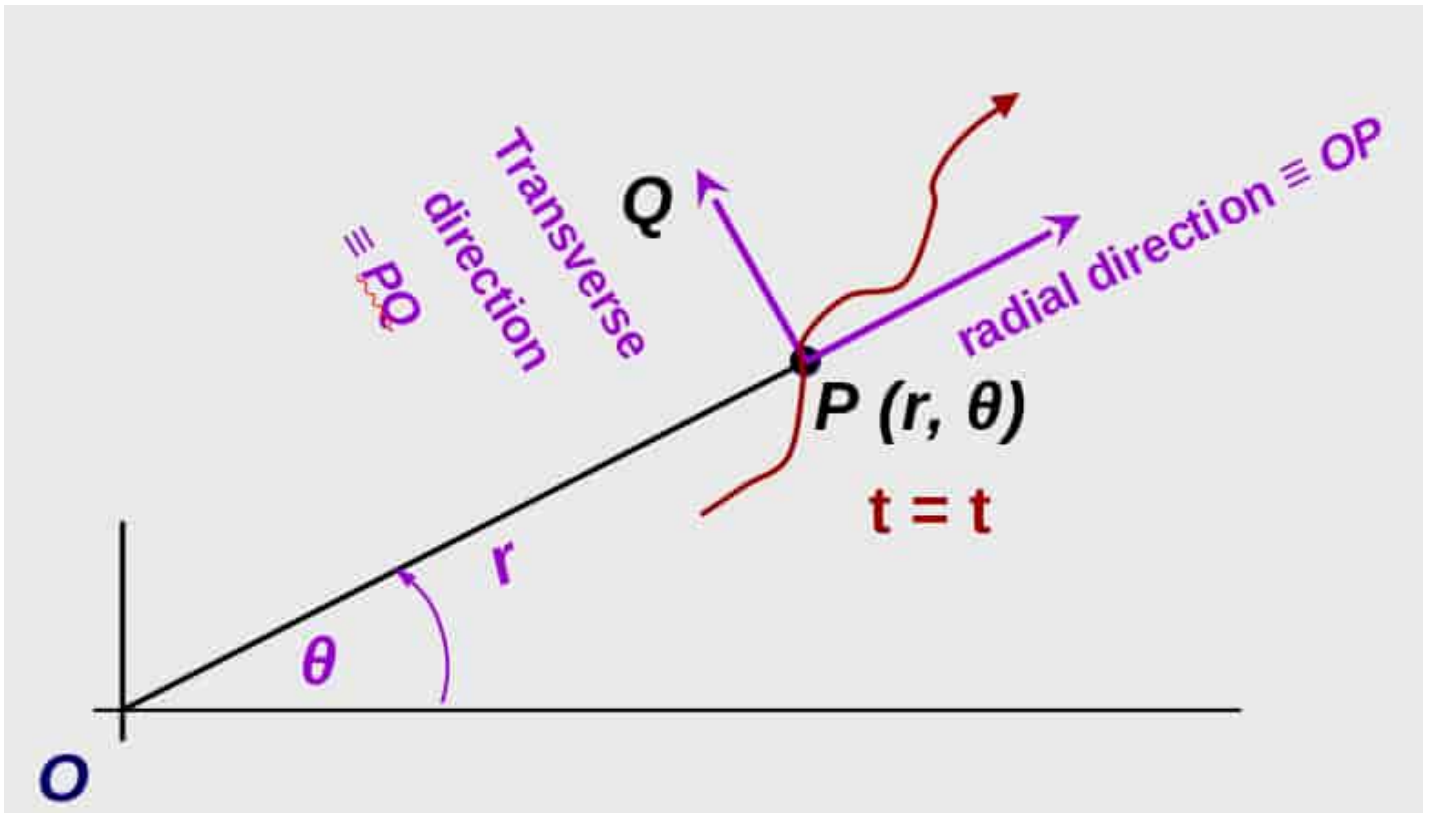
2D motion of a particle

Rectangular form

⚠ **TODO**

Finish this section

Polar form

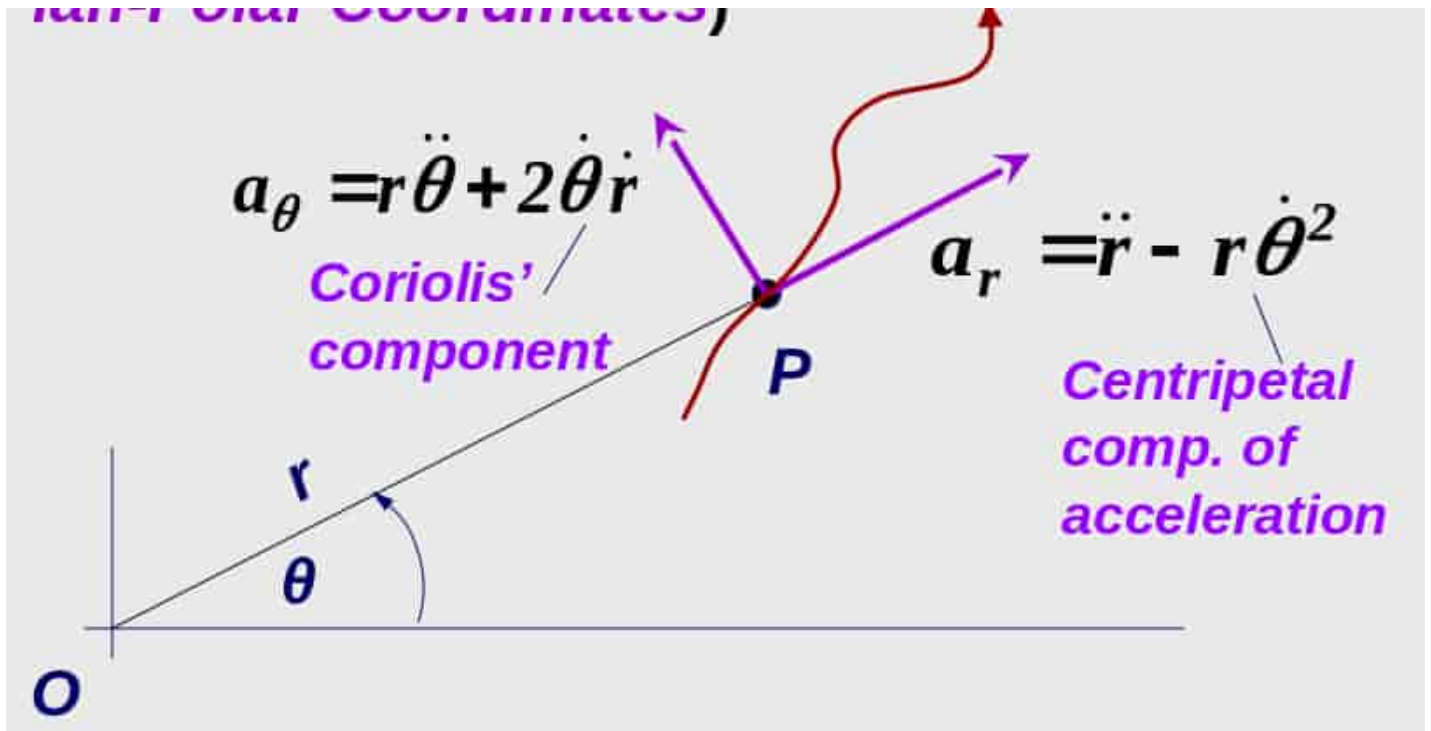


Velocities also have a transverse and radial components.

- Transverse component $v_\theta = \dot{\theta} \times r$
- Radial component $v_r = \dot{r}$

📌 **Note**

Right hand rule is used here to denote the direction of any rotary motions.



Acceleration also have a transverse and radial components.

- Transverse component
 - $a_\theta = r\ddot{\theta} + 2\dot{\theta}\dot{r}$
 - In vector equation: $\underline{a}_\theta = \underline{\ddot{\theta}} \times \underline{r} + 2(\underline{\dot{\theta}} \times \underline{\dot{r}})$
- Radial component
 - $a_r = \ddot{r} - r\dot{\theta}^2$
 - $\underline{a}_\theta = \underline{\ddot{r}} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

In the acceleration:

- **Coriolis' component of acceleration:** $2\dot{\theta}\dot{r}$
- **Centripetal component of acceleration:** $-r\dot{\theta}^2 = \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

Effects of Coriolis' component

- Objects reflect to the right in the northern hemisphere
- Objects reflect to the left in the southern hemisphere
- Maximum deflections occur at the poles. No deflection at the equator.

Unit vectors

Unit vectors in both transverse and radial directions are denoted by e_θ and e_r .

$$\dot{e}_r = \dot{\theta}e_\theta \quad \wedge \quad \dot{e}_\theta = -\dot{\theta}e_r$$

Velocity

$$v = \frac{d}{dt}(re_r) = \dot{r}e_r + r\dot{e}_r = \dot{r}e_r + r\dot{\theta}e_\theta$$

Acceleration

$$a = \frac{d}{dt}(r\dot{\theta}e_\theta) = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{\theta}\dot{r})e_\theta$$

2D kinematics of a rigid body

Rigid body

A solid body that doesn't deform.

Degrees of freedom

In the motion of a rigid body in 2D kinematics, there are 3 degrees of freedom.

- Movement along x direction
- Movement along y direction
- Rotation about z direction

In 3D, there are 6 degrees of freedom: movement and rotation along each direction.

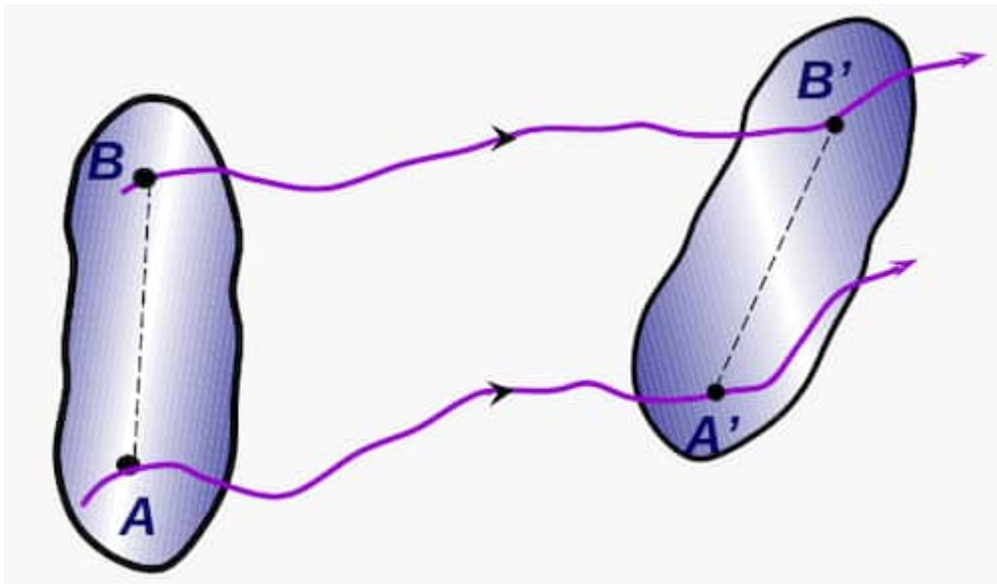
Translation

Movement that changes the position of an object. Translation can be done through a rectilinear or curvilinear path.

Rotation

Circular movement of an object about a fixed axis.

General 2D motion



$$\mathbf{v}_B = \mathbf{v}_A + \dot{\boldsymbol{\theta}} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \ddot{\boldsymbol{\theta}} \times \mathbf{r}_{B/A} + \dot{\boldsymbol{\theta}} \times (\dot{\boldsymbol{\theta}} \times \mathbf{r}_{B/A})$$

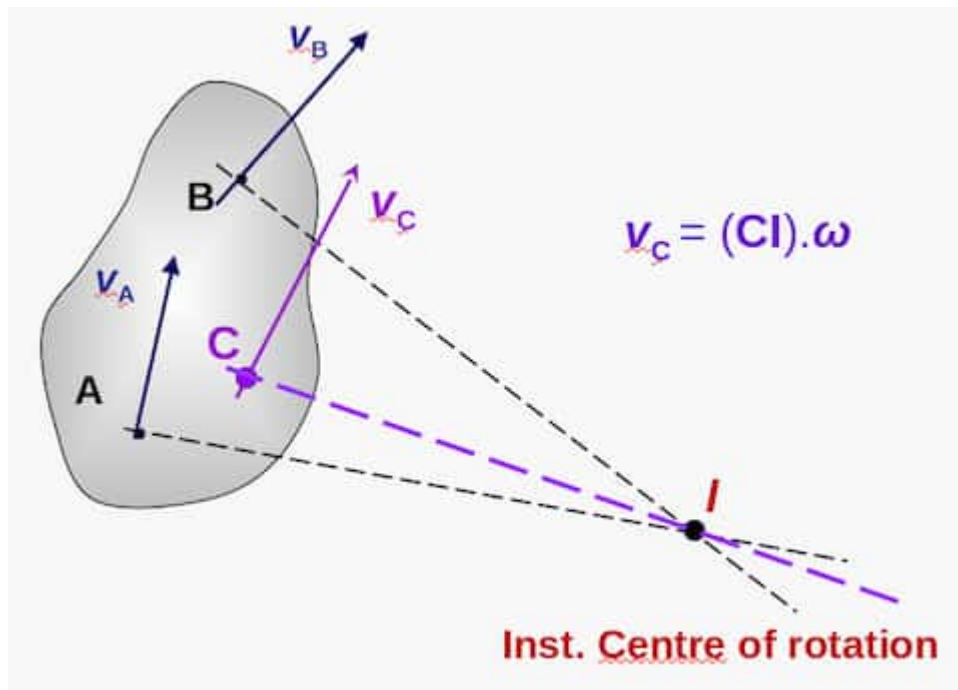
Instantaneous centre of rotation

The point that has zero velocity at a particular instant of time. This point might be changing throughout the motion. Denoted by

I

.

It can be imagined that the object is momentarily having a pure rotation about this centre I .



I can be found by drawing a line perpendicular at the velocity vectors at 2 different points and finding their intersection point.

Centrode

The locus of instantaneous centres during the motion.

Mechanisms

Mechanism

An assembly of rigid bodies or links designed to obtain a desired motion from an available motion while transmitting appropriate forces and moments. Motion of the links have definite relative motion with other links.

Simple mechanisms

- Lever
- Pulley
- Gear trains
- Belt and chain drive
- Four bar linkage

Other complex mechanisms

- Lock stitch mechanism (used in sewing machine)
- Geneva mechanism
Constant rotational motion to intermittent rotational motion. mostly used in watches.
- Scotch yoke mechanism
Constant rotational motion to linear motion (vice versa.). Mainly used as valve actuators in high pressure gas pipelines.
- Slider crank mechanism
Used in internal combustion engines

2D link mechanisms

Method of instantaneous centre of rotation

- Find the instantaneous centre of the rotation from known velocities at known points
- Use the instantaneous centre to find velocities at other points

Kinematic chain

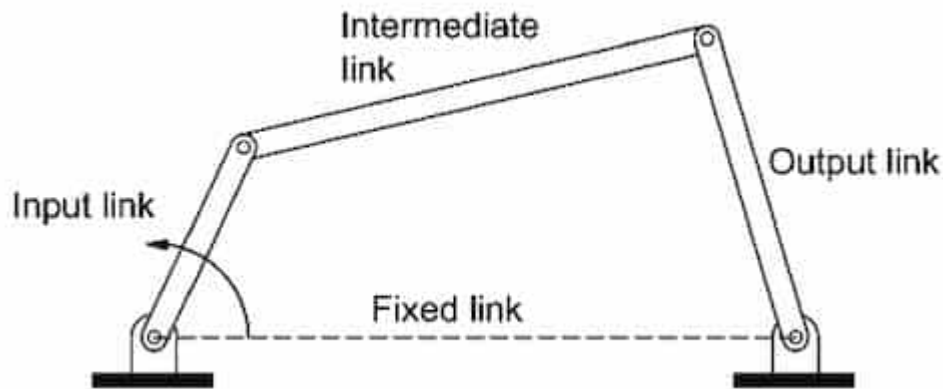
An arbitrary collection of links (forming a closed link) that is capable of relative motion and that can be made into a rigid structure by an additional single link.

Four-bar Linkage

Four bar-shaped members connected to each other in one plane.

Usually:

- 1 fixed link + 3 moving links
- 4 pin joints
- 2 moving pivots + 2 fixed pivots
- 4 turning pairs



- **input link** - usually denoted in the left.
- **output link** - usually denoted in the right.
- **coupler** - intermediate link
- **frame** - fixed link

Grashof's law

A four bar mechanism has at least one revolving link **if** $l_0 + l_3 \leq l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

Modes of motions

| Mechanism | Shortest link | Criteria |
|---------------|---------------|-----------------|
| Crank rocker | Input link | $s + l < p + q$ |
| Double crank | Fixed link | $s + l < p + q$ |
| Double rocker | Coupler link | $s + l < p + q$ |
| Change point | Any | $s + l = p + q$ |
| Triple rocker | Any | $s + l > p + q$ |

crank means a link that makes a full revolution. **rocker** means a link that doesn't make a full revolution.

Crank rocker mechanism

Shortest link rotates a full revolution. Output link oscillates.

Double crank mechanism

Shortest link is fixed. Both input and output links rotates a full revolution.

Double rocker mechanism

Shortest link make full resolution. Input and output links makes a full revolution.

Special cases

$$l_0 + l_3 = l_1 + l_2.$$

| Mechanism | Orientation |
|---|--|
| Parallelogram linkage or anti-parallelogram linkage | Equal links are opposite to each other |
| Deltoid linkage | Equal links are adjacent to each other |

Parallelogram linkage

Double crank mechanism. Opposite links are equal and parallel. Angular velocity of input crank & output crank is same. Orientation of the coupler doesn't change during the motion.

Anti-parallelogram linkage

Double crank mechanism. Angular velocity of input crank is different to output crank.

Deltoid linkage

- Longest link is fixed: crank rocker mechanism
- Shortest link is fixed: double crank mechanism

Non-Grashof's condition

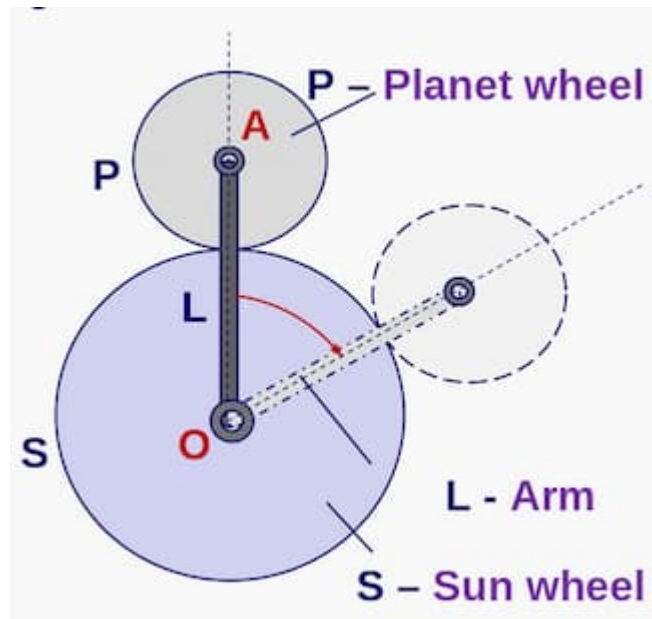
A four bar mechanism with the property **if** $l_0 + l_3 > l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

Three links are in oscillation.

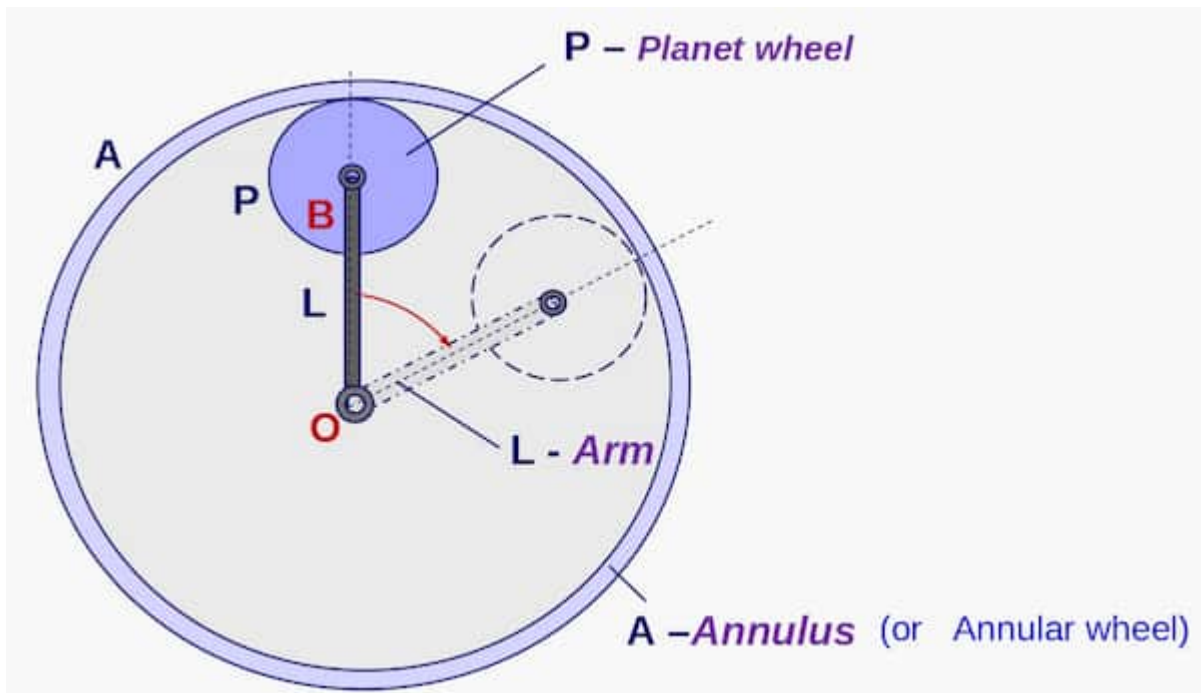
Epicyclic Gears

External



$$\omega_p = \left(1 + \frac{r_S}{r_P}\right)\omega_L - \left(\frac{r_S}{r_P}\right)\omega_S$$

Internal



$$\omega_p = \left(1 - \frac{r_A}{r_P}\right)\omega_L + \left(\frac{r_A}{r_P}\right)\omega_A$$

Mobility of Mechanisms

Lower Pair

TODO

Higher Pair

TODO

Grubler's Equation

A rigid body is constrained by a lower pair, which allows only rotational or sliding movement. It has one degree of freedom, and the two degrees of freedom are lost.

When a rigid body is constrained by a higher pair, it has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

- N - number of kinematic elements
- L - number of lower pairs
- H - number of higher pairs

$$F = 3(N - 1) - 2L - H = 1 \implies 3N - 2L + H = 4$$

Inversions of a mechanism

The inversions are obtained by making different kinematic element stationary (one at a time) while keeping the same set of kinematic pairs.