Summary | Electrical Fundamentals

Introduction

Be sure to revise the Electricity unit of G.C.E. (A/L) Physics.

Charge

- measured in Coulomb (C) = 6.25×10^{18} number of electrons
- quantized
- conserved

Time invariant charge is denoted as Q. And time varying charge is denoted as q.

Current

Amount of charges (in C) flowing through a point in unit time. Conventional current (opposite to electron flow) flows from positive to negative potentials.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

Time invariant current (DC) is denoted as I. And time varying current (AC) is denoted as i.

Voltage

Voltage at a point is the work that must be done against the electric field to move a unit positive charge from infinity to that point.

1 volt is the potential difference between 2 points when 1 joule of energy is used to move 1 coulomb of charge from one point to the other.

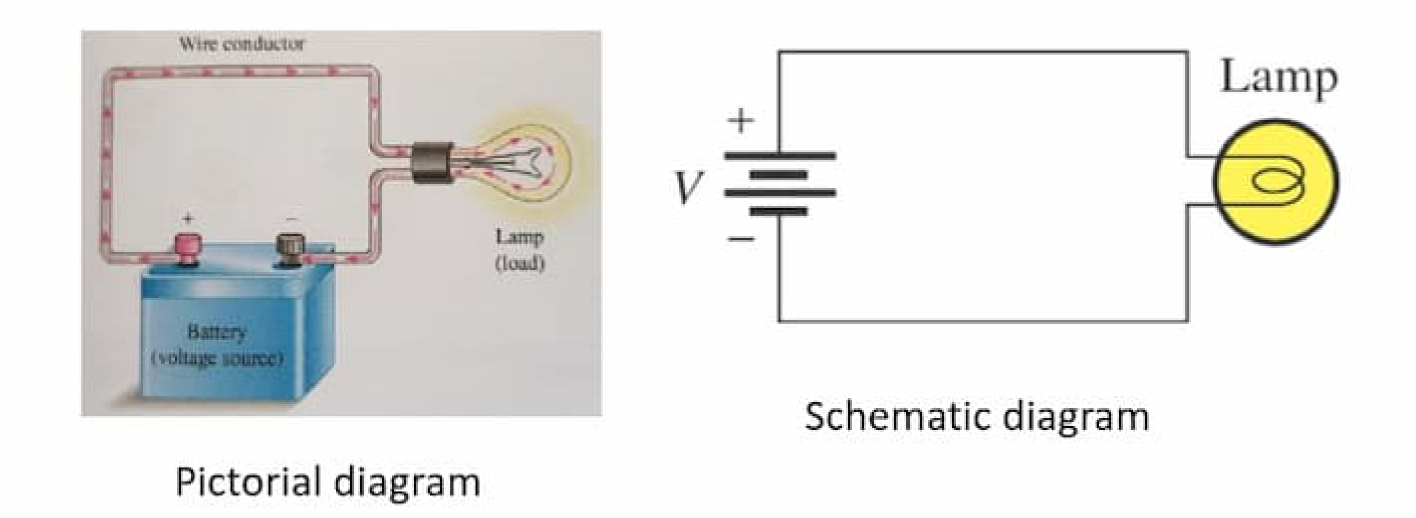
$$V = \frac{E}{Q}$$

Time invariant voltage is denoted as V. And time varying voltage is denoted as v.

Voltage difference is the work that must be done against the electric field to move a unit positive charge from one point to another.

$$V_{AB} = V_A - V_B$$

Electric Circuit



Types of circuits

- Closed circuit - the electricity flows
- Open circuit - the electricity doesn't flow. current = 0. ∞ resistance.
- Short circuit - very large current. 0 resistance.

Power

$$p = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}w}{\mathrm{d}q}\frac{\mathrm{d}q}{\mathrm{d}t} = vi$$

Total Work

$$w = \int_{t_0}^{t} p \,\mathrm{d}t = \int_{t_0}^{t} vi \,\mathrm{d}t$$

When v and i are constant

$$w = vi \int_{t_0}^{t} \mathrm{d}t = vi(t - t_0)$$

Electrical Load

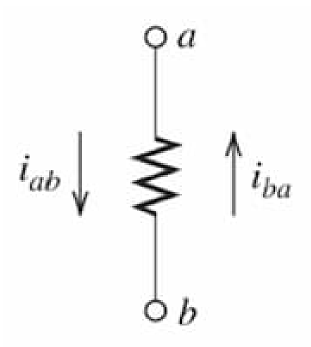
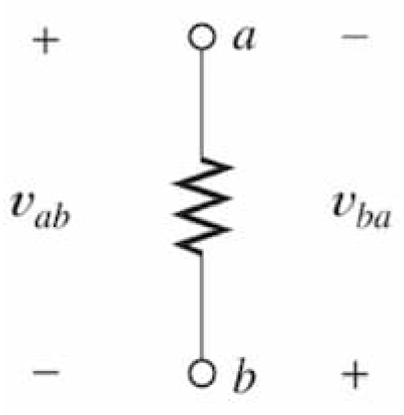
Something that consumes electrical energy.

Linear loads

Loads that can be expressed using a combination of resistors, capacitors and inductors only.

> **Note**
>
> If a AC sinusoidal voltage is applied across a load, current through the load will also be sinusoidal **iff** the load is linear.

Double subscript notation

-	Current	Voltage
Double subscript		
Equation	$i_{ab} = -i_{ba}$	$v_{ab} = -v_{ba} = v_a - v_b$
Description	Current is flowing **from point a to point b**	Voltage is **higher at point a and lower at point b**

Common Terms

Branch

A branch represents a single element, such as a resistor or a battery.

Node

A node is the point connecting more than 1 branches. Denoted by a dot.

> **Note**
>
> All points in a circuit that are connected directly by ideal conductors can be considered to be a single node.

Two terminal element

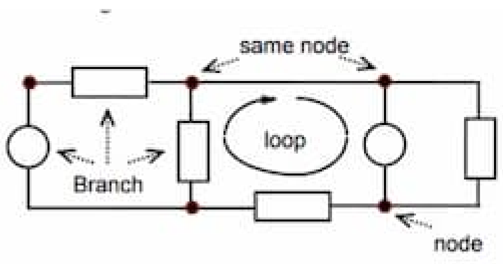
An element connected to two nodes. Branches are two terminal elements.

Loop

A loop is a closed path through a circuit in which no node is encountered more than once except for the same start/finish node.

Mesh

A mesh is a loop without having other loops inside it. Subset of loops.



Circuit elements

Two types of circuit elements.

- Active
- Passive

Active

Capable of generating electrical energy.

- Voltage sources
- Current sources

These can interchangeably be used.

Passive

Either consumes or stores electrical energy.

- Resistors
- Inductors
- Capacitors
- Any other elements

Voltage sources

- Batteries - electrochemical
- Solar cells - photo voltaic
- Generators - electromagnetic

Ideal voltage source

Constant voltage for any required currents. Does not exist.

Resistors

Resistance, in terms of physical dimensions:

$$R = \frac{\rho l}{A}$$

Here:

- l: length
- A: cross-sectional area
- ρ: resistivity

If a voltage V is applied across a conductor, then a given current I will flow through the conductor $V \propto I$. The proportionality constant is called resistance R.

$$R = \frac{V}{I}$$

Capacitors

Made of two conductive plates separated by an insulating (dielectric) layer.

Capacitance (C), in terms of physical dimensions:

$$C = \frac{\epsilon A}{d}$$

Here:

- d: distance between the plates
- A: area of a plate

In an ideal capacitor, the charge imbalance Q is proportional to the voltage V across the plates.

$$Q = CV$$

v and i

As C is constant, current i passing through the capacitor and the voltage v across the capacitor are related by:

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$$

Energy stored

Suppose voltage across an initially uncharged capacitor rises from 0 to V during a time period of t.

$$e = \int_0^t p\,dt = \int_0^t vi\,dt = C\int_0^v v\,dv$$

$$E = \frac{1}{2}CV^2$$

Inductors

When there is a current in the inductor, a magnetic field is created. Any change in current causes the magnetic field to change, this in turn induces a voltage across the inductor that opposes the original change in current.

A length of wire turned into a coil works as a inductor.

Inductance (L)

For an ideal inductor:

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Here the v is the voltage difference between the inductor, and i is the current through the inductor.

The polarity is such as to oppose the change in current.

Energy stored

Assume voltage across an inductor rises from 0 to i during a time period of t seconds.

$$e = \int_0^t p\,dt = \int_0^t vi\,dt = L\int_0^i i\,di$$

$$E = \frac{1}{2}Li^2$$

Kirchhoff Laws

Kirchhoff Current Law

The algebraic sum of all the currents entering and leaving a node is zero. Based on principle of conversation of charge.

$$\sum_{\text{node}} I = 0 \implies \sum_{\text{in}} I = \sum_{\text{out}} I$$

Kirchhoff Voltage Law

The algebraic sum of voltages around a loop is zero. Based on principle of conversation of energy.

$$\sum_{\text{node}} V = 0$$

Voltage division

Series connection is used to divide voltage. Potentiometeres are commonly used to create voltage divider circuits.

Current division

Parallel connection is used to divide current.

Introduction to Waves

Waveform

Obtained by plotting instantaneous values of a time-varying quantity against time.

Periodic Waveform

A pattern repeats after T time. Periodic time is T and frequency f is $\frac{1}{T}$.

Alternating Waveform

A waveform that changes in magnitude and direction with time. Is also a periodic waveform.

Sinusoidal Waves

Same as $\sin\theta$ vs θ (in rad). Also called sine waves or sinusoid.

$$y = Asin(\omega t + \phi)$$

When ϕ is:

- > 0 - the wave is said to be **leading** by ϕ
- $= 0$ - the wave is the **reference**
- < 0 - the wave is said to be **lagging** by ϕ

Sinusoidal voltages are be easily generated using rotating machines.

Complex Waveforms

Periodic non-sinusoidal waveforms can be split into its fundamental and harmonics.

Fundamental Waveform

$$f_0 = f_{\text{complex}}$$

Harmonics

Sine waves with higher frequencies which is a multiple of f_0.

$$f_{\text{harmonic}} = n \cdot f_0 \; ; n \in \mathbb{Z}$$

Harmonics are grouped into:

- **odd harmonic** when n is odd.
- **even harmonic** when n is even.

Definitions in AC Theory

Note

Only sinusoidal AC supply are considered in s1.

Say v is alternating as in $v = V_m sin(\omega t + \phi)$.

Peak value

Maximum instantaneous value. V_m in the example.

Peak-to-peak value

Maximum variation between maximum positive and negative instantaneous values. $2V_m$ in the example.

For a sinusoidal waveform, this is twice the peak value.

Mean value

$$v_{\text{mean}} = \frac{1}{T} \int_{T_0}^{T_0+T} v(t) \mathrm{d}t$$

Here:

- T_0 is the starting time of a cycle
- T is the periodic time

For any symmetric waveform, mean value is 0.

Average value

Mean value of the rectified version of a waveform.

For symmetric waveforms, half-cycle mean value is taken as the average value.

$$v_{\text{average}} = \frac{2}{T} \int_{T_0}^{T_0+\frac{T}{2}} v(t)\,\mathrm{d}t$$

For sinusoidal waveforms, from the example:

$$v_{\text{average}} = \frac{2}{T} \int_{T_0}^{T_0+\frac{T}{2}} V_m sin(\omega t + \phi)\,\mathrm{d}t$$

$$= \frac{2}{\pi} V_m = 0.637 V_m$$

Effective value or rms (root mean square) value

$$v_{\text{rms}} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} v(t)^2\,\mathrm{d}t}$$

For sinusoidal waveforms:

$$v_{\text{rms}} = V_m \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} sin^2(\omega t + \phi)\,\mathrm{d}t} = \frac{V_m}{\sqrt{2}}$$

Note

i_{rms} is the equivalent current that dissipates same amount of power across a resistor R in time T as $i(t)$. Similar for voltage.

> **Note**
>
> rms value is always used to express the magnitude of a time varying quantity.

Instantaneous power

$$P = vi = i^2 R$$

Form factor

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}} = \frac{V_m}{\sqrt{2}} \times \frac{2}{\pi V_m} = 1.111$$

Peak factor

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}} = V_m \times \frac{\sqrt{2}}{V_m} = 1.412$$

Phasor Representation

Phasor (phase vector) is a vector representing a sinusoidal function.

- Magnitude of the phasor: rms value of the wave
- Angle of the phasor: The angular position ϕ, with respect to a reference direction

Can also be represented by a complex number.

Representation

- Polar form: $A = |A| \angle \phi$
- Cartesian or rectangular form: $A = A_x + jA_y$

Here:

- $|A| = A_{\text{rms}} = \sqrt{A_x^2 + A_y^2}$
- $A_x = |A| \cos \phi$
- $A_y = |A| \sin \phi$
- $j = \sqrt{-1}$
- $\tan \phi = \frac{A_y}{A_x}$

Impedance & Admittance

Impedance (Z)

$$Z = \frac{V}{I} = R + jX$$

Here:

- R : Resistance
- X : Reactance

Admittance (Y)

Inverse of impedance.

$$Y = \frac{1}{Z} = \frac{I}{V} = G + jB$$

Here:

- G : Conductance
- B : Susceptance

From the definitions:

$$G = \frac{R}{R^2 + X^2} \ \wedge B = -\frac{X}{R^2 + X^2}$$

For simple circuit elements

Resistor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across a resistor with resistance R. From Ohm's law:

$$v = RI_m \sin(\omega t + \phi_0) \implies Z_R = R$$

No changes in frequency, phase angle. v is in phase with i. R doesn't have reactance.

Inductor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across an inductor with inductance L.

$$v = L\omega I_m \sin(\omega t + (\phi_0 + \frac{\pi}{2})) \implies Z_L = j\omega L$$

Reactance of the inductor is $X_L = L\omega$.

Note

v leads i by $\frac{\pi}{2}$. No changes in frequency.

Capacitor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across an capacitor with capacitance c.

$$v = \frac{I_m}{c\omega} \sin(\omega t + (\phi_0 - \frac{\pi}{2})) \implies Z_C = -j\frac{1}{c\omega}$$

Reactance of the capacitor (capacitive reactance) is $X_c = -\frac{1}{c\omega}$.

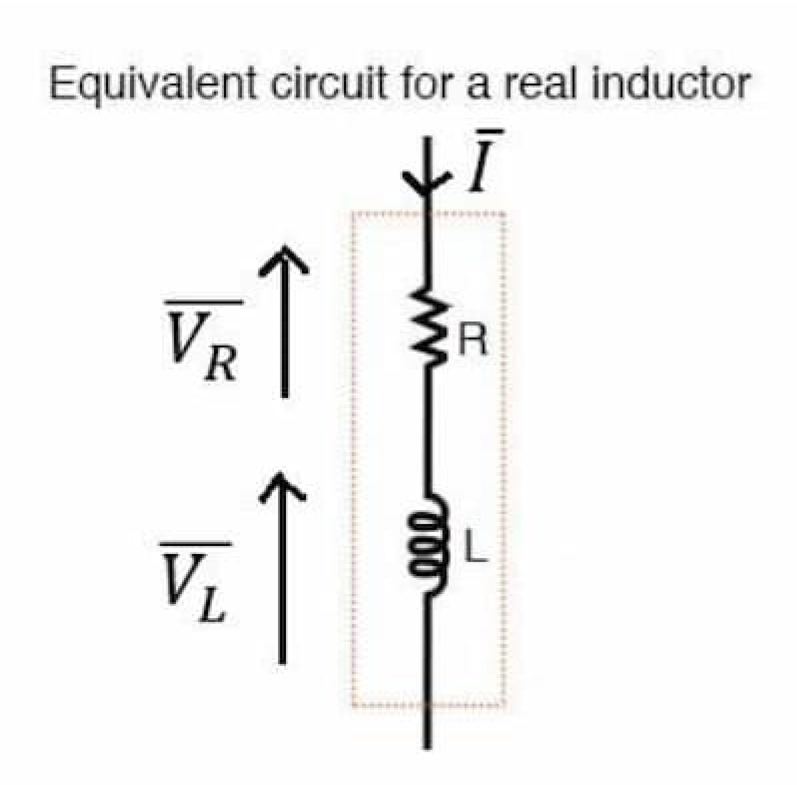
Note

v lags i by $\frac{\pi}{2}$. No changes in frequency.

> **Note**
>
> If v:
>
> - lags i - circuit is capacitive
> - leads i - circuit is inductive

For complex circuit elements

Real Inductor



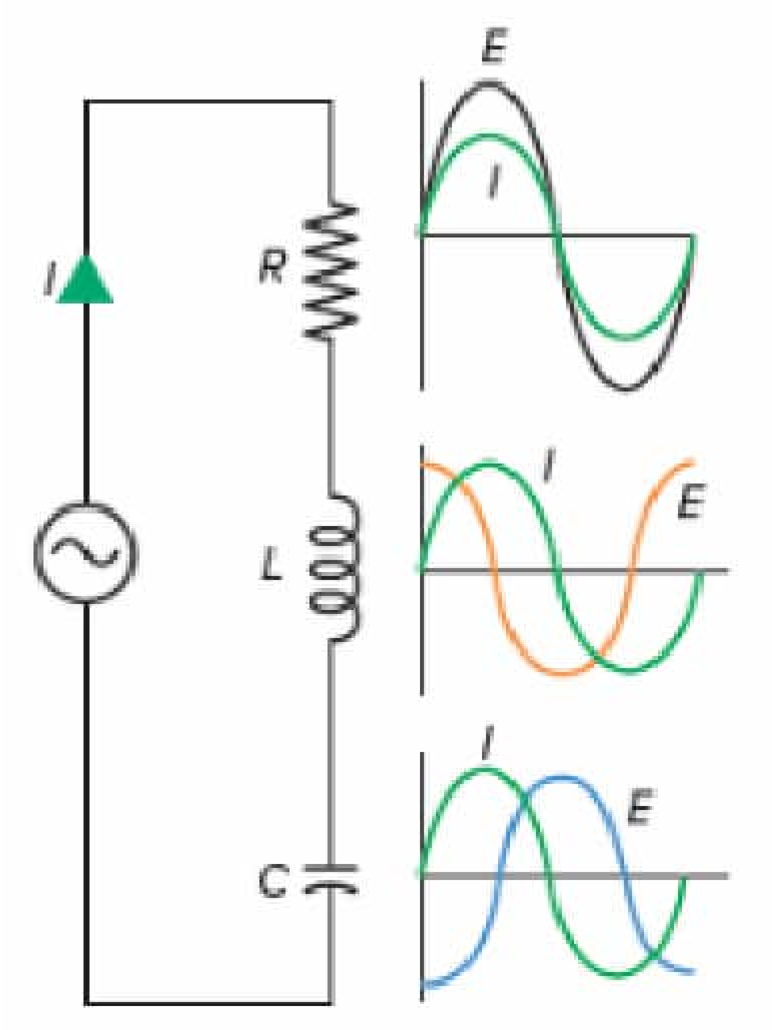
Take $\overline{I}$ as the reference. We get:

$$\overline{V} = \overline{I}(R + j\omega L)$$

From here $\overline{Z}$ can be written (in cartesian or polar form):

$$\overline{Z} = R + j\omega L = |\overline{Z}|\angle\phi$$

RLC series circuit



Complex impedances are added up to find the total impedance of a series circuit.

$$\overline{Z} = R + j(\omega L - \frac{1}{\omega C})$$

For a series circuit

Total impedance is the sum of each component’s impedance.

For a parallel circuit

Total admittance is the sum of each component’s admittance.

Power and Power factor

- In a purely resistive AC circuit, the energy delivered by the source will be dissipated in the form of the heat by the resistance.
- In a purely capacitive or purely inductive circuit, all of the energy will be stored during one half of each cycle, and then returned to the source during the other half cycle – there will be no net conversion to heat.
- When there is both a resistive component and a reactive component, some energy will be stored, and some will be converted to heat during each cycle.

Power equations

Purely resistive circuit

Suppose a circuit with load R resistance is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{R} \cos^2 (\omega t)$$

Always: $p(t) > 0$.

$$\text{Average power} = \frac{1}{2} \text{Peak power} = \frac{V_m^2}{2R}$$

Purely inductive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{2\omega L} \sin (2\omega t)$$

Purely capacitive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = -\frac{V_m^2 \omega C}{2} \sin(2\omega t)$$

Power of a general load

Consider a general load with both resistive and reactive components. Depending on how inductive or capacitive the reactive component, the phase shift between voltage and current phasor lies between 90° and -90°.

Suppose the circuit is supplied a voltage of $v(t) = V_m \cos(\omega t)$. And the current phasor shifts in θ phase angle.

$$i(t) = I_m \cos(\omega t - \theta)$$

This ends up with:

$$p(t) = \frac{1}{2} V_m I_m \left[\cos\theta + \cos\left(\omega t - \frac{\theta}{2}\right)\right]$$

Average of over 1 cycle

$$P_{\text{avg}} = \frac{1}{T} \int_{t_0}^{t_0+T} p(t)\,\mathrm{d}t = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

Types of power

Reactive Power

Power delivered to/from a pure energy storage element is known as reactive power.

- Average power consumed by a pure energy storage element is zero.
- Current associated with it is **not** 0. Transmission lines, transformers, fuses, etc. must all be designed to be capable of withstanding this current.
- Loads with energy storage elements will draw large currents and require heavy duty wiring even though little average power is consumed.
- In all electrical and electronic systems, it is the true power (the resistive power) that does the work, the reactive power simply shuttles back and forth between the source and the load.
- This means that the apparent power supplied is a combination of the true and the reactive power.

$$Q_{\text{reactive}} = V_{\text{rms}} I_{\text{rms}} \sin\theta$$

Active power

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

Apparent power

$$S = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

The apparent power is essentially the effective power that the source “sees”

Power factor

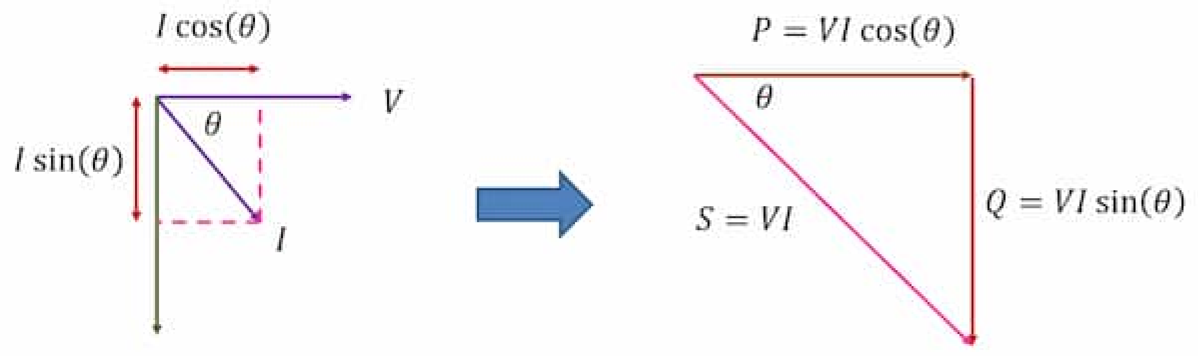
In the above equation of P_{avg}, the $\cos\theta$ is called the power factor.

$$\cos\theta = \frac{\text{Active power}}{\text{Apparent power}}$$

Power factor is:

- leading when I leads V
- lagging when I lags V

Power triangle



- Take V phasor as the reference.
- Draw V and I phasors.

Power systems

An electric power system consists of 3 principle sections

- Power stations: electricity is generated
- Transmission: voltage is stepped to high voltage
- Distribution: voltage is stepped down to medium voltage for distribution over a relatively small region

Variable load

Load on a power station changes with to uncertain demands of consumers. This is called the **variable load**.

Load vs time curve is called the **load curve**. Area under this curve is the **total energy requirement**.

Power grid

Nation-wide, massive, geographically distributed system for electrical power supply network.

Sri Lankan Voltage Levels

- High voltage $220\ \mathrm{kV}$
- Medium voltage $11\ \mathrm{kV}$
- Nominal voltage $230\ \mathrm{V}$
- Nominal line-to-line $400\ \mathrm{V}$

3-Phased System

Why 3-phase?

- The current can be distributed into 3 wires instead of just 1.
  There is a maximum limit of how much current a wire can carry.
- Economical as less amount of wires.
  3-phase system requires 4 wires (3 if balanced) while single phase system requires 6.

The phases are denoted by $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ in that order.

Balanced 3-phase

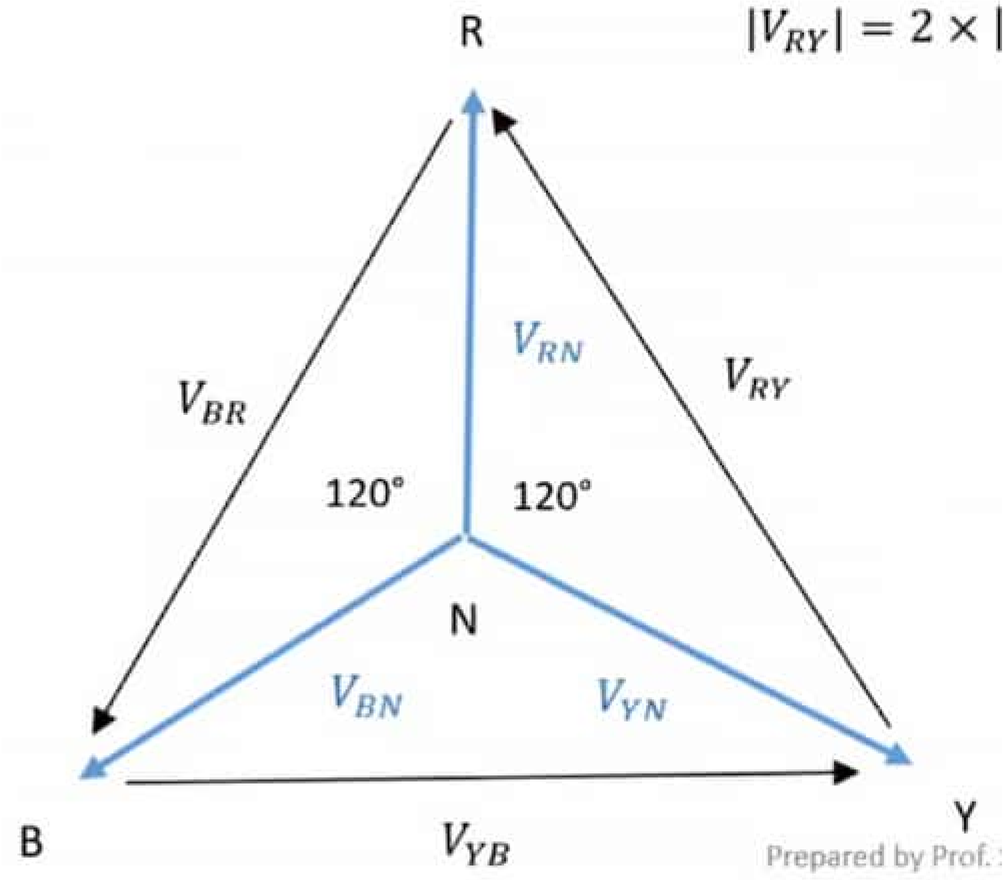
A 3-phase system is said to be balanced **iff**:

- Supply is balanced
- Loads are the same in each phase

Power source

A 3-phase power source which produces 3 phase voltages of equal rms value, but with 120° phase difference.

Phasor diagram



Phase voltage

Voltage between a phase wire and the neutral wire.

V_{RN}, V_{YN}, V_{BN} are the phase voltages.

Line-to-line voltage

Voltage between any 2 phase wires. Line-to-line voltages also have a 120° phase difference.

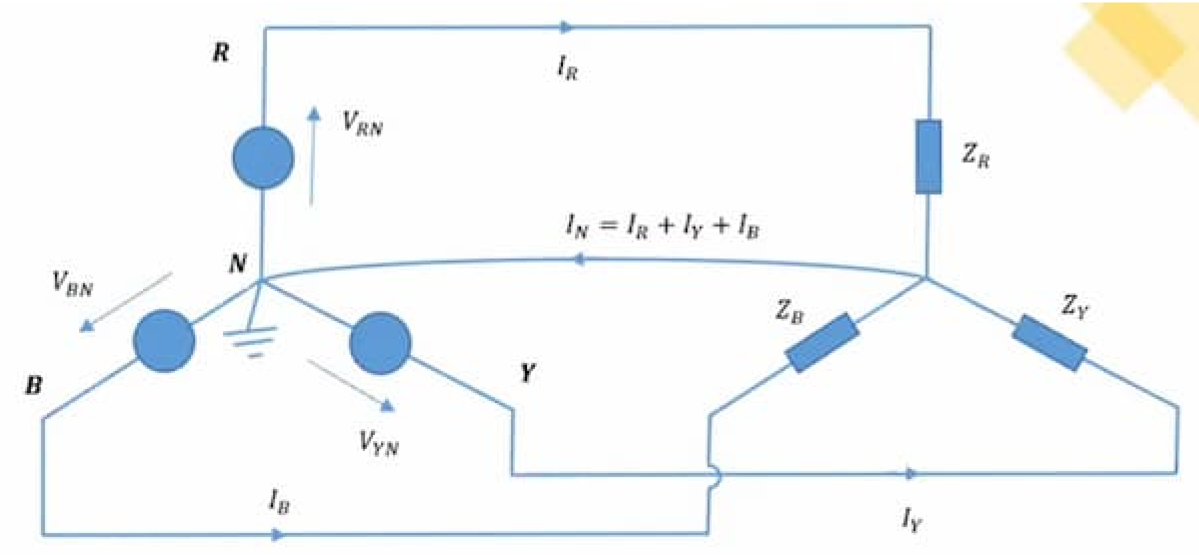
V_{RY}, V_{YB}, V_{BR} are the line-to-line voltages or line voltages.

$$\left|V_{\mathrm{BR}}\right| = 2 \times \left|V_{\mathrm{BN}}\right| \cos(30^{\circ}) = \sqrt{3}\left|V_{\mathrm{BN}}\right|$$

ⓘ **Note**

In a 3-phase system, line-to-line voltage is mentioned.

Analysis



$$I_N = E\left[\frac{1\angle 0^\circ}{z_R} + \frac{1\angle -120^\circ}{z_Y} + \frac{1\angle 120^\circ}{z_B}\right]$$

When the loads are balanced: $z_R = z_Y = z_B = z$, $I_N = 0$
In this case, neutral wire can be eliminated (it’s optional). We need to maintain $I_N = 0$ so that the voltage is equal to ground voltage in neutral wire. This makes sure there are no power losses in neutral wire.

Real-life Usage

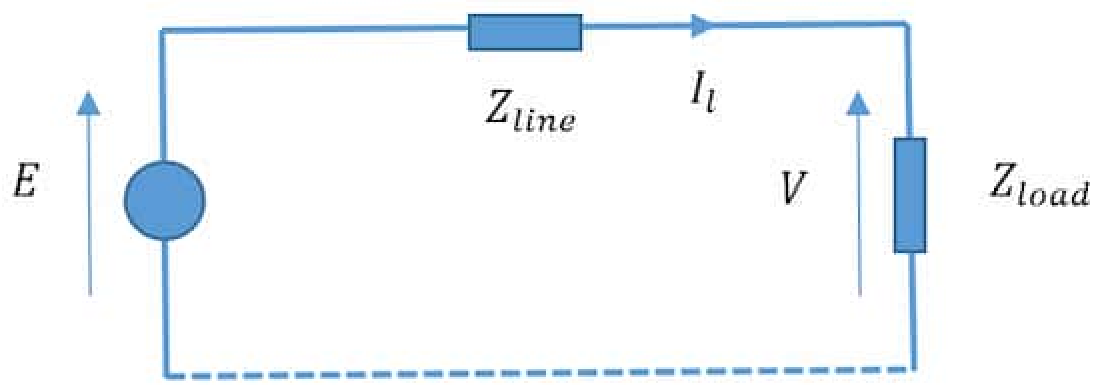
Most domestic loads are single-phase. In case of 3-phase domestic wiring, the single-phase loads are distributed among the 3 phases at the main distribution board.

Devices that have a 3-phase power input, doesn’t require a neutral line.

Per-phase Equivalent Circuit

Power, voltage, current, power factor are same for all 3 phases.

When a 3-phase system is balanced, it is sufficient to consider only a single phase. The diagram showing the single-phase equivalent of the power system using standard symbols.



Here:

- E - voltage across the source
- V - voltage across the load

$$\text{Per-phase power} = |V_p||I_l|\cos\theta = \frac{1}{3} \times \text{3-phase power} \quad \wedge \quad |V_l| = \sqrt{3}|V_p|$$

$$\implies \text{3-phase power} = \sqrt{3}|V_l||I_l|\cos\theta$$

Here:

- V_p - phase voltage
- V_l - line voltage
- I_l - line current
- $\cos\theta$ - power factor
- The power can either be source power, load power, transmission power losses.

Unbalanced 3-phase system

A 3-phase system becomes unbalanced, when load distribution among the phases is equal. $I_N \neq 0$.