

# Summary | Statics

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## Introduction

### Centroid / Centre of area

The point where the area of a figure is assumed to be concentrated.

Located on the axes of symmetry.

### First moment of area

Measure of spatial distribution of a shape in relation to an axis.

$$\text{About x-axis} = \int_A y \, dA = A\bar{x}$$

$$\text{About y-axis} = \int_A x \, dA = A\bar{y}$$

Here:

- $\bar{x}$  - Centroid's  $x$  coordinate
- $\bar{y}$  - Centroid's  $y$  coordinate
- $A$  - Total area

About an axis of symmetry, first moment of area is 0.

### Second moment of area

$$\text{About x-axis} = I_{xx} = I_x = \int_A y^2 \, dA$$

$$\text{About y-axis} = I_{yy} = I_y = \int_A x^2 \, dA$$

Always positive.

The product of moment of area about x,y axes

$$I_{xy} = \int_A xy \, dA$$

The polar moment of area about z axis

$$I_{zz} = J_0 = \int_A r^2 \, dA = I_{xx} + I_{yy}$$

Radius of gyration

About x-axis  $= r_x^2 = \frac{I_{xx}}{A}$

About y-axis  $= r_y^2 = \frac{I_{yy}}{A}$

About z-axis  $= r_z^2 = \frac{I_{zz}}{A}$

Derived Formulas for Common Shapes

Shape	Description	
Rectangle or Parallelogram	Base $b$ . Height $h$ . About centroidal axis parallel to base.	$\frac{bh^3}{12}$
Triangle	Base $b$ . Height $h$ . About base.	$\frac{bh^3}{12}$
Triangle	Base $b$ . Height $h$ . About centroidal axis parallel to	$\frac{bh^3}{36}$

Shape	Description	$I_{xx}$
	base.	
Circle	Diameter $d$ . About centroidal axis.	$\frac{\pi d^4}{64}$

## Parallel Axis Theorem

$$I_x = I_{x_1} + A\bar{y}^2$$

$$I_y = I_{y_1} + A\bar{x}^2$$

$$I_{xy} = I_{x_1y_1} + A\bar{x}\bar{y}$$

Here

- On LHS, the moments of area are about some  $x$ ,  $y$  axes.
- On RHS, the moments of area are about centroidal axes  $x_1$ ,  $y_1$  parallel to  $x$ ,  $y$ .
- $\bar{x}$  is the distance between  $x$  and  $x_1$  axes.
- $\bar{y}$  is the distance between  $y$  and  $y_1$  axes.

### **Note**

$I_x$  is at a minimum when the axis is through the centroid. Same for  $I_y$ .

## Perpendicular Axis Theorem

$$I_{zz} = I_{xx} + I_{yy} = I_{mm} + I_{nn}$$

$x$ ,  $y$ ,  $z$  are a set of axes.  $m$ ,  $n$ ,  $z$  are another set of axes.

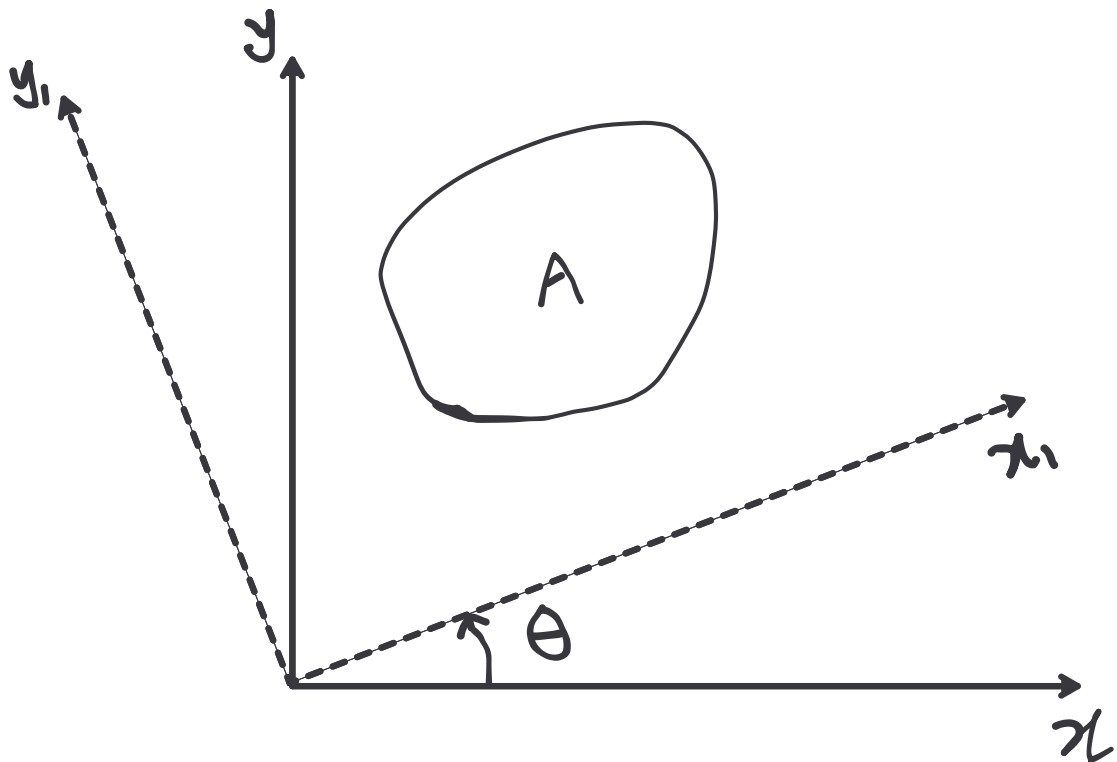
If  $I_{xx}$  is at maximum,  $I_{yy}$  will be at minimum.

# Transformation Law

The 2 sets of axes must share the origin.

## **Note**

Don't have to memorize this. Will be given on exams, if required.



$$I_{x_1x_1} = \frac{I_{xx} + I_{yy}}{2} + \left( \frac{I_{xx} - I_{yy}}{2} \right) \cos 2\theta - I_{xy} \sin 2\theta$$

$$I_{y_1y_1} = \frac{I_{xx} + I_{yy}}{2} - \left( \frac{I_{xx} - I_{yy}}{2} \right) \cos 2\theta + I_{xy} \sin 2\theta$$

$$I_{x_1y_1} = \left( \frac{I_{xx} - I_{yy}}{2} \right) \sin 2\theta + I_{xy} \cos 2\theta$$

# Principal Axes

The product of moment of area is 0 about principal axes.

$$I_{xy} = 0$$

There will be 2 directions of principal axes which are perpendicular to each other.

## **Note**

For a shape with more than 2 axis of symmetry, all axes through the centroid is a principal axis.

## Principal second moments of area

Second moments about the principal axes.

About principal axes second moments of area will be at minimum and maximum.

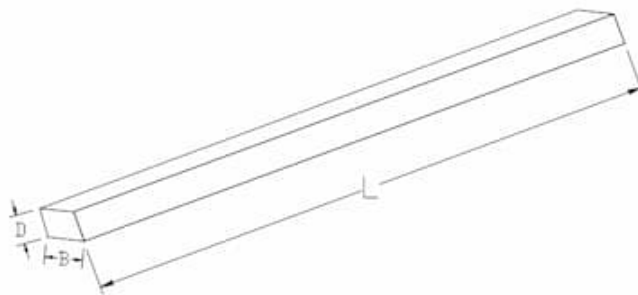
## Centroidal principal axes

Principal axes through the centroid.

## **Note**

Any axis of symmetry is a centroidal principal axis.

## Beams



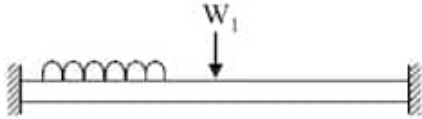
- long (  $L \gg B, D$  )
- axis of the beam is straight
- constant cross-section throughout its length

## Classified by supporting conditions

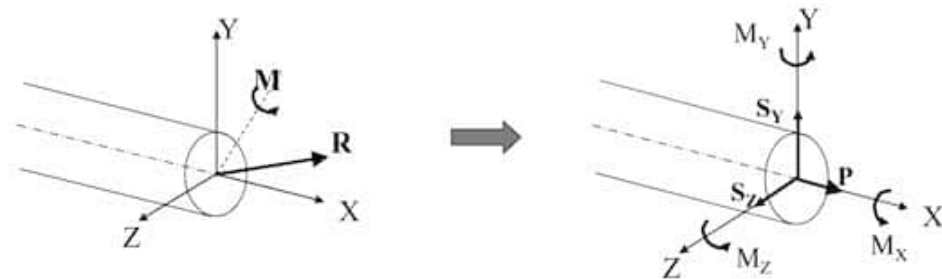
First 3 are the mandatory ones in s1.

u.d.l means uniformly distributed load.

Type	Image
Simply supported beam	
Cantilevered beam	
Overhanging beam	
Propped cantilevered beam	
Continuous beam	

Type	Image
Fixed beam	

## At a section



- $P$  - Normal force / Axial force
- $S_y, S_z$  - Shear forces along  $y$  and  $z$  axis
- $M_x$  - Twisting moment / Torque
- $M_y, M_z$  - Bending moments about  $y$  and  $z$  axis

## Degree of freedom

A plane member have 3 degrees of freedom. Any of the 3 can be restrained.

- Displacement in  $x$  -direction
- Displacement in  $y$  -direction
- Rotation about  $z$  -direction

## SFD & BMD

### Sign convention

- Bending moment
  - Hogging (curves upwards in the middle) is **(+) ve**
  - Sagging (curves downwards in the middle) is **(-) ve**
- Shear force
  - Clockwise shear is **(+) ve**.
  - Counterclockwise shear is **(-) ve**.

### Pure bending

A member is in pure bending when shear force is 0 and bending moment is a constant.

### Point of Contraflexure

The point about which bending moment is 0, and changes its sign through the point.

## Distributed load, shear force & bending moment

Suppose a beam is under a distributed load of  $w = f(x)$  per unit length.

$$\frac{dS}{dx} = -w$$

$$\frac{dM}{dx} = -S \quad \wedge \quad \frac{d^2M}{dx^2} = w$$

### Deflection of a beam

Suppose a simply supported beam is applied a load of  $W$  at mid-span.

$$S_{\max} = \frac{WL}{4I} \quad \wedge \quad D_{\max} = \frac{WL^3}{48EI}$$

Here:



- $S_{\max}$  - Maximum stress
- $D_{\max}$  - Deflection
- $W$  - Load
- $L$  - Span length
- $E$  - Young's modulus
- $I$  - Second moment of cross-sectional area

## Principle of Superposition

A beam with multiple loads can be split into multiple systems each with a single load. Reason for doing so is the ease of calculations.

Values will be the sum of each system's corresponding value.

## Structural Elements

3 types:

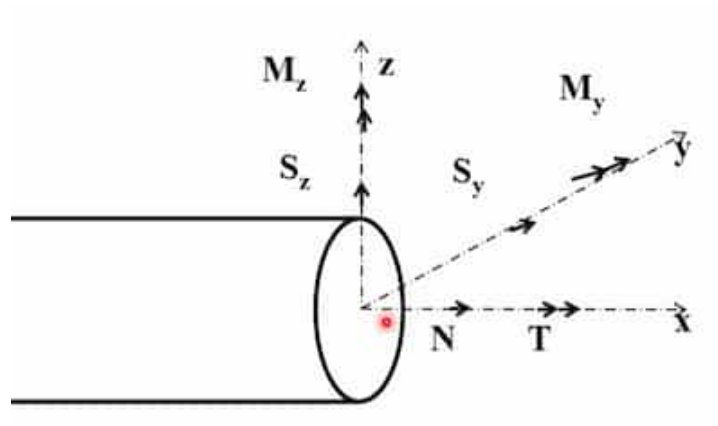
- Bars (1D)
- Plates and Shells (2D)
- Blocks (3D)

Bar elements are only focused for s1.

### Pin Joint

Doesn't exert a moment. Free rotations are allowed. When only pin joints are used, bars will have only axial forces.

## Bars



Here

- $N$  - Axial force
- $S_x, S_y$  - Shear force
- $M_x$

## Types of bars

### Axially loaded

Generally in trusses, **pin joints** are considered.

- Predominant tension - Ties
- Predominant compression - Struts

### Flexural

- Predominant bending - beams

### Torsional

- Predominant torque - shafts

## Trusses

Also known as Ties-Struts model.

## Definition

An assembly of members used to span long distances. Idealized as

- Connected by **frictionless pin joints** at their ends
- Developing axial forces

## Types

2 types:

- Plane truss (2D)
- Space truss (3D)

A truss requires 3 external reactions for equilibrium.

Predominant force is axial force.

## Advantages of truss

- High span
- Material efficiency

## Triangulation

To create a truss:

- Start with a triangle ( **3** bars and **3** joints)
- Add **2** more bars and **1** joint repeatedly

This type of truss is a **simple truss**.

## Simple (Closed) Truss

When a truss is only made of bars and joints.

## Open Truss

When a truss is pinned directly to a foundation. It has 1 member & 2 free joints less than a closed truss.

## Stability of trusses

When a truss is:

- unstable: it's called a mechanism
- stable: it's called a structure

### Stable truss

When the shape cannot be altered, the structure is **internally stable**.

### Stable & determinate (simply stiff)

**Determinate** means internal forces can be determined by laws of statics alone.

### Stable & indeterminate

**Indeterminate** means laws of statics alone are not sufficient to determine forces; relative stiffness of members will influence the solution (Indeterminate trusses are safer).

### Unstable truss

When the shape can be altered, the truss is called a mechanism.

## Necessary condition for being simply stiff

### ⓘ Note

These are necessary (but not sufficient) conditions.

Here:

- $m$  - Number of members (bars)
- $j$  - Number of joints

### For a 2D simple (closed) truss

- $m < 2j - 3$  - truss is unstable
- $m = 2j - 3$  - truss is determinate if stable
- $m > 2j - 3$  - truss is indeterminate if stable

### For a 2D open truss

- $m < 2j$  - truss is unstable
- $m = 2j$  - truss is determinate if stable
- $m > 2j$  - truss is indeterminate if stable

### For a 3D simple (closed) truss

$$m = 3j - 6$$

### For a 3D open truss

$$m = 3j$$

## Analysis of Trusses

Deviations from the ideal in real trusses.

- Loads are not applied only at joints; hence there is bending in members
- Joints are not perfectly pinned, so moments can be developed at joints

## Method of Joints

### Principle

Since the truss is in equilibrium, each pin joint must also be in equilibrium.

#### Note

2 equilibrium equations can be written at each joint – vertical & horizontal.

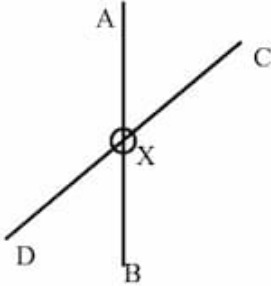
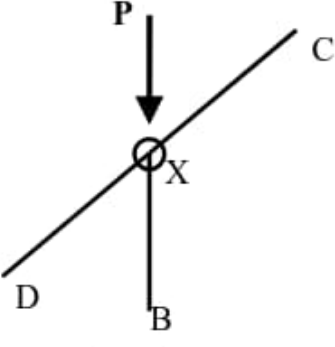
## Sign convention

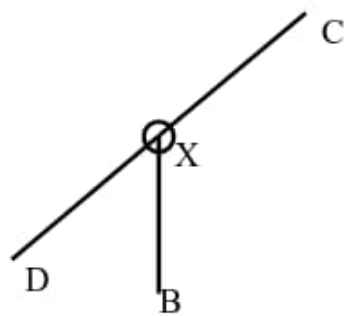
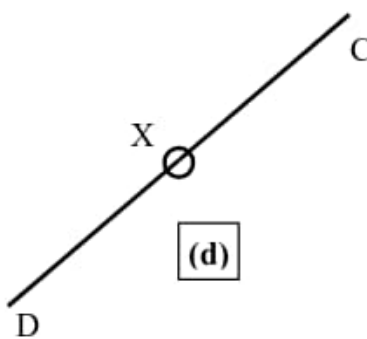
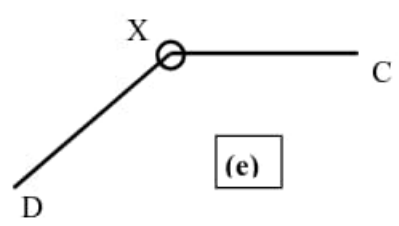
Forces acting on each joint is marked. Tensile forces are positive. Compressive forces are negative.

## Method

- Find external reactions using equilibrium equations for the entire truss.
- Start with a joint with only 2 unknown joint forces.
- Mark the forces (consider all forces are tensile) acting on the joint.
- Find the unknown forces at the selected joint, using 2 equilibrium equations for the joint.
- Go to all other joints in turn and find forces in all the members.

## Special cases

Case	Description
	$F_{AX} = F_{XB}, F_{DX} = F_{XC}$
	$F_P = F_{XB}, F_{DX} = F_{XC}$

Case	Description
	$F_{XB} = 0, F_{DX} = F_{XC}$
	$F_{DX} = F_{XC}$
	$F_{DX} = F_{XC} = 0$

## Method of Sections

### Principle

Since the truss is in equilibrium, each of its section must be in stable equilibrium.

## Method

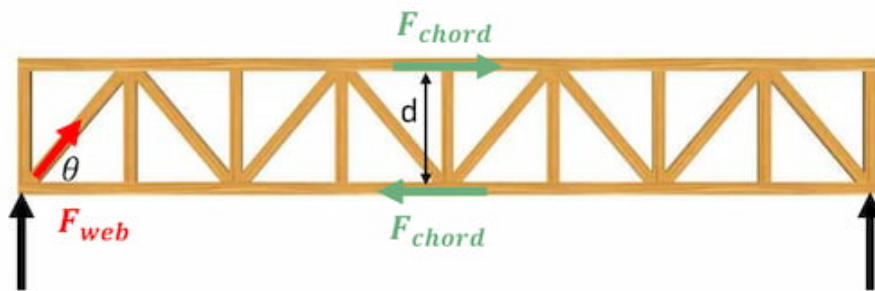
- Decide on which member's internal force must be calculated.
- Cut the truss through **3 or less** members including the target member.
- Internal forces in cut members become external forces. Can be represented as tensile forces.
- Use equilibrium equations for RHS or LHS section to find the internal forces.

## Beam Analogy (Approximate) method

In this method, the internal forces are found assuming the elongated truss is a beam.

### ⓘ For a simply supported beam

- Maximum bending moment is at mid-span:  $M_{\max} = \frac{wL^2}{8}$
- Maximum shear force is at the supports:  $\frac{wL}{2}$



Here:

- Chord members - horizontal members
- Web members - diagonal members
- $d$  - truss depth

In the truss,

- Bending moment is carried by chord members.

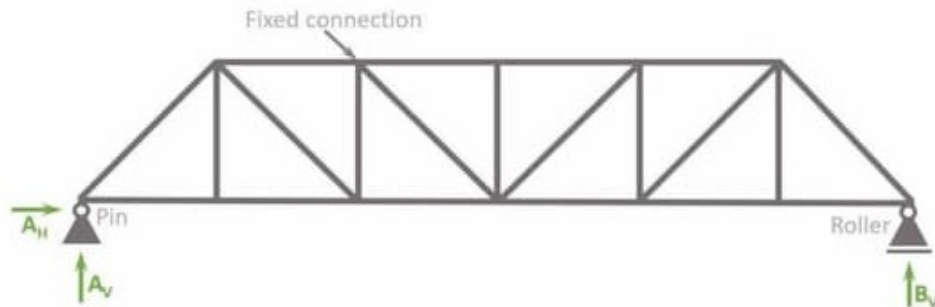
$$\text{Bending moment} = F_{\text{chord}} \times d$$

- Shear force is carried by vertical component of web member force



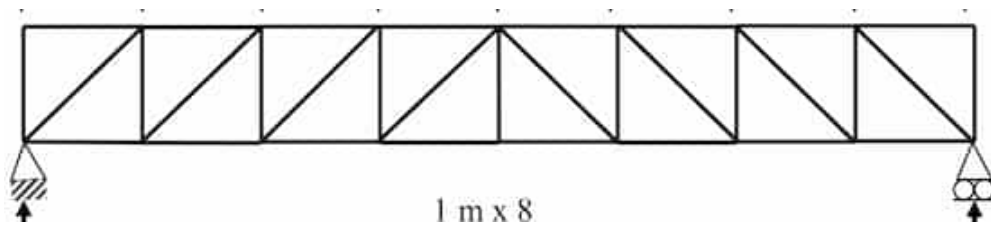
## Pratt & Howe type trusses

**Pratt type truss** is shown below.



Internal force in web members are tensile.

**Howe type truss** is shown below.



Internal force in web members are compressive.

Usually **Pratt type** is cost-efficient. To make sure a howe type truss is strong enough like pratt type, web members must be shorter and thicker.

## Indeterminate Trusses

When a truss is indeterminate, one or more compatibility equations (related to truss deformation) must be used.