Summary | Dynamics

Introduction

> ⚠ **Todo**
>
> This page is not very well organized yet. Let me know how it can be improved.

A branch of mechanics, which deals with motion of bodies.

2 parts:

- **Kinematics**: the study of geometric aspects of motion (not referencing the forces)
- **Kinetics**: the analysis of the forces that cause the motion

Kinematics of a particle

A particle has a mass and negligible size.

> ⓘ **Note**
>
> When bodies of finite size is of interest, the body might be considered as particles **provided** motion of the body is characterized by motion of its center of mass and any rotation of the body is neglected.

Rectilinear motion

When the motion of a particle is along a straight line.

Suppose x is the distance to the particle from a fixed point on its motion path.

- $\dot{x}$ is its instantaneous velocity.
- $\ddot{x}$ is its instantaneous acceleration.

Curvilinear motion

When the motion of a particle is along a curve (and not a straight line).

Suppose $\overline{r}$ is the position vector of the particle from a fixed point.

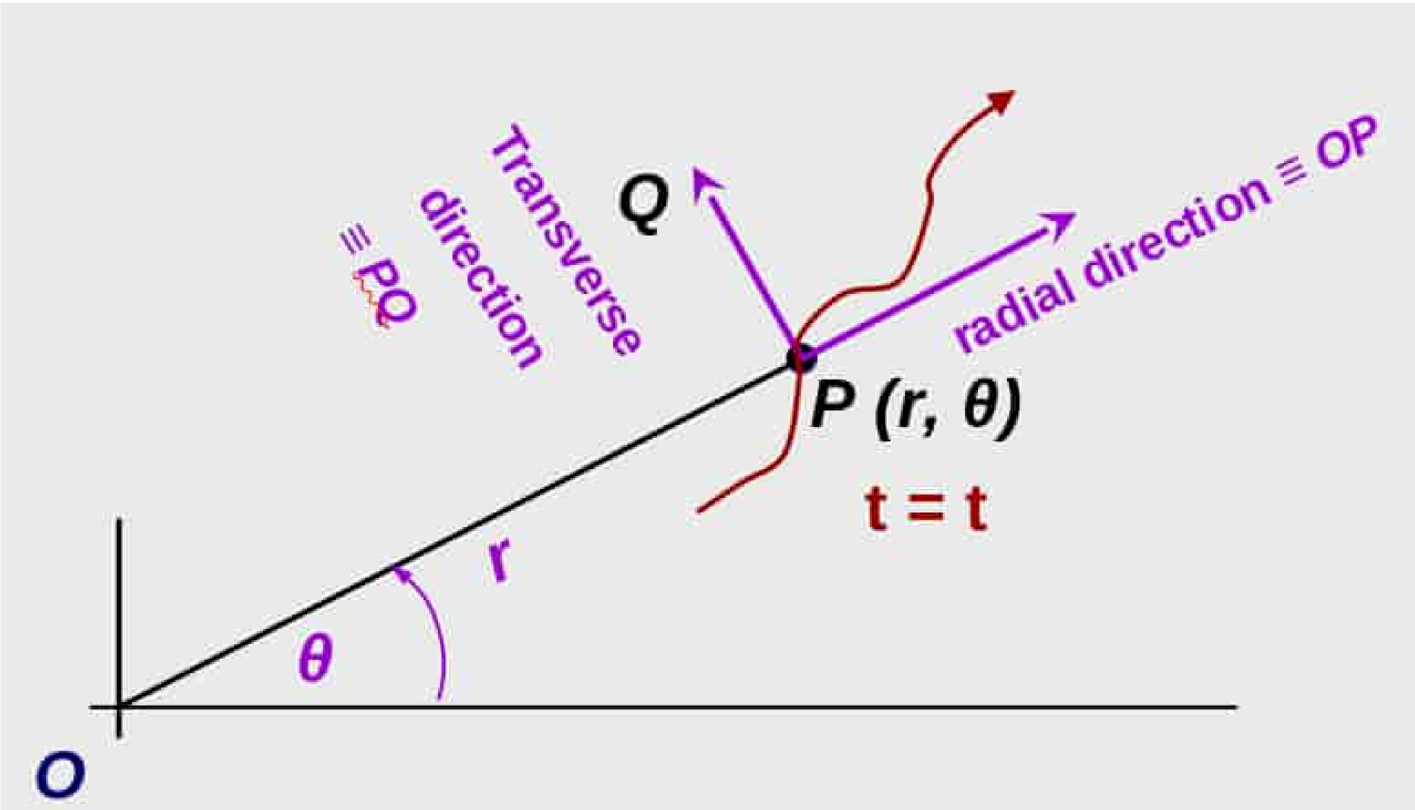
- Instantaneous velocity $v = \frac{\mathrm{d}r}{\mathrm{d}t}$
- Instantaneous speed $|v| = \frac{\mathrm{d}s}{\mathrm{d}t}$
- Instantaneous acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$

2D motion of a particle

Rectangular form

> ⚠ **TODO**
>
> Finish this section

Polar form



Velocities also have a transverse and radial components.

- Transverse component $v_\theta = \dot{\theta} \times r$
- Radial component $v_r = \dot{r}$

> ⓘ **Note**
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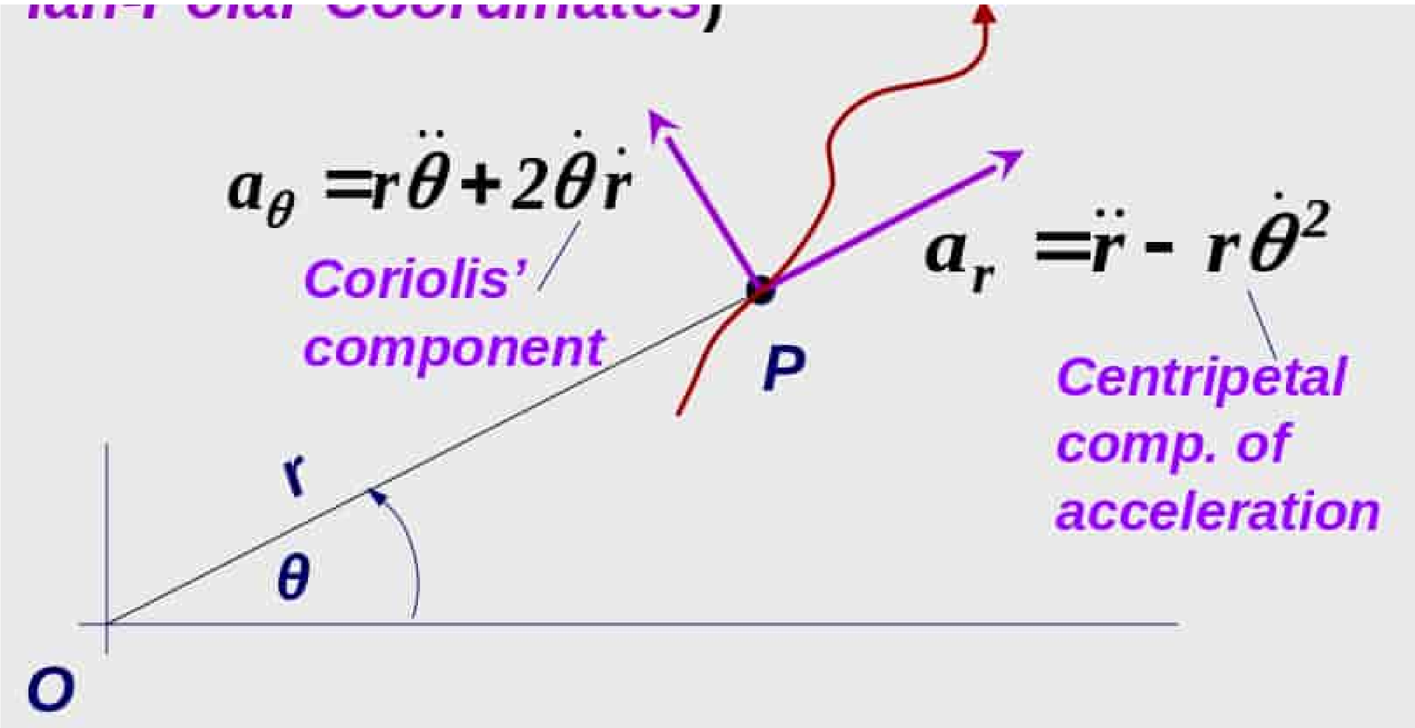
> Right hand rule is used here to denote the direction of any rotary motions.



Acceleration also have a transverse and radial components.

- Transverse component
  - $a_\theta = r\ddot{\theta} + 2\dot{\theta}\dot{r}$
  - In vector equation: $\underline{a_\theta} = \underline{\ddot{\theta}} \times \underline{r} + 2(\underline{\dot{\theta}} \times \underline{\dot{r}})$
- Radial component
  - $a_r = \ddot{r} - r\dot{\theta}^2$
  - $\underline{a_\theta} = \underline{\ddot{r}} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

In the acceleration:

- **Coriolis' component of acceleration**: $2\dot{\theta}\dot{r}$
- **Centripetal component of acceleration**: $-r\dot{\theta}^2 = \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

Effects of Coriolis' component

- Objects reflect to the right in the northern hemisphere
- Objects reflect to the left in the southern hemisphere
- Maximum deflections occur at the poles. No deflection at the equator.

Unit vectors

Unit vectors in both transverse and radial directions are denoted by e_θ and e_r.

$$\dot{e}_r = \dot{\theta} e_\theta \quad \wedge \quad \dot{e}_\theta = -\dot{\theta} e_r$$

Velocity

$$v = \frac{\mathrm{d}}{\mathrm{d}t}(r e_r) = \dot{r} e_r + r \dot{e}_r = \dot{r} e_r + r \dot{\theta} e_\theta$$

Acceleration

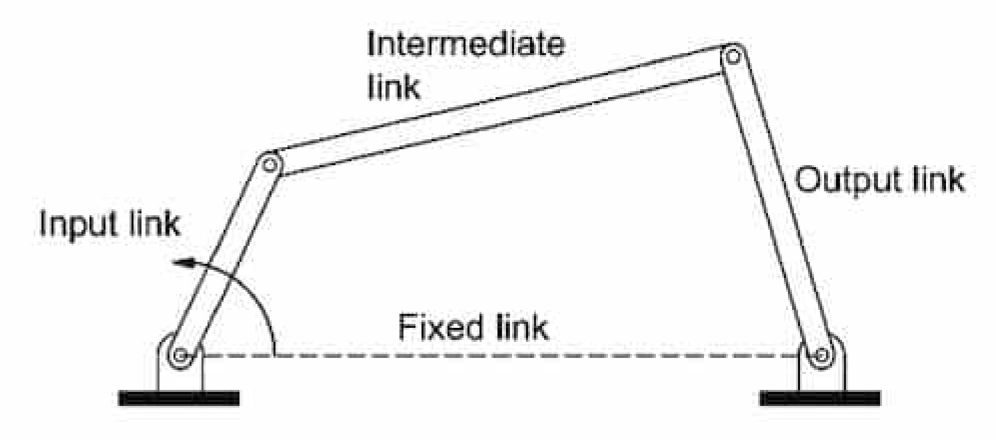
$$a = \frac{\mathrm{d}}{\mathrm{d}t}(r \dot{\theta} e_\theta) = (\ddot{r} - r\dot{\theta}^2) e_r + (r\ddot{\theta} + 2\dot{\theta}\dot{r}) e_\theta$$

Four bar linkage

Four bar-shaped members connected to each other in one plane.

Usually:

- 1 fixed link + 3 moving links
- 4 pin joints
- 2 moving pivots + 2 fixed pivots



- **input link** - usually denoted in the left.
- **output link** - usually denoted in the right.
- **coupler** - intermediate link
- **frame** - fixed link

Grashof's law

A four bar mechanism has at least one revolving link **if** $l_0 + l_3 \leq l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

Modes of motions

Mechanism	Action
Crank rocker	Shortest link is the input link
Double crank	Shortest link is the fixed link
Double rocker	Shortest link is the coupler link

crank means a link that makes a full revolution. **rocker** means a link that doesn't make a full revolution.

Crank rocker mechanism

Shortest link rotates a full revolution. Output link oscillates.

Double crank mechanism

Shortest link is fixed. Both input and output links rotates a full revolution.

Double rocker mechanism

Shortest link make full resolution. Input and output links makes a full revolution.

Special cases

$l_0 + l_3 = l_1 + l_2$.

Mechanism	Orientation
Parallelogram linkage or anti-parallelogram linkage	Equal links are opposite to each other
Deltoid linkage	Equal links are adjacent to each other

Parallelogram linkage

Double crank mechanism. Opposite links are equal and parallel. Angular velocity of input crank & output crank is same. Orientation of the coupler doesn't change during the motion.

Anti-parallelogram linkage

Double crank mechanism. Angular velocity of input crank is different to output crank.

Deltoid linkage

- Longest link is fixed: crank rocker mechanism
- Shortest link is fixed: double crank mechanism

Non-Grashof's condition

A four bar mechanism with the property **if** $l_0 + l_3 > l_1 + l_2$.

Here: l_0, l_1, l_2, l_3 are the length of four bars from shortest to longest.

Three links are in oscillation.

Mechanisms

Mechanism

An assembly of machine components (kinematic links) designed to obtain a desired motion from an available motion while transmitting appropriate forces and moments.

Simple mechanisms

- Lever
- Pulley
- Gear trains
- Belt and chain drive
- Four bar linkage

Other mechanisms

- Lock stitch mechanism (used in sewing machine)
- Geneva mechanism
  Constant rotational motion to intermittent rotational motion. mostly used in watches.
- Scotch yoke mechanism
  Constant rotational motion to linear motion (vice versa.). Mainly used as valve actuators in high pressure gas pipelines.
- Slider crank mechanism
  Used in internal combustion engines
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