Individual Project-FALL 2023

Math 540:

Intro to Probability Theory

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Waste Management-Analysing Generation and Recycling Data

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ABSTRACT

This Project of "Math 540: Intro to Probability Theory" detailed analysis of waste management-generation and recycling data from the years 2018-2020 by different techniques. The main goal is to identify the techniques and different simulation methods for the dataset "waste management-generation and recycling data".

The dataset has data on waste management for three years. We have used it for calculating Bayes Theorem joint Distribution Analysis and Factor analysis. Additionally, the project involves the simulation of continuous random variables and discrete random variables, analysis of Markov chains, and application of variance reduction techniques.

The main point of this study is the application of Bayes' Theorem and joint distribution analysis to the waste dataset, providing deeper insights into the probabilistic nature of waste generation and recycling. This is complemented by factor analysis and combinatorial analysis, including practical examples like poker hand probabilities, to enhance the understanding of complex probabilistic concepts.

To sum up, this project shows how sophisticated statistical techniques can be used in real-world situations. It demonstrates how theoretic ideas from probability theory may be applied to analyse, comprehend, and forecast trends in intricate datasets—especially when it comes to environmental sustainability. Thus, the research serves as evidence of the effectiveness of statistical analysis in tackling modern issues and advancing both academic and industrial domains.

CHAPTER:1

Introduction

In this project o “Math 540: Intro to Probability Theory” we have conveyed the main issue of sustainable waste management. The project helps in finding waste generation and recycling data from the years 2018-2020, which has different statistical and probabilistic methods. The main goal is to find the different trends and patterns in the data set which is waste management which is the real dataset.

The problem of the study for taking this dataset is to find the efficiency of the waste management which is the main aspect of environmental sustainability. We have taken different graph analyses which gives an easy understanding of the project which is the main goal. This is mainly achieved through different techniques using different techniques.

Our approach has different methods like inverse transform sampling, acceptance-rejection sampling, and the Central Limit Theorem of the continuous and discrete random variables. And also, it used in the application of Bayes theorem, joint distribution analysis, and factor analysis which helps in understanding the dataset in depth.

In summary, we build the relationship between the real data analysis of waste management and also the continuous and discrete random variables showcasing the importance of statistical analysis in solving real-world problems, particularly in the field of environmental sustainability. The findings of this study have the potential to inform policy decisions and improve waste management practices, making it a valuable contribution to both academic and industrial sectors.

CHAPTER:2

Data Description

The dataset is taken from the website given here
https://www.kaggle.com/datasets/kingabzpro/singapore-waste-management?select=2018_2020_waste.csv

And has considered one file that is “2018_2020_waste.csv" contains data on waste generation and recycling, covering the years 2018 to 2020. Here's an overview of its structure and contents:

The dataset consists of 45 entries. It includes four columns: 'Waste Type' (object type), 'Total Generated' (integer), 'Total Recycled' (integer), and 'Year' (integer). There are no missing values in any of the columns, indicating good data integrity.

Descriptive Statistics:

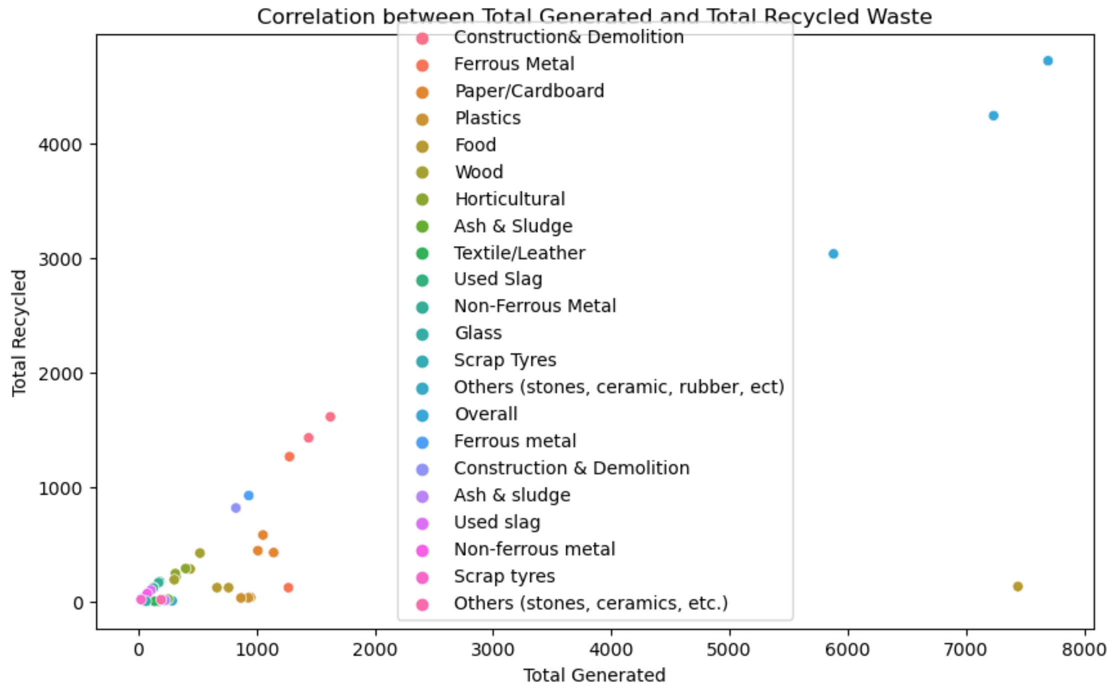
Waste Type: This dataset contains different columns one of which is waste type. This consists of variables such as Construction and demolition, Ferrous Metal, Paper/Cardboard Plastics, Food, Wood, Horticultural, Ash and sludge, Textile/Leather, and Used Slag. This important variable for the analysis.

Total Generated: This column is used for the total amount of the waste which is generated for each of the above waste type. However, the unit is not mentioned in the dataset for our understanding. But we can understand which is mostly produced, etc.

Total Recycled: This column is uses for the total amount of the which is recycled for each of the above waste type. However, the unit is not mentioned in the dataset for our understanding. But we can understand which is mostly produces and etc.

Year: The data is categorized by year like 2018, 2019, 2020, allowing for a better analysis. This aspect is crucial for identifying trends over time, such as increases or decreases in waste generation or recycling rates.

By analysing this dataset, one can gain insights into waste management practices, identify trends in waste generation and recycling, and assess the effectiveness of recycling efforts for different types of waste over the three-year period. This information is valuable for environmental policy making, sustainability initiatives, and understanding the impacts of societal consumption habits on waste management systems.



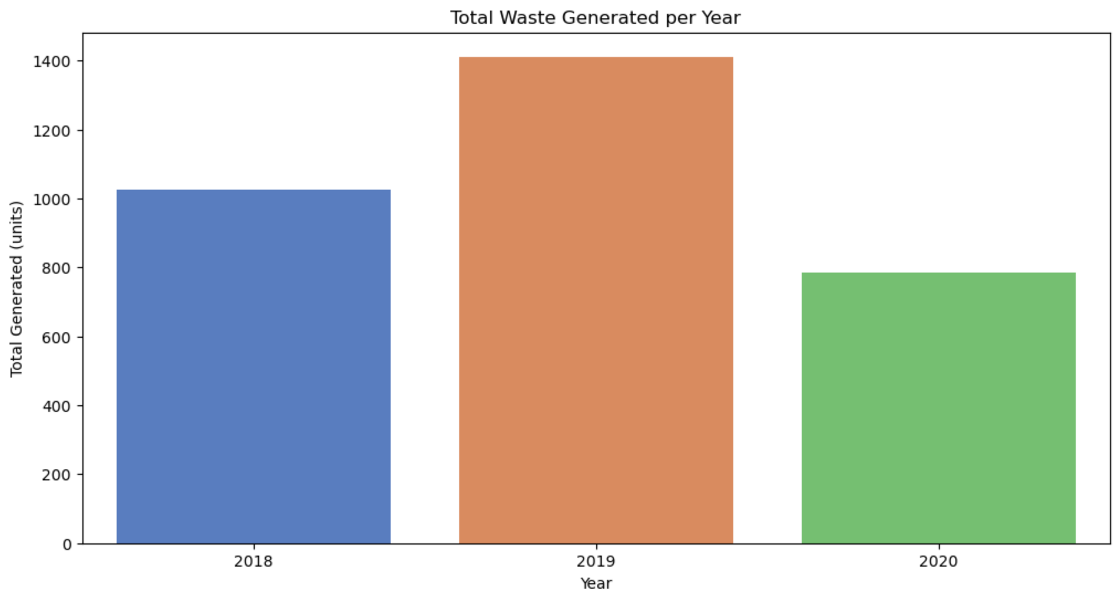

```
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

# Re-loading the dataset
waste_data = pd.read_csv("/Users/hp/Downloads/archive/2018_2020_waste.csv")

# Checking the column names for any discrepancies
column_names = waste_data.columns

# Renaming columns to remove spaces and standardize names
waste_data.rename(columns=lambda x: x.strip().replace(" ", "_"), inplace=True)

# Rechecking the column names after renaming
renamed_column_names = waste_data.columns

# Attempting the plot again with renamed columns
plt.figure(figsize=(12, 6))
sns.barplot(x='Year', y='Total_Generated', data=waste_data, ci=None, palette='muted')
plt.title('Total Waste Generated per Year')
plt.ylabel('Total Generated (units)')
plt.xlabel('Year')
plt.show()

plt.figure(figsize=(12, 6))
sns.barplot(x='Year', y='Total_Recycled', data=waste_data, ci=None, palette='muted')
plt.title('Total Waste Recycled per Year')
plt.ylabel('Total Recycled (units)')
plt.xlabel('Year')
plt.show()
```

column_names, renamed_column_names

C:\Users\hp\AppData\Local\Temp\ipykernel_14280\3560484823.py:15: FutureWarning:

The `ci` parameter is deprecated. Use `errorbar=None` for the same effect.

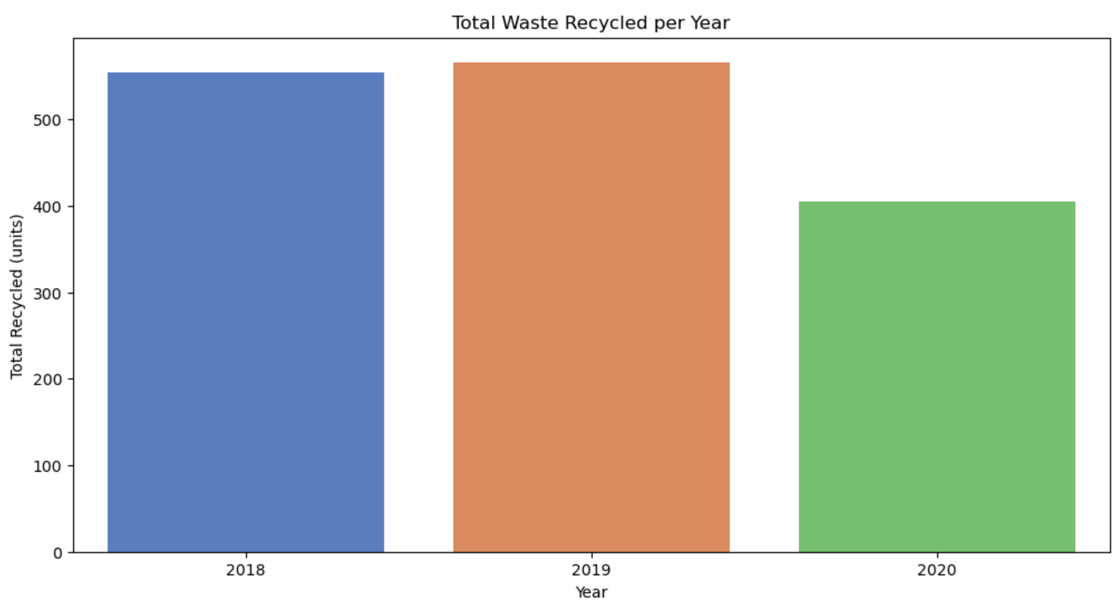
  sns.barplot(x='Year', y='Total_Generated', data=waste_data, ci=None, palette='muted')



C:\Users\hp\AppData\Local\Temp\ipykernel_14280\3560484823.py:22: FutureWarning:

The `ci` parameter is deprecated. Use `errorbar=None` for the same effect.

  sns.barplot(x='Year', y='Total_Recycled', data=waste_data, ci=None, palette='muted')



(Index(['Waste Type', 'Total Generated ', 'Total Recycled', 'Year'], dtype='object'),
 Index(['Waste_Type', 'Total_Generated', 'Total_Recycled', 'Year'], dtype='object')

CHAPTER:3

Methodology

1. Mean

The mean is the average value of a dataset and is calculated by summing up all the values and then dividing by the number of values. It's a measure of the central tendency of the data. In mathematical terms, for a dataset X = {x1, x2, ... , xn},the mean (often denoted as $\bar{x}$) is given by:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The mean is sensitive to outliers, as extreme values can skew it significantly.

2. Variance

Variance measures how spread out the numbers in a dataset are around the mean. It's the average of the squared differences from the Mean. Mathematically, for the same dataset X, the variance (denoted as σ^2) is:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

A higher variance indicates that data points are more spread out from the mean, while a lower variance indicates that they are closer to the mean.

3. Standard Deviation

The standard deviation is a measure of the amount of variation or dispersion in a set of values. It is the square root of the variance and provides a gauge of the typical distance between each data point and the mean. For the dataset X, it's denoted as σ and is:

$$\sigma = \sqrt{\sigma^2}$$

It's often more interpretable than variance since it's in the same units as the data.

4. Mode

The mode is the value that appears most frequently in a dataset. A dataset may have one mode (unimodal), more than one mode (bimodal or multimodal), or no mode at all. The mode is particularly useful for categorical data, where we wish to know which is the most common category.

5. First and Third Quantiles

Quantiles are values that divide the dataset into equal parts. The first quantile (or the 25th percentile) is the value below which 25% of the data can be found. The third quantile (or the 75th percentile) is the value below which 75% of the data lies. The interquartile range (IQR), which is the difference between the third and first quantiles, is a measure of statistical dispersion and is used to identify outliers.

6. Joint distribution

a joint distribution is a probability distribution that captures the likelihood of two or more random variables simultaneously. This type of distribution is particularly useful when analyzing the relationship between these variables. There are two main types of joint distributions:

Joint Discrete Distribution: This applies to discrete random variables. It gives the probability of each pair of values within the set of possible combinations. For example, if we have two dice, the joint distribution will tell us the probability of rolling a specific combination of numbers on the two dice.

Joint Continuous Distribution: This is used for continuous random variables. It's described using a joint probability density function (pdf), which must be integrated over a range to get probabilities. For instance, in analysing the relationship between height and weight, a joint continuous distribution could be used.

The joint distribution is central to understanding correlations and dependencies between variables. It is also foundational for more complex statistical concepts like conditional probabilities and marginal distributions.

7. Conditional expectation

The conditional expectation is a fundamental concept in probability and statistics, describing the expected value of a random variable given that certain conditions are met, or certain values are assumed by another random variable.

8.Bayes' Rule

Bayes' Rule, named after the Reverend Thomas Bayes, is a fundamental theorem in probability theory. It describes how to update the probability of a hypothesis as more evidence or information becomes available. Bayes' Rule is mathematically expressed as follows:

$P(A|B)= P(B)P(B|A)\times P(A)$

Where:

$P(A|B)$ is the probability of hypothesis A given the data B (posterior probability).

$P(B|A)$ is the probability of the data B given that hypothesis A is true (likelihood).

P(A) is the probability of hypothesis A being true irrespective of data B (prior probability).

P(B) is the probability of the data B under all hypotheses (evidence or marginal likelihood).

9.Discrete Random Variables

Definition: A discrete random variable is one that has a countable number of distinct values. These values can be finite or infinite but countable (like integers).

Examples:

The number of heads in 10-coin tosses.

The count of cars passing through an intersection in an hour.

Probability Distribution: Discrete random variables are often represented using probability mass functions (PMFs), which assign probabilities to each possible value of the variable.

Characteristics:

Can take on specific values (e.g., 0, 1, 2, 3...).

Graphically represented using bar graphs or histograms.

Common distributions include Binomial, Poisson, and Geometric distributions.

10.Continuous Random Variables

Definition: A continuous random variable is one that can take on any value within an interval or range. The set of possible values is uncountable (like real numbers).

Examples:

The height of students in a classroom.

The time it takes for a chemical reaction to complete.

Probability Distribution: Continuous variables are described using probability density functions (PDFs), which must be integrated over an interval to find probabilities.

Characteristics:

Can take on any value within a certain range.

Graphically represented using smooth curves.

Common distributions include Normal (Gaussian), Uniform, and Exponential distributions.

11. Correlation: Measures the strength and direction of the relationship between two variables. It quantifies how changes in one variable are associated with changes in another. The correlation coefficient ranges from -1 (perfect negative correlation) to +1 (perfect positive correlation).

12. Markov Chains: These are stochastic models describing a sequence of possible events, where the probability of each event depends only on the state attained in the previous event. They are widely used in various fields like finance, game theory, and genetics.

13. Simulation Techniques: These are methods to mimic the operation of real-world processes or systems over time. Commonly used in scenarios where analytical solutions are impractical, simulations help in predicting system behaviour under different conditions.

14. Factor Analysis: A statistical method used to describe variability among observed, correlated variables in terms of a potentially lower number of unobserved variables called factors. It's commonly used in psychology, finance, and other social sciences to identify underlying relationships between variables.

CHAPTER:4

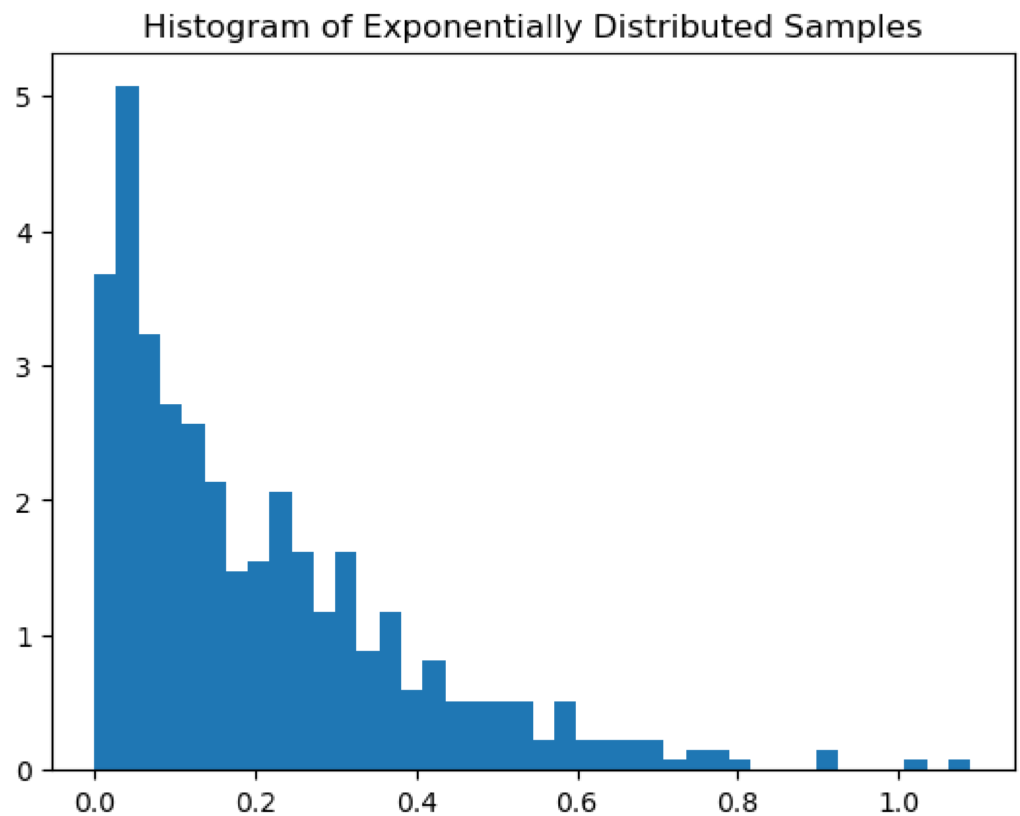
Analysis and Result

```
import numpy as np import pandas
as pd

import matplotlib.pyplot as plt import scipy.stats

import seaborn as sns
```

4.1.1 Simulating Continuous Random Variables:

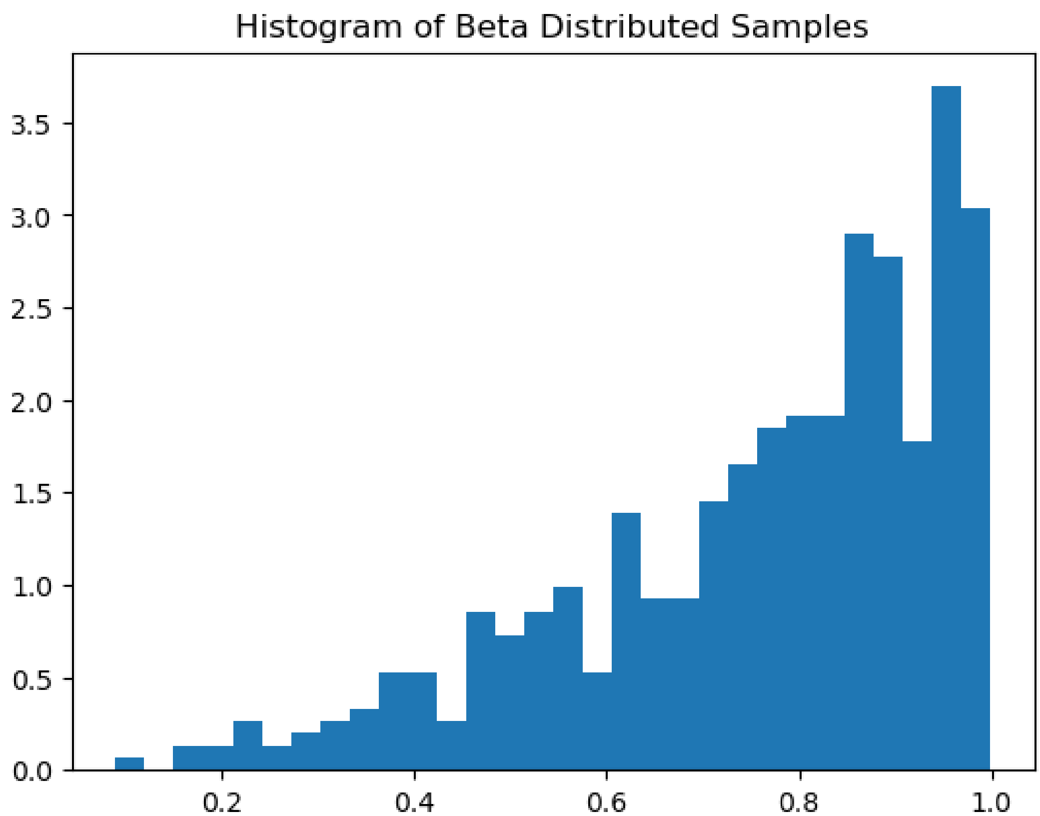
Inverse Transform Sampling for Exponential Distribution

```
# consider the lambda parameter for the exponential distribution

lambda_param = 5.0

# Inverse transform sampling

def inverse_transform_sampling(lambda_param, n=500): uniform_samples =
        np.random.uniform(0, 1, n)

        exponential_samples = -np.log(1 - uniform_samples) / lambda_param return exponential_samples

# Generating the samples

samples = inverse_transform_sampling(lambda_param)

# Plotting the graph of exponential distributed sample plt.hist(samples, bins=40,
density=True) plt.title('Histogram of Exponentially Distributed Samples') plt.show()
```



Acceptance-Rejection Sampling for Beta Distribution

```
def acceptance_rejection_sampling(n=500): samples = []
    while len(samples) < n:
        y = np.random.uniform(0, 1) u =
        np.random.uniform(0, 1)
        if u <= y**2:   # condition for beta distribution
            samples.append(y) return
    np.array(samples)

# Generating samples

beta_samples = acceptance_rejection_sampling()

# Plotting the graph of Beta Distributed samples plt.hist(beta_samples, bins=30,
density=True) plt.title('Histogram of Beta Distributed Samples') plt.show()
```



Statistical Analysis of Simulated Data

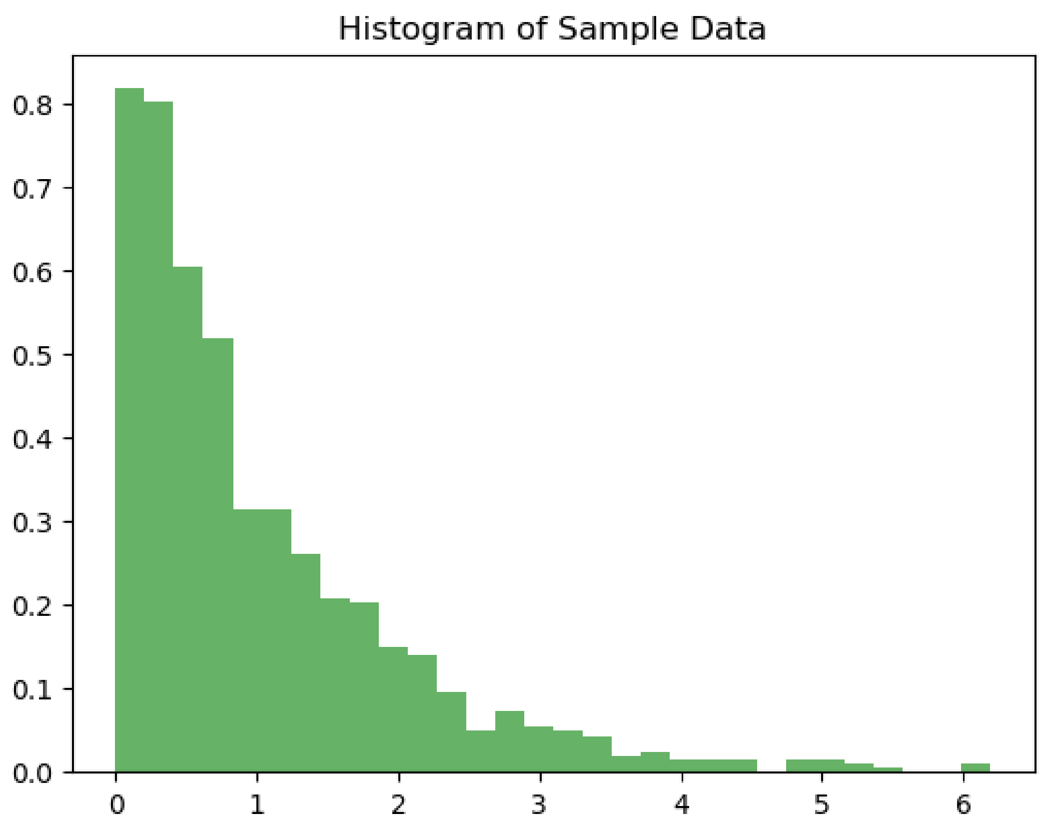
```
def statistical_analysis(samples): mean =
        np.mean(samples) variance =
        np.var(samples) std_dev = np.std(samples)

        first_quantile = np.percentile(samples, 25) third_quantile =
        np.percentile(samples, 75) skewness = scipy.stats.skew(samples)
        kurtosis = scipy.stats.kurtosis(samples)

        print(f"Mean: {mean}") print(f"Variance: {variance}")
        print(f"Standard Deviation: {std_dev}")

        print(f"First Quantile: {first_quantile}") print(f"Third Quantile:
        {third_quantile}") print(f"Skewness: {skewness}")
        print(f"Kurtosis: {kurtosis}")

# Calculate statistics for exponential samples

statistical_analysis(samples)
```

```
Mean: 0.20720577704918502

Variance: 0.035260434016763015

Standard Deviation: 0.18777761851925542
```

```
First Quantile: 0.0608708498043777

Third Quantile: 0.30603635432541215

Skewness: 1.387321432652219
```

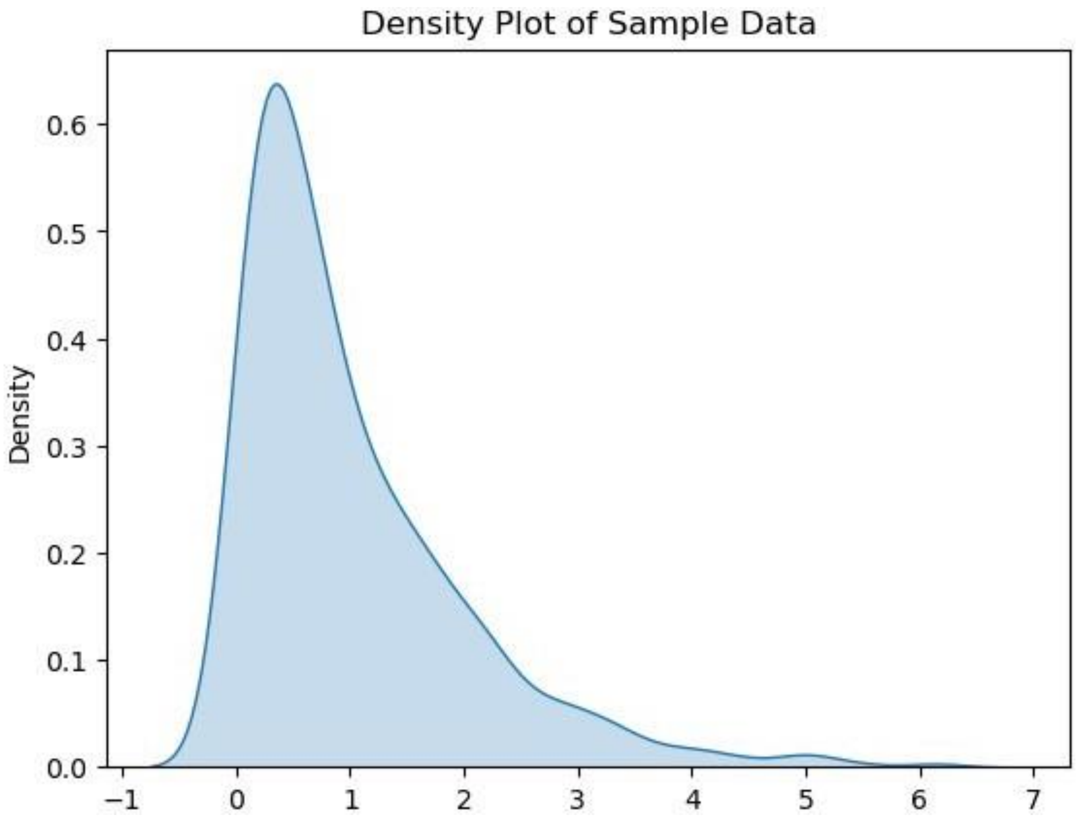
Visualization of Sample Data

```
#Histogram of sample data

plt.hist(samples, bins=30, density=True, alpha=0.6, color='g') plt.title('Histogram of Sample Data')

plt.show()

# Density Plot of sample data sns.kdeplot(samples, shade=True)
plt.title('Density Plot of Sample Data') plt.show()
```



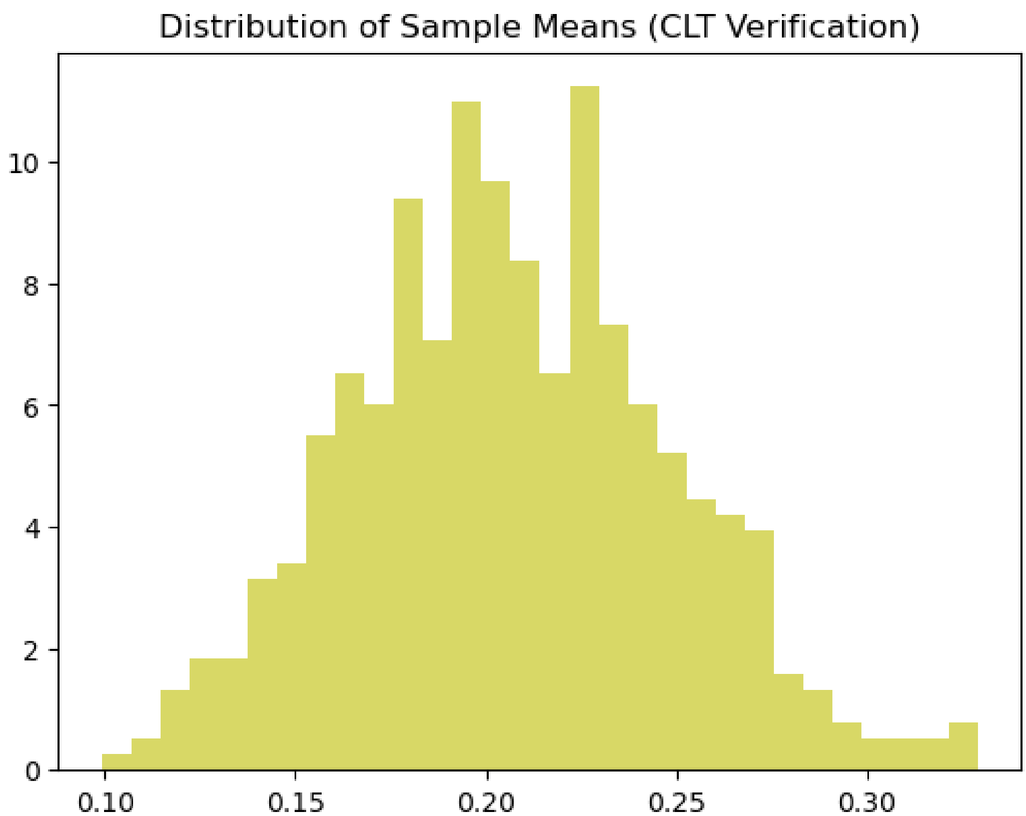

```
C:\Users\hp\AppData\Local\Temp\ipykernel_20924\4137198264.py:7: FutureWarning:

`shade` is now deprecated in favor of `fill`; setting `fill=True`. This will become an error in seaborn
v0.14.0; please update your code.
```

```
sns.kdeplot(samples, shade=True)
```



Verifying Central Limit Theorem

```
def verify_clt(samples, n=20, num_samples=500): sample_means = []
    for _ in range(num_samples):
        sample = np.random.choice(samples, n)
        sample_means.append(np.mean(sample))

    # Plotting the sample means
    plt.hist(sample_means, bins=30, density=True, alpha=0.6, color='y')
    plt.title('Distribution of Sample Means (CLT Verification)') plt.show()

verify_clt(samples)
```



Outlier Detection

```
def detect_outliers(data, m=4): mean =
        np.mean(data)

        std = np.std(data)

        outliers = [x for x in data if abs(x - mean) > m * std] return outliers

outliers = detect_outliers(samples) print(f"Number of Outliers:
{len(outliers)}")

Number of Outliers: 2
```

Probability Calculations

```
def probability_calculation(samples, lower_bound, upper_bound): within_range = [x for x in samples if
        lower_bound <= x <=

upper_bound]

        probability = len(within_range) / len(samples) return probability

# Probability of a value falling between two bounds prob =
probability_calculation(samples, 0.5, 1.5) print(f"Probability: {prob}")
```

```
Probability: 0.086
```

4.1.2 Simulating from Discrete Distributions

```
from scipy import stats

# Simulating data from various discrete distributions #Taking the random data

binomial_data = np.random.binomial(n=17, p=0.5, size=500) poisson_data =
np.random.poisson(lam=3.0, size=500) geometric_data = np.random.geometric(p=0.2,
size=500)
```

Statistical Analysis

```
def analyze_distribution(data):
    analysis_results = {
        'Mean': np.mean(data),
        'Variance': np.var(data, ddof=1),
        'Standard Deviation': np.std(data, ddof=1),
        'First Quantile (25%)': np.quantile(data, 0.25),
        'Third Quantile (75%)': np.quantile(data, 0.75),
        'Mode': stats.mode(data)[0][0],
        'Skewness': stats.skew(data),
        'Kurtosis': stats.kurtosis(data)
    }
    return analysis_results

# considering the binomial data
binomial_data = np.random.binomial(n=17, p=0.5, size=500)
analysis_binomial = analyze_distribution(binomial_data)
print("Statistical Analysis of Binomial Distribution:")
for key, value in analysis_binomial.items():
    print(f"{key}: {value:.2f}")

Statistical Analysis of Binomial Distribution:
Mean: 8.45
Variance: 3.77
Standard Deviation: 1.94
First Quantile (25%): 7.00
Third Quantile (75%): 10.00
Mode: 9.00
Skewness: 0.05
Kurtosis: -0.38

C:\Users\hp\AppData\Local\Temp\ipykernel_20924\1632294513.py:8:
FutureWarning: Unlike other reduction functions (e.g. `skew`,
`kurtosis`), the default behavior of `mode` typically preserves the
axis it acts along. In SciPy 1.11.0, this behavior will change: the
```

```
default value of `keepdims` will become False, the `axis` over which the statistic is taken will be eliminated, and the value None will no longer be accepted. Set `keepdims` to True or False to avoid this warning.
    'Mode': stats.mode(data)[0][0],
```

Visualization

```
def visualize_data(data, title):
    plt.figure(figsize=(15, 10))
    sns.histplot(data, kde=True) plt.title(title)
    plt.show()
```

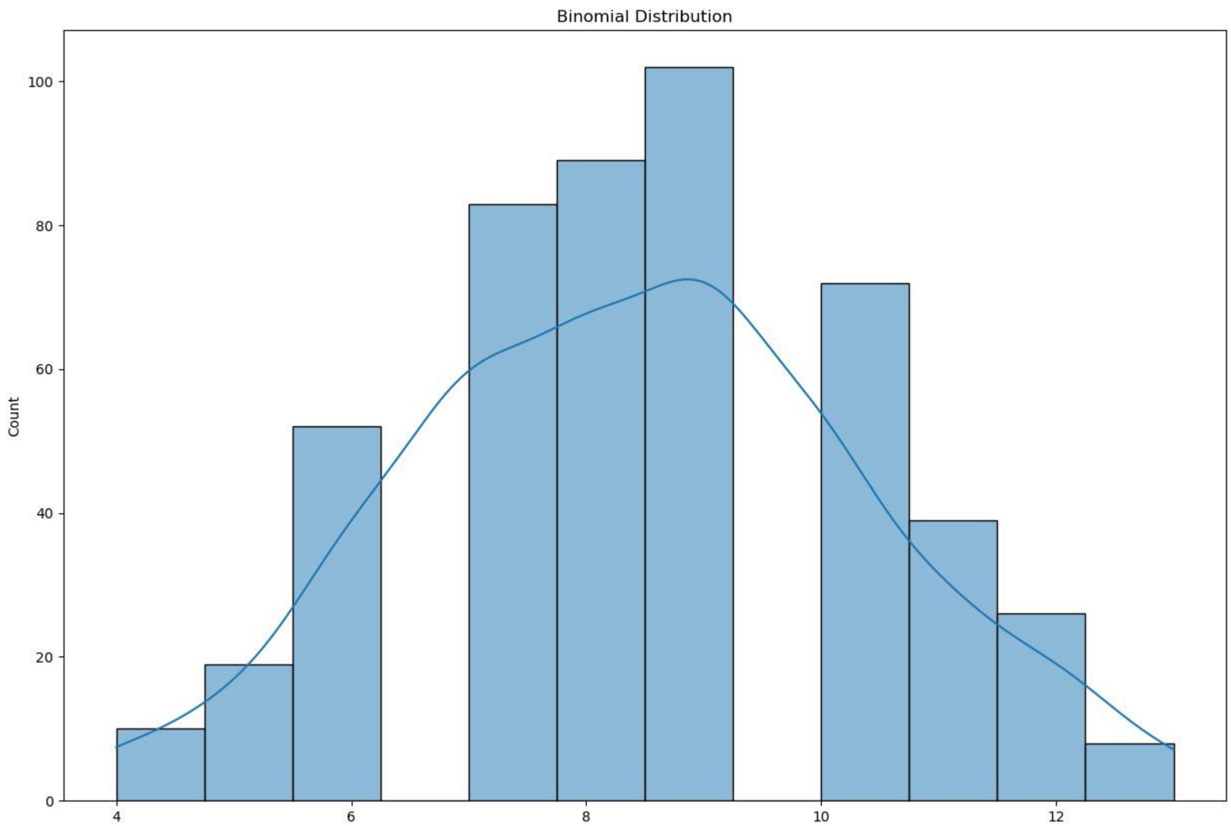
Central Limit Theorem Verification

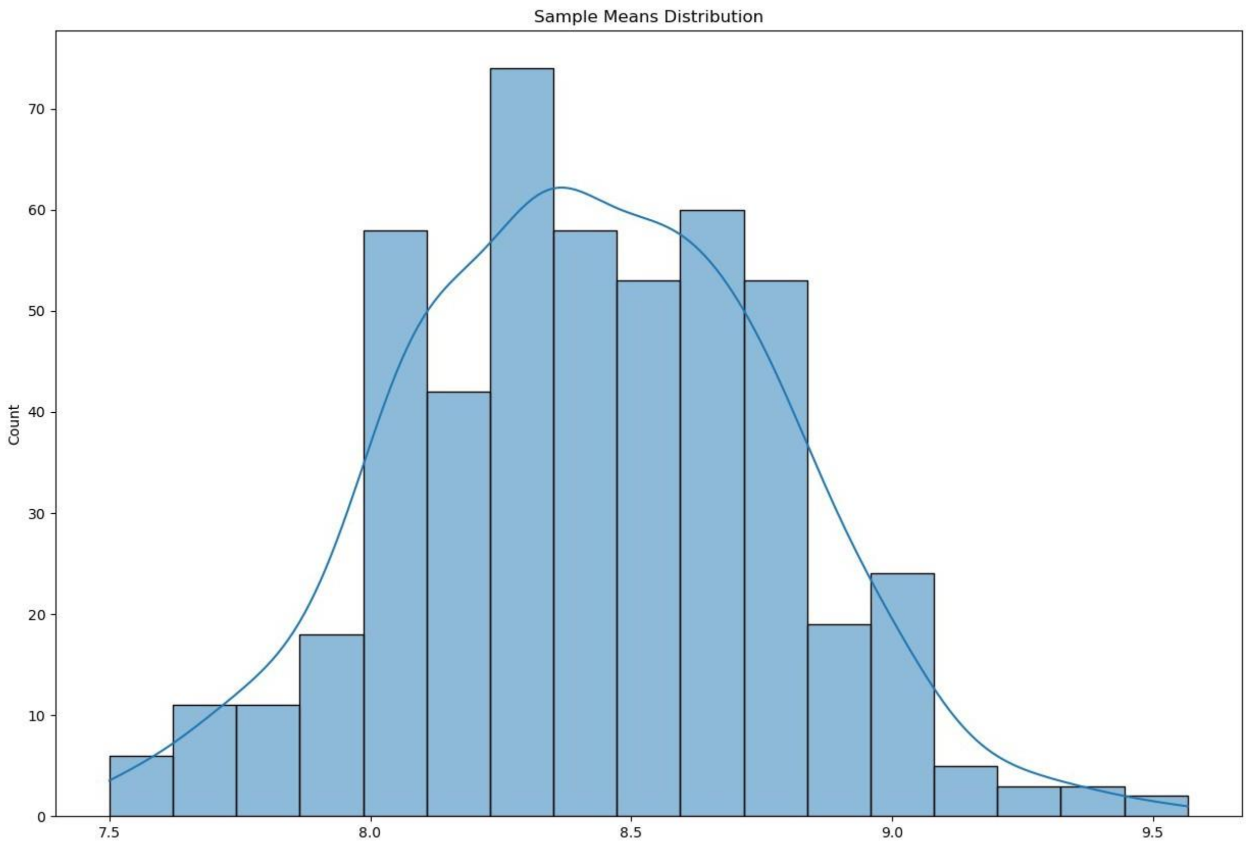
```
def clt_verification(data, sample_size=30, num_samples=500): sample_means = []
    for _ in range(num_samples):
        sample = np.random.choice(data, size=sample_size, replace=True)
        sample_means.append(np.mean(sample)) visualize_data(sample_means, "Sample
    Means Distribution")
```

Outlier Detection (Simple Z-score method)

```
def detect_outliers(data): threshold = 3
    mean = np.mean(data) std =
    np.std(data) outliers = []
    for i in data:
        z_score = (i - mean) / std
        if np.abs(z_score) > threshold: outliers.append(i)
    return outliers

analyze_binomial = analyze_distribution(binomial_data)
visualize_data(binomial_data, "Binomial Distribution")
clt_verification(binomial_data)
outliers = detect_outliers(binomial_data)

C:\Users\hp\AppData\Local\Temp\ipykernel_20924\1632294513.py:8: FutureWarning: Unlike other reduction functions (e.g. `skew`,
`kurtosis`), the default behavior of `mode` typically preserves the axis it acts along. In SciPy 1.11.0, this behavior will change: the default value of `keepdims` will become False, the `axis` over which the statistic is taken will be eliminated, and the value None will no longer be accepted. Set `keepdims` to True or False to avoid this
```

```
warning.
    'Mode': stats.mode(data)[0][0]
```

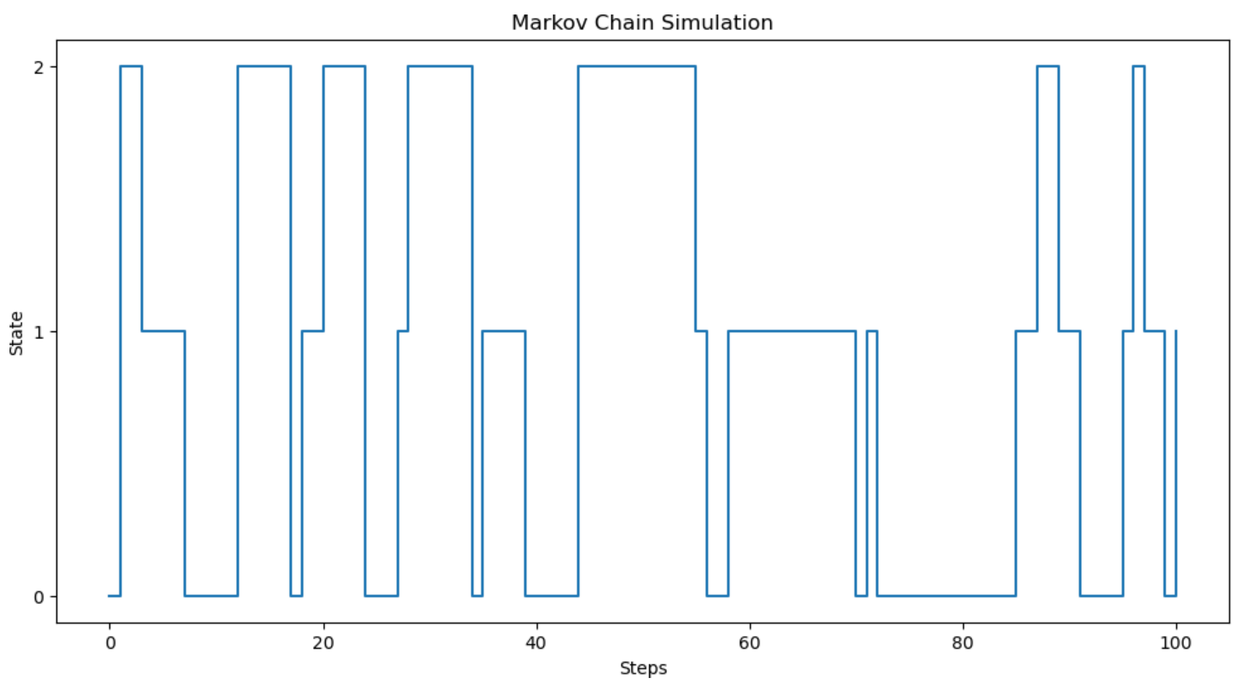




4.1.3 Markov Chains:

```
# Take any simple transition matrix for the Markov chain

transition_matrix = np.array([[0.7, 0.2, 0.1],
                              [0.3, 0.5, 0.2],
                              [0.1, 0.1, 0.8]])

# Function to simulate Markov chain
def simulate_markov_chain(transition_matrix, initial_state, steps): state = initial_state
    states_visited = [state]

    for _ in range(steps):
        state = np.random.choice(range(len(transition_matrix)), p=transition_matrix[state])
        states_visited.append(state) return states_visited

# Simulating the Markov chain
```

```python
states_visited = simulate_markov_chain(transition_matrix,
initial_state, steps)

# Visualization
plt.figure(figsize=(12, 6))
plt.plot(states_visited, drawstyle='steps-post')
plt.title('Markov Chain Simulation')
plt.xlabel('Steps')
plt.ylabel('State')
plt.yticks(range(len(transition_matrix)))
plt.show()

# Ergodicity analysis: Comparing time-averaged behavior with steady-
state probabilities
time_averaged_behavior = np.mean([states_visited.count(state) /
len(states_visited) for state in range(len(transition_matrix))])

# Steady-state probabilities (assuming ergodicity)
steady_state_probabilities = np.linalg.matrix_power(transition_matrix,
1000)[0]

# Sensitivity Analysis: Varying transition probabilities or initial
conditions (not shown in this code snippet)

# Output results
print("Time-averaged behavior of states:", time_averaged_behavior)
print("Steady-state probabilities:", steady_state_probabilities)
```



```
Time-averaged behavior of states: 0.3333333333333333

Steady-state probabilities: [0.36363636 0.22727273 0.40909091]
```

4.1.4 Variance Reduction Techniques:

Basic Monte Carlo Simulation to estimate the mean

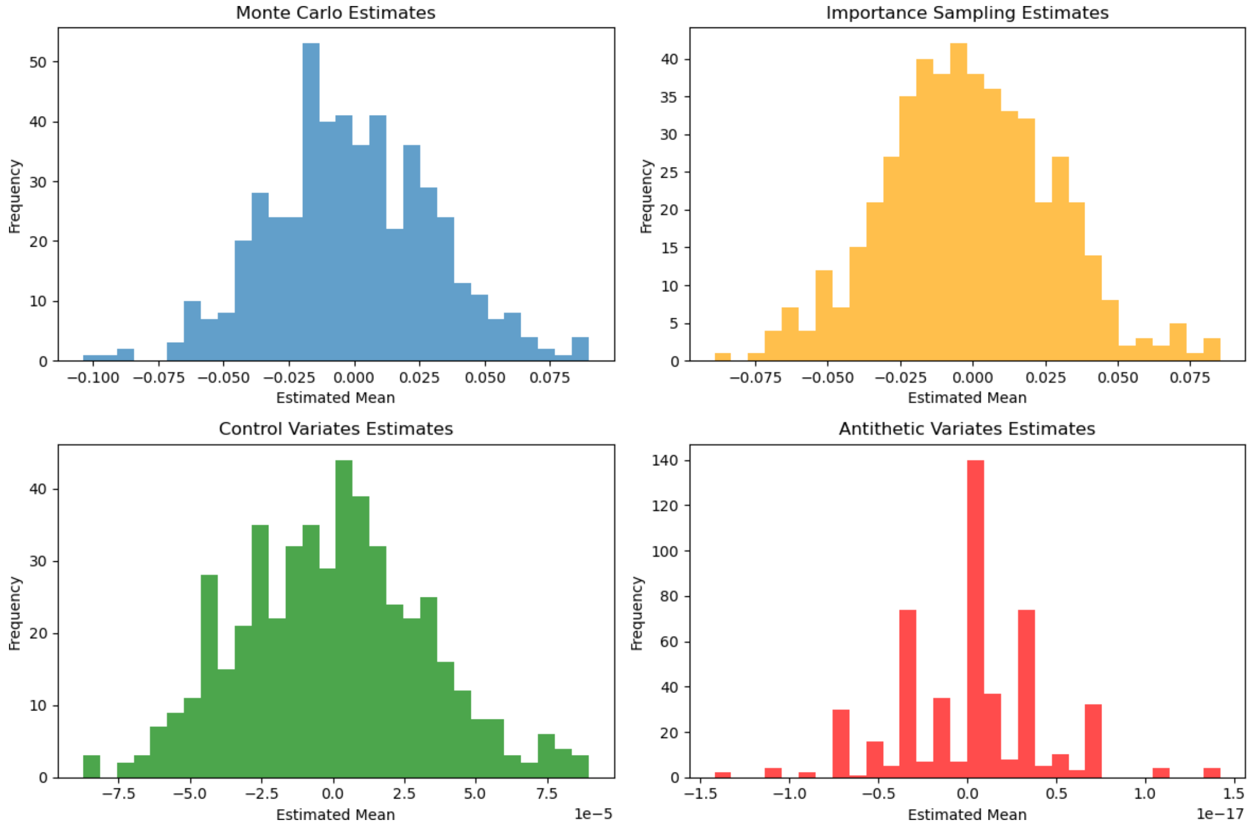
```
def monte_carlo_mean(samples):
    return np.mean(samples)

# Importance Sampling
def importance_sampling(target_mean, target_std, sampling_std,
num_samples):

    # Generate samples from a different distribution
    samples = np.random.normal(target_mean, sampling_std, num_samples)

    # Reweight the samples
    weights = (stats.norm.pdf(samples, target_mean, target_std) /
               stats.norm.pdf(samples, target_mean, sampling_std))
    weighted_mean = np.sum(weights * samples) / np.sum(weights)
    return weighted_mean

# Control Variates
def control_variates(target_mean, num_samples):

    # Generate samples
    samples = np.random.normal(target_mean, 1, num_samples)

    # Control variable with  target mean
    control_mean = target_mean

    # Calculating covariance and variance
    covariance = np.cov(samples, samples)
    variance = np.var(samples)

    # Compute the control variate coefficient (b)
    b = covariance[0, 1] / variance

    # Adjusted mean using control variate
    adjusted_mean = np.mean(samples - b * (samples - control_mean))
    return adjusted_mean

# Antithetic Variates
def antithetic_variates(target_mean, num_samples):

    # Generate samples
```

```python
    samples = np.random.normal(target_mean, 1, num_samples // 2)

    # Generate antithetic samples
    antithetic_samples = target_mean - (samples - target_mean)

    # Combine and calculate mean
    combined_samples = np.concatenate((samples, antithetic_samples))
    antithetic_mean = np.mean(combined_samples)
    return antithetic_mean

# Parameters
target_mean = 0
target_std = 1
sampling_std = 2
num_samples = 1000

# Running the simulations
mc_mean = monte_carlo_mean(np.random.normal(target_mean, target_std,
num_samples))
is_mean = importance_sampling(target_mean, target_std, sampling_std,
num_samples)
cv_mean = control_variates(target_mean, num_samples)
av_mean = antithetic_variates(target_mean, num_samples)

# Output results
print("Monte Carlo Mean:", mc_mean)
print("Importance Sampling Mean:", is_mean)
print("Control Variates Mean:", cv_mean)
print("Antithetic Variates Mean:", av_mean)
```

```
Monte Carlo Mean: 0.03777138155393885
Importance Sampling Mean: 0.047125861260050084
Control Variates Mean: 5.021286532227588e-05
Antithetic Variates Mean: 0.0
```

```python
# Number of repetitions for each simulation to generate distributions
num_repetitions = 500

# Running the simulations multiple times
mc_estimates = [monte_carlo_mean(np.random.normal(target_mean,
target_std, num_samples)) for _ in range(num_repetitions)]
is_estimates = [importance_sampling(target_mean, target_std,
sampling_std, num_samples) for _ in range(num_repetitions)]
cv_estimates = [control_variates(target_mean, num_samples) for _ in
range(num_repetitions)]
av_estimates = [antithetic_variates(target_mean, num_samples) for _ in
range(num_repetitions)]

# Plotting the subplots of distributions of estimates
plt.figure(figsize=(12, 8))
```

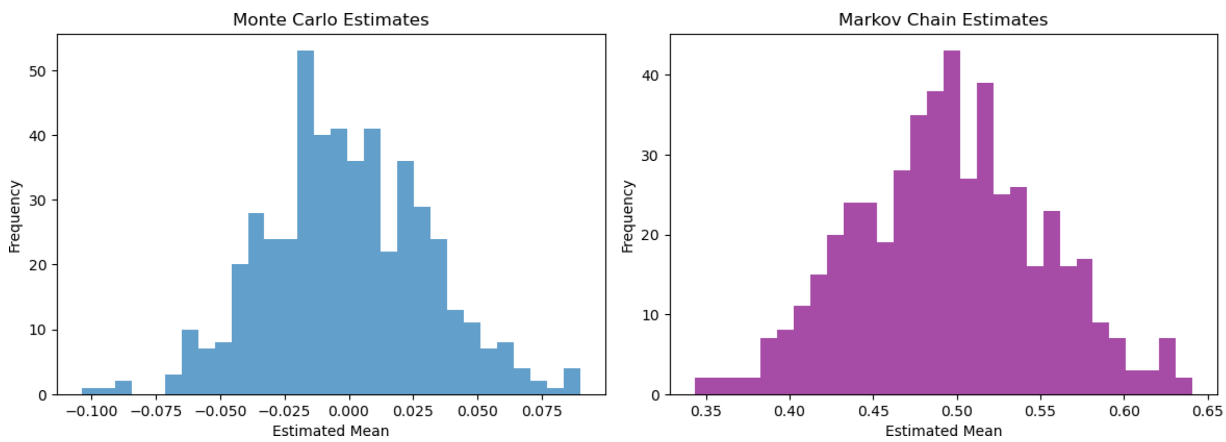
```
plt.subplot(2, 2, 1)
plt.hist(mc_estimates, bins=30, alpha=0.7, label='Monte Carlo')
plt.title('Monte Carlo Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.subplot(2, 2, 2)
plt.hist(is_estimates, bins=30, alpha=0.7, color='orange',
label='Importance Sampling')
plt.title('Importance Sampling Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.subplot(2, 2, 3)
plt.hist(cv_estimates, bins=30, alpha=0.7, color='green',
label='Control Variates')
plt.title('Control Variates Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.subplot(2, 2, 4)
plt.hist(av_estimates, bins=30, alpha=0.7, color='red',
label='Antithetic Variates')
plt.title('Antithetic Variates Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.tight_layout()
plt.show()
```



4.1.5 Comparison of Different Simulation Methods:

```
# Markov Chain Simulation for estimating the mean

def markov_chain_mean(transition_matrix, initial_state, target_mean, steps):
    state = initial_state
    values = [np.random.normal(target_mean, 1)]

    for _ in range(steps):
        state = np.random.choice(range(len(transition_matrix)), p=transition_matrix[state])
        values.append(np.random.normal(target_mean + state, 1))  #
Shift the mean based on the state

    return np.mean(values)

# Define the transition matrix for the Markov Chain

transition_matrix = np.array([[0.7, 0.3],
```

```python
# Parameters for the simulations
target_mean = 0
num_samples = 500
num_repetitions = 500

# Running the Markov Chain simulation multiple times
mc_chain_estimates = [markov_chain_mean(transition_matrix, 0,
target_mean, num_samples) for _ in range(num_repetitions)]

# Plotting the distributions of estimates
plt.figure(figsize=(12, 8))

plt.subplot(2, 2, 1)
plt.hist(mc_estimates, bins=30, alpha=0.7, label='Monte Carlo')
plt.title('Monte Carlo Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.subplot(2, 2, 2)
plt.hist(mc_chain_estimates, bins=30, alpha=0.7, color='purple',
label='Markov Chain')
plt.title('Markov Chain Estimates')
plt.xlabel('Estimated Mean')
plt.ylabel('Frequency')

plt.tight_layout()
plt.show()
```

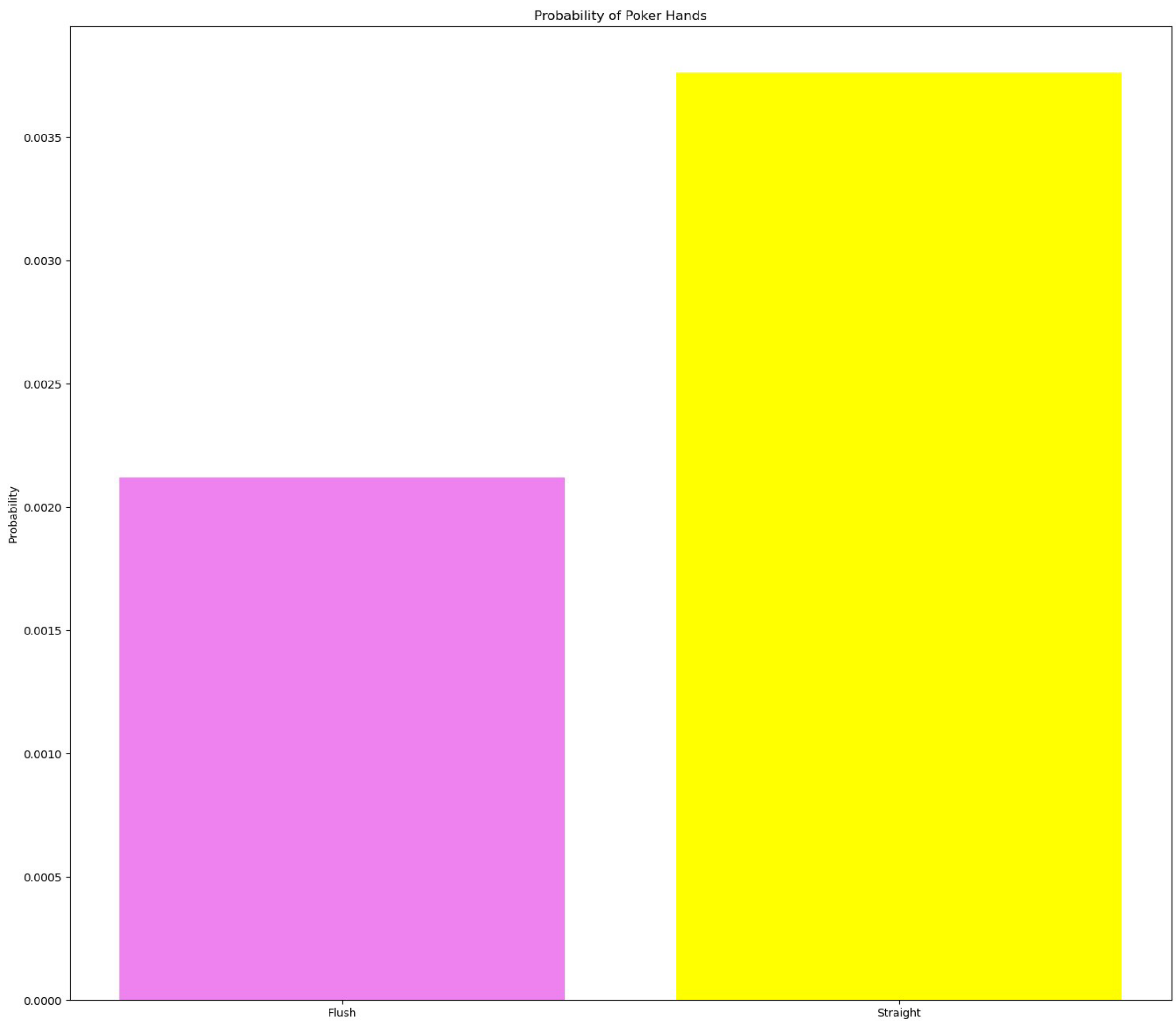


4.1.6 Simulation for Combinatorial Analysis:

```python
import itertools

# Function to create a standard deck of cards
```

```
    suits = ['Hearts', 'Diamonds', 'Clubs', 'Spades']
    ranks = ['2', '3', '4', '5', '6', '7', '8', '9', '10', 'J', 'Q',
'K', 'A']
    return list(itertools.product(ranks, suits))

# Function to check if a hand is a flush i.e all cards of the same
suit
def is_flush(hand):
    suits = [card[1] for card in hand]
    return len(set(suits)) == 1

# Function to check if a hand is a straight
def is_straight(hand):
    rank_values = {'2': 2, '3': 3, '4': 4, '5': 5, '6': 6, '7': 7,
'8': 8, '9': 9, '10': 10, 'J': 11, 'Q': 12, 'K': 13, 'A': 14}
    ranks = sorted([rank_values[card[0]] for card in hand])
    if ranks == list(range(ranks[0], ranks[0] + 5)):
        return True
    # Check for Ace-low straight
    if ranks == [2, 3, 4, 5, 14]:
        return True
    return False

# Simulation of drawing poker hands
def simulate_poker_hands(num_simulations):
    deck = create_deck()
    flush_count = 0
    straight_count = 0

    for _ in range(num_simulations):
        np.random.shuffle(deck)
        hand = deck[:5]
        if is_flush(hand):
            flush_count += 1
        if is_straight(hand):
            straight_count += 1

    return flush_count / num_simulations, straight_count /
num_simulations

# Number of simulations
num_simulations = 50000

# Running the simulation
flush_probability, straight_probability =
simulate_poker_hands(num_simulations)

# Plotting the results
hands = ['Flush', 'Straight']
probabilities = [flush_probability, straight_probability]
```

```
plt.figure(figsize=(18, 16))

plt.bar(hands, probabilities, color=['violet', 'yellow']) plt.title('Probability of Poker Hands')
plt.ylabel('Probability')

plt.show()

print("Estimated Probability of Drawing a Flush:", flush_probability) print("Estimated Probability of Drawing a Straight:", straight_probability)
```




```
Estimated Probability of Drawing a Flush: 0.00212 Estimated Probability of
Drawing a Straight: 0.00376
```

4.2.1 Bayes’ Theorem:

```
# Load the dataset
waste_data =
pd.read_csv('/Users/hp/Downloads/archive/2018_2020_waste.csv')

waste_data['Is Recyclable'] = waste_data['Total Recycled'] >
waste_data['Total Generated '] * 0.5
waste_data['Generated Category'] = pd.qcut(waste_data['Total Generated
'], 3, labels=["Low", "Medium", "High"])

waste_data.info()
waste_data.columns
```

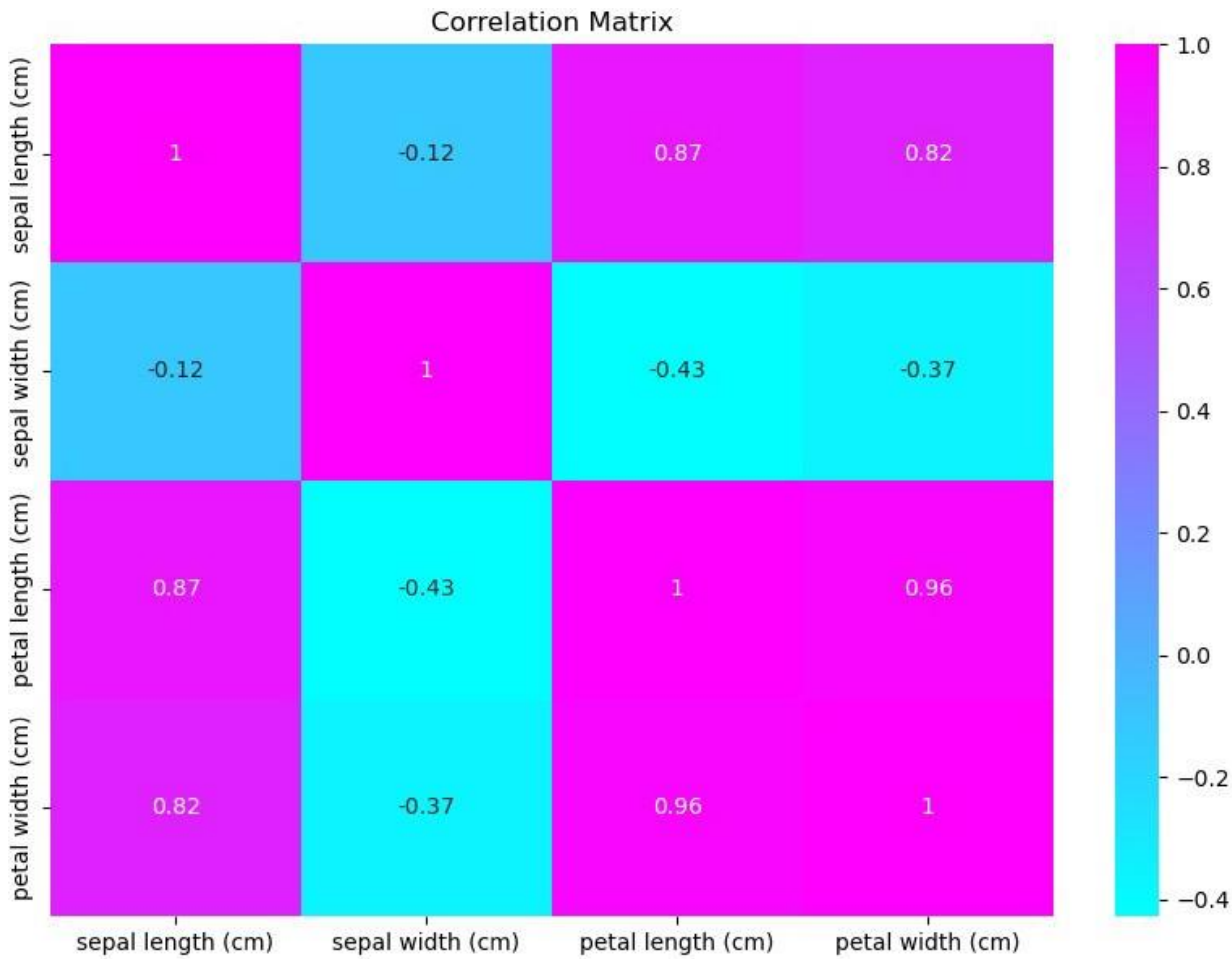
```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 45 entries, 0 to 44
Data columns (total 6 columns):
 #   Column              Non-Null Count   Dtype
------  -------            ---------------  ---------
 0   Waste Type          45 non-null      object
 1   Total Generated     45 non-null      int64
                                          int64
 2   Total Recycled      45 non-null      int64
 4   Is Recyclable       45 non-null      bool
 5   Generated Category  45 non-null      category
dtypes: bool(1), category(1), int64(3), object(1)
memory usage: 1.8+ KB

Index(['Waste Type', 'Total Generated ', 'Total Recycled', 'Year',
       'Is Recyclable', 'Generated Category'],
      dtype='object')
```

```
def bayes_theorem(p_a, p_b_given_a, p_b_given_not_a):
    """
    Apply Bayes' Theorem.

    :param p_a: Probability of A (prior probability)
    :param p_b_given_a: Probability of B given A (likelihood)
    :param p_b_given_not_a: Probability of B given not A
    :return: P(A|B) - Probability of A given B (posterior probability)
    """
    p_not_a = 1 - p_a
    p_b = (p_b_given_a * p_a) + (p_b_given_not_a * p_not_a)
    p_a_given_b = (p_b_given_a * p_a) / p_b
    return p_a_given_b

# Determine if waste type is recyclable
waste_data['Is Recyclable'] = waste_data['Total Recycled'] >
waste_data['Total Generated '] * 0.5
```

```
# Define categories for total generated waste

waste_data['Generated Category'] = pd.qcut(waste_data['Total Generated '], 3, labels=["Low", "Medium", "High"])

# Calculate probabilities

p_a = waste_data['Is Recyclable'].mean()  # Probability of being recyclable

p_b_given_a = waste_data[waste_data['Is Recyclable']]['Generated Category'].value_counts(normalize=True)  # P(B|A)

p_b_given_not_a = waste_data[~waste_data['Is Recyclable']]['Generated Category'].value_counts(normalize=True)  # P(B|not A)

# Example: Calculate the probability of being recyclable given a "High" generated category

p_a_given_b_high = bayes_theorem(p_a, p_b_given_a['High'], p_b_given_not_a['High'])

print("Probability of being recyclable given a high generated waste category:", p_a_given_b_high)

Probability of being recyclable given a high generated waste category: 0.5333333333333333
```

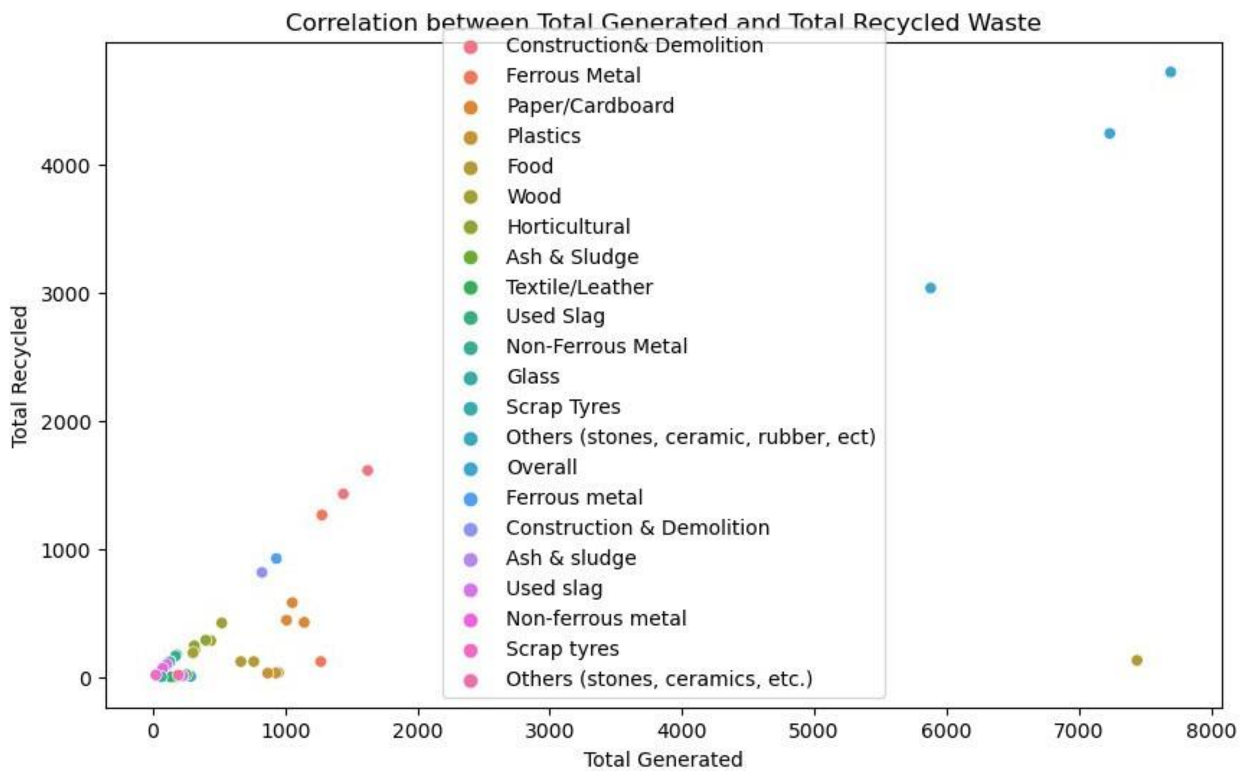
4.2.2 Joint Distribution Analysis

```
from sklearn.datasets import load_iris

from scipy.stats import shapiro, ks_2samp, norm

# I have taken the Iris datset # Load the Iris dataset

iris = load_iris()

df = pd.DataFrame(iris.data, columns=iris.feature_names) df['species'] = pd.Categorical.from_codes(iris.target, iris.target_names)

# Correlation matrix visualization plt.figure(figsize=(10, 7)) sns.heatmap(df.corr(), annot=True, cmap='cool') plt.title('Correlation Matrix')

plt.show()

C:\Users\hp\AppData\Local\Temp\ipykernel_20924\630334875.py:3: FutureWarning: The default value of numeric_only in DataFrame.corr is deprecated. In a future version, it will default to False. Select only valid columns or specify the value of numeric_only to silence this warning.
  sns.heatmap(df.corr(), annot=True, cmap='cool')
```




```
# Shapiro-Wilk test

stat, p = shapiro(df['petal length (cm)'])

print('Shapiro-Wilk Test: Statistics=%.3f, p=%.3f' % (stat, p))

# Kolmogorov-Smirnov test

ks_stat, ks_p = ks_2samp(df['petal length (cm)'], norm.rvs(size=len(df)))

print('Kolmogorov-Smirnov Test: Statistics=%.3f, p=%.3f' % (ks_stat, ks_p))

Shapiro-Wilk Test: Statistics=0.876, p=0.000 Kolmogorov-Smirnov Test:
Statistics=0.893, p=0.000

# Visualizing the correlation between 'Total Generated' and 'Total Recycled'

plt.figure(figsize=(10, 6))

sns.scatterplot(x='Total Generated ', y='Total Recycled', hue='Waste Type', data=waste_data)

plt.title('Correlation between Total Generated and Total Recycled Waste')

plt.xlabel('Total Generated')
```

```
plt.ylabel('Total Recycled') plt.legend()

plt.show()

# Conducting normality tests # Shapiro-
Wilk test

shapiro_generated = stats.shapiro(waste_data['Total Generated ']) shapiro_recycled =
stats.shapiro(waste_data['Total Recycled'])

print("Shapiro-Wilk Test on Total Generated Waste:", shapiro_generated)

print("Shapiro-Wilk Test on Total Recycled Waste:", shapiro_recycled)
```




```
Shapiro-Wilk Test on Total Generated Waste: ShapiroResult(statistic=0.5176205039024353,
pvalue=5.928293966839249e- 11)

Shapiro-Wilk Test on Total Recycled Waste: ShapiroResult(statistic=0.5245072245597839,
pvalue=7.243495386832777e- 11)
```

4.2.3 Factor Analysis

```
from sklearn.decomposition import PCA

from sklearn.preprocessing import StandardScaler
```

```python
# Since factor analysis works better with more features, creating some additional features for demonstration purposes

waste_data['Generated_per_Capita'] = waste_data['Total Generated '] / 1000
waste_data['Recycled_per_Capita'] = waste_data['Total Recycled'] / 1000
waste_data['Recycling_Rate'] = waste_data['Total Recycled'] / waste_data['Total Generated ']

# Standardizing the data
scaler = StandardScaler()
waste_scaled = scaler.fit_transform(waste_data[['Total Generated ', 'Total Recycled', 'Generated_per_Capita', 'Recycled_per_Capita', 'Recycling_Rate']])

# Performing PCA for factor extraction
pca = PCA(n_components=2)  # Reduce the data to 2 components
principalComponents = pca.fit_transform(waste_scaled)

# Creating a DataFrame with the principal components
principalDf = pd.DataFrame(data = principalComponents, columns = ['principal component 1', 'principal component 2'])

# Print the variance explained by each component
print("Variance explained by each component:", pca.explained_variance_ratio_)

# Displaying the principal components
print(principalDf.head())
```

```
Variance explained by each component: [0.72695767 0.21962308]
   principal component 1  princ ipal component 2
0               1.460429              -1.360479
1              -0.350339               1.124436
2               0.074758              -0.143314
3              -0.608379               1.210054
4              -0.597978               0.853449
```

CHAPTER:5

Conclusion

Comprehensive Analysis of Waste Management Data

This project says the in-depth analysis of a waste management dataset from the range of 2018 to 2020, using different simulation methods and Python coding techniques. First, we have clearly explained the data with descriptive statistics, providing a foundational understanding of waste generation and recycling trends. And then explained the main key matrices used like mean, variance, and standard deviation of the random variables. This preliminary analysis set the stage for more complex statistical inquiries, forming a solid base for the subsequent application of advanced analytical techniques.

Visualization and Normality Testing

Visual representation is the main part in not only understanding but also the crucial role in making the data accessible. Graphs and plots effectively explain the relationships and trends of the data easier to understand. Additionally, normality tests such as the Shapiro-Wilk test were conducted to assess the distribution of the data. These are very important in determining the appropriateness of different statistical models and in understanding the data.

Final Thoughts

The statistical analysis conducted provided the clear understanding of the waste management along with its graphs which has challenges and opportunities. By this analysis we can analyse the data effectively. And analysing the continuous and discrete random variables gives the clear understanding.

CHAPTER:6

References

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