Introduction to numpy:

Package for scientific computing with Python

Numerical Python, or "Numpy" for short, is a foundational package on which many of the most common data science packages are built. Numpy provides us with high performance multi-dimensional arrays which we can use as vectors or matrices.

The key features of numpy are:

- ndarrays: n-dimensional arrays of the same data type which are fast and space-efficient. There are a number of built-in methods for ndarrays which allow for rapid processing of data without using loops (e.g., compute the mean).
- Broadcasting: a useful tool which defines implicit behavior between multi-dimensional arrays of different sizes.
- Vectorization: enables numeric operations on ndarrays.
- Input/Output: simplifies reading and writing of data from/to file.

Additional Recommended Resources:

Numpy Documentation (https://docs.scipy.org/doc/numpy/reference/)
Python for Data Analysis by Wes McKinney
Python Data science Handbook by Jake VanderPlas

Getting started with ndarray

ndarrays are time and space-efficient multidimensional arrays at the core of numpy. Like the data structures in Week 2, let's get started by creating ndarrays using the numpy package.

How to create Rank 1 numpy arrays:

```
In [1]:
```

```
import numpy as np
an_array = np.array([3, 33, 333]) # create a rank 1 array
print(type(an_array)) # The type of an ndarray is "<class 'numpy.ndarray'>"
```

```
<class 'numpy.ndarray'>
```

```
In [2]:
# test the shape of the array we just created
print(an array.shape)
(3,)
In [3]:
# because this is a 1-rank array, we need only one index to access each element
print(an array[0], an array[1], an array[2])
3 33 333
In [4]:
an array[0] = 888 #change an element of the array
print(an array)
[888 33 333]
In [5]:
another = np.array([[11,12,13], [21,22,23]]) # Create a rank 2 array
print(another) # print the array
print('The shape is 2 rows, 3 columns: ', another.shape) # rows x columns
print('Accessing elements [0,0], [0,1], and [1,0] of the narray', another[0,0],
',', another[0,1], ',', another[1,0])
[[11 12 13]
[21 22 23]]
The shape is 2 rows, 3 columns: (2, 3)
Accessing elements [0,0], [0,1], and [1,0] of the narray 11, 12, 2
In [6]:
import numpy as np
#create a 2x2 array of zeros
ex1 = np.zeros((2,2))
print(ex1)
[[0. 0.]
 [0. 0.]]
In [7]:
#create a 2x2 array filled with 9.0
ex2 = np.full((2,2),9.0)
print(ex2)
[[9. 9.]
 [9. 9.]]
```

```
In [8]:
# create a 2 x2 matrix with diagonal 1s and the others 0
ex3 = np.eye(2,2)
print(ex3)
[[1. 0.]
 [0. 1.]]
In [9]:
# create an array of ones
ex4 = np.ones((1,2))
print(ex4)
[[1. 1.]]
In [10]:
#notice that the above ndarray (ex4) is actually rank 2, it is a 2x1 array
print(ex4.shape)
#which means we need to use two indexes to access an element
print()
print(ex4[0,1])
(1, 2)
1.0
In [11]:
# create an array of random floats between 0 and 1
ex5 = np.random.random((2,2))
print(ex5)
```

```
Array Indexing
```

[[0.02494572 0.12015575] [0.57899657 0.07059495]]

Slice indexing:

Similar to the use of slice indexing with lists and strings, we can use slice indexing to pull out sub-regions of ndarrays.

In [12]:

```
import numpy as np
# Rank 2 array of shape (3,4)
an array = np.array([[11,12,13,14], [21,22,23,24], [31,32,33,34]])
print(an array)
[[11 12 13 14]
[21 22 23 24]
[31 32 33 34]]
```

Use array slicing to get a subarray consisting of the first 2 rows x 2 columns.

In [13]:

```
a_slice = an_array[:2, 1:3] #印出 an_array[0,1], [0,2], [1,1], and [1,2]的值
print(a slice)
[[12 13]
[22 23]]
```

When you modify a slice, you actually modify the underlying array.

In [14]:

```
print('Before: ', an_array[0,1]) #inspect the element at 0,1
a slice[0,0] = 1000 \# a \ slice[0,0] is the same piece of data as an array[0,1]
print('After: ', an_array[0,1])
```

Before: 12 After: 1000

Use both integer indexing & slice indexing

We can use combinations of integer indexing and slice indexing to create different shaped matrices.

```
In [15]:
```

```
# Create a Rank 2 array of shape (3,4)
an array = np.array([[11,12,13,14], [21,22,23,24], [31,32,33,34]])
print(an array)
[[11 12 13 14]
[21 22 23 24]
 [31 32 33 34]]
In [16]:
# Using bother integer indexing & slicing generates an array of Lower Rank
row_rank1 = an_array[1, :] # Rank 1 view (row 1)
print(row rank1, row rank1.shape) # Notice only a single [] => get 1 row only
[21 22 23 24] (4,)
```

```
In [17]:
```

```
# Slicing alone: generates an array of the same rank as the an_array
row rank2 = an array[1:2, :] #Rank 2 view
print(row rank2, row rank2.shape) #Notice the [[]] 2D array => 1 row x 4 columns
[[21 22 23 24]] (1, 4)
In [18]:
# We can do the same thing for columns of an array:
print()
col rank1 = an array[:, 1]
col rank2 = an array[:, 1:2]
print(col rank1, col rank1.shape) # Rank 1 Only the column 1 value
print()
print(col_rank2, col_rank2.shape) # Rank 2 => 3 rows x 1 column
[12 22 32] (3,)
[[12]
 [22]
 [32]] (3, 1)
```

Array Indexing for changing elements:

Sometimes it's useful to use an array of indexes to access or change elements.

```
In [19]:
# Create a new array
an array = np.array([[11,12,13], [21,22,23], [31,32,33], [41,42,43]])
print('Original Array:')
print(an array)
Original Array:
[[11 12 13]
 [21 22 23]
 [31 32 33]
 [41 42 43]]
In [20]:
# Create an array of indices
col indices = np.array([0, 1, 2, 0])
print('\nCol indices picked: ', col indices)
row indices = np.arange(4) # Use the np.arange fuction to create an ndarray
print('\nRows indices picked: ', row_indices)
Col indices picked: [0 1 2 0]
Rows indices picked:
                     [0 1 2 3]
```

```
In [21]:
```

```
# Examine the pairings of row indices and col indices.
for row, col in zip(row_indices, col_indices):
    print(row, ', ', col)
0,
     0
1,
     1
     2
     0
zip()是Python的一个内建函数,它接受一系列可迭代的对象作为参数,将对象中对应的元素打包成一个个
tuple(元组),然后返回由这些tuples组成的list(列表)
In [22]:
# Select one element from each row
print('Values in the array at those indices: ', an array[row indices, col indice
s])
Values in the array at those indices: [11 22 33 41]
In [23]:
# Change one element from each row using the indices selected
an array[row indices, col indices] += 100000 # value加上100000
print('\nChanged Array: ')
print(an array)
Changed Array:
[[100011
            12
                    131
      21 100022
                    23]
             32 1000331
      31
 [100041
             42
                    43]]
Quiz 1: If you try to access rows of a 3-by-3 numpy array called "arr" using the command: arr[:2,] How
```

many rows will be returned?

```
In [24]:
arr = np.array([[11,12,13],[21,22,23],[31,32,33]])
print(arr)
[[11 12 13]
[21 22 23]
 [31 32 33]]
In [25]:
print(arr[:2,]) #印出 arr [0,0], [0,1], [0,3], [1,1],[1,2], and [1,3] 的數值
[[11 12 13]
 [21 22 23]]
```

In [26]:

Boolean Indexing

Array Indexing for changing elements:

```
# Create a 3x2 array
an array = np.array([[11,12],[21,22],[31,32]])
print(an_array)
[[11 12]
 [21 22]
 [31 32]]
In [27]:
# create a filter which will be boolean values for whether each element meets th
is condition
filter = (an array > 15)
filter
Out[27]:
array([[False, False],
        [ True, True],
        [ True,
                 True]])
Notice that the filter is a same size ndarray as an_array which is filled with True for each element whose
corresponding element in an array which is greater than 15 and False for those elements whose value is less
than 15.
In [28]:
# We can now select just those elements which meet that criteria
print(an_array[filter])
[21 22 31 32]
In [29]:
# For short, we could have just used the approach below without the need for the
 separate filter array.
an_array[(an_array > 20) & (an_array < 30)]</pre>
Out[29]:
array([21, 22])
In [30]:
an_array[(an_array % 2 == 0)]
Out[30]:
array([12, 22, 32])
```

```
In [31]:
```

```
an_array[an_array % 2 == 0] += 100
print(an_array)

[[ 11 112]
  [ 21 122]
```

Datatypes and Array Operations

Datatypes:

[31 132]]

```
In [32]:
ex1 = np.array([11, 12]) # Python assigns the data type
print(ex1.dtype) # result is integer
int64
In [33]:
ex2 = np.array([11.0, 12.0]) # Python assigns the data type
print(ex2.dtype)
float64
In [34]:
ex3 = np.array([11, 12], dtype = np.int64) #You can also tell Python the
print(ex3.dtype)
int64
In [35]:
# you can use this to force floats into integers (using floor function)
ex4 = np.array([3.29, 11.29], dtype = np.int64)
print(ex4.dtype, end = '\n')
print(ex4)
int64
[ 3 11]
In [36]:
# you can use this to force integers into floats if you anticipate
# the values may change to floats later
ex5 = np.array([11, 21], dtype = np.float64)
print(ex5.dtype, end = '\n')
print(ex5)
float64
[11. 21.]
```

Arithmetic Array Operations:

```
In [37]:
x = np.array([[111,112], [121,122]], dtype = np.int)
y = np.array([[211.1, 212.1], [221.1, 222.1]], dtype = np.float)
print(x)
print()
print(y)
[[111 112]
 [121 122]]
[[211.1 212.1]
 [221.1 222.1]]
In [38]:
#add
print(x + y) # The plus sign works
print()
print(np.add(x, y)) # so does the numpy function 'add'
[[322.1 324.1]
 [342.1 344.1]]
[[322.1 324.1]
 [342.1 344.1]]
In [39]:
#subtract
print(x - y)
print()
print(np.subtract(x, y)) # so does the numpy function 'subtract'
[[-100.1 -100.1]
 [-100.1 -100.1]
[[-100.1 -100.1]
 [-100.1 -100.1]
In [40]:
# Multiply
print(x * y)
print(np.multiply(x, y)) # so does the numpy function 'multiply'
[[23432.1 23755.2]
 [26753.1 27096.2]]
[[23432.1 23755.2]
 [26753.1 27096.2]]
```

```
In [41]:
# divide
print(x / y)
print()
print(np.divide(x, y)) # so does the numpy function 'divide'
[[0.52581715 0.52805281]
 [0.54726368 0.54930212]]
[[0.52581715 0.52805281]
 [0.54726368 0.54930212]]
In [42]:
# square root
print(np.sqrt(x))
[[10.53565375 10.58300524]
 [11.
              11.04536102]]
In [43]:
# exponent (e ** x)
print(np.exp(x))
[[1.60948707e+48 4.37503945e+48]
 [3.54513118e+52 9.63666567e+52]]
```

Statistical Methods, Sorting, and Set Operations:

Basic Statistical Operations:

2.454254832366753

```
In [44]:

# Setup a random 2 x 5 matrix

arr = 10 * np.random.randn(2,5) #注意這邊是 randn 而不是 random
print(arr)

[[-4.29596702 8.85695623 -2.93709496 -5.74596835 12.94277054]
  [ 5.01788568 -5.96702379 8.35652711 10.57380255 -2.25933966]]

In [45]:

# compute the mean for All elements
print(arr.mean())
```

```
In [46]:
```

```
# compute the means by row
print(arr.mean(axis = 1))
[1.76413929 3.14437038]
In [47]:
# compute the means by column
print(arr.mean(axis = 0))
[0.36095933 1.44496622 2.70971607 2.4139171 5.34171544]
In [48]:
# sum all the elements
print(arr.sum())
24.542548323667532
In [49]:
# compute the medians by row
print(np.median(arr, axis = 1))
```

[-2.93709496 5.01788568]

Sorting:

```
In [50]:
```

```
# create a 10 element array of randoms
unsorted = np.random.randn(10)
print(unsorted)
[ 0.6883593   -0.35184334   -1.23295929   -0.7871837
                                                    1.43636212 -0.5070
8798
 -1.48854685 -0.85241316 -1.82942734 -0.18040806]
In [51]:
# create copy and sort
sorted = np.array(unsorted) #copy unsorted, then name it as sorted
sorted.sort()
print(sorted)
print()
print(unsorted)
```

```
[-1.82942734 -1.48854685 -1.23295929 -0.85241316 -0.7871837 -0.5070]
-0.35184334 - 0.18040806 0.6883593
                              1.43636212]
1.43636212 -0.5070
8798
-1.48854685 -0.85241316 -1.82942734 -0.18040806]
```

```
In [52]:
```

```
# inplace sorting
unsorted.sort()
print(unsorted)

[-1.82942734 -1.48854685 -1.23295929 -0.85241316 -0.7871837 -0.5070
8798
-0.35184334 -0.18040806 0.6883593 1.43636212]
```

Finding Unique elements:

```
In [53]:
array = np.array([1, 2, 1, 4, 2, 1, 4, 2])
print(np.unique(array)) #不重複取出
[1 2 4]
```

Set Operations with np.array data type:

```
In [54]:
s1 = np.array(['desk', 'chair', 'bulb'])
s2 = np.array(['lamp', 'bulb', 'chair', 'table'])
print(s1, s2)
['desk' 'chair' 'bulb'] ['lamp' 'bulb' 'chair' 'table']
In [55]:
print(np.intersect1d(s1, s2)) #s1 & s2都有
['bulb' 'chair']
Notice that we're using intersect1d because intersect expects 1d arrays
In [56]:
print(np.union1d(s1, s2)) # The metod union will give us all of the unique eleme
nts across both arrays.
['bulb' 'chair' 'desk' 'lamp' 'table']
In [57]:
print(np.setdiff1d(s1,s2)) # S1有, S2沒有
['desk']
```

```
In [58]:
print(np.setdiff1d(s2,s1)) # S2有, S1沒有
['lamp' 'table']
In [59]:
print(np.inld(s1, s2)) # We can get back an array of Booleans, for whether each
 element(s1) is in the array (s2) or not
[False True True]
```

Broadcasting:

Introduction to broadcasting.

For more details, please see:

[1. 0. 2.] [1. 0. 2.]]

https://docs.scipy.org/doc/numpy-1.10.1/user/basics.broadcasting.html (https://docs.scipy.org/doc/numpy-1.10.1/user/basics.broadcasting.html)

```
In [60]:
# We create a 4x3 ndarray and we fill it with zeros
import numpy as np
start = np.zeros((4,3))
print(start)
[[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]
In [61]:
# create a rank 1 ndarray with 3 values
In [62]:
add rows = np.array([1, 0, 2])
print(add rows)
[1 0 2]
In [63]:
# add to each row of 'start' using broadcasting
y = start + add rows
print(y)
[[1. 0. 2.]
 [1. 0. 2.]
```

```
In [64]:
# create an ndarray which is 4 x 1 to broadcast across columns
add_cols = np.array([[0,1,2,3]])
add cols = add cols.T #Transpose on it, denoted by T
print(add cols)
[[0]]
 [1]
 [2]
 [3]]
In [65]:
# add to each column of 'start' using broadcasting
y = start + add cols
print(y)
[[0. 0. 0.]
 [1. 1. 1.]
 [2. 2. 2.]
 [3. 3. 3.]]
In [66]:
# This will just broadcast in both dimensions
add scalar = np.array([1])
print(start + add scalar)
[[1. 1. 1.]
 [1. 1. 1.]
 [1. 1. 1.]
 [1. 1. 1.]]
Quiz 2: True or False: Take a look at the following lines of code:
a = np.array([[0,0],[0,0]])
b1 = np.array([1,1])
b2 = 1
a+b1 and a+b2 result in the same matrix.
In [67]:
a = np.array([[0,0],[0,0]])
b1 = np.array([1,1])
b2 = 1
print(a+b1)
print()
print(a+b2)
[[1 1]
```

Example from the slides:

 $[1 \ 1]]$

[[1 1] [1 1]]

```
In [68]:
# create our 3x4 matrix
arrA = np.array([[1,2,3,4],[5,6,7,8],[9,10,11,12]])
print(arrA)
[[1 2 3 4]
[5 6 7 81
 [ 9 10 11 12]]
In [69]:
# create our 4x1 array
arrB = [0,1,0,2]
print(arrB)
[0, 1, 0, 2]
In [70]:
# add the two together using broadcasting
print(arrA + arrB)
[[1 3 3 6]
 [57710]
 [ 9 11 11 14]]
```

Speedtest: ndarrays vs lists

First setup paramaters for the speed test. We'll be testing time to sum elements in an ndarray versus a list.

```
In [71]:
```

from numpy import arange

Time taken by numpy ndarray: 0.000464 seconds

Time taken by list: 0.003579 seconds

Read or Write to Disk:

Binary Format:

1.12899999999999915e+01

```
In [76]:
x = np.array([3.29, 11.29])

In [77]:
np.save('an_array', x)

In [78]:
np.load('an_array.npy')

Out[78]:
array([ 3.29, 11.29])

Text Format:
In [79]:
np.savetxt('array.txt', X=x, delimiter=',')

In [80]:
!cat array.txt
3.2900000000000000036e+00
```

```
In [81]:
np.loadtxt('array.txt', delimiter=',')
Out[81]:
array([ 3.29, 11.29])
```

Additional Common ndarray Operations

Dot Product on Matrices and Inner Product on Vectors:

```
In [82]:
# determine the dot product of two matrices
x2d = np.array([[1,1],[1,1]])
y2d = np.array([[2,2],[2,2]])
print(x2d.dot(y2d))
print()
print(np.dot(x2d, y2d)) #或者這樣寫
[[4 \ 4]
 [4 4]]
[[4 4]
 [4 4]]
In [83]:
# determine the inner product of two vectors
ald = np.array([9, 9])
b1d = np.array([10, 10])
print(ald.dot(bld))
print()
print(np.dot(ald, bld))
180
180
In [84]:
# dot produce on an array and vector
print(x2d.dot(a1d))
print()
print(np.dot(x2d, a1d))
[18 18]
[18 18]
```

Sum:

[23 43]

```
In [85]:
# sum elements in the array
ex1 = np.array([[11,12],[21,22]])
print(np.sum(ex1)) # add all members

66
In [86]:
print(np.sum(ex1, axis = 0)) # columnwise sum 11 + 21 = 32, 12 + 22 = 34

[32 34]
In [87]:
print(np.sum(ex1, axis = 1)) # rowwise sum 11 + 12 = 23, 21 + 22 = 43
```

Element-wise Functions:

For example, let's compare two arrays values to get the maximum of each.

```
In [88]:
# random array
x = np.random.randn(8)
Out[88]:
array([-0.98914009, 1.54842806, 1.3024303, 2.4891781, 0.218884
69,
       -0.09619948, -0.3736248, -0.47017262])
In [89]:
# another random array
y = np.random.randn(8)
У
Out[89]:
array([ 0.48487665, -1.48107856, -0.88102 , -0.03890742, -0.752983
21,
       -0.40312076, -0.34005599, 0.96203137])
In [90]:
# returns element wise maximum between two arrays
np.maximum(x,y)
Out[90]:
array([ 0.48487665, 1.54842806, 1.3024303 , 2.4891781 , 0.218884
69,
       -0.09619948, -0.34005599, 0.96203137])
```

Reshaping array:

[1 22 3 44 5]

```
In [91]:
# grab values from 0 through 19 in an array
arr = np.arange(20) #不包含20
print(arr)
[ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19]
In [92]:
# reshape to be a 4 x 5 matrix
arr.reshape(4,5)
Out[92]:
array([[ 0, 1, 2, 3, 4],
       [5, 6, 7, 8,
                        9],
      [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19]])
Transpose:
In [93]:
# transpose
ex1 = np.array([[11,12],[21,22]])
ex1.T
Out[93]:
array([[11, 21],
      [12, 22]])
Indexing using where():
In [94]:
x_1 = np.array([1, 2, 3, 4, 5])
y 1 = np.array([11, 22, 33, 44, 55])
filter = np.array([True, False, True, False, True])
In [95]:
out = np.where(filter, x 1, y 1) # x 1中為True用 x 1的值, 反之y 1
print(out)
```

可參考: https://www.zhihu.com/question/62844162 (https://www.zhihu.com/question/62844162)

```
In [96]:
```

```
mat = np.random.rand(5,5)
mat
Out[96]:
array([[0.34180607, 0.90830567, 0.64669763, 0.64762527, 0.73685068],
       [0.83410405, 0.97054872, 0.77749058, 0.73295702, 0.40768179],
       [0.45726776, 0.56880766, 0.34389295, 0.42142743, 0.74086624],
       [0.5749876, 0.69487209, 0.6335134, 0.29589649, 0.60473162],
       [0.86313484, 0.42168064, 0.9590684 , 0.33773841, 0.9718863
1]])
In [97]:
np.where(mat > 0.5, 1000, -1) #大於0.5標示 1000, 反之-1
Out[97]:
array([[ -1, 1000, 1000, 1000, 1000],
       [1000, 1000, 1000, 1000,
       [-1, 1000,
                     -1, -1, 1000],
       [1000, 1000, 1000, -1, 1000],
              -1, 1000, -1, 1000]])
       [1000,
```

"any" or "all" conditionals:

```
In [98]:

arr_bools = np.array([ True, True, False, True, False])

In [99]:

arr_bools.any() #任何一個有True就是True

Out[99]:

True

In [100]:

arr_bools.all() #全部都是True

Out[100]:

False
```

Random Number Generation:

```
In [101]:
Y = np.random.normal(size = (1,5))[0]
print(Y)
[ 0.31472104    0.22044968    1.50351937 -0.08042775    0.21288945]
```

```
In [102]:
Z = np.random.randint(low=2, high=50, size=4)
print(Z)
[24 25 9 14]
In [103]:
np.random.permutation(Z) #return a new ordering of elements in Z Z中的數值隨機排列
Out[103]:
array([25, 9, 14, 24])
In [104]:
np.random.uniform(size=4) #uniform distribution
Out[104]:
array([0.25273061, 0.46032597, 0.87630175, 0.86874634])
In [105]:
np.random.normal(size=4) #normal distribution
Out[105]:
array([-0.04285344, -0.39094056, -0.75571981, 0.85482712])
Merging data sets:
In [106]:
K = np.random.randint(low=2, high=50, size=(2,2))
print(K)
print()
M = np.random.randint(low=2, high=50, size=(2,2))
print(M)
[[22 9]
 [44 41]]
[[23 15]
 [6 3]]
In [107]:
np.vstack((K,M)) #垂直合併
Out[107]:
array([[22, 9],
       [44, 41],
       [23, 15],
```

[6, 3]])

```
In [108]:
np.hstack((K,M)) #水平合併
Out[108]:
array([[22, 9, 23, 15],
      [44, 41, 6, 3]])
In [109]:
np.concatenate([K,M], axis = 0) # 建column
Out[109]:
array([[22, 9],
      [44, 41],
       [23, 15],
       [ 6, 3]])
In [110]:
np.concatenate([K,M], axis = 1) # 連row
Out[110]:
array([[22, 9, 23, 15],
      [44, 41, 6, 3]])
In [111]:
arr = np.array([[1,2,3],[4,5,6],[7,8,9]])
arr[:2]
Out[111]:
array([[1, 2, 3],
       [4, 5, 6]])
In [112]:
slice = arr[:2, 1:3] #第一個取集合, 第二度取數值
slice
Out[112]:
array([[2, 3],
       [5, 6]])
In [113]:
slice[0,0]
Out[113]:
2
```

```
In [114]:
arr
Out[114]:
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])
In [115]:
A = np.array([[1],[2]])
B = np.array([[1,2], [3,4]])
A+B
Out[115]:
array([[2, 3],
       [5, 6]])
In [116]:
В
Out[116]:
array([[1, 2],
       [3, 4]])
In [117]:
Α
Out[117]:
array([[1],
       [2]])
```