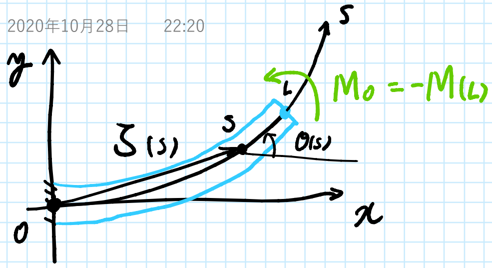
# Elastica cantilever

2020年10月28日 22:20




- $$\left.\begin{aligned} \zeta(s) &:= \int_0^s e^{i\theta(s')} ds', \\ M(s) &= -EI\,\partial_s \theta(s) \quad . \end{aligned}\right\}$$
- $M(s) = -M_0, \ s \in [0, L],$
- $\theta(s)|_{s=0} = 0.$

$$\Rightarrow \ M_0 = EI\,\partial_s\theta(s),$$

$$\Leftrightarrow \ \partial_s\theta(s) = \frac{M_0}{EI},$$

$$\Leftrightarrow \ \theta(s) = \frac{M_0}{EI}s + C_0,$$

- $\theta(0) = C_0 = 0,$

$$\Rightarrow \ \theta(s) = \frac{M_0}{EI}s \ .$$

$$\Rightarrow \ \zeta(s) = \int_0^s e^{i\theta(s')}ds'$$

$$= \int_0^s e^{i\frac{M_0}{EI}s'}ds' = \left.\frac{EI}{iM_0} e^{i\frac{M_0}{EI}s'}\right|_0^s$$

$$= -i\frac{EI}{M_0}\left(e^{i\frac{M_0}{EI}s} - 1\right) = -i\frac{EI}{M_0}\left(\cos\frac{M_0}{EI}s + i\sin\frac{M_0}{EI}s - 1\right)$$

$$= \frac{EI}{M_0}\sin\frac{M_0}{EI}s + i\frac{EI}{M_0}\left(1 - \cos\frac{M_0}{EI}s\right)$$

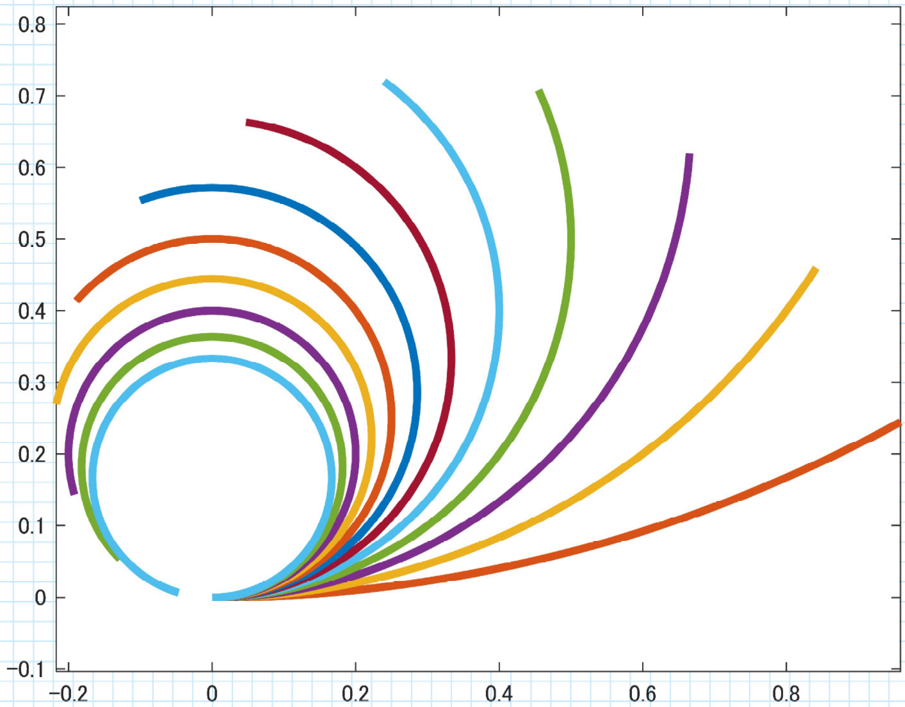
$$\Rightarrow \left.\begin{aligned} w(s) = \mathrm{Im}(\zeta(s)) &= \frac{EI}{M_0}\left(1 - \cos\frac{M_0}{EI}s\right) \\ &=: \frac{L}{m_0}\left(1 - \cos\frac{m_0 s}{L}\right), \quad \left(m_0 := \frac{M_0 L}{EI}\right), \\ v(s) = \mathrm{Re}(\zeta(s)) &= \frac{L}{m_0}\sin\frac{m_0 s}{L} \ . \end{aligned}\right\}$$

$$\Rightarrow \ w(s)^2 + v(s)^2 = \frac{L^2}{m_0^2}\left(1 - 2\cos\frac{m_0}{L}s + \cos^2\frac{m_0}{L}s\right) + \frac{L^2}{m_0^2}\sin^2\frac{m_0}{L}s$$

$$\Rightarrow \quad w^2(s) + v(s)^2 = \frac{L^2}{m_0^2}\left(1 - 2a\frac{m_0}{L}s + a^2\frac{m}{L}s\right) + \frac{L^2}{m_0^2} a'^2 \frac{m_0}{L} s$$

$$= 2\frac{L^2}{m_0^2} - 2\frac{L^2}{m_0^2} a \frac{m_0}{L} s$$

$$= 2\frac{L^2}{m_0^2}\left(1 - a\frac{m_0}{L}s\right) = 2\frac{L}{m_0} w(s)$$

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◎ Linear Case

$$\left.\begin{aligned} & EI\frac{\partial^2 w}{\partial x^2} = -M(x), \\ & M(x) = -M_0, \ x \in [0, L]. \end{aligned}\right\}$$

$$\Rightarrow \quad EI\frac{\partial^2 w}{\partial x^2} = M_0$$

$$\Leftrightarrow \quad \frac{\partial^2 w}{\partial x^2} = \frac{M_0}{EI} =: \frac{m_0}{L}$$

$$\Leftrightarrow \quad \frac{\partial w}{\partial x} = \frac{m_0}{L}x + C_0$$

$$\text{B.C.}: \left.\frac{\partial w}{\partial x}\right|_{x=0} = C_0 = 0$$

$$\Rightarrow w = \frac{m_0}{2L}x^2 + C_1$$

$$\text{B.C.}: \left. w \right|_{x=0} = C_1 = 0$$

$$\Rightarrow w = \frac{m_0}{2L}x^2$$