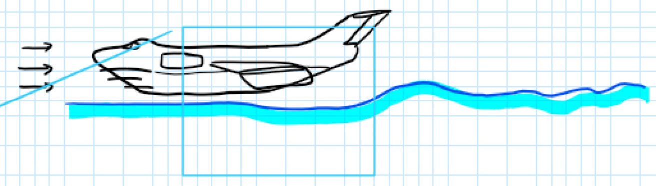
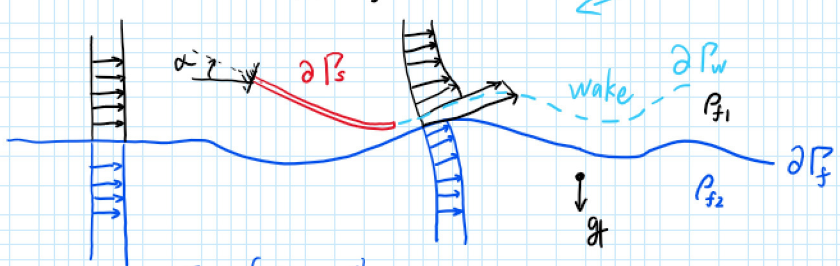
Water surface and flexible sheet

2021年2月17日　16:52



$$\Delta\Phi_1 = \Delta\{U_\infty x + \phi_1\} = 0,$$
$$\Leftrightarrow \Delta\phi_1 = 0.$$

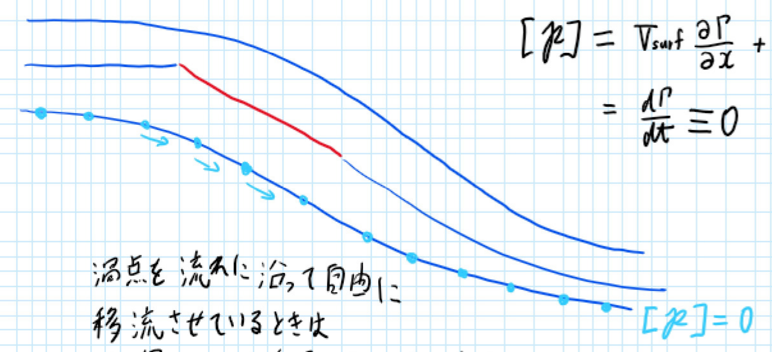


$$\Delta\Phi_2 = \Delta\{U_\infty x + \phi_2\} = 0,$$
$$\Leftrightarrow \Delta\phi_2 = 0.$$

- PDE : $\Delta\phi_1 = 0,\ \Delta\phi_2 = 0.$
- BC : (i) $\mathbf{n}^T\cdot(\mathbf{u}_1 - \mathbf{u}_2) = 0$,

$$\Leftrightarrow \mathbf{n}^T(\nabla\phi_1 - \nabla\phi_2) = 0,\ x \in \partial\Gamma_f.$$

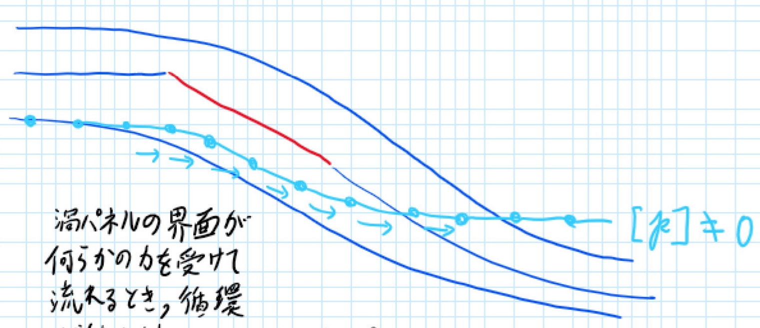
(a)



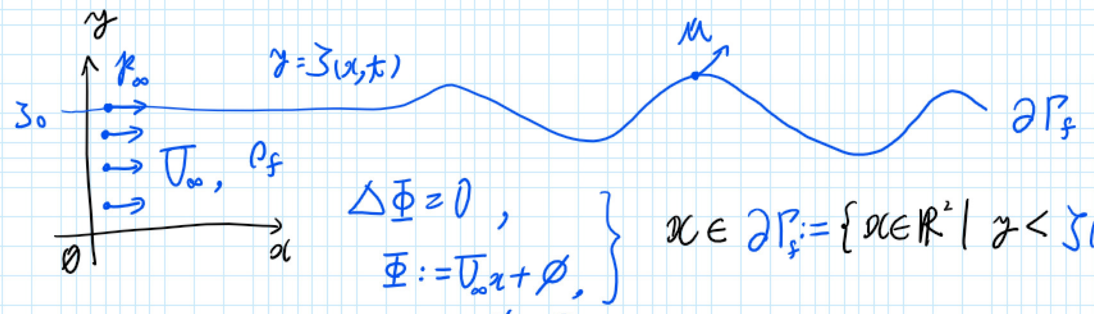
$$[p] = V_{surf}\frac{\partial\Gamma}{\partial x} + \frac{\partial\Gamma}{\partial t}$$
$$= \frac{d\Gamma}{dt} \equiv 0$$

渦点を流れに沿って自由に
移流させているときは
循環は0になる. ⇒ 圧力差0.
(流れは流線を横切らない)

(b)



渦パネルの界面が
何らかの力を受けて
流れるとき, 循環
が値を持つ. ⇒ 圧力差≠0.



$$\left.\begin{aligned}&\Delta\Phi = 0,\\ &\Phi := U_\infty x + \phi,\\ &\Rightarrow \Delta\phi = 0.\end{aligned}\right\}\quad x \in \partial\Gamma_f := \{x \in \mathbb{R}^2 \mid y < \zeta(x,t)\},$$

- Kinematic B.C.

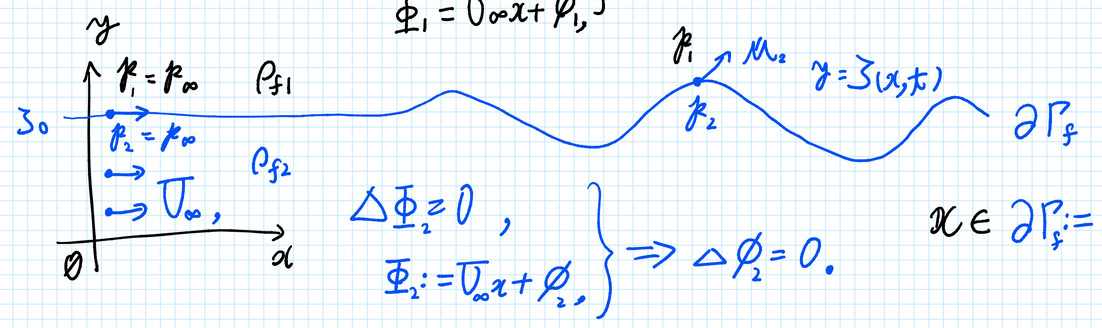
$$\partial\Gamma_f : y = \zeta(x,t) \Leftrightarrow F(x,t)\big|_{\partial\Gamma_f} := \{y - \zeta(x,t)\}\big|_{\partial\Gamma_f} \equiv 0,$$
$$\Leftrightarrow \frac{d}{dt}F(x,t)\Big|_{\partial\Gamma_f} \equiv 0,$$
$$\Leftrightarrow \frac{\partial F}{\partial t}\Big|_{\partial\Gamma_f} + \frac{\partial F}{\partial x}\Big|_{\partial\Gamma_f}\cdot\frac{dx}{dt}\Big|_{\partial\Gamma_f} = 0,$$
$$\Leftrightarrow -\frac{\partial\zeta}{\partial t}\Big|_{\partial\Gamma_f} + \begin{bmatrix}-\frac{\partial\zeta}{\partial x} & 1\end{bmatrix}\Big|_{\partial\Gamma_f}\cdot\begin{bmatrix}u\\ v\end{bmatrix}\Big|_{\partial\Gamma_f} = 0,$$
$$\Leftrightarrow v\big|_{\partial\Gamma_f} = \left\{\frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x}\right\}\Big|_{\partial\Gamma_f},$$
$$\Leftrightarrow \boxed{\frac{\partial\Phi}{\partial y} = \frac{\partial\zeta}{\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial\zeta}{\partial x},\ x \in \partial\Gamma_f.} \quad \text{—— ①}$$

- Dynamic B.C. (Unsteady Bernoulli's principle)

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}\|\mathbf{u}\|^2 + \frac{p}{\rho_f} + gy = f(t),$$
$$\Rightarrow \left\{\frac{\partial\Phi}{\partial t} + \frac{1}{2}\|\mathbf{u}\|^2 + \frac{p}{\rho_f} + gy\right\}\Big|_{\partial\Gamma_f} = \frac{1}{2}U_\infty^2 + \frac{p_\infty}{\rho_f} + g\zeta_0$$
$$\Leftrightarrow \left\{\frac{\partial\Phi}{\partial t} + \frac{1}{2}\|\mathbf{u}\|^2\right\}\Big|_{\partial\Gamma_f} + g\{\zeta|_{\partial\Gamma_f} - \zeta_0\} = \frac{1}{2}U_\infty^2 + \frac{1}{\rho_f}\{p_\infty - p|_{\partial\Gamma_f}\},$$
$$\Leftrightarrow \left\{\frac{\partial\Phi}{\partial t} + \frac{1}{2}\|\mathbf{u}\|^2\right\}\Big|_{\partial\Gamma_f} + g\{\zeta|_{\partial\Gamma_f} - \zeta_0\} = \frac{1}{2}U_\infty^2, \qquad [p|_{\partial\Gamma_f} \equiv p_\infty]$$
$$\Leftrightarrow \boxed{\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left\{\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2\right\} + g(\zeta - \zeta_0) = \frac{1}{2}U_\infty^2,\quad x \in \partial\Gamma_f} \quad \text{—— ②}$$

$$② \Rightarrow \zeta = -\frac{1}{g}\left[\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left\{\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2\right\} + \frac{1}{2}U_\infty^2\right] + \zeta_0,\ x \in \partial\Gamma_f \quad \text{—— ②'}$$

$$①, ②' \Rightarrow \frac{\partial\Phi}{\partial y} = \frac{\partial\zeta}{\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial\zeta}{\partial x},\ x \in \partial\Gamma_f,$$
$$= -\frac{1}{g}\left[\left\{\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x\partial t} + \frac{\partial\Phi}{\partial y}\frac{\partial^2\Phi}{\partial y\partial t}\right\} + \frac{\partial\Phi}{\partial x}\left\{\frac{\partial^2\Phi}{\partial x\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial\Phi}{\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\right\}\right]$$
$$= -\frac{1}{g}\left[\frac{\partial^2\Phi}{\partial t^2} + 2\frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x\partial t} + \frac{\partial\Phi}{\partial y}\left\{\frac{\partial^2\Phi}{\partial y\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x\partial y}\right\} + \left(\frac{\partial\Phi}{\partial x}\right)^2\frac{\partial^2\Phi}{\partial x^2}\right]$$
$$= -\frac{1}{g}\left[\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial\Phi}{\partial x}\left\{2\frac{\partial^2\Phi}{\partial x\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x^2}\right\} + \frac{\partial\Phi}{\partial y}\left\{\frac{\partial^2\Phi}{\partial y\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial x\partial y}\right\}\right]$$



$$\left.\begin{aligned}&\Delta\Phi_1 = 0,\\ &\Phi_1 = U_\infty x + \phi_1,\end{aligned}\right\} \Rightarrow \Delta\phi_1 = 0$$

$$\left.\begin{aligned}&\Delta\Phi_2 = 0,\\ &\Phi_2 := U_\infty x + \phi_2,\end{aligned}\right\} \Rightarrow \Delta\phi_2 = 0.$$

$$x \in \partial\Gamma_f := \{x \in \mathbb{R}^2 \mid y < \zeta(x,t)\},$$

- Dynamic B.C. (Unsteady Bernoulli's principle)

$$\left.\begin{aligned}&\frac{\partial\Phi_1}{\partial t} + \frac{1}{2}\|u_1\|^2 + \frac{p_1}{\rho_{f1}} + gy = f(t),\\ &\frac{\partial\Phi_2}{\partial t} + \frac{1}{2}\|u_2\|^2 + \frac{p_2}{\rho_{f2}} + gy = f(t),\\ &p_1|_{\partial\Gamma_f} = p_2|_{\partial\Gamma_f}.\end{aligned}\right\}$$

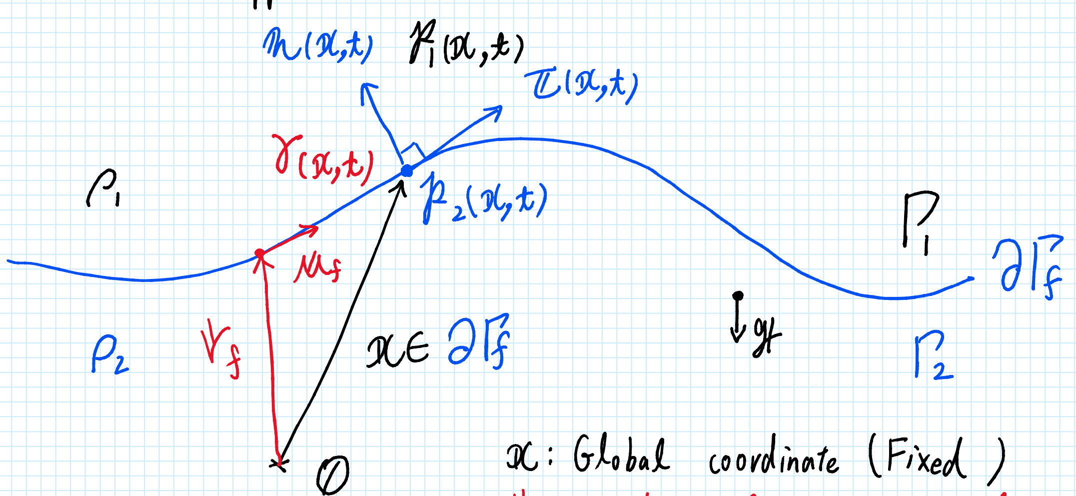
$$\Leftrightarrow \left.\begin{aligned}&\left\{\frac{\partial\Phi_1}{\partial t} + \frac{1}{2}\|u_1\|^2 + \frac{p_1}{\rho_{f1}} + gy\right\}\Big|_{\partial\Gamma_f} = \frac{1}{2}U_\infty^2 + \frac{p_\infty}{\rho_{f1}} + g\zeta_0,\\ &\left\{\frac{\partial\Phi_2}{\partial t} + \frac{1}{2}\|u_2\|^2 + \frac{p_2}{\rho_{f2}} + gy\right\}\Big|_{\partial\Gamma_f} = \frac{1}{2}U_\infty^2 + \frac{p_\infty}{\rho_{f2}} + g\zeta_0,\\ &p_1|_{\partial\Gamma_f} = p_2|_{\partial\Gamma_f}.\end{aligned}\right\}$$

$$\Leftrightarrow \left.\begin{aligned}&\left\{\frac{\partial\Phi_1}{\partial t} + \frac{1}{2}\|u_1\|^2\right\}\Big|_{\partial\Gamma_f} - \frac{1}{2}U_\infty^2 + g(\zeta-\zeta_0) = \frac{1}{\rho_{f1}}\left(p_\infty - p_1|_{\partial\Gamma_f}\right),\\ &\left\{\frac{\partial\Phi_2}{\partial t} + \frac{1}{2}\|u_2\|^2\right\}\Big|_{\partial\Gamma_f} - \frac{1}{2}U_\infty^2 + g(\zeta-\zeta_0) = \frac{1}{\rho_{f2}}\left(p_\infty - p_2|_{\partial\Gamma_f}\right),\\ &p_1|_{\partial\Gamma_f} = p_2|_{\partial\Gamma_f}.\end{aligned}\right\}$$

$$\Leftrightarrow \left\{\frac{\partial\Phi_2}{\partial t} + \frac{1}{2}\|u_2\|^2\right\}\Big|_{\partial\Gamma_f} - \frac{1}{2}U_\infty^2 + g(\zeta-\zeta_0) = \frac{\rho_{f1}}{\rho_{f2}}\left[\left\{\frac{\partial\Phi_1}{\partial t} + \frac{1}{2}\|u_1\|^2\right\}\Big|_{\partial\Gamma_f} - \frac{1}{2}U_\infty^2 + g(\zeta-\zeta_0)\right],$$

$$\Leftrightarrow \left[\frac{\partial}{\partial t}\left\{\Phi_2 - \frac{\rho_{f1}}{\rho_{f2}}\Phi_1\right\} + \frac{1}{2}\left\{\|u_2\|^2 - \frac{\rho_{f1}}{\rho_{f2}}\|u_1\|^2\right\}\right]\Big|_{\partial\Gamma_f} - \frac{1}{2}\left(1 - \frac{\rho_{f1}}{\rho_{f2}}\right)U_\infty^2 + g\left(1 - \frac{\rho_{f1}}{\rho_{f2}}\right)(\zeta - \zeta_0) = 0.$$

- Vortex sheet approximation



x: Global coordinate (Fixed)
r_f: Coordinate for computational point ($d_t r_f = u_f$)

$$\left.\begin{aligned}&\circ\ p_1(x,t) = p_2(x,t),\\ &\circ\ \gamma(x,t) := -(u_1 - u_2)^T\cdot\tau,\\ &\circ\ u_f := \frac{1}{2}(u_1 + u_2),\end{aligned}\right\}\quad x \in \partial\Gamma_f,$$

$$\circ\ \left.\begin{aligned}&\rho_n \frac{d}{dt}\left\{u_n|_{x=r(t)}\right\} = -\nabla p_n|_{x=r(t)} - \rho_n g,\\ &\frac{dr}{dt} = u_n(r,t),\end{aligned}\right\},\quad n \in \{1,2\},$$

$$\Rightarrow \rho_1 \underbrace{D_t}_{\text{[Material derivative]}} u_1|_{x=r(t)} - \rho_2 D_t u_2|_{x=r(t)} = -(\nabla p_1 - \nabla p_2)|_{x=r(t)} - (\rho_1-\rho_2)g,\quad r(t) \in \partial\Gamma_f,$$

$$\Leftrightarrow \{\rho_1 D_t u_1 - \rho_2 D_t u_2\}^T\cdot\tau = -(\nabla p_1 - \nabla p_2)^T\cdot\tau - (\rho_1-\rho_2)g^T\cdot\tau,$$

$$\Leftrightarrow \frac{1}{2}\left\{(\rho_1+\rho_2)\underbrace{D_t}_{} u_1 + (\rho_1-\rho_2)D_t u_1 - (\rho_1+\rho_2)D_t u_2 + (\rho_1-\rho_2)D_t u_2\right\}^T\cdot\tau = -\partial_s(p_1-p_2) - (\rho_1-\rho_2)\,g\,e_y^T\cdot\tau,$$

$$\left[D_t f := \frac{d}{dt}\left\{f(x,t)|_{x=r(t)}\right\}\right]$$

$$\Leftrightarrow \frac{1}{2}\left\{(\rho_1+\rho_2)D_t(u_1-u_2) + (\rho_1-\rho_2)D_t(u_1+u_2)\right\}^T\cdot\tau = -\partial_s(p_1-p_2) - (\rho_1-\rho_2)\,g\,e_y^T\cdot\tau, \quad \text{— ③}$$

$$\left.\begin{aligned}&\circ\ \gamma = -(u_1-u_2)^T\cdot\tau \Leftarrow u_1 - u_2 = -\gamma\cdot\tau,\\ &\circ\ u_f = \frac{1}{2}(u_1+u_2),\end{aligned}\right\},\quad x \in \partial\Gamma_f,$$

$$\Rightarrow \left.\begin{aligned}&u_1 - u_2 = -\gamma\tau,\\ &u_1 + u_2 = 2u_f,\end{aligned}\right\}$$

$$\Leftrightarrow\quad u_1 = u_f - \frac{1}{2}\gamma\tau,\quad u_2 = u_1 + \gamma\tau = u_f + \frac{1}{2}\gamma\tau,$$

• $D_t u_n(\bar{x},t) = d_t\{u_n(x,t)|_{x=r_n(\bar{x},t)}\} = d_t u_n(x,t)|_{x=r_n(\bar{x},t)} + \partial_x u_n(x,t)|_{x=r_n(\bar{x},t)}\cdot \underbrace{u_n(r_n(\bar{x},t),t)}_{[\partial_t r_n(\bar{x},t) = u_n(r_n(\bar{x},t),t)]},\ n\in\{1,2\}.$

$$\Rightarrow \begin{cases} D_t u_1 = d_t u_1|_{x=r_1(\bar{x},t)} + \partial_x u_1\cdot\{u_f - \frac{1}{2}\gamma\tau\}|_{x=r_1(\bar{x},t)} \\ \quad = \{d_t u_1 + \partial_x u_1\cdot u_f\}|_{x=r_1(\bar{x},t)} - \frac{1}{2}\gamma\,\partial_x u_1\cdot\tau|_{x=r_1(\bar{x},t)} \\ \quad = \{d_t u_1 + \partial_x u_1\cdot u_f\}|_{x=r_1(\bar{x},t)} - \frac{1}{2}\gamma\,\partial_s u_1|_{x=r_1(\bar{x},t)} \\ \quad = \left[d_t\{u_1|_{x=r_f(\bar{x},t)}\} - \frac{1}{2}\gamma\,\partial_s u_1|_{x=r_f(\bar{x},t)}\right]\Big|_{r_f=r_1(\bar{x},t)}, \qquad \text{④} \\ D_t u_2 = d_t u_2|_{x=r_2(\bar{x},t)} + \partial_x u_2\cdot\{u_f + \frac{1}{2}\gamma\tau\}|_{x=r_2(\bar{x},t)} \\ \quad = \left[d_t\{u_2|_{x=r_f(\bar{x},t)}\} + \frac{1}{2}\gamma\,\partial_s u_2|_{x=r_f(\bar{x},t)}\right]\Big|_{r_f=r_2(\bar{x},t)}. \qquad \text{⑤} \end{cases}$$

• $D_t u_f = D_t\{\frac{1}{2}(u_1+u_2)\} = \frac{1}{2}(D_t u_1 + D_t u_2)$

$$= \frac{1}{2}\left[d_t\{u_1|_{x=r_f(\bar{x},t)}\} + d_t\{u_2|_{x=r_f(\bar{x},t)}\} - \frac{1}{2}\gamma\left(\partial_s u_1|_{x=r_f(\bar{x},t)} - \partial_s u_2|_{x=r_f(\bar{x},t)}\right)\right]$$

$$= \frac{1}{2}\left[d_t\{u_1|_{x=r_f(\bar{x},t)}\} + d_t\{u_2|_{x=r_f(\bar{x},t)}\} - \frac{1}{2}\gamma\,\partial_s\{(u_f - \frac{1}{2}\gamma\tau) - (u_f + \frac{1}{2}\gamma\tau)\}|_{x=r_f(\bar{x},t)}\right]$$

$$= d_t\{\frac{1}{2}(u_1+u_2)|_{x=r_f(\bar{x},t)}\} + \frac{1}{4}\gamma\,\partial_s(\gamma\tau)|_{x=r_f(\bar{x},t)}$$

$$= d_t\{u_f|_{x=r_f(\bar{x},t)}\} + \frac{1}{4}\gamma\,\partial_s(\gamma\tau)|_{x=r_f(\bar{x},t)}$$

$$\Rightarrow D_t u_f\cdot\tau = d_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{4}\gamma\,\partial_s(\gamma\tau)\cdot\tau\,|_{x=r_f(\bar{x},t)}$$

$$= d_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{4}\gamma\{\partial_s\gamma\cdot\tau + \gamma\underbrace{\partial_s\tau\}\cdot\tau}_{[\partial_s\tau\perp\tau]}\,|_{x=r_f(\bar{x},t)}$$

$$= d_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{4}\gamma\,\partial_s\gamma\,|_{x=r_f(\bar{x},t)}$$

$$= d_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{8}\partial_s(\gamma^2)\,|_{x=r_f(\bar{x},t)}.$$

③: $\frac{1}{2}\{(\rho_1+\rho_2)\underbrace{D_t(u_1-u_2)}_{⑥} + (\rho_1-\rho_2)\underbrace{D_t(u_1+u_2)}_{⑦}\}^T\cdot\tau = -\partial_s\underbrace{(p_1-p_2)}_{[p_1=p_2,\ r_f\in\partial\Gamma_f]} - (\rho_1-\rho_2)\,g\,e_y^T\cdot\tau,$

④,⑤,⑥: $D_t(u_1-u_2) = \left[d_t\{u_1|_{x=r_f(\bar{x},t)}\} - \frac{1}{2}\gamma\,\partial_s u_1|_{x=r_f(\bar{x},t)}\right] - \left[d_t\{u_2|_{x=r_f(\bar{x},t)}\} + \frac{1}{2}\gamma\,\partial_s u_2|_{x=r_f(\bar{x},t)}\right]$

$$= d_t\{-\gamma\tau\,|_{x=r_f(\bar{x},t)}\} - \gamma\,\partial_s u_f|_{x=r_f(\bar{x},t)},$$

④,⑤,⑦: $D_t(u_1+u_2) = 2D_t u_f,$

$$\Rightarrow \frac{1}{2}\left[D_t(u_1-u_2) + \alpha D_t(u_1+u_2)\right]^T\cdot\tau = -\alpha\, g\, e_y^T\cdot\tau,\quad \alpha := \frac{\rho_1-\rho_2}{\rho_1+\rho_2},$$

$$\Leftrightarrow \frac{1}{2}\left[d_t\{-\gamma\tau\,|_{x=r_f(\bar{x},t)}\}^T\cdot\tau - \gamma\,\partial_s u_f|_{x=r_f(\bar{x},t)}^T\cdot\tau + 2\alpha D_t u_f^T\cdot\tau\right] = -\alpha\, g\, e_y^T\cdot\tau,$$

$$\Leftrightarrow \frac{1}{2}\left[d_t\{-\gamma\tau\,|_{x=r_f(\bar{x},t)}\}^T\cdot\tau - \gamma\,\partial_s u_f^T\cdot\tau\right] = -\alpha D_t u_f^T\cdot\tau - \alpha\, g\, e_y^T\cdot\tau,$$

$$\Leftrightarrow d_t\{\gamma\,|_{x=r_f(\bar{x},t)}\}\,\tau^T\tau + \gamma\, d_t\{\tau\,|_{x=r_f(\bar{x},t)}\}^T\cdot\tau + \gamma\,\partial_s u_f^T\cdot\tau = 2\alpha D_t u_f^T\cdot\tau + 2\alpha\, g\, e_y^T\cdot\tau,$$

$$\Leftrightarrow d_t\{\gamma\,|_{x=r_f(\bar{x},t)}\} + \gamma\, d_t\{\tau\,|_{x=r_f(\bar{x},t)}\}^T\cdot\tau + \gamma\,\partial_s u_f^T\cdot\tau = 2\alpha D_t u_f^T\cdot\tau + 2\alpha\, g\, e_y^T\cdot\tau,$$

$$\Leftrightarrow d_t\{\gamma\,|_{x=r_f(\bar{x},t)}\} + \gamma\,\frac{1}{2}d_t\{\underbrace{\tau^T\cdot\tau}_{[=\|\tau\|^2=1]}\,|_{x=r_f(\bar{x},t)}\} + \gamma\,\partial_s u_f^T\cdot\tau = 2\alpha D_t u_f^T\cdot\tau + 2\alpha\, g\, e_y^T\cdot\tau,$$

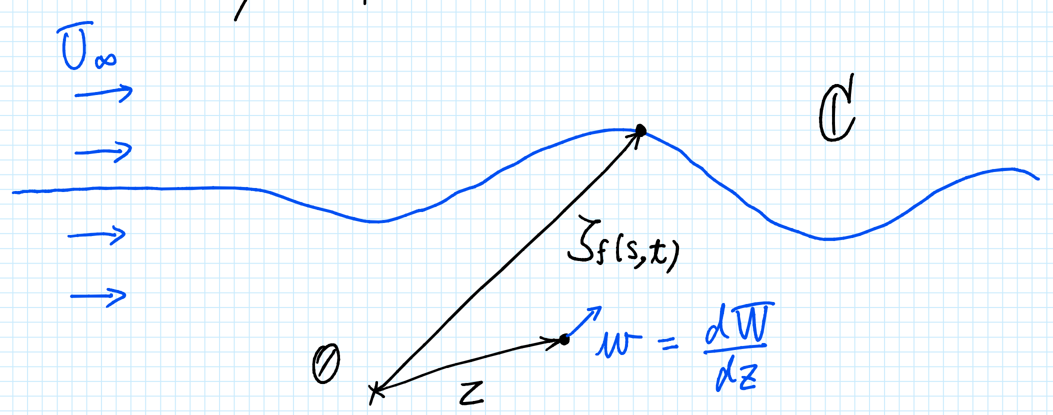
$$\Longleftrightarrow \boxed{\partial_t\{\gamma|_{x=r_f(\bar{x},t)}\} + \gamma\,\partial_s u_f^{T}\cdot\tau = 2\alpha\, D_t u_f^{T}\cdot\tau + 2\alpha\, g\, e_y^{T}\tau},$$

$$\Longleftrightarrow \partial_t\{\gamma|_{x=r_f(\bar{x},t)}\} + \gamma\,\partial_s u_f^{T}\cdot\tau = 2\alpha\left[\partial_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{8}\partial_s(\gamma^2)\right] + 2\alpha\, g\, e_y^{T}\tau.$$

∘ $\partial_t r_f = u_f$

$$= U_\infty e_x + \int_{\partial\Gamma_f} K(r_f - x(s^\circ))\,\gamma(x(s^\circ),t)\,ds^\circ,\quad K(x) := \frac{1}{2\pi}\frac{e_z\times x}{\|x\|^2},$$

∘ Complex velocity potential



• $W = \phi + j\psi = U_\infty z + \frac{1}{j2\pi}\int_{\partial\Gamma_f} \ln(z - \zeta_f(s^\circ,t))\,\gamma(s^\circ,t)\,ds^\circ,\quad z\in\mathbb{C}.$

∘ $\partial_t\zeta_f = \mathrm{Re}(\partial_t\zeta_f) + j\,\mathrm{Im}(\partial_t\zeta_f)$

$$= \overline{w(\zeta_f)},$$

$$w(\zeta_f) = \partial_z W(\zeta_f)$$

$$= U_\infty + \frac{1}{j2\pi}\int_{\partial\Gamma_f}\frac{\gamma(s^\circ,t)}{z - \zeta_f(s^\circ,t)}\,ds^\circ\Big|_{z=\zeta_f(s,t)}$$

$$= U_\infty + \frac{1}{j2\pi}\int_{\partial\Gamma_f}\frac{\gamma(s^\circ,t)}{\zeta_f(s,t) - \zeta_f(s^\circ,t)}\,ds^\circ.$$

$$\partial_t\{\gamma|_{x=r_f(\bar{x},t)}\} = -\gamma\,\partial_s u_f^{T}\cdot\tau + 2\alpha\left[\partial_t\{u_f|_{x=r_f(\bar{x},t)}\}\cdot\tau + \frac{1}{8}\partial_s(\gamma^2)\right] + 2\alpha\, g\, e_y^{T}\tau.$$

$$\Rightarrow \begin{cases} \circ\ \partial_t\gamma_j = -\gamma\,\partial_s u_{f;j,k}\tau_k + 2\alpha\left[\partial_t u_{f;j,k}\tau_k + \frac{1}{8}\partial_s(\gamma^2)_j\right] + 2\alpha g\, e_k\tau_{j,k} \\ \circ\ u_{f;j,1} + j\,u_{f;j,2} = \overline{\left(\sum_{l=1}^{N}\frac{1}{j2\pi}\frac{\gamma_l}{z_j - z_l}\Delta s_l\right)} \end{cases}$$

$$\Rightarrow \partial_t u_{f;j,1} + j\,\partial_t u_{f;j,2} = \overline{\left(\sum_{l=1}^{N}\left\{-\frac{1}{j2\pi}\frac{\partial_t z_j - \partial_t z_l}{(z_j - z_l)^2}\gamma_l + \frac{1}{j2\pi}\frac{\partial_t\gamma_l}{z_j - z_l}\right\}\Delta s_l\right)}$$

$$= \left(\sum_{l=1}^{N}\overline{\partial_t\{K(z_j - z_l)\}}\,\gamma_l\Delta s_l\right) + \left(\sum_{l=1}^{N}\overline{K(z_j - z_l)}\,\partial_t\gamma_l\Delta s_l\right)$$

$$\Rightarrow \partial_t u_{f;j,k}\tau_k = \mathrm{Re}\left(\partial_t z_j\cdot\bar{\tau}\right) = \mathrm{Re}\left(\left[\sum_{l=1}^{N}\overline{\partial_t\{K(z_j - z_l)\}}\,\gamma_l\Delta s_l\right]\cdot\bar{\tau}\right)$$

$$+ \mathrm{Re}\left(\left[\sum_{l=1}^{N}\overline{K(z_j - z_l)}\,\partial_t\gamma_l\Delta s_l\right]\cdot\bar{\tau}\right)$$

$$\Rightarrow \partial_t\gamma_j - 2\alpha\,\mathrm{Re}\left(\left[\sum_{l=1}^{N}\overline{K(z_j - z_l)}\,\partial_t\gamma_l\Delta s_l\right]\cdot\bar{\tau}\right) = -\gamma\,\partial_s u_f^{T}\cdot\tau + 2\alpha\left[\mathrm{Re}\left(\left[\sum_{l=1}^{N}\overline{\partial_t\{K(z_j - z_l)\}}\,\gamma_l\Delta s_l\right]\cdot\bar{\tau}\right) + \frac{1}{8}\partial_s(\gamma^2)_j\right] + 2\alpha g\, e_k\tau_{j,k}$$