EKF for attitude estimation

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- Euler parameter

$$\boldsymbol{\varepsilon} := [x_0\ \boldsymbol{x}] \in \mathbb{R}^4,$$

- Euler parameter vs Angular velocity

$$d_T \boldsymbol{\varepsilon} = \frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T \boldsymbol{\omega}^{\circ} \in \mathbb{R}^4,$$

where $\underbrace{(\cdot)^{\circ} \text{ represents vector on local coordinate}}_{[\text{ Axes on IMU sensor } O^{\circ}\text{-}xyz\ ]}$ and,

$$\mathbb{L}(\boldsymbol{\varepsilon}) := [-\boldsymbol{x} \quad x_0\mathbb{I} - \tilde{\boldsymbol{x}}],$$

$$\tilde{\boldsymbol{x}} := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

- Angular velocity model

$$\underbrace{\boldsymbol{\omega}_m^{\circ}}_{[\text{Measured}]} = \boldsymbol{\omega}^{\circ} + \underbrace{\boldsymbol{\omega}_b^{\circ}}_{[\text{Bias}]} + \boldsymbol{w}_m \in \mathbb{R}^3,$$

$$d_T \boldsymbol{\omega}_b^{\circ} = A_\omega \boldsymbol{\omega}_b^{\circ} + \boldsymbol{w}_b \in \mathbb{R}^3,$$

where $\boldsymbol{w}_m \sim \mathcal{N}(0, \sigma_m)$, $\boldsymbol{w}_b \sim \mathcal{N}(0, \sigma_b)$ and

$$A_\omega := \begin{bmatrix} -\beta_x & 0 & 0 \\ 0 & -\beta_y & 0 \\ 0 & 0 & -\beta_z \end{bmatrix}.$$

※ Angular velocity $\boldsymbol{\omega}^{\circ}$ are treated as external input to EKF rather than measurements, and their noises enter the filter as process noise rather than as measurement noise.

- Process model

$$\left.\begin{aligned} d_T \boldsymbol{\varepsilon} &= \frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T \boldsymbol{\omega}^{\circ} \\ &= \frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T(\boldsymbol{\omega}_m^{\circ} - \boldsymbol{\omega}_b^{\circ}) - \frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T \boldsymbol{w}_m, \\ d_T \boldsymbol{\omega}_b^{\circ} &= A_\omega \boldsymbol{\omega}_b^{\circ} + \boldsymbol{w}_b, \end{aligned}\right\}$$

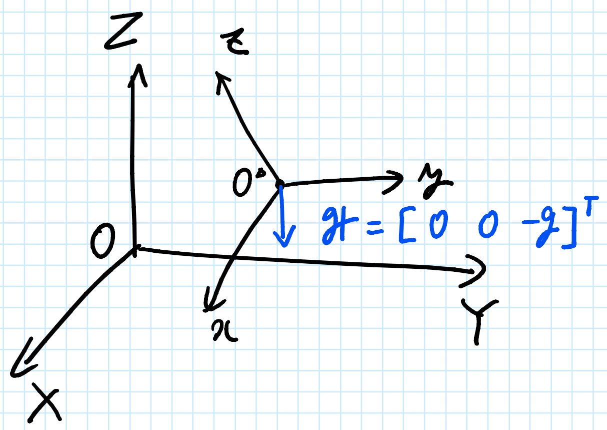
$$\Rightarrow\ d_T \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\omega}_b^{\circ} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T(\boldsymbol{\omega}_m^{\circ} - \boldsymbol{\omega}_b^{\circ}) \\ A_\omega \boldsymbol{\omega}_b^{\circ} \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}\mathbb{L}(\boldsymbol{\varepsilon})^T & \mathbb{O} \\ \mathbb{O} & \mathbb{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{w}_m \\ \boldsymbol{w}_b \end{bmatrix} \in \mathbb{R}^7.$$

$$\Rightarrow\ d_T \boldsymbol{x} = f(\boldsymbol{x}, \boldsymbol{u}) + h(\boldsymbol{x})\boldsymbol{w},$$

where

$$\boldsymbol{x} := [\boldsymbol{\varepsilon}^T\ \boldsymbol{\omega}_b^{\circ T}]^T \in \mathbb{R}^7,$$

$$\boldsymbol{u} := \boldsymbol{\omega}_m^{\circ} \in \mathbb{R}^3.$$



- Measurement model

$$\boldsymbol{a}^{\circ} = \mathbb{A}(\boldsymbol{\varepsilon})\boldsymbol{g} + \boldsymbol{v} \in \mathbb{R}^3,$$

where $\boldsymbol{g} := [0\ \ 0\ \ -g]^T$, $\boldsymbol{v} \sim \mathcal{N}(0, \sigma_v)$ and,

$$\mathbb{A}(\boldsymbol{\varepsilon}) := \mathbb{I} + 2\tilde{\boldsymbol{x}}(\tilde{\boldsymbol{x}} + x_0\mathbb{I}) \in \mathbb{R}^{3\times 3}$$

$$A(\varepsilon) := \mathbb{I} + 2\tilde{k}(\tilde{k} + k_0 \mathbb{I}) \in \mathbb{R}^{3\times 3}.$$

$$\Rightarrow \quad y = h(x) + v,$$

where

$$h(x) := A(\varepsilon)\, g \in \mathbb{R}^3 .$$

- Extended Kalman filter

$$\left.\begin{aligned} d_t x &= f(x, u) + b(x) w, \\ y &= h(x) + v. \end{aligned}\right\}$$

$$\Rightarrow \quad \left.\begin{aligned} x_{k+1} &= x_k + f(x_k, u_k)\Delta T + b(x_k) w_k, \\ y_{k+1} &= h(x_k) + v_k . \end{aligned}\right\}$$

$$\Rightarrow \quad \left.\begin{aligned} x_{k+1} &= f_d(x_k, u_k) + b(x_k) w_k, \\ y_{k+1} &= h(x_k) + v_k . \end{aligned}\right\}$$

where,

$$f_d(x_k, u_k) := x_k + f(x_k, u_k)\Delta T,$$

[EKF]

- $\hat{x}_k^- = f_d(\hat{x}_{k-1}, u_{k-1})$,
- $A_{k-1} = \partial_x f_d(x, u_{k-1}) \big|_{x=\hat{x}_{k-1}}$,

  $C_k^T = \partial_x h(x) \big|_{x=\hat{x}_k^-}$,
- $\mathbb{P}_k^- = A_{k-1} \mathbb{P}_{k-1} A_{k-1}^T + b\, w_k w_k^T b^T$,
- $\mathbb{K}_k = \mathbb{P}_k^- C_k (C_k^T \mathbb{P}_k^- C_k + v_k v_k^T)^{-1}$,
- $\hat{x}_k = \hat{x}_k^- + \mathbb{K}_k \{ y_k - h(x_k^-) \}$,
- $\mathbb{P}_k = \{ \mathbb{I} - \mathbb{K}_k C_k^T \} \mathbb{P}_k^-$.

$$\Rightarrow \quad \circ\ A_{k-1} = \partial_x f_d(x, u_{k-1}) \big|_{x=\hat{x}_{k-1}},$$

$$= \partial_x \{ x + f(x, u_k)\Delta T \} \big|_{x=\hat{x}_{k-1}}$$

$$= \mathbb{I} + \Delta T \frac{\partial}{\partial [\varepsilon^T\ \omega_b^{\triangle T}]} \begin{bmatrix} \frac{1}{2} \mathbb{L}(\varepsilon)^T (\omega_{m,k}^{\triangle} - \omega_b^{\triangle}) \\ A_\omega \omega_b^{\triangle} \end{bmatrix} \Bigg|_{x=\hat{x}_{k-1}}$$

$$= \mathbb{I} + \Delta T \begin{bmatrix} \frac{1}{2} \partial_\varepsilon \{ \mathbb{L}(\varepsilon)^T (\omega_{m,k}^{\triangle} - \omega_b^{\triangle}) \} & -\frac{1}{2} \mathbb{L}(\varepsilon)^T \\ 0 & A_\omega \end{bmatrix} \Bigg|_{x=\hat{x}_{k-1}} \in \mathbb{R}^{7\times 7},$$

$$= \mathbb{I} + \Delta T \begin{bmatrix} \cdots & \cdots \\ 0 & \mathbb{A}_\omega \end{bmatrix} \Bigg|_{x = \hat{x}_{k-1}} ,$$

where

$$\frac{1}{2} \partial_{\varepsilon} \{ \mathbb{L}(\varepsilon)^T (\hat{\omega}_{m,k} - \hat{\omega}_{b}) \} = \frac{1}{2} \begin{bmatrix} 0 & -(\hat{\omega}_{m1,k} - \hat{\omega}_{b1}) & -(\hat{\omega}_{m3,k} - \hat{\omega}_{b2}) & -(\hat{\omega}_{m3,k} - \hat{\omega}_{b3}) \\ \hat{\omega}_{m1,k} - \hat{\omega}_{b1} & 0 & \hat{\omega}_{m3,k} - \hat{\omega}_{b3} & -(\hat{\omega}_{m2,k} - \hat{\omega}_{b2}) \\ \hat{\omega}_{m2,k} - \hat{\omega}_{b2} & -(\hat{\omega}_{m3,k} - \hat{\omega}_{b3}) & 0 & \hat{\omega}_{m1,k} - \hat{\omega}_{b1} \\ \hat{\omega}_{m3,k} - \hat{\omega}_{b3} & \hat{\omega}_{m2,k} - \hat{\omega}_{b2} & -(\hat{\omega}_{m1,k} - \hat{\omega}_{b1}) & 0 \end{bmatrix} .$$

- $$C_k^T = \partial_x h(x) \big|_{x = \hat{x}_k^-} ,$$

$$= \partial_x \, \mathbb{A}(\varepsilon)\, g \big|_{x = \hat{x}_k^-}$$

$$= \frac{\partial}{\partial [\, \varepsilon^T \;\; \hat{\omega}_b^T \,]} \mathbb{A}(\varepsilon)\, g \Big|_{x = \hat{x}_k^-}$$

$$= \begin{bmatrix} \partial_{\varepsilon} \{ \mathbb{A}(\varepsilon)\, g \} & 0 \end{bmatrix} \Big|_{x = \hat{x}_k^-} .$$

$$= \begin{bmatrix} m_g & \widetilde{(m_g + k_0}\, g) + (k^T g)\mathbb{I} - g k^T & 0 \end{bmatrix} \Big|_{x = \hat{x}_k^-} \in \mathbb{R}^{3 \times 7} ,$$

where

$$m_g := \tilde{g}\, k .$$