SC602 Simulations Report

Rudraksh Kuchiya
Mechanical Engineering
Roll: 20D110021
Email: 20d110021@iitb.ac.in

***Abstract*—The following report summarizes the results for simulation and control design of a Two-link manipulator and rigid-body rotational dynamics. The Two-link manipulator is firstly governed by passivity based design and then by backstepping control design. The control law for Rigid body rotational dynamics is derived from the passivity based design.**

I. TWO LINK MANIPULATOR

A. The Model

The dynamics of a two-link manipulator in joint space is given by:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D\dot{q} + g(q) = u$$

Where $q \in \mathbb{R}^2$ are the generalized coordinates, $\dot{q}$ are the generalized velocities. $0 < M(q) \in \mathbb{R}^{2\times 2}$, $C(q, \dot{q}) \in \mathbb{R}^{2\times 2}$, $0 \leq D \in \mathbb{R}^{2\times 2}$ and $g(q) \in \mathbb{R}^2$ are the coriolis, centrifugal, viscous damping and gravity respectively.

B. The Control Design

Denote $e = q - q_r$, where q_r is a constant reference. Consider the control input as: $u = g(q) - K_p e + \nu$ for feedback passivation so dynamics in presence of feedback is modified as:

$$M(q)\ddot{e} + C(q, \dot{q})\dot{e} + D\dot{e} + K_p e = \nu$$

Now, consider the storage function as:

$$V(e, \dot{e}) = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \frac{1}{2}e^T K_p e$$

For this storage function we had proven that the modified dynamics is passive with the output $y = \dot{e}$, also that the (ν, y) pair is zero-state observable. Thus, We choose $\nu = -k_1 tanh(\dot{e})$ because it satisfies the property that $\phi(0) = 0$ and also bounds the control. As our objective is to track **constant** reference, we get $e = q - q_r \implies \dot{e} = \dot{q} \implies \ddot{e} = \ddot{q}$.

Now the final model for the Two-Link Manipulator can be constructed as

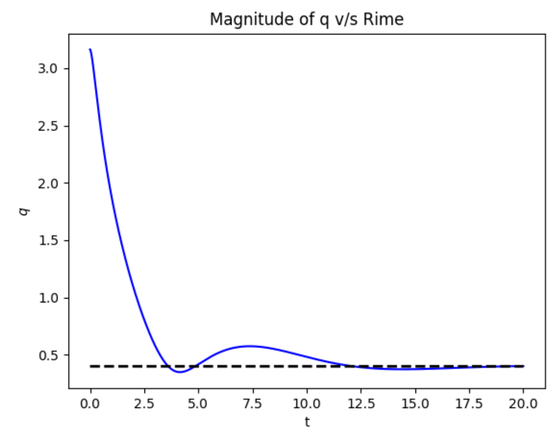
$$\ddot{q} = M(q)^{-1}(-k_1 tanh(\dot{q}) - C(q, \dot{q})\dot{q} - D\dot{q} - K_p e)$$

$$\dot{q}_{\text{next}} = \dot{q}_{\text{previous}} + \ddot{q}\Delta t$$

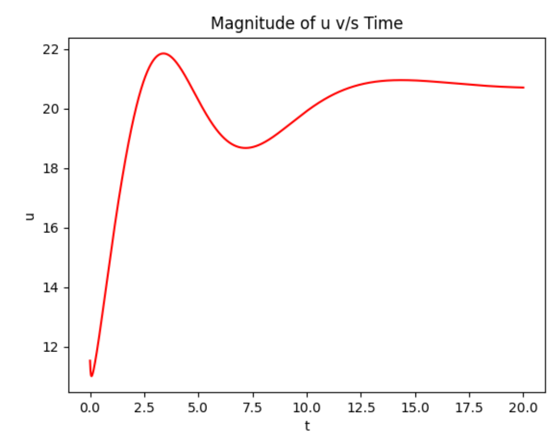
$$q_{\text{next}} = q_{\text{previous}} + \dot{q}\Delta t$$

C. Results

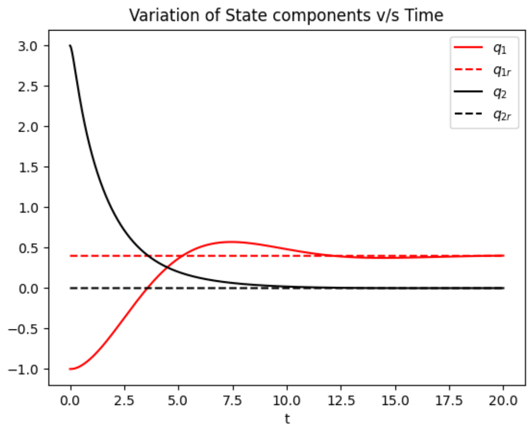
The model was simulated with the design parameters using $m_1 = 2.5kg, m_2 = 1.5kg, l = 0.4m, g = 9.81m/s^2$ and the constant reference $q_r = \begin{bmatrix}0.4 & 0\end{bmatrix}^T$.



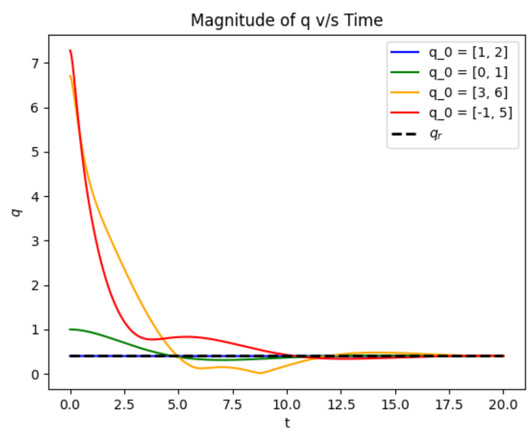
The above plot is when the initial state of the system $q_0 = \begin{bmatrix}-1 & 3\end{bmatrix}^T$. It is evident from the plot that the state q converges to q_r in approximately 15 iterations.



From the above plot it can be observed that the control input rises and then stabilizes around 20 N-m, which is a fairly reasonable value of torque and can be provided by an actuator.



Above is the plot of the components of the state q_1 and q_2 with respect to time. As we can see that the initial state of the system was $q = \begin{bmatrix}-1 & 3\end{bmatrix}^T$, q_1 starts from -1 and q_2 starts from 3, and since the reference is $q_r = \begin{bmatrix}0.4 & 0\end{bmatrix}^T$, q_1 stabilizes at 0.4 and q_2 stabilizes at 0.



The above plot shoes convergence for different values of initial input.

Now, when we vary the Control Law Gain (i.e. $k/_s$), we observe that when the gain is increased the control input components oscillate at a very high frequency about some mean, this results in a high frequency vibration of the states as well. This is called chattering and this is undesirable as well as a prominent problem in many control designs. This leads us to conclude that increasing the . And, when the value of the gain k1 increases chattering in our system the gain is decreased, t the response of system becomes sluggish as in the system takes more time to stabilize and there is significant overshoot in the states of the system. This again is not always desirable because we may have a restricted configuration space which would not always allow the room for overshoot. The settling time also is an important design consideration because faster the settling time the better behaviour of the system

D. conclusion

We were successfully able to implement a passivity based design and control the system and make it it track a constant reference. We also observed the convergence for different initial inputs. And in the end we discussed about the variation with variation in the control law gain.

II. SCALAR MANIPULATOR

A. The Model

This is just a scalar version of the Two-Link Manipulator as discussed in the previous section. The only difference is, in addition to the previous controller design, this is augmented with an integrator.and hence the dynamics of the system is given by:

$$m(q)\ddot{q} + c(q, \dot{q})\dot{q} = \xi$$

$$\dot{\xi} = \tau$$

where $q \in \mathbb{R}$ is the state (angle in this case) and $\tau \in \mathbb{R}$ is the control torque. Further, $m(q) = p_1 + 2p_3 cos(q) > 0$ and $c(q, \dot{q}) = -p_3 sin(q)\dot{q}$. We want this system to track a constant reference q_r. To do so we use the backstepping based design to derive a control torque from the non-augmented system.

B. The Control Design

We know that the control dynamics of a non-augmented system is given by:

$$m(q)\ddot{q} + c(q, \dot{q})\dot{q} = \tau$$

Now, in order to track the constant reference, we denote the error term as $e = q - q_r$ and we choose our Control Lyapunov Function as:

$$V_0(e, \dot{e}) = \frac{e^2}{2} + \frac{\dot{e}^2}{2}$$

It can be verified that this function satisfies all the properties it needs to. Now, as we have to track a constant reference, i.e. q_r is constant with reference to time, thus $\dot{(e)} = \dot{(q)}$, this implies that:

$$V_0 = \frac{e^2}{2} + \frac{\dot{q}^2}{2}$$

Now, as per theory, for a Lyapunov based control τ, we need $\dot{V_0} < 0$

$$\therefore e\dot{e} + \dot{q}\ddot{q} < 0$$

$$\implies e\dot{q} + \dot{q}\left(\frac{\tau - c(q, \dot{q})}{m(q)}\right) < 0$$

$$\dot{q}\left(q - q_r + \frac{\tau - c(q, \dot{q})}{m(q)}\right) < 0$$

Note that if we choose a τ such that the term $q - q_r + \frac{\tau - c(q,\dot{q})}{m(q)} = -\dot{q}$, we get $\dot{V_0} = -\dot{q}^2 < 0$.

Now, we augment this system with an integrator to obtain the originally required design

$$m(q)\ddot{q} + c(q,\dot{q})\dot{q} = \xi$$

$$\dot{\xi} = \tau$$

For this system, we choose our Control Lyapunov Function as V_1:

$$V_1 = V_0 + \frac{1}{2}(\xi - k_0)^2$$

Computing $\dot{V}_1$, we get:

$$\dot{V}_1 = \dot{V}_0 + (\xi - k_0)(\dot{\xi} - \dot{k}_0)$$

$$\implies \dot{V}_1 = e\dot{q} + \dot{q}\ddot{q} + (\xi - k_0)(\tau - \dot{k}_0)$$

Substituting $\dot{q}$ from the dynamics,

$$\implies \dot{V}_1 = \dot{q}\left(q - q_r + \frac{\xi - c(q,\dot{q})}{m(q)}\right) + \\ (\xi - c(q,\dot{q})\dot{q} - m(q)(q_r - q - \dot{q}))\,(\tau - \dot{k}_0)$$

$$\implies \dot{V}_1 = \dot{q}\left(q - q_r + \frac{\xi - c(q,\dot{q})}{m(q)}\right) + \\ m(q)\left(\frac{\xi - c(q,\dot{q})\dot{q}}{m(q)} - q_r + q + \dot{q}\right)(\tau - \dot{k}_0)$$

$$\implies \dot{V}_1 = \dot{q}\left(q - q_r + \frac{\xi - c(q,\dot{q})}{m(q)} + \dot{q} - \dot{q}\right) + \\ m(q)\left(\frac{\xi - c(q,\dot{q})\dot{q}}{m(q)} - q_r + q + \dot{q}\right)(\tau - \dot{k}_0)$$

$$\implies \dot{V}_1 = -\dot{q}^2 + \left(\frac{\xi - c(q,\dot{q})\dot{q}}{m(q)} + q - q_r + \dot{q}\right) \\ (\dot{q} + m(q)\tau - m(q)\dot{k}_0)$$

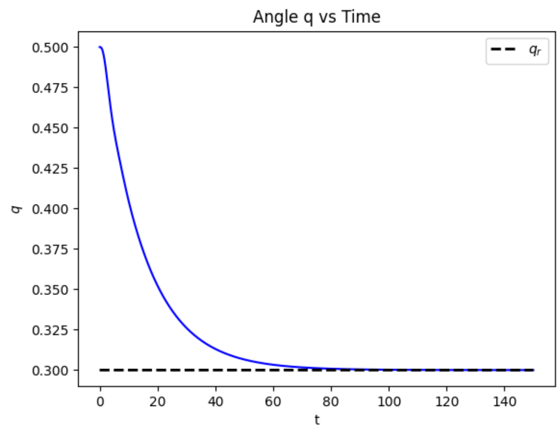
Now, as the system is based upon Lyapunov Based Control Design, we require the CLF to be negative, i.e. $\dot{V}_1 < 0$. Note that the first term in the above equation is $\dot{V}_0$ which is always negative, so we just need to choose a τ such that the second term also becomes negative. Therefore,

$$\dot{q} + m(q)\tau - m(q)\dot{k}_0 = -\left(\frac{\xi - c(q,\dot{q})\dot{q}}{m(q)} + q - q_r + \dot{q}\right)$$

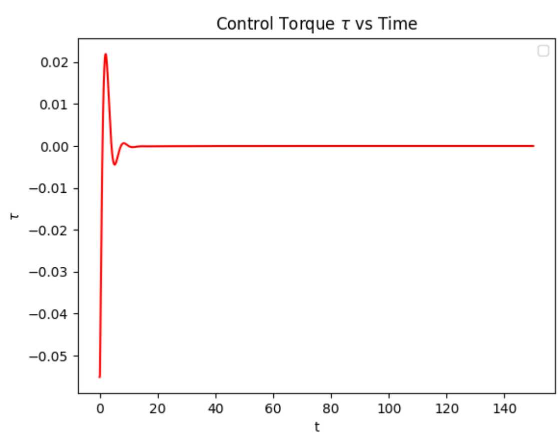
ensures that $\dot{V}_1$ is always negative. Solving for the control torque τ from this equation gives us,

$$\tau = \dot{k}_0 - \frac{2\dot{q}}{m(q)^2} - \frac{\xi}{m(q)^2} + \frac{c(q,\dot{q})\dot{q}}{m(q)^2} + \frac{q_r - q}{m(q)^2}$$

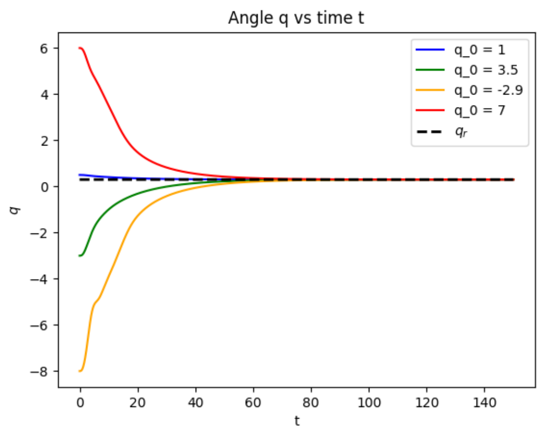
C. Results



The problem requires us to track a constant referen , $q_r = 0.3\ rad$. We choose the initial value to be $q_0 = 0.5$. The above plot shows the variation of q with respect to time. Note that, the graph converges to q_r in about 80 iterations. To obtain this behaviour the variation of the control torque with respect to time is plotted below.



We can see that τ starts with a negative value as to decrease $q(t)$. Then as to avoid the overshoot it increases to a positive value before attaining 0 N-m. The control input is fairly feasible and can be achieved easily.



The above plot depicts convergence of q to q_r for different initial values.

D. Conclusion

We were successfully able to implement a back-stepping based design and derive a control torque τ and make it it track a constant reference. We also observed the convergence for different initial inputs. And in the end we discussed about the variation with variation in the control law gain.

III. RIGID BODY ROTATIONAL DYNAMICS

A. The Model

The rigid body rotation is governed with the following dynamics:

$$\dot{\rho} = [I + [\rho]_\times + \rho\rho^T]\omega$$

$$J\dot{\omega} = -(\omega \times J\omega) + u$$

With

$$y = \omega$$

Where $\rho \in \mathbb{R}^3$ is the modified Rodrigues parameter, $J = J^T > 0$ is the inertia matrix, $\omega \in \mathbb{R}^3$ is the angular velocity and $u \in \mathbb{R}^3$ is the thrust. Note that the system is zero state observable.

B. The Control Design

We know that the candidate storage function $V(\omega) = \frac{1}{2}\omega^T J\omega$, the driver system is passive with (u, y). And, also that the dynamics for this system $\dot{\rho} = [I + [\rho]_\times + \rho\rho^T]\omega$ is cascaded with the passive driver system.

For making the system stable, we find a Positive Definite Function W(p) such that the whole system acts passively with respect to $y = \omega$. And thus from this passive output we derive the ρ dynamics

For the selection of the control u, consider the total Lyapunov function $U(\omega, \rho)$ such that:

$$U(\omega, \rho) = V(\omega) + W(\rho)$$

Taking the time derivative on both sides,

$$\dot{U}(\omega, \rho) = \dot{V}(\omega) + \dot{W}(\rho)$$

We know that $\dot{V}(\omega) lequ^T y$ with $y = \omega$, since the driver system is passive. Hence we get,

$$\dot{U}(\omega, \rho) \leq uy^T + \frac{\partial W(\rho)}{\partial \rho}\left([I + [\rho]_\times + \rho\rho^T]\omega\right)$$

$$\therefore \dot{U}(\omega, \rho) \leq y^T\left(u + \left(\frac{\partial W(\rho)}{\partial \rho}[I + [\rho]_\times + \rho\rho^T]\right)^T\right)$$

Thus for the entire system to be passive, we need

$$\dot{U}(\omega, \rho) \leq y^T \nu$$

For some control ν, Therefore comparing the two equations above, we obtain,

$$u = \nu - \left(\frac{\partial W(\rho)}{\partial \rho}[I + [\rho]_\times + \rho\rho^T]\right)^T$$

Now we can use the Barbashin-Krasovskii-LaSalle theorem in conjunction with zero state observability to prove that the system is indeed stable in the sense of Lyapunov. For this particular dynamics a choice of $W(\rho)$ that works is:

$$W(\rho) = k_1 ln(1 + \rho^T\rho)$$

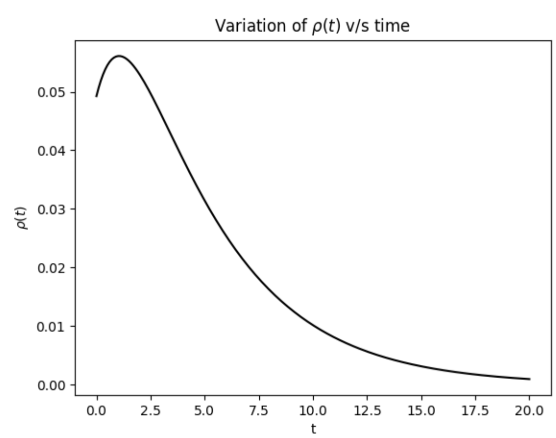
On choosing $\nu = -k_2 tanh(\omega)$, the control input u becomes

$$u = -k_2 tanh(\omega) - \left(\frac{\partial W(\rho)}{\partial \rho}[I + [\rho]_\times + \rho\rho^T]\right)^T$$

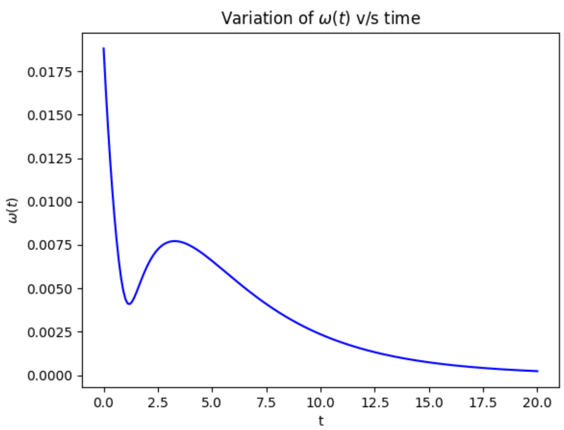
C. Results

The designed controller was tested with the following initial values: $\rho(0) = \begin{bmatrix}-0.02 & 0 & 0.045\end{bmatrix}^T, \omega(0) = \begin{bmatrix}0.004 & -0.007 & 0.017\end{bmatrix}^T rad/s$. With,

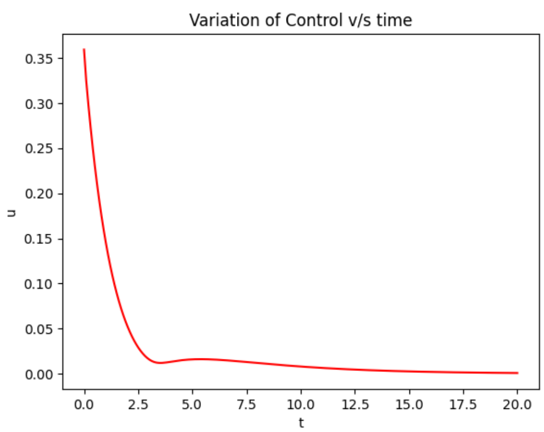
$$J = \begin{bmatrix} 20.21 & 1.2 & 0.9 \\ 1.2 & 17.21 & 1.4 \\ 0.9 & 1.4 & 15.21 \end{bmatrix}$$



The plot above shows the stabilization of the magnitude of the modified Rodrigues parameter. Similarly the next plot shows the stabilization of the magnitude of the angular velocity vector



The variation of the magnitude of the control input u with respect to time is plotted below.



The variation of the components of the control also stabilizes at 0, which is the result of zero state observability that the control input stabilizes to 0 once the system parameters stabilize at 0. All the plots so far were for gain values ks = 1.2 and kc = 14.

Increasing the value of ks reduces the settling time but increases the overshoot. Reducing the value of ks reduces the overshoot but significantly increases the settling time.

Upon increasing the value of kc we observe that the overshoot decreases but the settling time increases. And upon decreasing the value of kc, it increases the overshoot but decreases the settling time.

It is really interesting to observe the pattern here, increasing only k1 and decreasing only k2 has the exact same effect on the system, i.e. increasing overshoot and reducing settling time.

And decreasing only k1 and increasing only k2 also has the exact same effect on the system, i.e. decreasing the overshoot and increasing the settling time. Using this information we can find that the system's behaviour is desirable for $\frac{k_2}{k_1} \approx 10$.

D. Conclusion

We were successfully able to implement a passivity based design and derive a control torque and make it it track a constant reference. We also observed the convergence for different initial inputs. And in the end we discussed about the variation with variation in K_p and k_c