EE23BTECH11209 - K S Ballvardhan*

CHAPTER 15

17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

Solution:

Parameter	Description	Value
F	frequency of source	430
$f_c(n)$	frequency of closed end pipe	?
$f_o(n)$	frequency of open end pipe	?
l	length of pipe	0.2m
v	speed of sound	$340ms^{-1}$

TABLE I
INPUT VALUES

Fundamental frequency of a pipe with one closed end is given by,

$$f_c(o) = \frac{v}{4l} \tag{1}$$

where, v is speed of sound and l is length of the tube. n^{th} harmonic frequency of closed-end pipe is given by,

$$f_c(n) = \frac{v(2n+1)}{4l} \quad n = 1, 2, \ldots \tag{2}$$

From Table I:

$$\frac{v(2n-1)}{4l} = F \tag{3}$$

$$2n + 1 = \frac{F(4l)}{v} \tag{4}$$

$$\implies 2n - 1 \approx 1.01 \tag{5}$$

$$\therefore n \approx 1.005 \tag{6}$$

For the Fundamental frequency, the open pipe and source are in resonance.

If the pipe is open at both ends, its fundamental frequency is given by,

$$f_c(o) = \frac{v}{2l} \tag{7}$$

n^{th} harmonic frequency of open-end pipe is given by,

$$f_o(n) = \frac{v(n)}{2l} \quad n = 1, 2, \ldots \tag{8}$$

From Table I:

$$\frac{v(n)}{2l} = F \tag{9}$$

$$n = \frac{F(2l)}{v} \tag{10}$$

$$\implies n \approx 0.5 \tag{11}$$

So, source and the same pipe with two open ends can never be in resonance.