EE23BTECH11209 - K S Ballvardhan*

Exercise 9.5

5. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Solution:

Parameter	Value	Description
$x_1(0)$	2	First term
d_1	2	Common difference
$x_1(n)$	[2+2n]u(n)	General term of the series
$x_2(0)$	5	First term
d_2	10	Common difference
$x_2(n)$	[5+5n]u(n)	General term of the series

TABLE I
Parameter Table1

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{1}$$

By the problem there are two APs,

$$\implies X(z) = \frac{2}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2} + \frac{5}{1 - z^{-1}} + \frac{10z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

$$= \frac{2}{(1 - z^{-1})^2} + \frac{5 + 5z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$\implies Y(z) = X(z)\, U(z) \tag{5}$$

$$Y(z) = \left(\frac{2}{(1 - z^{-1})^2} + \frac{5 + 5z^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{6}$$

$$= \frac{2}{(1 - z^{-1})^3} + \frac{5 + 5z^{-1}}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{7}$$

$$Y(z) = Y_1(z) + Y_2(z) \tag{8}$$

Using Contour Integration to find the inverse Z-transform,

$$y_1(49) = \frac{1}{2\pi j} \oint_C Y_1(z)\, z^{48}\, dz \tag{9}$$

$$= \frac{1}{2\pi j} \oint_C \frac{2z^{48}}{(1 - z^{-1})^3}\, dz \tag{10}$$

$$y_2(9) = \frac{1}{2\pi j} \oint_C Y_2(z)\, z^{8}\, dz \tag{11}$$

$$= \frac{1}{2\pi j} \oint_C \frac{\left(5 + 5z^{-1}\right) z^{8}}{(1 - z^{-1})^3}\, dz \tag{12}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R_1 = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{13}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{2z^{51}}{(z - 1)^3}\right) \tag{14}$$

$$= \lim_{z \to 1} \frac{d^2}{dz^2} (z^{51}) \tag{15}$$

$$= 2550 \tag{16}$$

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{(5z + 5)\, z^{10}}{(z - 1)^3}\right) \tag{17}$$

$$= \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2} (5z^{11} + 5z^{10}) \tag{18}$$

$$= 500 \tag{19}$$

$$\therefore y(60) = R_1 + R_2 \tag{20}$$

$$= 2550 + 500 \tag{21}$$

$$= 3050 \tag{22}$$

$$\boxed{y(60) = 3050} \tag{23}$$

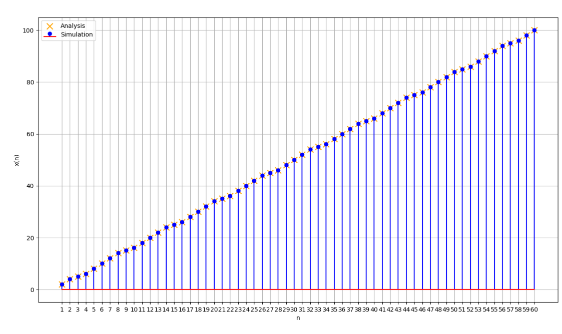

Fig. 1. x(n) vs n