EE23BTECH11209 - K S Ballvardhan*

CHAPTER 15

17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

Solution:

Parameter	Description	Value
F	frequency of source	430
$f_c(n)$	frequency of closed end pipe	?
$f_o(n)$	frequency of open end pipe	?
l	length of pipe	0.2m
v	speed of sound	$340ms^{-1}$

TABLE I
INPUT VALUES

Fundamental frequency of a pipe with one closed end is given by,

$$f_c(o) = \frac{v}{4l} \tag{1}$$

where, v is speed of sound and l is length of the tube. n^{th} harmonic frequency of closed-end pipe is given by,

$$f_c(n) = \frac{v(2n+1)}{4l} \quad n = 1, 2, \ldots \tag{2}$$

From Table I:

$$\frac{v(2n-1)}{4l} = F \tag{3}$$

$$2n + 1 = \frac{F(4l)}{v} \tag{4}$$

$$\implies 2n - 1 \approx 1.01 \tag{5}$$

$$\therefore n \approx 1.005 \tag{6}$$

Hence, for the Fundamental frequency or first harmonic mode, the closed organ pipe and source are in almost near resonance(but not in perfect resonance).

If the pipe is open at both ends, its fundamental frequency is given by,

$$f_c(o) = \frac{v}{2l} \tag{7}$$

n^{th} harmonic frequency of open-end pipe is given by,

$$f_o(n) = \frac{v(n)}{2l} \quad n = 1, 2, \ldots \tag{8}$$

From Table I:

$$\frac{v(n)}{2l} = F \tag{9}$$

$$n = \frac{F(2l)}{v} \tag{10}$$

$$\implies n \approx 0.5 \tag{11}$$

So, the source and the pipe with both ends are open can never be in resonance.