

EE23BTECH11209 - K S Ballvardhan*

EXERCISE 9.5

5. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

Solution:

Parameter	Value	Description
$x_1(0)$	2	First term
d_1	2	Common difference
$x_1(n)$	$[2+2n]u(n)$	General term of the series
$x_2(0)$	5	First term
d_2	10	Common difference
$x_2(n)$	$[5+5n]u(n)$	General term of the series

TABLE I
PARAMETER TABLE I

For an AP,

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (1)$$

By the problem there are two APs,

$$\Rightarrow X(z) = \frac{2}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{5}{1-z^{-1}} + \frac{10z^{-1}}{(1-z^{-1})^2} \quad (2)$$

$$= \frac{2}{(1-z^{-1})^2} + \frac{5+5z^{-1}}{(1-z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z)U(z) \quad (5)$$

$$Y(z) = \left(\frac{2}{(1-z^{-1})^2} + \frac{5+5z^{-1}}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right) \quad (6)$$

$$= \frac{2}{(1-z^{-1})^3} + \frac{5+5z^{-1}}{(1-z^{-1})^3}, \quad |z| > 1 \quad (7)$$

$$Y(z) = Y_1(z) + Y_2(z) \quad (8)$$

Using Contour Integration to find the inverse Z-transform,

$$y_1(49) = \frac{1}{2\pi j} \oint_C Y_1(z) z^{48} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \frac{2z^{48}}{(1-z^{-1})^3} dz \quad (10)$$

$$y_2(9) = \frac{1}{2\pi j} \oint_C Y_2(z) z^8 dz \quad (11)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(5+5z^{-1})z^8}{(1-z^{-1})^3} dz \quad (12)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R_1 = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{2z^{51}}{(z-1)^3} \right) \quad (14)$$

$$= \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{51}) \quad (15)$$

$$= 2550 \quad (16)$$

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(5z+5)z^{10}}{(z-1)^3} \right) \quad (17)$$

$$= \frac{1}{2} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (5z^{11} + 5z^{10}) \quad (18)$$

$$= 500 \quad (19)$$

$$\therefore y(59) = R_1 + R_2 \quad (20)$$

$$= 2550 + 500 \quad (21)$$

$$= 3050 \quad (22)$$

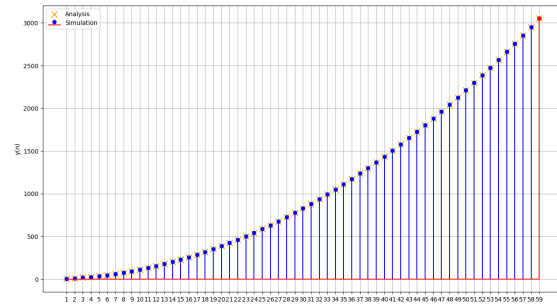


Fig. 1. $y(n)$ vs n