

EE23BTECH11209 - K S Ballvardhan*

EXERCISE 5.3

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

Parameter	Value	Description
$x_1(0)$	3	First term
d_1	3	Common difference
$x_1(n)$	$[3+3n]u(n)$	General term of the series

TABLE I
PARAMETER TABLE I

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{3}{1 - z^{-1}} + \frac{3z^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$= \frac{3}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (5)$$

$$Y(z) = \left(\frac{3}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (6)$$

$$= \frac{3}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (7)$$

Using Contour Integration to find the inverse Z-transform,

$$y(11) = \frac{1}{2\pi j} \oint_C Y(z) z^{10} dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \frac{3z^{10}}{(1 - z^{-1})^3} dz \quad (9)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (10)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{3z^{13}}{(z-1)^3} \right) \quad (11)$$

$$= \frac{3}{2} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{13}) \quad (12)$$

$$= 234 \quad (13)$$

$$\therefore y(11) = 234 \quad (14)$$

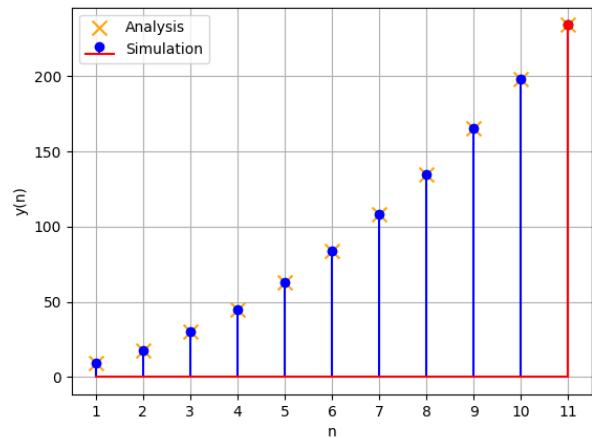


Fig. 1. $y(n)$ vs n