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$y'' + 4y' + 4y = d(t-2) \quad y'(0) = y(0) = 0.$

step1. $\mathcal{L}[y'' + 4y' + 4y]$

$= s^2 Y(s) + 4sY(s) + 4Y(s) \quad (Y(s) = \mathcal{L}[y(t)])$

$(\because y'(0) = y(0) = 0)$

$\mathcal{L}[d(t-2)] = e^{-2s}$

step 2. $Y(s) = e^{-2s} \cdot \frac{1}{s^2+4s+4}$

$= e^{-2s} \cdot \frac{1}{(s+2)^2}$

step 3. $\mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[e^{-2s} \frac{1}{(s+2)^2}\right]$

$\left( \frac{e^{-2s}}{(s+2)^2} \ldots \quad \frac{A}{s+2} + \frac{B}{(s+2)^2} \Rightarrow A(s+2) + B = e^{-2s} \right.$

$\frac{d}{ds}\left(A(s+2)+B\right) = \frac{d}{ds}\left(e^{-2s}\right) \Rightarrow A = -2e^{-2s}$

$\left. \therefore A = -2e^{-2s},\ B = (2s+5)e^{-2s} \ (\because A(s+2)+B = e^{-2s}) \right)$

$= \mathcal{L}^{-1}\left[e^{-2s} \frac{1}{(s+2)^2}\right] = \mathcal{L}^{-1}\left[e^{-2s} \cdot \frac{-2e^{-2s}}{s+2} + e^{-2s} \frac{2s \cdot e^{-2s}}{(s+2)^2} + e^{-2s} \frac{5e^{-2s}}{(s+2)^2}\right]$

$= \mathcal{L}^{-1}\left[-2\frac{e^{-4s}}{s+2}\right] + \mathcal{L}^{-1}\left[\frac{2s \cdot e^{-4s}}{(s+2)^2}\right] + \mathcal{L}^{-1}\left[\frac{5e^{-4s}}{(s+2)^2}\right]$

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$(s+4)(s^2+9) = (s+4)(s+3i)(s-3i)$ 이므로.

Decomposition $\Rightarrow F(s) = \frac{A}{s+4} + \frac{Bs+C}{s^2+9}$

$(s^2+9)A + (s+4)(Bs+C) = s.$

$As^2 + 9A + Bs^2 + 4Bs + Cs + 4C = s$

- $A+B=0$ ( $s^2$의 계수 $=0$ )
- $4B+C=1$ ( $s$의 계수 $=1$ )
- $9A+4C=0$ (상수$=0$).

$\Rightarrow A = -\frac{4}{25}.\ B = \frac{4}{25}.\ C = \frac{9}{25}$

$\Rightarrow \mathcal{L}^{-1}\left[\frac{(-\frac{4}{25})}{s+4}\right] + \mathcal{L}^{-1}\left[\frac{(\frac{4}{25}s)}{s^2+9}\right] + \mathcal{L}^{-1}\left[\frac{\frac{9}{25}}{s^2+9}\right].$

$\Rightarrow \left( \begin{array}{l} -\frac{4}{25}\mathcal{L}^{-1}\left[\frac{1}{s-(-4)}\right] = -\frac{4}{25}e^{-4t} \\ \frac{4}{25}\mathcal{L}^{-1}\left[\frac{s}{s^2+3^2}\right] = \frac{4}{25}\cos(3t) \\ \frac{9}{25}\cdot\frac{1}{3}\cdot\mathcal{L}^{-1}\left[\frac{3}{s^2+3^2}\right] = \frac{3}{25}\cdot\sin(3t) \end{array} \right)$

$\therefore \mathcal{L}^{-1}[F(s)] = -\frac{4}{25}e^{-4t} + \frac{4}{25}\cos 3t + \frac{3}{25}\sin 3t.$

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$L\frac{di}{dt} + Ri(t) + \frac{q(t)}{C} = E(t)$ 이므로.

$L = 0.1H,\quad R = 4\Omega,\quad C = 1/30\,F \quad i(0) = 0$

$\mathcal{L}\left[0.1\frac{di}{dt} + 4i(t) + \frac{1}{30}\int_0^t i(\tau)d\tau\right] = \mathcal{L}[E]$

$\left(\because q(t) = \int_0^t \frac{dq}{d\tau}d\tau = \int_0^t i(\tau)d\tau\right)$

$\Rightarrow 0.1(sI - \overset{0}{i(0)}) + 4I + \frac{1}{\frac{1}{30}} I \frac{1}{s} = 150 \cdot \frac{(1-e^{-3s})}{s}$

$\Rightarrow \left(0.1s + 4 + 30\frac{1}{s}\right) I = \frac{150}{s}(1-e^{-3s})$

$\Rightarrow I = 150(1-e^{-3s})\frac{1}{(0.1s^2+4s+30)}$

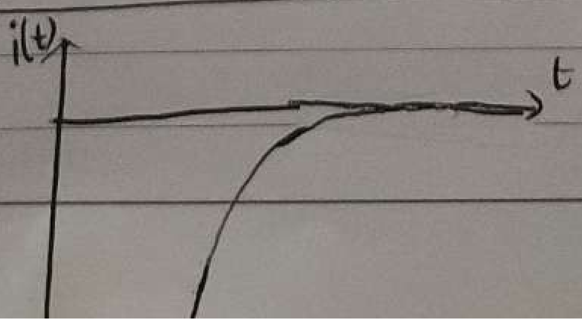
$I(s) = \frac{15}{s^2+40s+300} - \frac{15\cdot e^{-3s}}{s^2+40s+300}$

$= \frac{15}{(s+20)^2-100} - \frac{15\, e^{-3s}}{(s+20)^2-100}$

$= \frac{15}{10}\frac{10}{(s-(-20))^2-(10)^2} - \frac{15}{10}\cdot\frac{e^{-3s}\cdot 10}{(s-(-20))^2-(10)^2}$

$\mathcal{L}^{-1}[I(s)] = \frac{3}{2}e^{-20t}\sinh 10t - \frac{3}{2}\cdot e^{-20t}\cdot \sinh 10(t+3)$ ... (a)

$= \frac{3}{2}e^{-20t}\left(\sinh(10t) - \sinh 10(t+3)\right)$




... (b)

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$$f(t) = \begin{cases} \sin t & (0 \le t \le \pi) \\ 0 & (\pi < t < 2\pi) \end{cases} \quad . \quad f(t+2\pi) = f(t)$$

주기 $T = 2\pi$.

i) $f_1(t) = \sin t$ 부분을 생각하면,

$\mathcal{L}[f(t)] = F_1(s) = \underbrace{\int_0^{\pi} e^{-st} \cdot \sin t \, dt}$. ( ∵ Laplace transform definition)

이때, $\sin t = \dfrac{e^{jt} - e^{-jt}}{2j}$ ( ∵ 삼각함수 항등식 ) 이므로

$$F_1(s) = \int_0^{\pi} e^{-st} \left( \frac{e^{jt} - e^{-jt}}{2j} \right) dt.$$

$$= \frac{1}{2j} \int_0^{\pi} \left( e^{(j-s)t} - e^{(-j-s)t} \right) dt$$

$$= \frac{1}{2j} \left[ \frac{1}{j-s} e^{(j-s)t} + \frac{1}{j+s} \cdot e^{(-j-s)t} \right]_0^{\pi}$$

$$= \frac{1}{2j} \left( \frac{1}{j-s} \left( e^{(j-s)\pi} - 1 \right) + \frac{1}{j+s} \left( e^{(-j-s)\pi} - 1 \right) \right)$$

ii) $f_2(t) = 0$ 일 때를 생각하면,

$\mathcal{L}[f_2(t)] = F_2(s) = \int_{\pi}^{2\pi} e^{(-st)} \cdot 0 \, dt = 0$

$$\therefore F(s) = F_1(s) + F_2(s)$$

$$= \frac{1}{2j} \left( \frac{1}{j^2 - s^2} \left( (j+s) e^{(j-s)\pi} + (j-s) e^{(-j-s)\pi} - 2j \right) \right)$$