HW 5 DE. 경 신후. 21900136.

(a) $y'' - 5y' + 6y = f(t)$. $\quad f(t) = \begin{cases} 1 & (0 \le t < 2) \\ -1 & (t \ge 2) \end{cases}$.

- $f(t)$ 식변환 $\rightarrow$ $f(t) = 1 - 2 \cdot u_2(t)$ $\quad (t \ge 0)$.
- Laplace transform $\rightarrow$ $\mathcal{L}[y'' - 5y' + 6y] = \mathcal{L}[1 - 2u_2(t)]$.
- Left hand side: $\mathcal{L}[y''] - 5 \cdot \mathcal{L}[y'] + 6\mathcal{L}[y]$

$$= (s^2 \mathcal{L}[y] - sy(0) - y'(0))$$
$$- 5(s\mathcal{L}[y] - y(0))$$
$$+ 6(\mathcal{L}[y]).$$
$$= (s^2 - 5s + 6)\mathcal{L}[y] - 1 \quad (\because y(0) = 0,\ y'(0) = 1)$$

- right hand side: $\mathcal{L}[1] - 2 \cdot \mathcal{L}[u_2(t) \cdot 1]$

$$= \mathcal{L}[1] - 2 \cdot e^{-2s}\mathcal{L}[1]$$
$$= 1/s - 2e^{-2s}(1/s) \quad (\because \mathcal{L}[1] = \tfrac{1}{s}).$$

$$(s^2 - 5s + 6)\mathcal{L}[y] - 1 = 1/s - 2 \cdot e^{-2s}(1/s)$$

$$\Rightarrow \mathcal{L}[y] = \frac{1}{(s-2)(s-3)} + \frac{1}{s(s-2)(s-3)} - 2\frac{e^{-2s}}{s(s-2)(s-3)}$$

$$\Rightarrow \mathcal{L}^{-1}[\mathcal{L}[y]] = y(t) = \mathcal{L}^{-1}\left[\frac{1}{(s-2)(s-3)}\right] + \mathcal{L}^{-1}\left[\frac{1}{s(s-2)(s-3)}\right] - 2\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s(s-2)(s-3)}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{-1}{s-2} + \frac{1}{s-3}\right] + \mathcal{L}^{-1}\left[\frac{1}{6}\cdot\frac{1}{s} + \frac{7}{6}\cdot\frac{1}{s-2} + \left(-\frac{4}{3}\right)\cdot\frac{1}{s-3}\right] - 2\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s(s-2)(s-3)}\right]$$

$$= -e^{2t} + e^{3t} + \frac{1}{6} + \frac{7}{6}e^{2t} - \frac{4}{3}e^{3t} - 2\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s(s-2)(s-3)}\right]$$

$$= -\frac{1}{3}e^{3t} + \frac{1}{6}e^{2t} + \frac{1}{6} - 2\left(u_2(t) f(t-2)\right)$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s-2)(s-3)}\right] = \frac{1}{6} + \frac{7}{6}e^{2t} - \frac{4}{3}e^{3t} = f(t)$$

$$\boxed{= -\frac{1}{3}e^{3t} + \frac{1}{6}e^{2t} + \frac{1}{6} - 2u_2(t)\left(\frac{1}{6} + \frac{7}{6}e^{2(t-2)} - \frac{4}{3}e^{3(t-2)}\right)}$$

(b) $y'' + 4y' + 5y = g(t)$, $g(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$.

- $g(t) = t - t\,u_1(t) \quad (t \ge 0)$

- Laplace transform $\longrightarrow$ $\mathcal{L}[y''] + 4\mathcal{L}[y'] + 5\mathcal{L}[y] = \mathcal{L}[t] - \mathcal{L}[t\,u_1(t)]$

- left hand side : $\left(s^2\mathcal{L}[y] - s\,y(0) - y'(0)\right)$

$$+ 4\left(s\mathcal{L}[y] - y(0)\right) + 5\mathcal{L}[y]$$

$$= (s^2+4s+5)\mathcal{L}[y] - s - 4$$

- right hand side : $\mathcal{L}[t] - \mathcal{L}[t\,u_1(t)]$

$$= \frac{1}{s^2} - e^{-s}\mathcal{L}[t+1] \quad (\because \text{제 1 이동정리 변형})$$

$$= \frac{1}{s^2} - e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$$

$$\Rightarrow (s^2+4s+5)\mathcal{L}[y] - s - 4 = \frac{1}{s^2} - e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$$

$$\Rightarrow \mathcal{L}[y] = \frac{1}{s^2(s^2+4s+5)} + \frac{s}{s^2(s^2+4s+5)} + \frac{4}{s^2(s^2+4s+5)}$$

$$- \frac{e^{-s}}{s^2(s^2+4s+5)} - \frac{e^{-s}}{s(s^2+4s+5)}$$

$$= \frac{5 - e^{-s}}{s^2(s^2+4s+5)} + \frac{(1-e^{-s})}{s(s^2+4s+5)}$$

$$\mathcal{L}^{-1}\left[\mathcal{L}[y]\right] = y(t) = \mathcal{L}^{-1}\left[\frac{5-e^{-s}}{s^2((s+2)^2+1)}\right] + \mathcal{L}^{-1}\left[\frac{1-e^{-s}}{s((s+2)^2+1)}\right]$$

$$\cdots ?$$