(21900136, 김신후)

1.

(a)

$$4y''-4y'-3y=\cos 2x \quad \cdots ①$$

$$\begin{cases} y = A\cos 2x + B\sin 2x \\ y' = -2A\sin 2x + 2B\cos 2x \\ y'' = -4A\cos 2x - 4B\sin 2x \end{cases}$$ ①에 대입하면,

$$4(-4A\cos 2x - 4B\sin 2x)$$
$$-4(-2A\sin 2x + 2B\cos 2x)$$
$$-3(A\cos 2x + B\sin 2x) = \cos 2x$$

$$\Rightarrow (-19A-8B)\cos 2x + (8A-19B)\sin 2x = \cos 2x$$

$$\Rightarrow \begin{cases} -19A-8B=1 \\ 8A-19B=0 \end{cases} \Rightarrow \begin{matrix} A=-\frac{19}{425} \\ B=-\frac{8}{425} \end{matrix}$$

특수해 : $y_p = -\frac{19}{425}\cos 2x - \frac{8}{425}\sin 2x$

(b)

동차인 경우의 일반해 : $y = c_1 e^{\frac{3}{2}x} + c_2 e^{-\frac{1}{2}x}$

방정식의 일반해 : $y = c_1 e^{\frac{3}{2}x} + c_2 e^{-\frac{1}{2}x} - \frac{19}{425}\cos 2x - \frac{8}{425}\sin 2x$

$$y' = \frac{3}{2}c_1 e^{\frac{3}{2}x} - \frac{1}{2}c_2 e^{-\frac{1}{2}x} - \frac{38}{425}\sin 2x + \frac{16}{425}\cos 2x$$

$$\begin{cases} y(0) = c_1 + c_2 - \frac{19}{425} = 1 \\ y'(0) = \frac{3}{2}c_1 - \frac{1}{2}c_2 + \frac{16}{425} = 2 \end{cases} \Rightarrow \begin{cases} c_1 = \frac{528}{425} \\ c_2 = -\frac{84}{425} \end{cases}$$

IVP solution : $y = \frac{528}{425} e^{\frac{3}{2}x} - \frac{84}{425} e^{-\frac{1}{2}x} - \frac{19}{425}\cos 2x - \frac{8}{425}\sin 2x$.

2. (a) $y'' - 6y' + 9y = (x+2)e^{3x}$

$y_1 = e^{3x}$, $y_2 = xe^{3x}$.

$y_1' = 3e^{3x}$, $y_2' = e^{3x} + 3xe^{3x}$

∘ $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$= e^{3x}(e^{3x} + 3xe^{3x}) - xe^{3x} \cdot (3e^{3x}) = e^{6x}$

∘ y''의 계수가 1이므로. $r = (x+2)e^{3x}$.

$$y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$$

$$= -e^{3x} \int xe^{3x}(x+2)e^{3x} \cdot e^{-6x} dx + xe^{3x} \int e^{3x}(x+2)e^{3x} \cdot e^{-6x} dx$$

$$= -e^{3x} \int x(x+2) dx + xe^{3x} \int x+2 \, dx$$

$$= e^{3x}\left(-\frac{1}{3}x^3 - x^2 + \frac{1}{2}x^3 + 2x^2\right)$$

$$= e^{3x}\left(\frac{1}{6}x^3 + x^2\right)$$

(b)

$$y = y_h + y_p = c_1 e^{3x} + c_2 x e^{3x} + e^{3x}\left(\frac{1}{6}x^3 + x^2\right)$$

$$y' = 3c_1 e^{3x} + c_2(e^{3x} + 3xe^{3x}) + \sim$$

$$\begin{cases} y(0) = c_1 = 2 \\ y'(0) = 3c_1 + c_2 = 1 \end{cases} \Rightarrow \begin{cases} c_1 = 2 \\ c_2 = -5 \end{cases}$$

$$\therefore y = 2e^{3x} - 5xe^{3x} + e^{3x}\left(\frac{1}{6}x^3 + x^2\right)$$