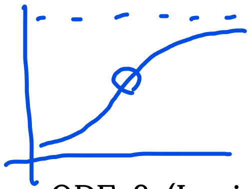
ex:



Section 1-5 Application of First Order ODE 2 (Logistic growth model.)

1. Logistic growth model.

1) Exponential growth.

← 증가하는 모델인데 증가함수가 Exponential 함수이다.

무한히 증가하는 건 비현실적

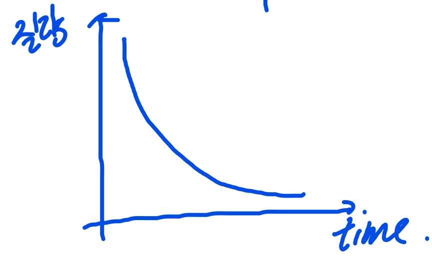
$y = population$

$\frac{dy}{dt} = ry$

$r : rate\, of\, growth\, (r>0)$

$$\int \frac{1}{y} dy = \int r dt$$
$$\ln|y| = rt + C$$
$$y = \pm e^{C} e^{rt} = Ae^{rt}$$
$$y(t) = y_0 e^{rt}$$

(if r<0) exponential decay.



2) Growth rate depend on the population.

$$r \rightarrow h(y)$$

← r이 커지면 y가 줄어들기 때문에. r을 y에 대한 함수로 적어야한다.

$$\frac{dy}{dt} = h(y)y$$

h의 조건.

Choice of h

① $h(y) \simeq r > 0 \;\; y \ll 1$

② h decrease as y growth larger.

③ $h \cancel{>} 0, \;\; y \gg 1$ ← y가 너무 크면 h가 줄어들어야한다.

h<0.

$$h(y) = r - ay \;\; (a : positive\ constant)$$

← 가장 간단한 감소 함수 (일차함수).

$$\frac{dy}{dt} = (r - ay)y$$

Logistic growth model. (Verhulst equation)

$$\frac{dy}{dt} = r(1 - \frac{a}{r}y)y$$

$$\frac{r}{a} = k$$

$$\frac{dy}{dt} = r(1 - \frac{y}{k})y$$

$r : \int rinsic\, growth\, rate$

Simple solution. : $y_0 = const$

$$0 = r(1 - \frac{y}{k})y \quad \text{Equilibrium solution.}$$
$$\Rightarrow y = 0\, or\, y = k$$

↖ 이 경우에는 y가 변하지 않는다.

(해양 모델에 따르면 ...)

의미는?.

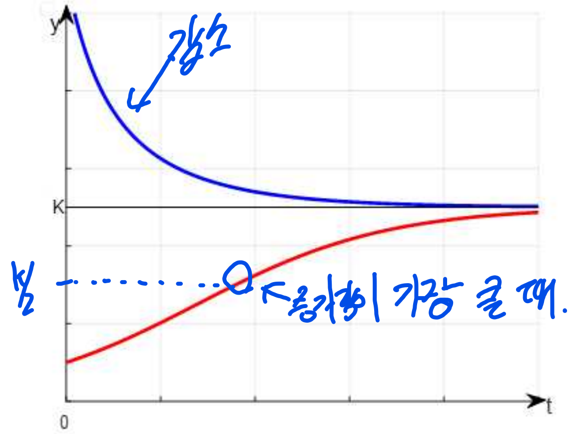


그림 1

변곡점 : y의 증가량이 가장클 때

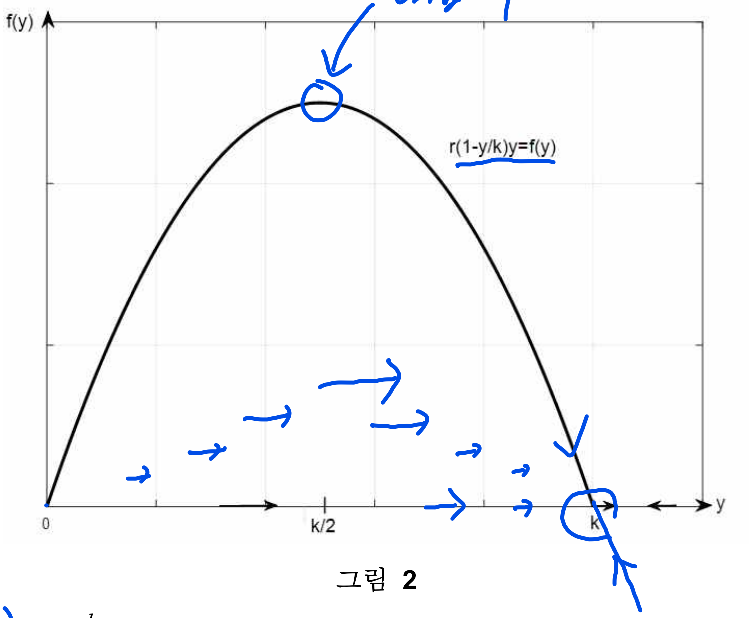


그림 2

$f(y) = \frac{dy}{dt} > 0$, $0 < y < k$ y moves from left to right

$f(y) = \frac{dy}{dt} < 0$, $y > K$ y moves from right to left

초기값 y에 따라 y값이 커지거나, 작아지거나 결정된다.

$$\text{Concavity} \quad \frac{d^2y}{dt^2} = \frac{d}{dt} f(y) = f'(y) \frac{dy}{dt}$$

$$f(y) = r(1 - \frac{y}{k} y) = f'(y) f(y)$$

$$\frac{d^2y}{dt^2} > 0 \quad f' > 0 \,\text{and}\, f > 0 \;\Rightarrow\; 0 < y < \frac{k}{2}$$

$$or\, f' > 0 \,\text{and}\, f < 0 \;\Rightarrow\; y > k$$

$$\frac{d^2y}{dt^2} < 0 \quad f' > 0 \,\text{and}\, f < 0$$

$$\text{or} f' < 0 \,\text{and}\, f > 0 \quad \frac{k}{2} < y < k$$

$k : Saturation \leq vel / Carrying\ capacity$

$$y_0 < k \Rightarrow y(t) < k$$

$$\lim_{t\to\infty} y(t) = k$$

How to solve it

$$\int \frac{1}{(1-\frac{y}{k})y} dy = \int r\, dt$$

$$\int \frac{\frac{1}{k}}{1-\frac{y}{k}} + \frac{1}{y} = ry + C$$

$$\ln|y| - \ln\left|1-\frac{y}{k}\right| = rt + C$$

$$\ln\left|\frac{y}{1-\frac{y}{k}}\right| = rt + C$$

$$\frac{y}{1-\frac{y}{k}} = \pm e^c e = Ae^{rt}$$

$$\frac{y_0}{1-\frac{y_0}{k}} = A$$

$$\Rightarrow \frac{y}{1-\frac{y}{k}} = \frac{y_0}{1-\frac{y_0}{k}} e^{rt}$$

$$y = X(1-\frac{y}{k})$$

$$(1+\frac{1}{k}X)y = X$$

$$y = \frac{X}{1+\frac{1}{k}X} = \frac{1}{\frac{1}{k}+\frac{1}{X}}$$

$$y = \frac{1}{\frac{1}{k} + \frac{1-\frac{y_0}{k}}{y_0} e^{-rt}}$$

$$= \frac{y_0 k}{y_0 + (k-y_0)e^{-rt}}$$

$$\lim_{t\to\infty} y(t) = \frac{y_0 k}{y_0} = k$$

Q: Are $y=0$ and $y=k$ singular solutions?

2. Modified logistic model

Fox squirrel is a small mammal native to the Rocky Mt. They are very territorial. So, if their population is large, their rate of growth decreases and become negative. On the other hand. If the population is too small, fertile adults run the risk of not being able to find suitable mates, so again the rate of growth is negative.

$\Rightarrow$ Model

N: $Carrying\ capacity$ $\rightarrow$ Too big population.

M: $Sparsity\ consistant$ $\rightarrow$ Too small population.

$$\frac{dy}{dt} = g(y)$$

$$\begin{aligned} g'(y) < 0 &\quad \text{if } y < N \\ &\quad or\ \ y < M \\ g'(y) > 0 &\quad \text{if } M < y < N \end{aligned}$$

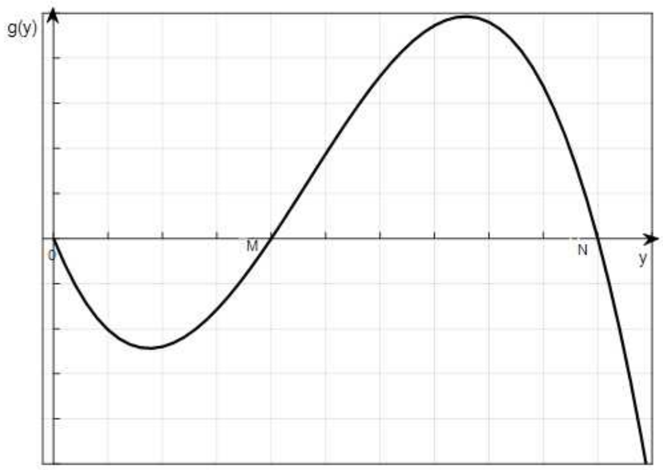


그림 3

$$\frac{dy}{dt} = g(y) = ky(1-\frac{y}{N})(somthing)$$

$$\begin{aligned} (something) > 0 &\quad y > M \\ (something) < 0 &\quad y < M \end{aligned}$$

$$(somthing) = (\frac{y}{M} - 1)$$

$$\frac{dy}{dt} = ky(1-\frac{y}{N})(\frac{y}{M} - 1) \quad (k>0)$$

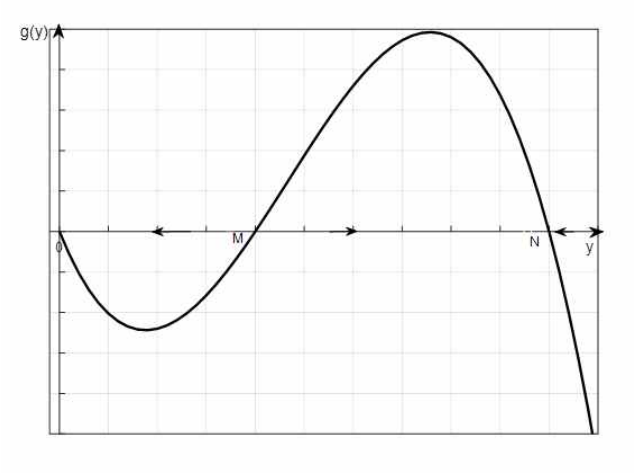


그림 **4**

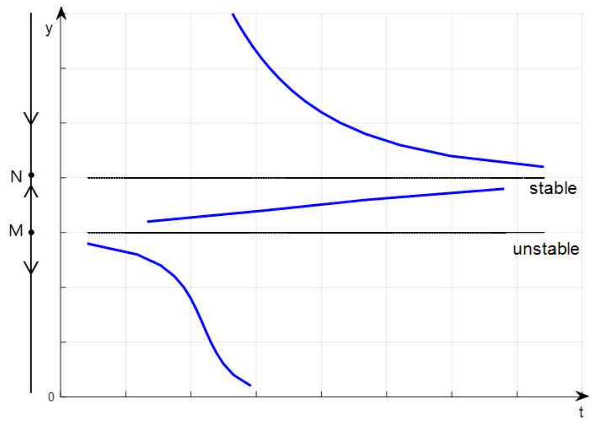


그림 **5**