Exercise 1

Determine the time complexity of following algorithm

Matrix multiplication : Iterative solution

Square-Matrix-Multuply (A, B)

```
n = A.rows
let C be a new n x n matrix
for i = 1 to n
    for j = 1 to n
        c_ij = 0
        for k = 1 to n
            c_ij = c_ij + a_ik * b_kj
return C
```

Exercise 2

Matrix multiplication : Divide-and-Conquer

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{11} & C_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} * \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Exercise 2

Square-Matrix-Multuply-Recursive (A, B)

n = A.rows

let C be a new n x n matrix

if n == 1

    $c_{11} = a_{11} * b_{11}$

else partition A, B, and C four matrices

    C_{11} = Square-Matrix-Multuply-Recursive (A_{11}, B_{11})

        + Square-Matrix-Multuply-Recursive (A_{12}, B_{21})

    C_{12} = Square-Matrix-Multuply-Recursive (A_{11}, B_{12})

        + Square-Matrix-Multuply-Recursive (A_{12}, B_{22})

    C_{21} = Square-Matrix-Multuply-Recursive (A_{21}, B_{11})

        + Square-Matrix-Multuply-Recursive (A_{22}, B_{21})

    C_{22} = Square-Matrix-Multuply-Recursive (A_{21}, B_{12})

        + Square-Matrix-Multuply-Recursive (A_{22}, B_{22})

return C

Exercise 2

Express time complexity T(n) of divide-and-conquer algorithm as recurrence equation. Then solve the equation with

(a)Recursion tree method

(b)Master theorem method

Exercise 3

$$T(n) = \begin{cases} \Theta(1) & \text{if } n=1 \\ 2T(n/2) + n\lg(n) & \text{if } n>1 \end{cases}$$

(1) With recursion tree method

(2) With master theorem method

Exercise 4

Solve the following recurrence equation with master theorem.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n=1 \\ 4T(n/4) + \lg(n) & \text{if } n>1 \end{cases}$$

Exercise 5

Express time complexity *T*(*n*) of selection sort algorithm as recurrence equation.

Exercise 6

- Prove $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1) / 6$