

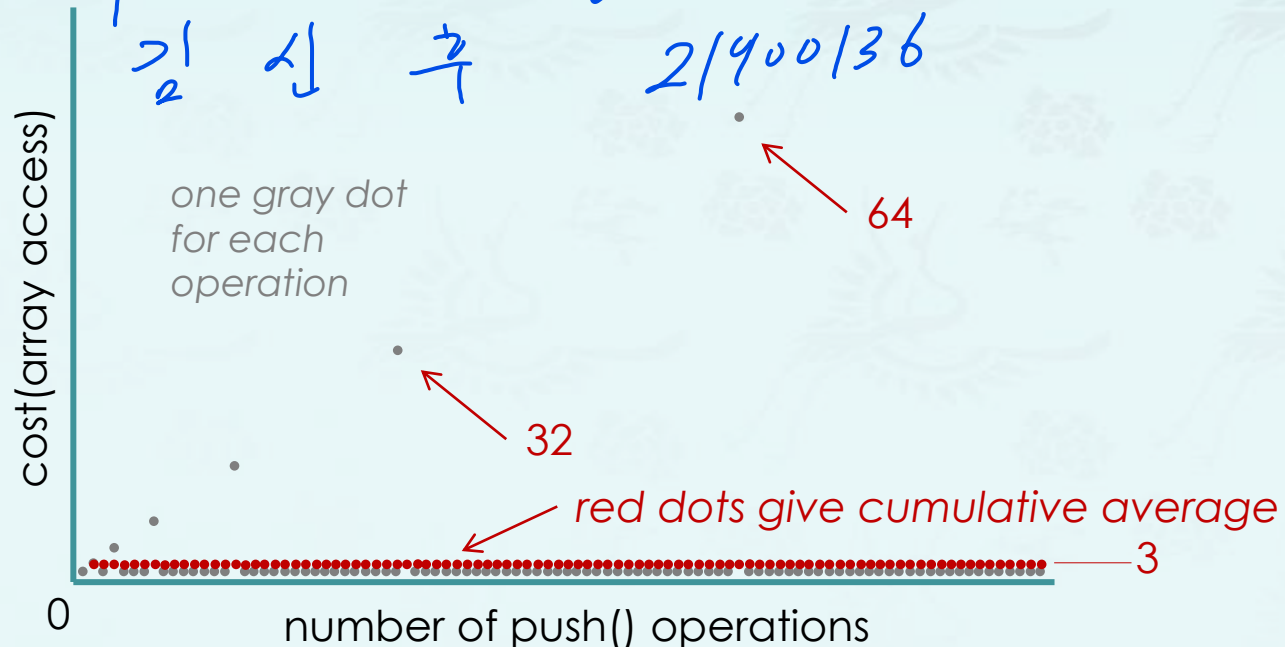
## Stack: Quiz

- The time complexity  $T(N)$  of inserting first  $N$  items by using `resize(capacity * 2)` may be expressed in an open form:

$$T(N) = N + (1 + 2 + 4 + 8 + \dots + N)$$

- Rewrite  $T(N)$  shown above in a closed form.

*On my honor, I pledge that I have neither received nor provided improper assistance in the completion of this assignment.*



$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

$$1 + 2 + 4 + \dots + 2^n = \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$$

$$\begin{aligned} T(N) &= N + 1 + 2 + 4 + \dots + 2^k \quad (2^k = N, k \in \text{real number}) \\ &= N + 2^{k+1} - 1 \quad (\because \sum_{j=0}^k 2^j = 2^{k+1} - 1) \\ &= N + 2^{k+1} \\ &= 2N + 1 \quad (\because 2^k = N) \end{aligned}$$

$$\begin{aligned} \text{Therefore, } T(N) &= N + (1 + 2 + 4 + \dots + N) \\ &= ? \quad 2N + 1 \end{aligned}$$

The time complexity of the algorithm is  $O(n)$ .