Experimental Investigation of Slosh Dynamics of Liquid-filled Containers

by N. C. Pal, S. K. Bhattacharyya and P. K. Sinha

ABSTRACT—This paper is concerned with the experimental studies on the sloshing response of liquid-filled containers. A three-dimensional finite element analysis is carried out for the numerical simulation of this problem. The effects of sloshing are computed in the time domain using Newmark's time integration scheme. A simple experimental setup is designed and fabricated in-house to conduct experiments for measuring some of the basic parameters of sloshing. A sensor device is especially developed to record the free-surface wave heights. Each wave height sensor is a capacitance probe that detects the change in level of liquid (water) precisely with no time lag. The sensors are used in conjunction with a signal-processing unit in which the capacitance values are transduced to a voltage signal between 0 V and 10 V. These wave height sensors simultaneously record the slosh wave height near the periphery of the container wall from 16 predetermined locations to give the free-surface profiles of liquid at desired time steps. The experimental results are compared with those obtained from the present theoretical analysis, and good agreements are observed.

KEY WORDS—Slosh frequency, slosh displacement, hydrodynamic pressure, damping

Sloshing is a phenomenon of great engineering importance, as may be observed in nearly all liquid-filled containers ranging from the common teacup to the huge anti-roll passive tank stabilization systems employed in oceangoing ships, which at one time or another possess the common feature of an unrestrained liquid-free surface. This particular feature leads to the occurrence of a phenomenon that constitutes the focal point of interest in the motions of these containers and has remained a challenging problem in the field of mechanics. Its resolution is, therefore, of significant practical importance to many engineering disciplines, such as aerospace, civil, mechanical and marine.

This is a well-recognized problem in aerospace technology, particularly in liquid propellant launch vehicles, which have an enormous percentage of their initial weight as fuel. The dynamic forces resulting from the motions of these large liquid masses can be very substantial, even beyond the capabilities of the control system to counteract them or the structure to resist them. The problem becomes more complex because of the consumption of propellants during flight, when significant changes in the various frequencies take place rather rapidly.

The first analytical solution of such a dynamic problem in a liquid-filled container was due to Westergaard,[1] who presented the pressure on a rectangular vertical dam subjected to horizontal acceleration. The first study of a related problem in the field of aerospace engineering was due to Graham and Rodriguez,[2] who investigated the aircraft dynamics that were affected by the motion of the fuel in partially filled tanks. A rather comprehensive view of the general subject on slosh dynamics was presented by Abramson.[3] Analytical and experimental studies on the sloshing of liquid in annular rigid tanks subjected to both harmonic and arbitrary ground motions were carried out by Aslam *et al.*[4] Washizu *et al.*[5] obtained the time histories of slosh height of the liquid in a two-dimensional rigid open container subjected to lateral translation taking the free-surface nonlinearity into account. An experimental and analytical study was carried out by Kobayashi *et al.*[6] to determine the liquid natural frequencies and the resulting slosh forces in horizontal cylindrical tanks. Okamoto and Kawahara[7] carried out both numerical and experimental investigations on large-amplitude sloshing waves in a two-dimensional rigid container. The sloshing of liquid in an arbitrary rigid vertical cylindrical tank subjected to both harmonic and irregular base motions was described by Isaacson and Ryu.[8] Pal *et al.*[9,10] recently carried out analytical studies on the coupled slosh dynamic behavior of liquid-filled laminated composite containers.

The present study is concerned with the experimental investigation of sloshing problems in three-dimensional containers. A simple experimental setup is designed and fabricated to conduct experiments for measuring some of the basic parameters of sloshing and to verify certain parametric relationships with numerical computations. A sensor system comprising liquid wave height sensors is developed inhouse to measure the sloshing displacement from various locations on the liquid-free surface and to obtain the free-surface profiles at different time steps. The paper offers a detailed description of the experimental setup and the measurement techniques used in the present investigation.

The numerical simulation of such an initial boundary value problem (shown in Fig. 1), defined by the governing differential equation based on the Eulerian approach, is carried

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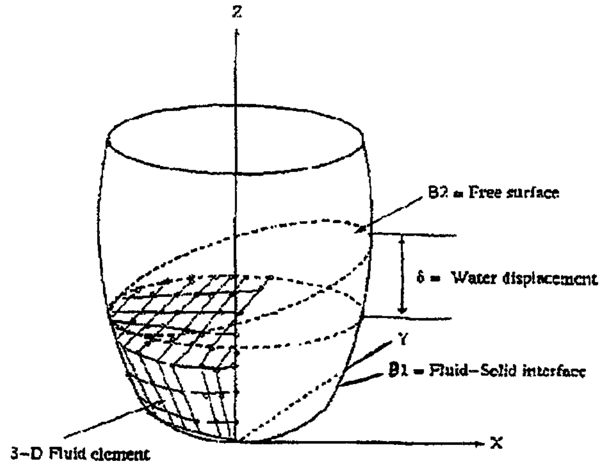

Fig. 1—Problem geometry and boundaries of fluid inside a container

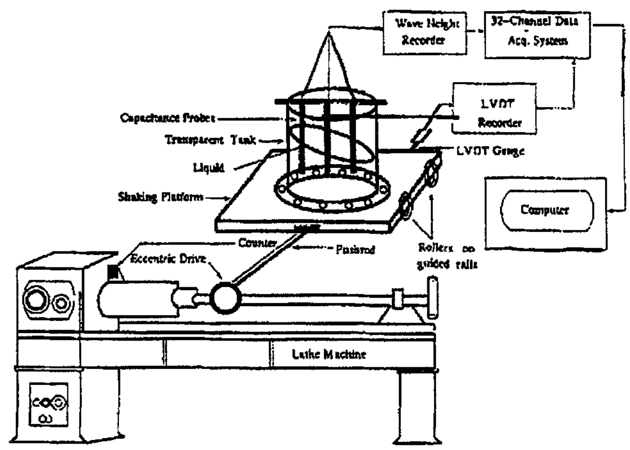

Fig. 2—Schematic diagram of the driving mechanism for the tank-excitation systems and the associated instrumentation for experimental slosh measurement

out using the finite element technique. The finite element discretization of the complete fluid domain is made assuming fluid velocity potential as the nodal unknowns. The discretized form of the governing fluid equation is then derived involving the fluid stiffness matrix, fluid mass matrix and fluid force vector due to external excitation. These equations are solved to determine liquid slosh frequencies, slosh response, hydrodynamic pressure distribution and other associated dynamic characteristics of liquid in rigid containers. The computed numerical results for liquid natural frequencies, mode shapes and forced response characteristics of containers with various geometries are found to be in good agreement with the analytical and experimental results available in the literature.

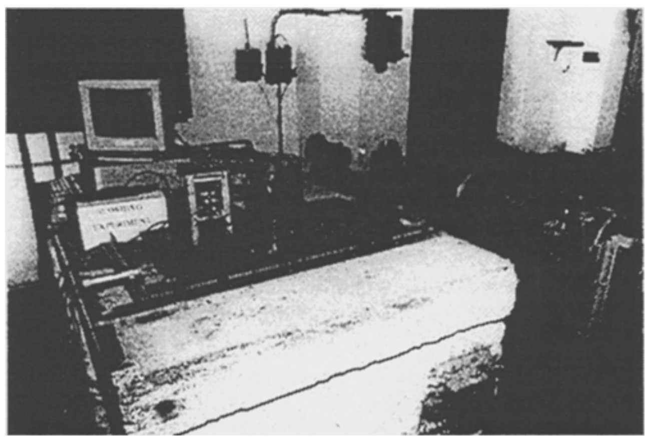

Fig. 3—Photograph of the experimental arrangements and instrumentation with a mechanically manipulated lathe machine as the source of excitation

Experimental Determination of Liquid Slosh Parameters

Experimental Setup

The experimental setup for measuring slosh-induced parameters comprises a shaking platform for supporting and exciting the test containers and the associated instrumentation for capturing the desired response. Review of literature reveals a variety of experimental techniques for supporting and oscillating tank models (laboratory scale models) to induce sloshing motions. In the present investigation, a lathe machine is mechanically manipulated and transformed into a shaking table with the help of suitable fixture attachments. A plate, acting as a platform, is supported on rollers with guided movement using spring attachments. A cam mechanism is used to transform the circular motion into a linear movement of the platform. A cylindrical tank model made of Perspex is fixed to the platform through a fixing arrangement. It may be noted that this transparent cylindrical tank, which is partially filled with water, is primarily aimed at visual observation of slosh modes. The schematic diagram of the setup is shown in Fig. 2. The driving mechanism shown in the figure is a mechanical system that permits variations in the amplitude and frequency of oscillations. The frequency and the amplitude of displacement of the shaking platform are recorded by a linear variable differential transducer along with an amplifying recorder and a 32-channel dynamic data acquisition system. The photograph of the experimental arrangements and instrumentation is shown in Fig. 3. Similar conditions of excitations are also feasible through the use of electromagnetic or hydraulic shakers as the driving source. The second arrangement is more effective in generating various slosh modes. The photograph of one such arrangement with an electromagnetic shaker as the source of excitation is shown in Fig. 4.

Measurement of Slosh Frequencies

The response of a liquid due to the excitation of a container is of significance when the motion of the container is essentially periodic and the period coincides with the first natural frequency of the liquid. Experimental results of previous research reveal that although substantial liquid amplitudes may be attained at a higher frequency of excitation (second and higher natural modes), the forces associated with these liquid motions are generally of secondary importance. Thus, the fundamental problem in analyzing liquid sloshing is the establishment of the characteristics of the first lateral mode, that is, its frequency and mode shape.

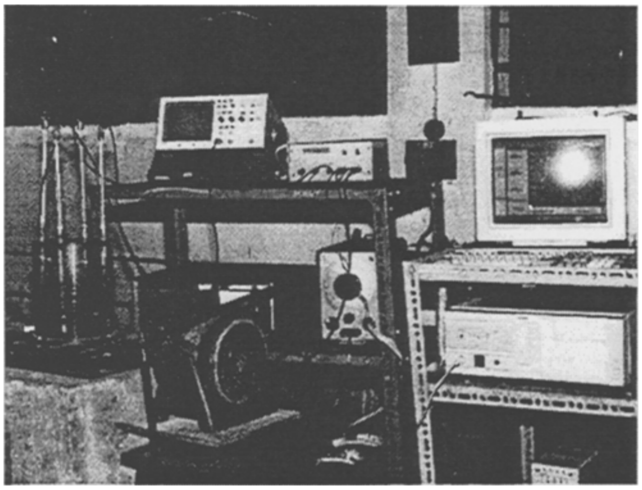

Fig. 4—Photograph of the experimental arrangements and instrumentation with an electromagnetic shaker as the source of excitation

The general technique to measure the liquid natural frequency is to oscillate the tank at low amplitudes and record the frequency at which the amplitude of the undistorted wave shape reaches a maximum without rotation. Above the natural frequency, the fluid surface and particles will exhibit a tendency to rotate, and, during this rotation, a pronounced increase in wave amplitude is possible. However, the rotary motions are also associated with the problem of nonlinear sloshing, and therefore sufficient care is taken to avoid any confusion while measuring the slosh frequency at different liquid depths.

Measurement of Slosh Displacement

An in-house-developed sensor system comprising liquid wave height sensors is employed for the measurement of slosh displacement from various locations on the liquid-free surface. Each wave height sensor is a capacitance probe that detects the change in level of liquid (water) precisely with no time lag. The sensors are mounted parallel to, but offset slightly and insulated from, the walls of the tank at the location of the anti-nodes of the sloshing liquid. The sensors are used in conjunction with a signal-processing unit in which the capacitance values are transduced to a voltage signal between 0 V and 10 V. The capacitance variation is a function of the submerged area of the probe and, hence, varies linearly with the wave height. This was verified with proper calibration. These probes yield electric outputs proportional to the difference in height of the liquid surface at the desired points and are self-compensating so as to maintain a constant zero level as the tank is drained. The electrical signals are continuously recorded with the help of a 32-channel data acquisition system.

Measurement of Slosh Damping

As mentioned earlier, propellant sloshing is a potential source of disturbance in liquid-fueled space vehicles. The motion of the fluid in launch vehicle tanks if unchecked is capable of exerting large upsetting forces and moments on the vehicle. The general procedure for limiting these inputs, and thus minimizing the requirements of the vehicle control system to overcome them, is to install slosh baffles in the tanks. This part of the experiment is aimed at observing the extent of slosh suppression through the use of baffles. The two basic techniques for measuring the extent of slosh suppression induced by the slosh baffles in small laboratory model tanks are the logarithmic decay method and the forced-response method.

The logarithmic decay method involves the measurement of the rate of decay of the fluid oscillation in a given mode—invariably the first mode, since higher modes are so heavily damped that the oscillations are neither of substantial interest nor amenable to accurate measurement, especially if the tank is fitted with even a minimum of sloshing baffles. The rate of decay may be interpreted in terms of the natural logarithm of the ratio of the forces imposed by the fluid on the tank during consecutive oscillations or of the ratio of the amplitude of the fluid motions during consecutive oscillations. In either case, a transducer is used to obtain an effective measure of the magnitude of the sloshing fluid, and the signal is fed into a readout system.

Numerical Simulation

A three-dimensional numerical analysis was also carried out to support the experimental findings. The detailed description of the analysis procedures that is based on Galerkin's finite element formulation of the governing differential equation is reported elsewhere.[9,10] The numerical simulation procedure is briefly presented here.

Figure 1 shows the problem geometry of a typical three-dimensional liquid-filled container under forced excitation. The base of the container is assumed to be rigid. The contained fluid is assumed to be incompressible and inviscid, resulting in an irrotational flow field. Hence, a single velocity potential ϕ exists at every point that satisfies the Laplace equation. This equation is solved using the finite element technique with the appropriate time-dependent boundary conditions on the interface boundary B_1 and on the free-surface boundary B_2 (see Fig. 1). The unknown field variable ϕ is expressed in terms of fluid finite element shape functions and the time-dependent nodal values of the field variable. The finite element form of the governing equation is obtained by vanishing the total weighted interior and boundary residuals.

Two types of elements, namely, eight-noded isoparametric quadrilateral axisymmetric ring elements and trilinear brick elements, are employed to discretize the fluid in the three-dimensional domain, depending on the tank geometry, with velocity potential ϕ as the nodal unknown. Using the standard finite element procedures,[11] the fluid stiffness matrix, free-surface mass matrix and fluid load vector at the fluid-solid interface are obtained. The assemblage of these matrices results in a set of linear, coupled, second-order differential equations. Numerical condensation technique[12] is applied to obtain the condensed dynamic equilibrium equation of motion of the fluid inside a rigid container. These equations are solved employing Newmark's step-by-step integration scheme for obtaining the dynamic responses. Liquid slosh frequencies and free-surface slosh modes for tanks of various geometries are obtained after solving the free-vibration problem with homogeneous boundary conditions but nonzero initial conditions. The developed computer codes generate numerical data for slosh frequency, sloshing displacement, free-surface profiles and hydrodynamic pressure variation at different time steps.

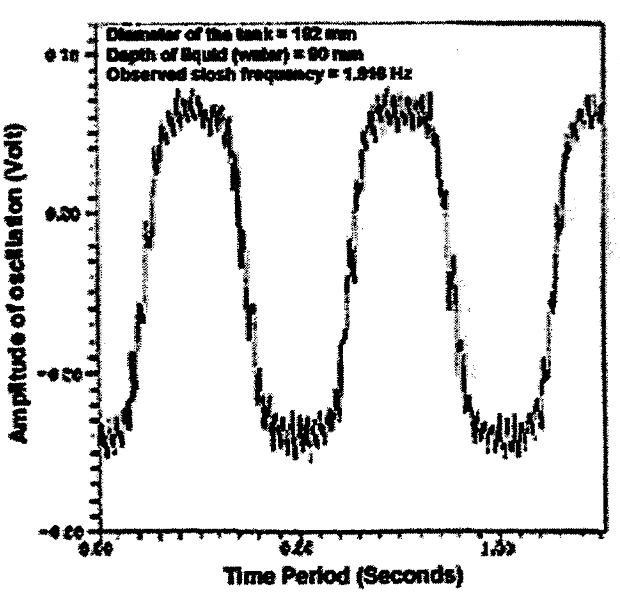

Fig. 5—Recorded motion of the shaking table supporting and oscillating a cylindrical tank

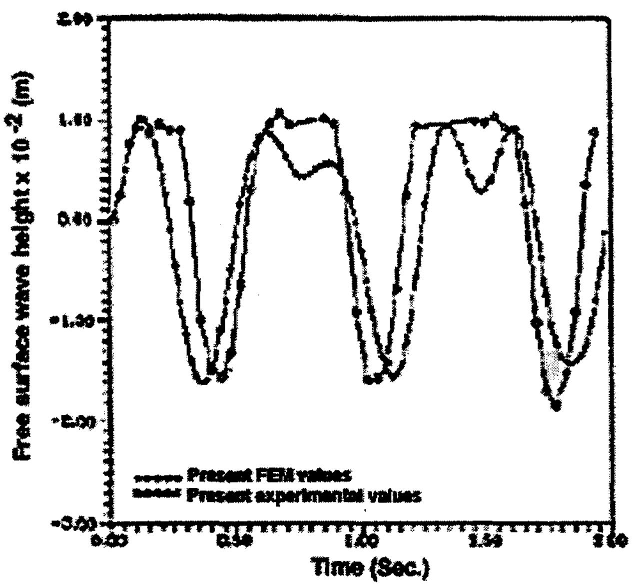

Fig. 6—Comparison of experimental and theoretical (finite element method [FEM]) results of the slosh response near the wall of a cylindrical container

Experimental Results and Discussion

Slosh Frequency

The experimentally measured values of the first slosh frequency (Hz) for the test cylinder for different depths of liquid (water) are presented in Table 1. The frequencies are obtained from the recorded motion of the shaking platform as explained above. One typical record of the tank base motion is shown in Fig. 5 to supplement the tabular data. It may be observed that for different depths of liquid, the experimental values differ within 10 percent from the analytical[3] and the present numerical finite element results. The present numerical modeling is based on a linearized free-surface boundary condition, and the fluid viscosity effects are not considered in the formulation. Moreover, the recorded motion of the shaking platform is captured only after confirmation of a particular slosh mode through visual observation. In the context of the above, the present results may be regarded as satisfactory considering these limitations of experimental measurements and the approximations made during the numerical simulation.

Slosh Displacement

Figure 6 illustrates the comparison between the present experimental and analytical results of the free-surface slosh response near the wall of a cylindrical container. The depth of liquid (water) for this test case is 90 mm. The container is subjected to sinusoidal horizontal base excitation with 0.005 m amplitude of displacement and a frequency of 1.2 Hz. A good agreement between the experimental data and present finite element method results is observed. Figure 7 illustrates the free-surface profiles of the liquid at different time steps obtained from the present experimentation. The profiles are obtained by simultaneous recording of the slosh wave height near the periphery of the container wall from 16 predetermined locations. The corresponding theoretical results using the present finite element method are illustrated in Fig. 8. It is interesting to note that the free-surface profiles obtained from both experimental and finite element methods match extremely well.

Slosh Damping

In the present investigation, the free-surface slosh response is measured using the wave height sensors for both baffled and unbaffled conditions of the test cylinder. One ring baffle of 25 mm width and 6 mm thickness is used in the experiment to suppress the sloshing of the liquid in the test cylinder. A schematic diagram of the cylinder with one ring baffle is shown in Fig. 9. A typical experimental result illustrated in Fig. 10 clearly indicates a significant amount of slosh reduction due to the presence of a baffle. It is further observed that the attenuation of the slosh amplitude is similar in both conditions. This is primarily due to the fact that the baffle is not flexible enough to induce coupled interaction and, thereby, cause the rate of decay of slosh amplitude to change.

Conclusions

Adequate understanding of any complex physical phenomenon such as sloshing is enhanced to a great extent by the use of experimental techniques. Experimental studies allow researchers to check the validity of assumptions of the mathematical model and to employ the model effectively for design applications. The present paper deals with an overall perspective of the sloshing problem, with numerical and experimental solutions to some of the practical cases. A simple experimental setup is designed and fabricated in-house to conduct experiments for measuring some of the basic parameters of sloshing. A sensor device is especially developed to record the free-surface wave heights. Each wave height sensor is a capacitance probe that detects the change in level of liquid (water) precisely with no time lag. The sensors are used in conjunction with a signal-processing unit in which the capacitance values are transduced to a voltage signal between 0 V and 10 V. The recorded amplitudes of the sloshing liquid from 16 predetermined locations provide the free-

TABLE 1—EXPERIMENTAL RESULTS OF THE FIRST SLOSH FREQUENCY (Hz) FOR A LIQUID-FILLED CIRCULAR CYLINDRICAL TANK

Radius of Tank (mm)	Depth of Water (mm)	Analytical (Abramson[3])	Present Theoretical (Finite Element Method)	Present Experimental
96.00	30.00	1.573	1.574	1.513
96.00	60.00	1.974	1.976	1.873
96.00	90.00	2.114	2.117	1.916
96.00	120.00	2.161	2.164	1.969
96.00	150.00	2.176	2.180	2.078

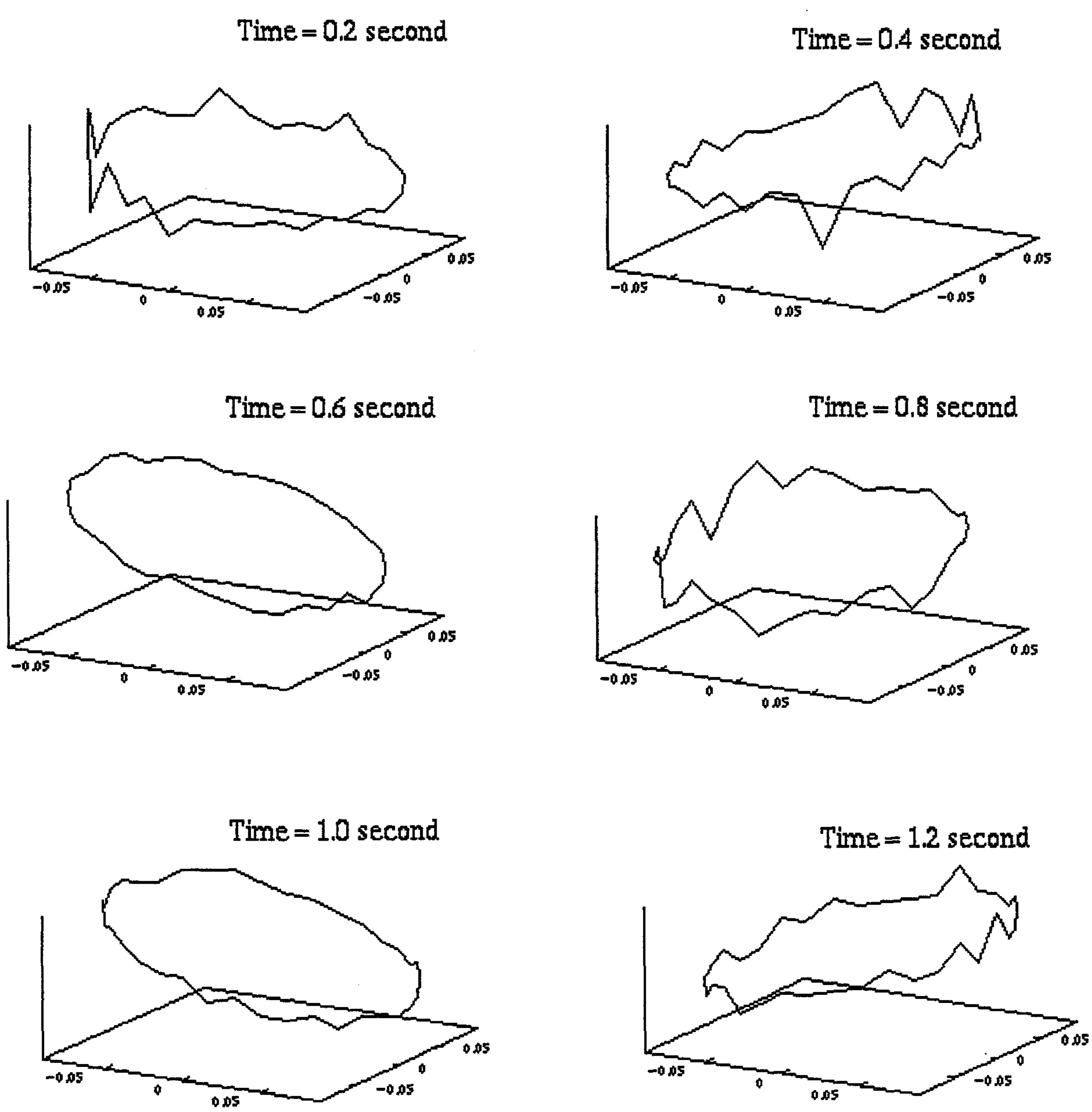

Fig. 7—Free-surface profiles of liquid at different time steps for a circular cylindrical container due to harmonic base excitation (experimental results)

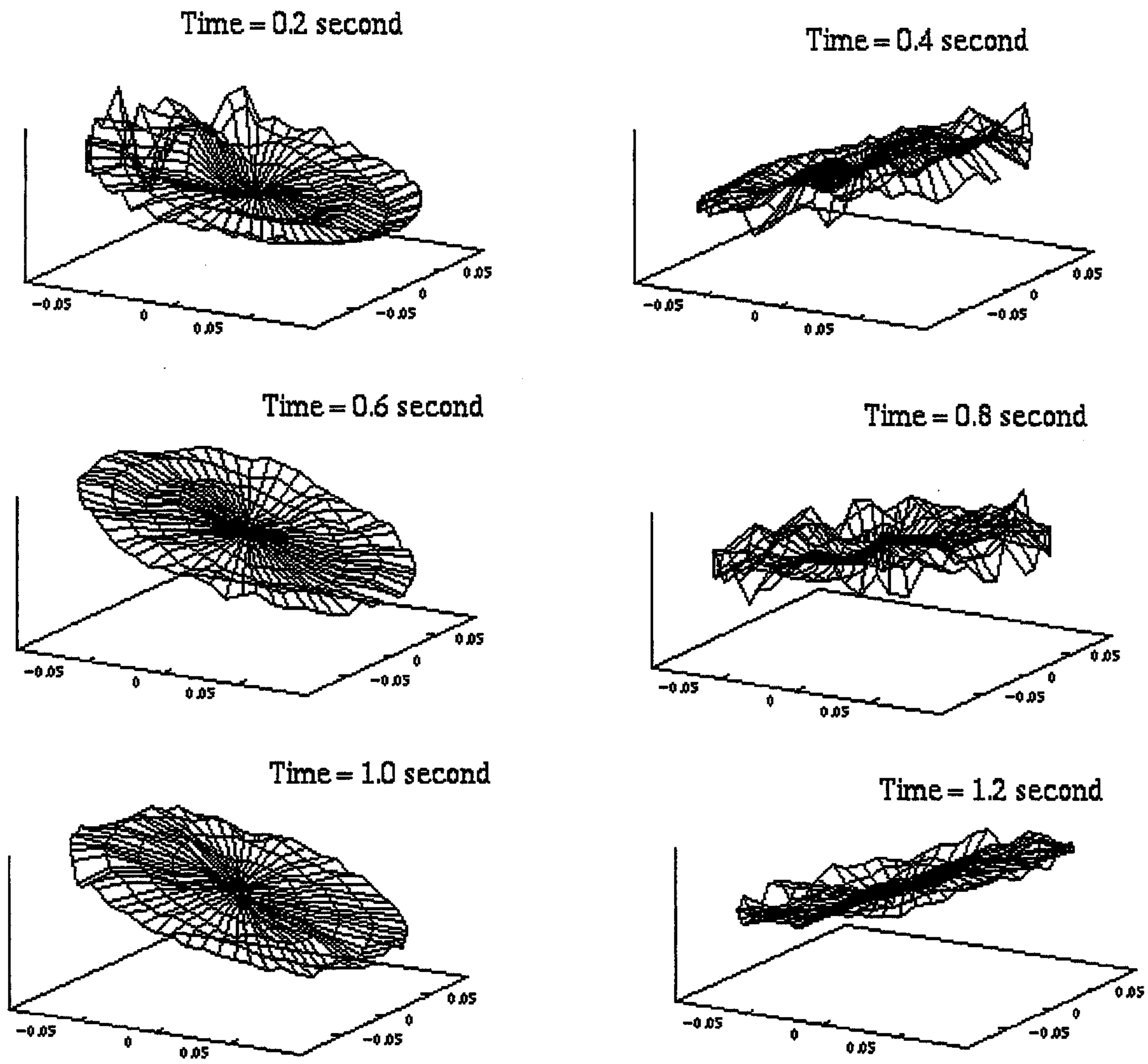

Fig. 8—Free-surface profiles of liquid at different time steps for a circular cylindrical container due to harmonic base excitation (finite element method results)

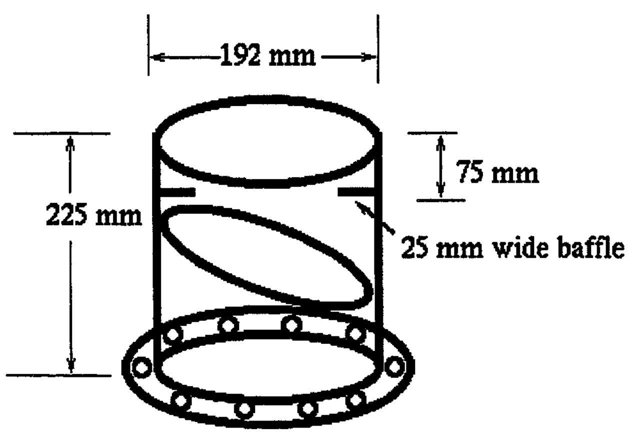

Fig. 9—Schematic diagram of the Perspex test cylinder with one ring baffle (6 mm thick)

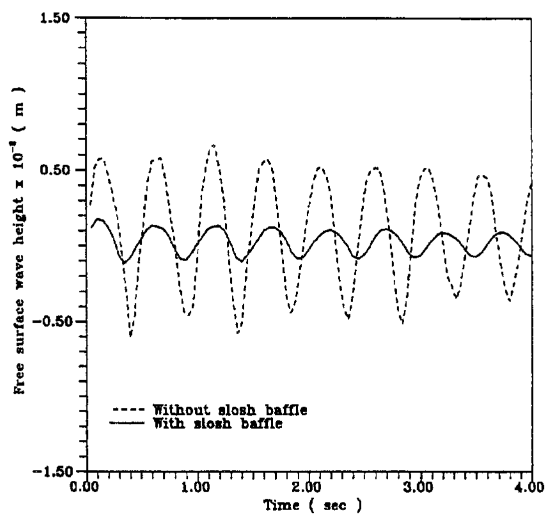

Fig. 10—Experimental observation of the attenuation of liquid oscillation in a cylindrical container

surface profiles at different time steps for the test cylinder. Measurement of free-surface liquid response in absence of any excitation source is done to assess possible slosh suppression techniques from the rate of decay of the slosh amplitude. A three-dimensional finite element analysis is also carried out to support the experimental findings.

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