AIAA Atmospheric Flight Mechanics Conference and Exhibit
15 - 18 August 2005, San Francisco, California

AIAA 2005-5923

Novel Methods for Slosh Parameter Estimation Using Pendulum Analogy

Digvijay D. Odhekar*
P. S. Gandhi† and Keyur B. Joshi‡
Indian Institute of Technology Bombay, Mumbai,India-400076

Liquid slosh is modeled as simple pendulum. Parameter estimation strategies for this model are developed using translational and pitching excitations. Translational excitation and quick stop gives the parameters like natural frequency, slosh mass and hinge point location. Moment of inertia of rigid mass of liquid is estimated by giving pitching excitation to the tank. It is observed that moment of inertia of rigid liquid mass has a strong dependency on frequency of excitation.

Nomenclature

d	Diameter of tank,m
F_x	Slosh force in X direction,N
f_n	First natural frequency of sloshing,Hz
g	Gravitational acceleration,m/s^2
h	Liquid depth,m
I_{GT}	Moment of inertia of tank,kgm^2
I_{GR}	Moment of inertia of rigid liquid mass,kgm^2
l_{pe}	Pendulum length,m
l_h	Hinge point location of pendulum,m
l_{hn}	non-dimensional hinge location= l_h/d
m_s	Slosh mass (Pendulum mass),kg
m_{sf}	Slosh mass fraction = m_s/M
M	Total mass of the liquid,kg
M_y	Moment in Y-direction due to sloshing,Nm
M_{yp}	Moment in Y-direction due to sloshing because of pitching excitation,Nm
r	Radius of the tank,m
sh	Slosh wave height,m
θ	Pitching excitation of the tank
ϕ	Pendulum angle (rad)
ϕ_0	Pendulum angle at quick stop $=tan^{-1}\left(\frac{sh}{d}\right)$
$\dot{\phi}$	Pendulum angular velocity,rad/sec
ζ	Damping factor
ω_n	natural frequency,rad/s
β	Boundary layer thickness
ν	Kinematic viscosity (m^2/sec)
λ	non-dimensional frequency parameter $= \sqrt{\omega_n^2 r/g}$
c_t	Torsional damping co-efficient = $2m_s l_{pe}^2 \omega_n \zeta$

*Student,M.Tech, Indian Institute of Technology Bombay, Mumbai, India.digvijayodhekar@yahoo.co.in
†Asst. Professor,Mechanical Engg., Indian Institute of Technology Bombay, Mumbai India. gandhi@iitb.ac.in
‡Student,M.Tech and Research Associate, Indian Institute of Technology Bombay, Mumbai, India.keyurjoshi@iitb.ac.in

Introduction

SLOSH dynamics has well established importance in rocketry and space science. Study of slosh has been done at various levels in the past and engineers and researchers have tried to come out with simple models of slosh in order to include slosh forces in the control systems and implement better control. Equivalent mechanical models for sloshing liquid are dealt with in great detail in the literature[1–4] as they help in simplifying control system design.

Slosh parameter estimation using pendulum model is studied in the past.[1,2] Franklin et al.[1] gives empirical formulae for pendulum model parameters. Experimental estimates of slosh mass and hinge location are also obtained from maximum force and moment value after quick stop. Unruh et al.[2] has given estimates of slosh mass and hinge location. They have used circular fit technique around resonant peak to estimate the parameters. Gangadharan et al.[3] has developed a 3-D rotor model to predict nutational time constant for the spinning space craft. They have estimated parameters like spring constant and damping by using nonlinear least square technique. Moment of inertia of rigid liquid mass was estimated by Widmayer et al. and Dodge et al.[5,6] In both cases, moment of inertia was estimated from observed natural frequency and known system stiffness.

This work focuses on development of novel parameter identification strategies based on pendulum model analogy of slosh.

- Strategy I : Translational excitation and quick stop
  This strategy has been worked out along with pendulum model. It has given the estimates of slosh mass,natural frequency and hinge location closer to the theoretical predictions of.[1]
- Strategy II : Pitching excitation

This strategy is used to estimate moment of inertia of rigid liquid mass. It uses the same pendulum model that is used for Strategy I with appropriate modifications.

B. Strategy I

Tank is given lateral excitation using translational actuation (see appendix for details about slosh rig). Force and moment data is recorded by the six axis force sensor mounted beneath the tank.The liquid slosh results in forces in X & Z directions and moment in Y direction. The pendulum model is used to find the analytical expressions for these forces and moments in terms of model parameters like m_s, ω_n and l_h. The error between experimental and model response will be minimized to find the parameters. The recorded in force data in X direction is used to find the slosh mass (m_s) and the recorded moment about Y-axis is used to find the hinge location (l_h). The schematic diagram of the sloshing as simple pendulum model is given in Fig.1.

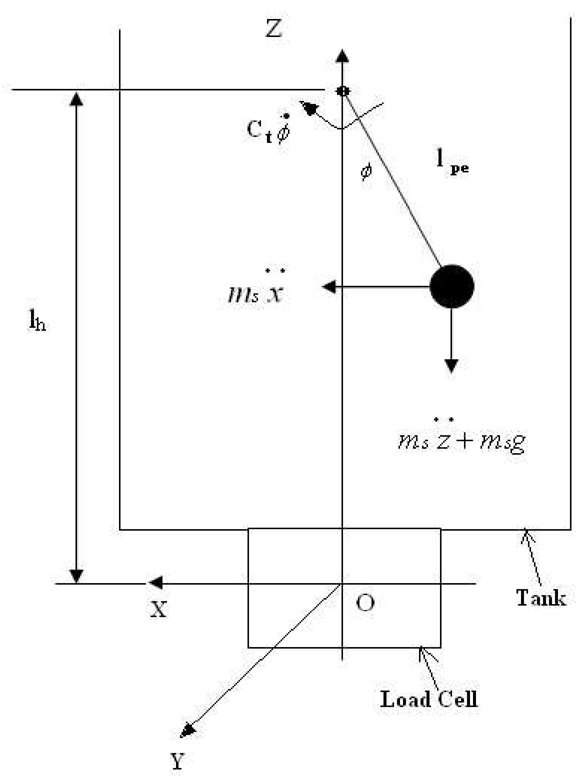

Fig. 1 Simple Pendulum Model for Slosh Rig

The analytical equation used for calculating the force in X direction is given by,

$$F_x = \frac{c_t}{l_{pe}}\dot{\phi}\cos\phi + gm_s\sin\phi\cos\phi + m_s l_{pe}\dot{\phi}^2\sin\phi \tag{1}$$

and the moment in Y direction in is given by,

$$\begin{aligned} M_y =& F_x(l_h - l_{pe}\cos\phi) - c_t\dot{\phi} - (gm_s - \frac{c_t}{l_{pe}}\dot{\phi}\sin\phi \\ & - gm_s\sin^2\phi + m_s l_{pe}\dot{\phi}^2\cos\phi)l_{pe}\sin\phi \end{aligned} \tag{2}$$

I. Pendulum Motion after quick stop

After the tank is quick stopped, sloshing motion of liquid will be free under-damped vibration. This motion is assumed to be viscously damped and it is given by,

$$\phi = C_1 e^{-\zeta\omega_n t}\cos(\sqrt{1-\zeta^2}\omega_n t + \psi) \tag{3}$$

with initial condition $\dot{\theta} = 0$ at the quick stop ($t = 0$). differentiating ,we have,

$$\dot{\phi} = (-\zeta\omega_n)\phi - (\sqrt{1-\zeta^2}\omega_n)C_1 e^{-\zeta\omega_n t} sin(\sqrt{1-\zeta^2}\omega_n t + \psi) \tag{4}$$

II. Parameter Estimation Process

The procedure of estimating the parameters of the pendulum model is presented here.

Experimental Procedure

The tank is given sinusoidal translational excitation with rotary degree of freedom locked. Frequency is slowly increased to a value below first mode resonance frequency of slosh as the pendulum model is not valid near resonance frequency.[1] Once the sloshing motion of liquid achieves steady state, amplitude of sloshing liquid is noted. The tank is then quick-stopped at zero velocity. The readings of forces and moments as a result of liquid sloshing after quick-stop are recorded.

Estimation of Frequency

Since the signals are recorded after quick stop, oscillations of liquid being free vibration the FFT of force or moment signal gives natural frequency.

From the natural frequency we can find the pendulum length by,

$$\omega_n = \sqrt{\frac{g}{l_{pe}}} \tag{5}$$

Estimation of Damping Factor

Estimation of damping factor is done by calculating the amount of decay over a certain number of cycles.

Estimation of Slosh Mass and Hinge Location

The estimation of parameters is done by using optimization tools available in MATLAB. Slosh mass is estimated first using Eq.(1) and recorded data. Using estimation of slosh mass, frequency and damping factor, hinge location is identified using Eq.(2) and recorded data. The procedure for optimization is summarized below.

The error is the difference between experimental and simulated values. The objective function for optimization is the sum of squares of error at each time instant. i.e. objective function,

$$f = \Sigma_i (ExperimentalResponse - modelResponse)^2 \quad i = 1, 2, ...Ndatapoints \tag{6}$$

This optimization problem is solved using a constrained optimization function 'fmincon' from MATLAB optimization toolbox.

III. Results and Discussion

The results of the experiments are plotted along with the results of the[1] in Fig2,3 and 4. The experimental results are within 10% of theroretical estimates of.[1]

Parameter estimation is done using large number of data points over sometime after quick-stop. It is observed that the results for slosh mass and hinge location obtained with this strategy are better than those obtained by comparing maximum force and moment value after quick stop. This is because the high frequency components excited because of quick stop will die out after sometime. Also the proposed strategy uses all the data values as against the only first maximum amplitude value.

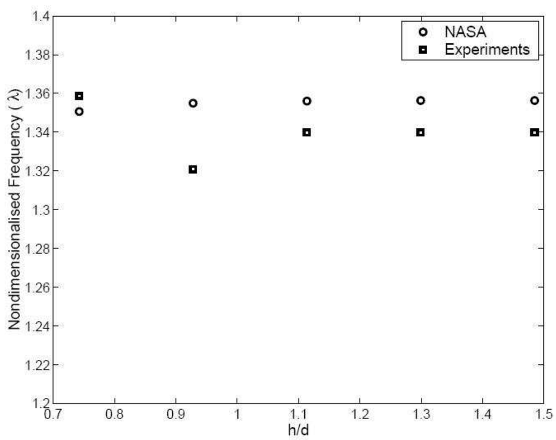

Fig. 2 Nondimensional Frequency Vs Liquid depth ratio ($\frac{h}{d}$) for Cylindrical Tank

C. Strategy II

In this strategy, moment of inertia of rigid liquid mass is estimated by giving pitching excitation to the tank. The same pendulum model that is used for Strategy I is used here with appropriate modifications. Free body diagram of tank with liquid for pitching excitation is shown in Figure 5. The pendulum parameters evaluated by linear excitation and quick-stop are used to simulate the pendulum motion in pitching excitation.

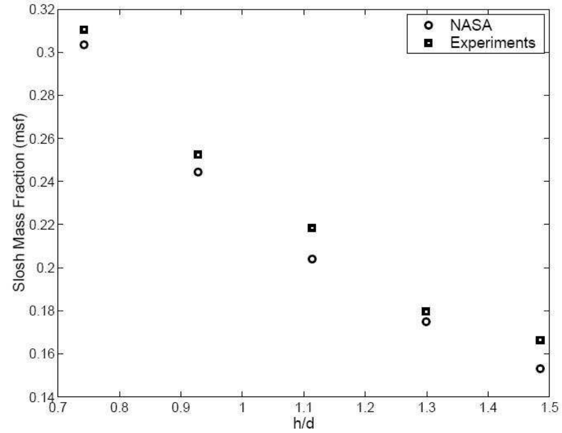

Fig. 3 Slosh mass fraction Vs Liquid depth ratio ($\frac{h}{d}$) for Cylindrical Tank

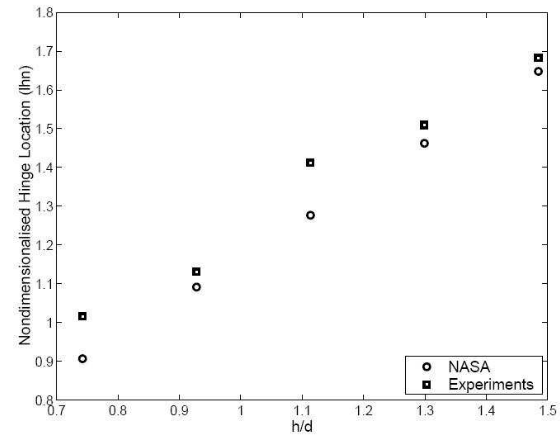

Fig. 4 Nondimensional hinge location Vs Liquid depth ratio ($\frac{h}{d}$) for Cylindrical Tank

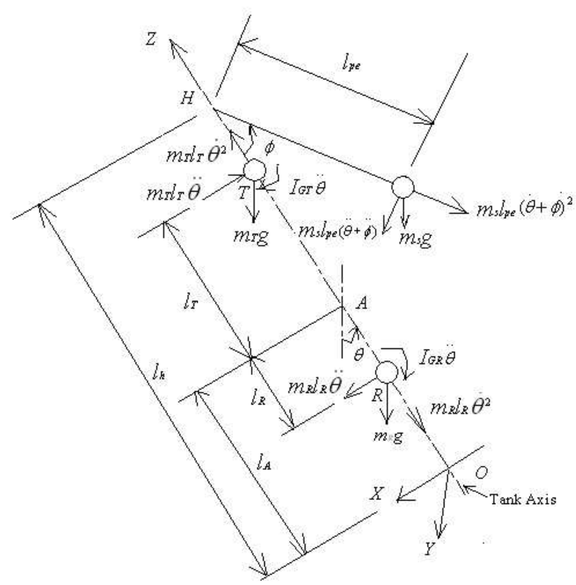

Fig. 5 Free Body Diagram for Pitching Excitation

Here, three masses are considered, namely, tank mass m_T pendulum mass m_s and rigid liquid mass m_R. T, S and R are CGs of tank mass, pendulum mass and rigid liquid mass respectively. I_{GT} is mass moment of inertia of tank and I_{GR} is mass moment of inertia of rigid liquid mass about its CG. ϕ is the pendulum angle relative to the tank axis.

Moment about Y axis recorded on the sensor in this case is given by,

$$\begin{aligned} M_{yp} =& (m_R l_R \ddot{\theta} + m_R g \sin\theta)(l_A - l_R) \\ &+ (-m_T l_T \ddot{\theta} + m_T g \sin\theta)(l_A + l_T) \\ &- (m_s l_{pe} (\dot{\theta} + \dot{\phi})^2 + m_s g \cos(\theta + \phi)) l_h \sin\phi \\ &- (I_{GT} + I_{GR})\ddot{\theta} - c_t \dot{\phi} \end{aligned} \tag{7}$$

Now, the pendulum motion ϕ can be simulated by writing down the system's equations of motion. It is given by,

$$\begin{aligned} \ddot{\phi} =& \frac{I_s}{I_{total} I_s - I_s^2} [(m_T l_T + m_R l_R) g \sin\theta \\ &+ m_s g l_{pe} \sin(\theta + \phi)] \\ &- \frac{I_{total}}{I_{total} I_s - I_s^2} [c_t \dot{\phi} + m_s g l_{pe} \sin(\theta + \phi)] \end{aligned} \tag{8}$$

where, $I_s = m_s l_{pe}^2$
$I_{total} = I_{GT} + m_T l_T^2 + I_{GR} + m_R l_R^2 + I_s$

Thus from these equations we can find the analytical time history of moment. This can be compared to experimental results to find the error which is minimized to estimate moment of inertia of rigid liquid mass I_{GR}. This optimization problem is solved using Levenberg-Marquardt algorithm of nonlinear least squares function in MATLAB optimization toolbox.

I. Results and Discussion

In order to determine the effect of frequency and amplitude on estimated inertia, experiments were carried out at different frequency of excitation for the same liquid mass. The results are shown in Figure 6. Rigid liquid inertia was observed to be practically independent of amplitude of excitation. But it was having strong dependency on frequency of excitation. Also, at higher frequencies, amplitude of excitation also has some effect on inertia.

Figure 7 shows the variation of inertia with liquid mass. Theoretical values of inertia are also plotted. It can be seen on the graph that liquid inertia values are different from theoretical (assuming it as solid) values. In all these experiments, slosh wave amplitude (ϕ) was kept constant by changing the amplitude (θ_0) and frequency (ω) of excitation.

In order to study the effect of viscosity on inertia of rigid liquid mass, experiments were carried out with oil. Table 1 lists the oil and water properties. Figure 8 shows the effect of viscosity on inertia for the same volume of liquid in tank. The ratio of rigid liquid inertia to solid inertia is plotted against frequency of excitation. It can be seen that the ratio is higher for oil. This can be attributed to the fact that boundary layer formed on container walls is thicker for oil than water for same excitation frequency. This is evident from the equation

$$\beta = \sqrt{\frac{2\nu}{\omega}}, \tag{9}$$

where,
β= boundary layer thickness
ν= kinematic coefficient of viscosity
ω = circular frequency

Table 1 Properties of Oil and Water

Liquid	Density (Kgm^3)	Dynamic Viscosity (Ns/m^2)
Water	1000	0.0008
Oil	860	0.1

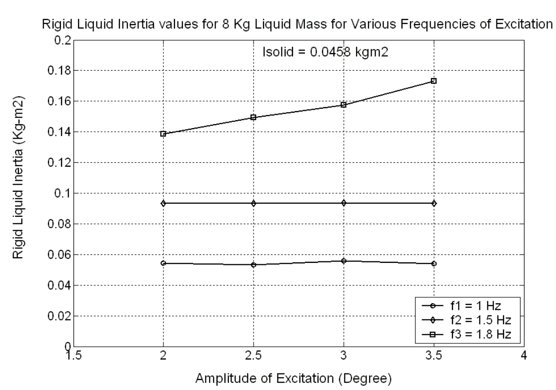

Fig. 6 Liquid Inertia Vs Excitation Amplitude (for 8 Kg Liquid Mass)

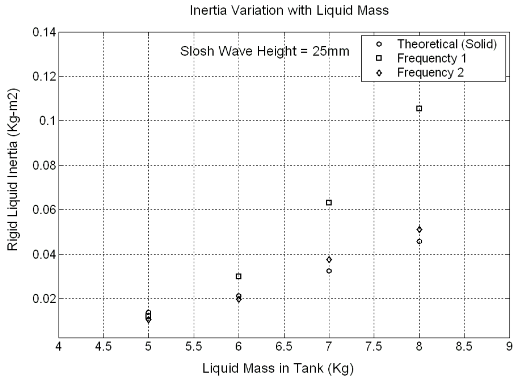

Fig. 7 Variation of Inertia with Frequency

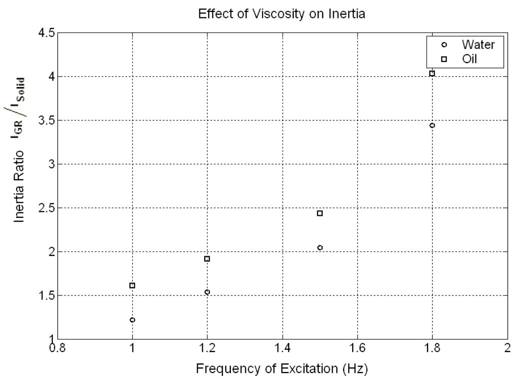

Fig. 8 Effect of Viscosity on Rigid Liquid Inertia

Acknowledgement

This work is supported by Indian Space Research Organization (ISRO) under Project Code 03IS001 through STC program at IIT Bombay. The authors would like to acknowledge the help rendered by Prof Ananthkrishnan of Aerospace department and Jatinder Mohan who built the slosh test rig.

References

[1] Abramson, H., "The Dynamic Behavior of Liquids in Moving Containers," , No. NASA SP-106, 1966.

[2] Unruh, J., Kana, D., Dodge, F., , and Fey, T., "Digital Data Analysis Techniques for Extraction of Fuel Slosh Parameters," *Journal of Spacecraft*, Vol. 23, No. 2, March-April 1986, pp. 171–177.

[3] Gangadharan, S., Sudermann, J., Marlowe, A., and Njenga, C., "Parameter Estimation of Spacecraft Fuel Slosh Model," *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Vol. AIAA Paper 2004-1965, 19-22 April 2004.

[4] Abramson, H., Chu, W., and Ransleben, G., "Representation of Fuel Sloshing in Cylindrical tanks by an Equivalent Mechanical Model," *ARS Journal*, Vol. 31, December 1961, pp. 1697–1705.

[5] Widmayer, E. and Reese, J., "Moment of Inertia and Damping of Fluid in Tanks Undergoing Pitching Oscillations," Rsearch Memorandum RM L53E01a, NACA National Advisory Comittee on Aeronautics, Washiogton, 1953.

[6] Dodge, F. and Kana, D., "Moment of Inertia of and Damping of Liquids in Baffled Cylindrical Tanks," Tech. Rep. NASA CR-383, NASA National Aeronautics and Space Administration, February 1966.

Appendix

The slosh rig

The Rig has two ball-screw slides, each driven by separate PMDC servo motor, giving it the capability of providing the tank with only translation, only pitching about a fixed axis or both these motion simultaneously. The pitching motion is achieved through a slider-crank mechanism that converts linear motion of one of the ball-screw into pitching motion of the cradle that carries tank. As an extra feature the rig has provision for choosing the angle between the plane of the translation and that of pitching excitation.

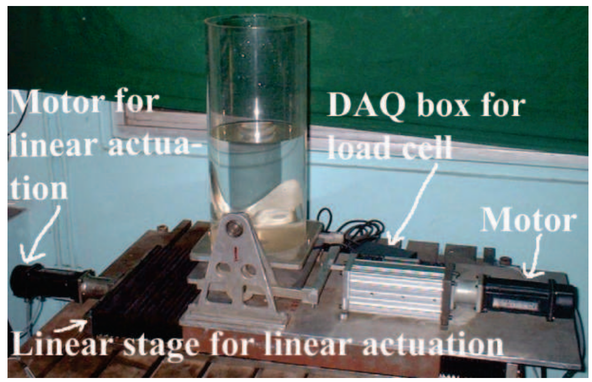

Fig. 9 The Slosh Rig

Force and moment data are measured through a six-axis force sensor situated beneath the tank. The force sensor is sensitive enough to sense second mode of sloshing. The data acquisition and control implementation is done through a sophisticated dSPACE control card that allows real-time interface with SIMULINK models.