Experimental Testing of Liquid Slosh Suppression in a Suspended Container with Compound-Pendulum Dynamics

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***Abstract*—Liquid slosh is a concern for industries that move liquids in rapid or precise operations. The majority of existing control techniques utilize feedback control to reduce liquid slosh. On the other hand, open-loop input shaping has shown to be robust and effective in minimizing liquid slosh without the use of sensors. This paper examines through experimentation the use of a wide range of input shapers to reduce liquid slosh. The container used in the experiments is suspended from an overhead trolley and exhibits compound-pendulum dynamics. Test results demonstrate the effectiveness and robustness of input shaping in reducing liquid slosh and the ability to provide fast settling times over a broad range of frequency variations. The results also provide a thorough comparison of several different types of input shaping. These results will help guide engineers working on slosh-control problems.**

***Index Terms*—Input Shaping, Liquid Slosh, Compound Pendulum.**

I. Introduction

In industries that move liquids, slosh can decrease both the safety and efficiency of operations. Suspended liquid containers, such as those used in transferring molten metal exhibit oscillations in two forms when moved. One type of oscillation is induced by the pendulum swing of the container and the other by the slosh of the liquid in the container. Previous research was performed on the control of liquid slosh by feedback methods [1]–[3]; however, these methods require sensors to detect the liquid motion. The sensing problem can be very challenging. Earlier work investigated the use of command shaping in suppressing slosh [3]–[11]. In [10], input shaping was implemented to suppress slosh in a horizontally moving liquid container and [11] implemented robust input shapers on a suspended container with simple pendulum dynamics. This paper further investigates the use of input shapers on a suspended container, but with compounded-pendulum dynamics.

Input shaping reduces oscillation by properly shaping the reference command. The method convolves a series of impulses with the original reference command to produce a new command that results in a reduced residual vibration and a relatively short increase in rise time. Input shapers differ in the number of impulses and in their impulses magnitudes and timings. Multiple input shapers can be convolved together to suppress residual vibration over a broader range of frequencies. Such input shapers will be robust, but they will result in a further increase in rise time. Figure 1 illustrates the process of input shaping [12]–[17]. The intent of this paper is to present a simple, cost effective, and robust command-generation technique to move a complex non-linear system such as the one under investigation.

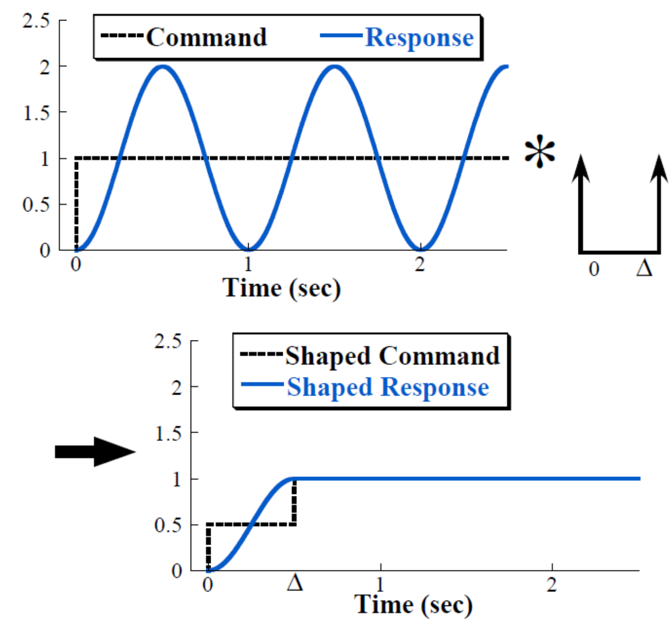

Fig. 1. Input-Shaping Process

II. Experimental Setup

The test apparatus is shown in Fig. 2. It is a cuboid container made from Acrylic glass that is suspended from an overhead

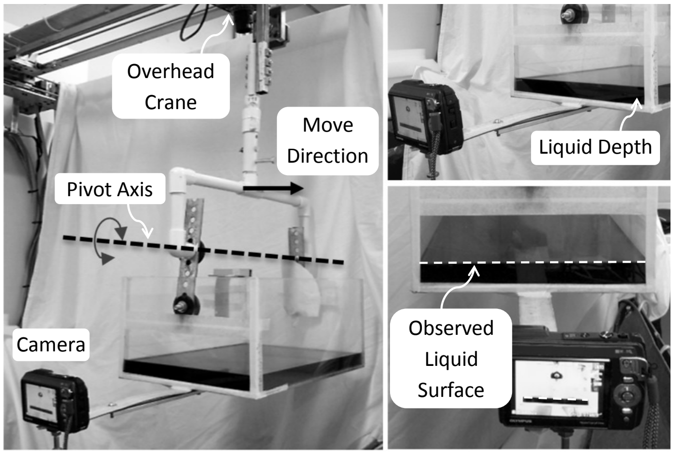

Fig. 2. Experiment Setup

crane trolley. The container pivots on a rod via a pair of ball bearings that allow it to swing with minimum friction. The overhead crane is controlled with a PLC and a computer GUI [18]. Only the trolley portion of the overhead crane is utilized in this research to provide movement of the container. The liquid waves travel along the short side of the cuboid container. A camera is placed at a fixed distance from the short side of the cuboid. The camera is fixed in relation to the container and captures the relative sloshing of liquid within the swinging container.

Image frames are extracted from the videos and analyzed using MATLAB. Figure 3 shows an extracted image from the test video captured from a typical response (top), and a plot of the response amplitude, measured in pixels, for each image frame (bottom). This process measures displacement of the liquid on the far right end of the container.

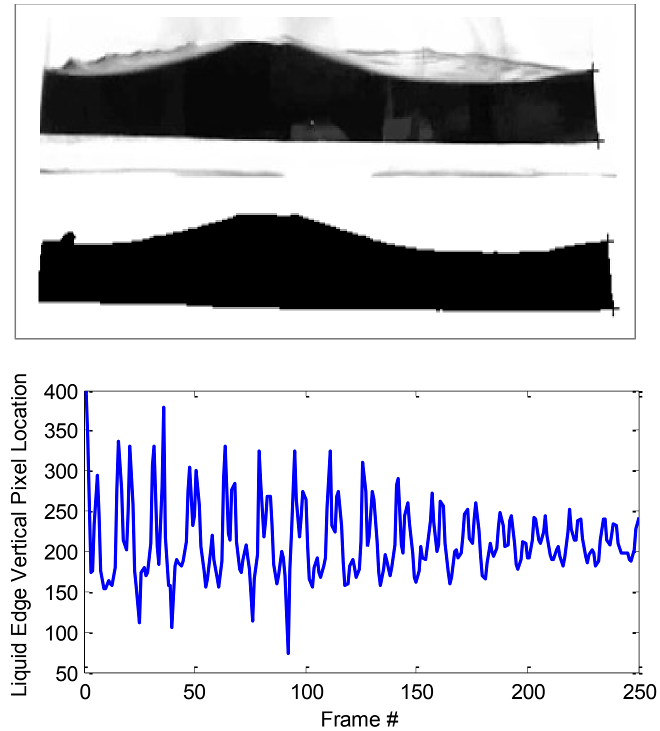

Fig. 3. Image Processing using MATLAB

TABLE I
NATURAL FREQUENCIES OF THE CONTAINER PENDULUM SYSTEM

Liquid Depth (cm)	Calculated w_n (rad/s)	Measured w_n (rad/s)	Measured ζ
2.0	6.27	6.07	0.025
3.0	6.33	6.02	0.050
4.0	6.35	6.01	0.024
5.0	6.36	5.90	0.035

III. SYSTEM MODEL

A. Compounded Pendulum

Because the model investigated here has compound pendulum dynamics, changes in the liquid depth result in changes to the effective pendulum length and thus change the natural frequency of the container motion.

The effective pendulum length of the container, L_{eff}, depends on the Radius of Gyration, k_O and the Center of Mass, r_G, of the container assembly.

$$L_{eff} = \frac{k_o^2}{r_G} \tag{1}$$

The natural frequency of the container can be computed with the pendulum frequency equation [19]:

$$w_n = \sqrt{\frac{g}{L_{eff}}} \tag{2}$$

B. Experimental Verification of Vibration Frequency and Damping Ratio

To ensure that accurate pendulum natural frequencies are used when designing the input shapers, the natural frequencies of the container system were measured experimentally. This was performed by measuring the damped free-vibration response of the container at different liquid depths.

Table I shows the experimentally measured values of the pendulum natural frequency at four liquid depths. The calculated and measured values are within 10% of each other. Note that the frequency changes very little as the liquid depth changes. The table also lists the measured damping ratio of the compound-pendulum container at these four liquid depths.

IV. SLOSH DYNAMICS

The calculations for the liquid slosh frequencies are based on earlier work that models the liquid as a system of damped harmonic oscillators in the case of one-dimensional motion, each introducing a vibration mode [20]–[23]. Such a model is illustrated in Fig. 4.

The equation for calculating the ith sloshing frequency is [22]:

$$w_i^2 = (2i-1)\frac{g}{a}tanh((2i-1)\frac{h}{a}) \tag{3}$$

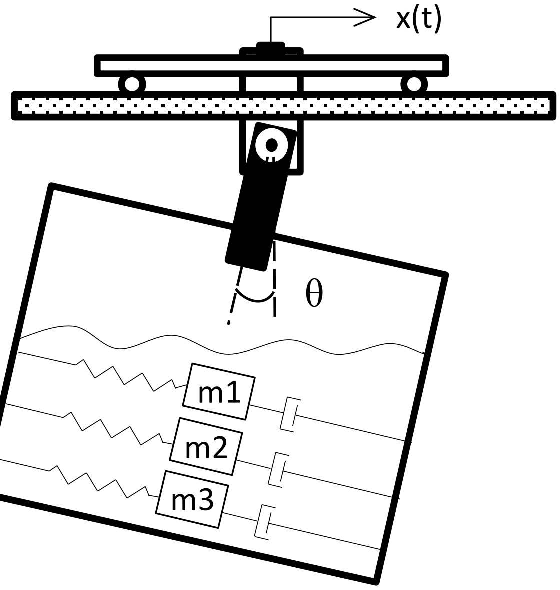

Fig. 4. Representation of the Suspended Liquid Container

where w_i is the i^{th} sloshing frequency, a is the liquid length, g is the acceleration of gravity and h is the liquid height. The damping ratio ζ is usually about 0.01 for water.

In this paper, the input shapers are designed primarily based on the container pendulum frequencies and the liquid's 1^{st} sloshing frequencies. For some input shapers the 2^{nd} sloshing frequencies are also considered. The oscillation of the container and the shaking of the apparatus during motion may introduce noise, seen as additional vibration frequencies. However, the experimental results clearly show the amplitude of the response at the calculated 1^{st} and 2^{nd} slosh frequencies to be dominant, as seen in Fig. 5. Figure 5 shows the FFT of the slosh response to a single mode input shaper; a Zero Vibration (ZV) input shaper designed for the liquid's 1^{st} mode at a 3.5 cm liquid depth. The previously calculated 1^{st} and 2^{nd} slosh frequencies for this liquid depth are 7.0 rad/s and 17.7 rad/s respectively. The container pendulum frequency at this depth is 6 rad/s. The dotted line in the figure is the sensitivity curve for the ZV input shaper used.

Table II lists the calculated 1^{st} and 2^{nd} slosh frequencies for different liquid depths. Figure 6 shows a plot of the two frequencies versus each depth in addition to the container pendulum frequencies that were derived experimentally. There is little variation in the container pendulum frequency across the liquid depths compared to the change in slosh frequencies.

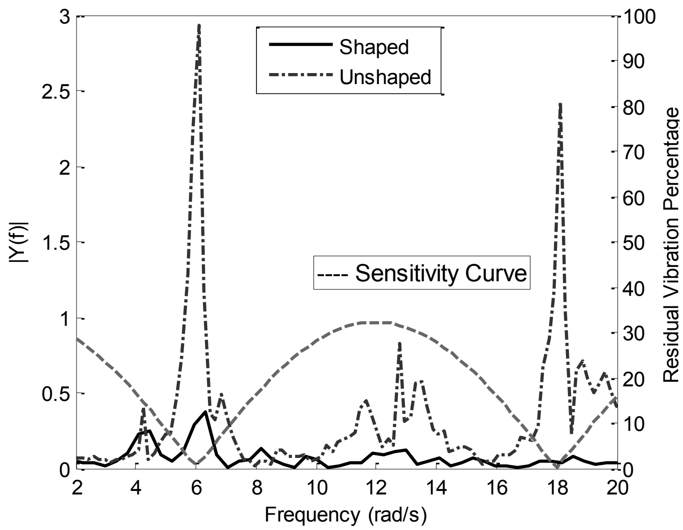

Fig. 5. FFT Plot of Unshaped and ZV Shaped Residual Amplitudes and the ZV Shaper Sensitivity Curve. 3.5cm liquid depth.

TABLE II
LIQUID SLOSHING FREQUENCIES; FIRST AND SECOND MODES

Liquid Depth (cm)	1.5	2	2.5	3	3.5	4	4.5
w_1 **(rad/s)**	4.7	5.4	6.0	6.6	7.0	7.5	7.8
w_2 **(rad/s)**	13.6	15.2	16.3	17.1	17.7	18.1	18.4
Liquid Length — Along travel direction = 25.4 cm							

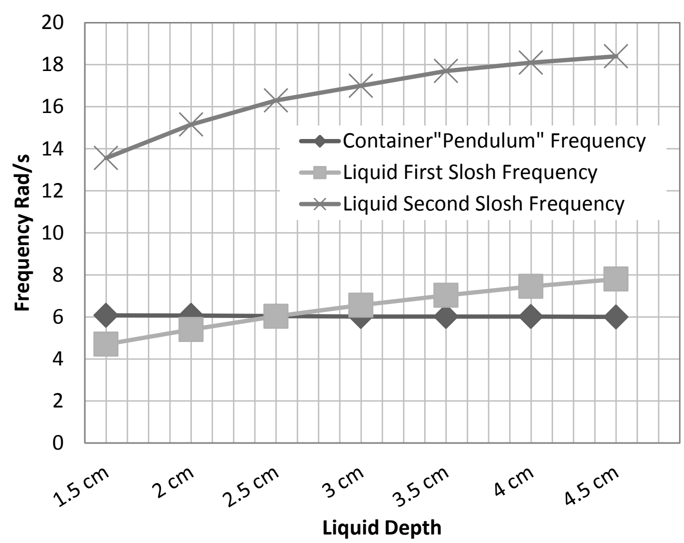

Fig. 6. Model Frequencies vs. Depth

V. FREQUENCY SPECIFIC INPUT SHAPERS TEST

A. Input Shapers

For the experiments discussed in this section, sixteen (16) different input shapers were tested at three liquid depths: 2 cm, 3 cm & 4 cm. The shapers were designed using the calculated liquid slosh frequencies and the experimentally-obtained container pendulum frequencies. During all of the tests the container was moved a distance of 63 cm.

Table III lists the sixteen input shapers. Each input shaper is designed for one or more of the three listed vibration modes. Columns 2-4 list the input shaper types used for the respective vibration modes. As an example, input shaper 5 is a 3-mode input shaper designed by convolving three ZV shapers, one

TABLE III
FREQUENCY SPECIFIC INPUT SHAPERS

Shaper	$w_{container}$	$w_{liquid1^{st}}$	$w_{liquid2^{nd}}$
1	ZV		
2		ZV	
3	ZV	ZV	
4		ZV	ZV
5	ZV	ZV	ZV
6	ZVD		
7		ZVD	
8	ZVD	ZV	
9	UMZV		
10		UMZV	
11	UMZVD		
12		UMZVD	
13	UMZV	ZV	
14	5% EI		
15		5% EI	
16	5% EI	5% EI	

ZV - Zero Vibration [13], [23]
ZVD - Zero Vibration Derivative [13], [14]
UMZV - Unity Magnitude Zero Vibration [13], [14]
UMZVD - Unity Magnitude Zero Vibration Derivative [13]
5% EI - Extra Insensitive - 5% Tolerable Vibration [13], [14], [23]

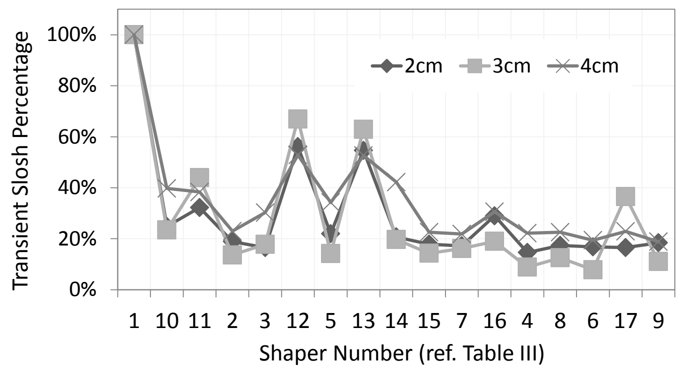

Fig. 7. Peak Transient Slosh as Percentage of Maximum Unshaped Transient Response

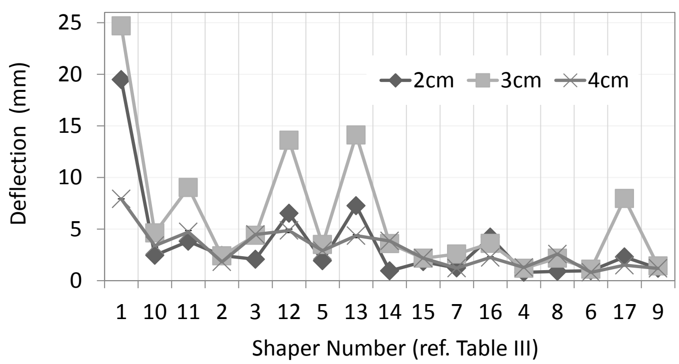

Fig. 8. Peak Residual Slosh

designed for the container pendulum frequency, one for the 1^{st} slosh frequency and one for the 2^{nd} slosh frequency. The ZVD [12] and EI [17] shapers are robust shapers that are effective even when there is significant modelling errors. The UM shapers [16] contain negative impulses and provide faster motion. [10], [13], [14]

B. Results

The transient and residual responses of the liquid slosh are analyzed individually. Lower transient vibration amplitudes can help reduce operational risks when transferring dangerous liquids, while lower residual slosh vibration amplitudes can help reduce the overall transfer durations and make the transfer operations more efficient and accurate. Figure 7 shows the maximum transient liquid slosh as a percentage of the unshaped maximum transient response. Figure 8 shows the maximum residual liquid slosh in mm. The order in which the input shapers are listed in the figures correspond to the shaper durations, with the UMZV shaper for the container pendulum frequency mode being the shortest and the 2-mode shaper; a ZVD shaper for the container pendulum frequency mode convolved with a ZV shaper for the liquid 1^{st} slosh frequency mode, being the longest. Each case was recorded and analyzed using three trials and the average results are presented. The results vary within $\pm 5\%$ of the average for each trial.

Using the 3-mode ZV input shaper the transient response amplitude of the liquid slosh was reduced to under 20% of the unshaped response. The same input shaper resulted in a residual slosh amplitude of less than 2 mm. The highest slosh amplitudes are observed at the 2-3 cm liquid depths. At these depths the 1^{st} slosh frequency coincides with the container pendulum frequency, as was shown in Fig. 6.

Depending on the depth, the shaper that resulted in the least amount of slosh differs; however, the shapers that are designed for the container pendulum frequency mode generally produced the best results due to the fact that the liquid container oscillation is minimized. Therefore, the container motion cannot transfer substantial energy to the sloshing modes. The shaper that produced the best overall slosh reduction was the 3-mode ZV shaper designed to suppress all 3 vibration modes.

The reduction in vibration amplitudes is gained at the relatively small expense of longer rise times. Figure 9 shows the liquid Settling Time for each depth and input shaper. The Settling Time is defined as the time required for the liquid surface deflection to reach and remain below 1.5 mm after the container comes to a complete stop. A zero settling time means that the deflection is already below 1.5 mm when the container has stopped. The figure also shows, on the right-hand vertical axis, the move durations, defined as the time required to travel the distance of 63 cm. The 3-mode ZV shaper increased the move duration by only 1.5 sec and resulted in a 28 sec reduction in Settling Time for the highest vibration amplitude case (3 cm liquid depth). As noted earlier, it can also be

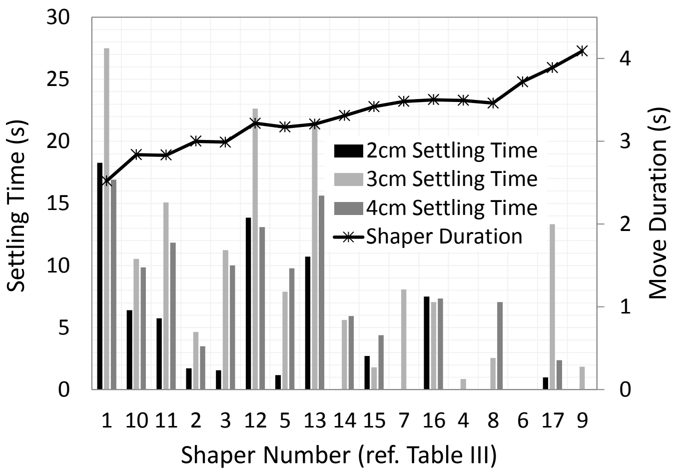

Fig. 9. Settling Time Residual (Time to remain below 1.5 mm deflection)

TABLE IV
ROBUST INPUT SHAPERS

Shaper	$W_{container}$	$W_{liquid1^{st}}$	$W_{liquid2^{nd}}$
1	ZV		
2	ZVD		
3	ZV	ZV	
4	ZV	ZV	ZV
5	ZVDD		
6	5% EI		
7	5% EI 2-hump		

ZV-Zero Vibration [13], [23]
ZVD-Zero Vibration Derivate [13], [14]
ZVDD-Zero Vibration Double Derivate [14]
5% EI-Extra Insensitive with 5% Vibration [13], [14], [23]
5% EI 2-hump-Extra Insensitive 2-hump - 5% Tol Vib [13], [14]
Nominal frequency used for each of the three modes

seen here that the input shapers designed for the container pendulum mode provided the fastest settling times.

The data in Fig. 9 also show that using a multi-mode input shaper that targets the container pendulum mode and the liquid slosh modes is the most effective method in reducing slosh. The ZV shaper that targets the container pendulum mode plus the 1^{st} slosh mode and the ZV shaper that targets all three modes illustrate this good result. The settling time is nearly zero for both shapers.

VI. Robust Input Shapers Test

A. Input Shapers

A second experimental investigation sought to rigorously investigate the robustness to changes in liquid depth. Seven input shapers were tested at seven liquid depths ranging from 1.5 cm to 4.5 cm. The tested shapers are listed in Table IV. The shapers were designed using a nominal container pendulum frequency and nominal liquid slosh frequencies. The nominal frequencies were chosen to be equal to the frequencies at which the liquid exhibits maximum sloshing from the first experimental protocol described in the previous section. In our particular model, these frequencies correspond to a depth that is half the liquid depth range examined.

The purpose of the robustness tests is to observe the liquid slosh levels at different depths in response to a set of fixed input shapers. This represents a practical scenario where a preprogrammed input shaper is used to command an overhead crane motion, while the liquid depth in the container is time varying.

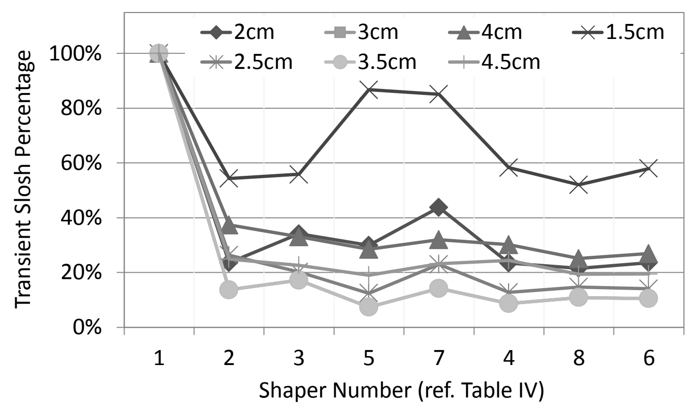

Fig. 10. Peak Transient Slosh as Percentage of Maximum Unshaped Transient Response

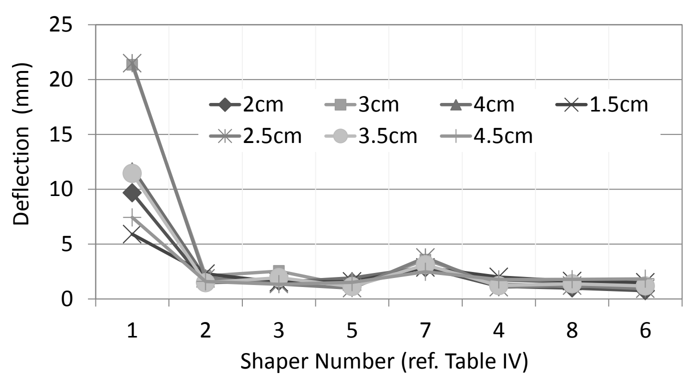

Fig. 11. Peak Residual Slosh

B. Results

Figure 10 shows the maximum transient liquid slosh with the robust input shapers as a percentage of the unshaped maximum transient response. Figure 11 shows the maximum residual liquid slosh in mm. The order of the input shapers in the figures correspond to the shaper durations, with the Zero Vibration shaper being the fastest and the Zero Vibration Double Derivative shaper being the slowest. Each test was preformed three times and the average results are presented.

The seven input shapers resulted in roughly the same reduction in the transient response amplitude. The percentage transient slosh response reduction provided by the input shapers varies from one depth to another because the base

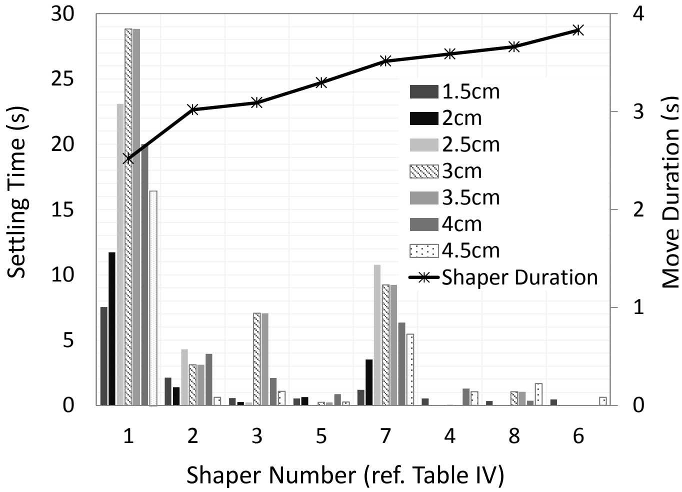

Fig. 12. Settling Time Residual (Time to remain below 1.5 mm deflection)

comparison (the unshaped slosh response) itself varies greatly from one depth to another. The seven input shapers resulted in a residual slosh amplitude of less than 5 mm and the amplitude exhibited at all liquid depths and from each input shaper only varied slightly from one another.

The Settling Time of the liquid slosh was reduced significantly as a result of the input shapers, as shown in Fig. 12. The 3-mode ZV shaper, for example, produced a maximum of 1.6 mm maximum deflection and settling times of less than 1 sec. This is a reduction of 28 sec from the unshaped motion for a 3 cm depth and at a cost of only 0.8 sec added to the total move time.

VII. Conclusions

The fluid-sloshing system investigated here is a compound pendulum that has a natural frequency dependent on the depth of liquid within the container. However, the natural frequency of the pendulum does not change significantly compared to the sloshing frequency changes that occur with changes in liquid depth. The liquid slosh response is highly dependent on the ability to keep the container free from vibration by designing input shapers that target the container pendulum frequency. Once the container oscillation is kept at a minimum, convolving that base input shaper with additional shapers that target the liquid slosh modes can further reduce the total slosh. The increase in travel time as a result of implementing the input shapers is negligible compared to the reduction of slosh and settling time. This adds to the efficiency, as well as to the safety of liquid-handling operations.

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