Passive and Active Transformations

1. The Difference Between Passive and Active Transformations

To show this, consider a scalar field $\phi(x)$ defined on spacetime points x^μ.

Passive viewpoint: The coordinates are transformed,

$$x^\mu \to x'^\mu = x^\mu + \epsilon^\mu,$$

while the physical situation remains unchanged. The field transforms as

$$\phi'(x') = \phi(x).$$

Expressing the transformed field at the new coordinate x', we have

$$\phi'(x') = \phi(x' - \epsilon).$$

Active viewpoint: The coordinates are fixed, but the field configuration is transformed by shifting its argument,

$$\phi(x) \to \phi'(x) = \phi(x - \epsilon).$$

In this case, you stay at the same spacetime point, but the field itself shifts.

Comparison: Both viewpoints give the same mathematical expression for the transformed field at the fixed coordinate x,

$$\phi'(x) = \phi(x - \epsilon).$$

The difference lies only in the interpretation:

- *Passive:* Change of coordinate labels, field values remain fixed.
- *Active:* Fixed coordinates, but the field values are shifted.

Conclusion: The active and passive viewpoints are mathematically equivalent descriptions of the same physical transformation. Observables depending on ϕ evaluated at spacetime points remain consistent between these perspectives.

In the active view, you define a new field configuration and examine what it assigns to the fixed point x. You're not claiming the original field value changed — you're defining a new field and describing how it differs from the original.
In the passive view, you're shifting the coordinate system and ensuring that the field value at a physical point remain the same under the change of labels.

The active transformation is saying: the new field, as seen by the old observer, is a transformed version of the old observer's field; i.e. the coordinates are the same, and the field itself is transformed. The passive viewpoint is saying: transform coordinates. the new observer assigns the same value to the new field that the old observer assigned to the old field; i.e. the field stays the same, and the coordinates are what change. You're shifting the coordinate system and ensuring that the field value at a physical point remains the same under the change of basis.

Notice that the passive and active viewpoints both lead to the same mathematical identity: $\phi'(x) = \phi(x - \epsilon)$.