Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

3C1 SIGNALS AND SYSTEMS LABORATORY

Department of Electronic and Electrical Engineering

(MATLAB e-Report submission)

Aim:

1. To determine the frequency response of a linear time-invariant (LTI) system
2. To examine the steady-state response and the transient response of an LTI system
3. To investigate the relationship between the values of poles and zeros of the transfer function and the response of the system

Submitted by:

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Title:

To understand Signals using Sine Wave and Linear-Time Invariant systems by implementation in MATLAB programming software using various data inputs as provided.

Abstract:

Provisional data as categorized in subdivisions of Part 2: Signals and Part 3: LTI Systems were performed on MATLAB and the results as Figures and Outputs obtained are shown in the report.

Introduction:

The experiment performed on MATLAB to understand a deterministic or stochastic signal using Sine Wave. And again, to understand LTI Systems, their Gain and Phase in the signal and Characterising the Transient Response of a System.

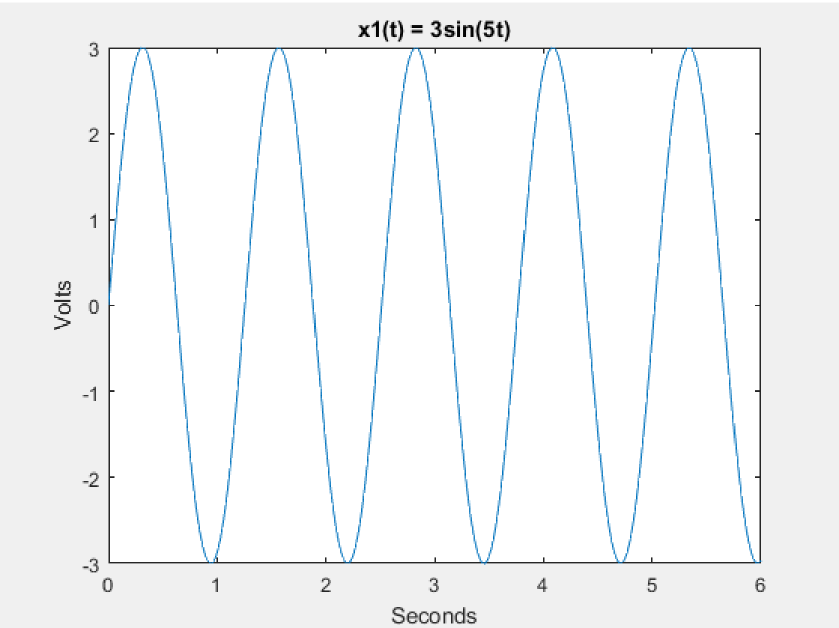
Experiment Performed:

Part 2: Signals

2.1 The Sine wave: a deterministic signal

2.1.1. Using MATLAB, generate the deterministic signal x1(t) = 3sin5t and plot it over the range t = 0: 6 seconds. This is a sine wave. Label the x-axis as Seconds and the y-axis as Volts. Let us assume this signal is the measurement of the voltage from an AC power supply. Thus, the plot you have made is the plot which you would see if you hooked up an oscilloscope to the terminals of the power supply to measure voltage.

Solution:



*(**figure 2.1.1** – func. x1(t) in MATLAB plot)*

2.1.2. What is the maximum and minimum value of the sinusoid (in Volts), the frequency of the sinusoid in Hertz, and the period of the wave in seconds?

Solution:

V_{max} = +3 V,

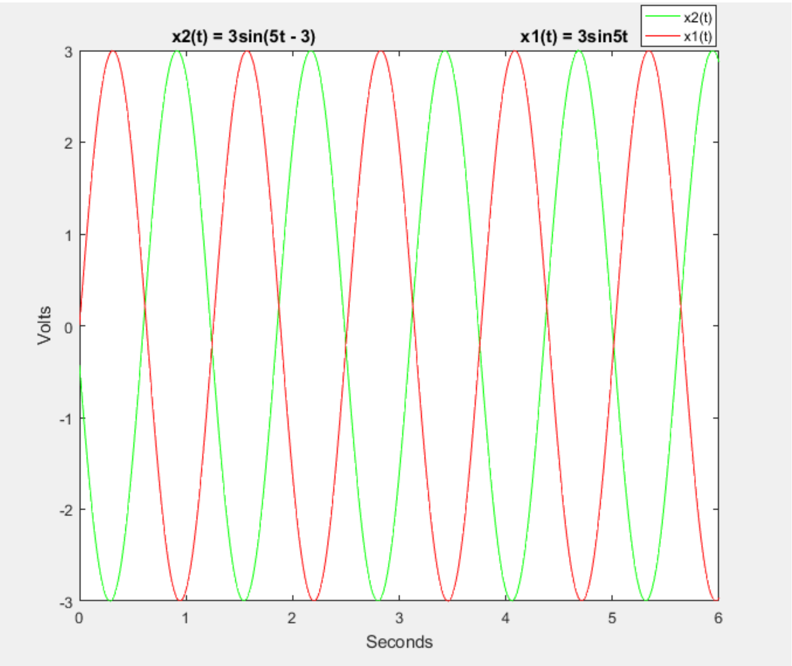
V_{min} = -3 V,

f = (5/2π) Hz or 0.795 Hz,

T = 1.256 s

2.1.3. Use MATLAB to plot the graph of x2(t) = Asin(ω1t + ϕ) with A = 3, ω1 = 5, and ϕ = −3 (green colour), label the axes as previously and show x1 (in red) on the same plot. Two lines should now be plotted on the graph. What is the frequency of x2 in Hertz? What is the difference between the signals x2 and x1? Is there a phase lag? Is this a delay or an advance?

Solution:

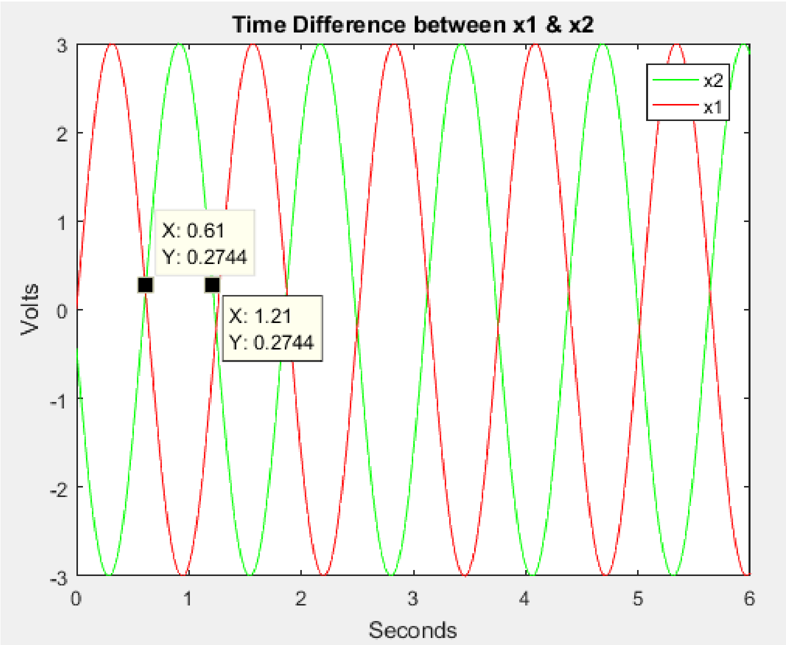


*(**figure 2.1.3** – func. x1(t) and x2(t) in MATLAB plot)*

f	=	(5/2π) Hz or 0.795 Hz,	
x_2 is	**delayed** by **0.6 s**		to x_1,
phase lag	=	3 rad	

2.1.4. By how much is x_2 delayed (in seconds) with respect to x_1? Clearly indicate which points you used to calculate the delay. How does this value relate to the constant offset term, ϕ in the argument for the sin function in x_2? Show exactly how ϕ can be used to calculate the phase lag in seconds. (Hint: ϕ has units of radians.)

Solution:

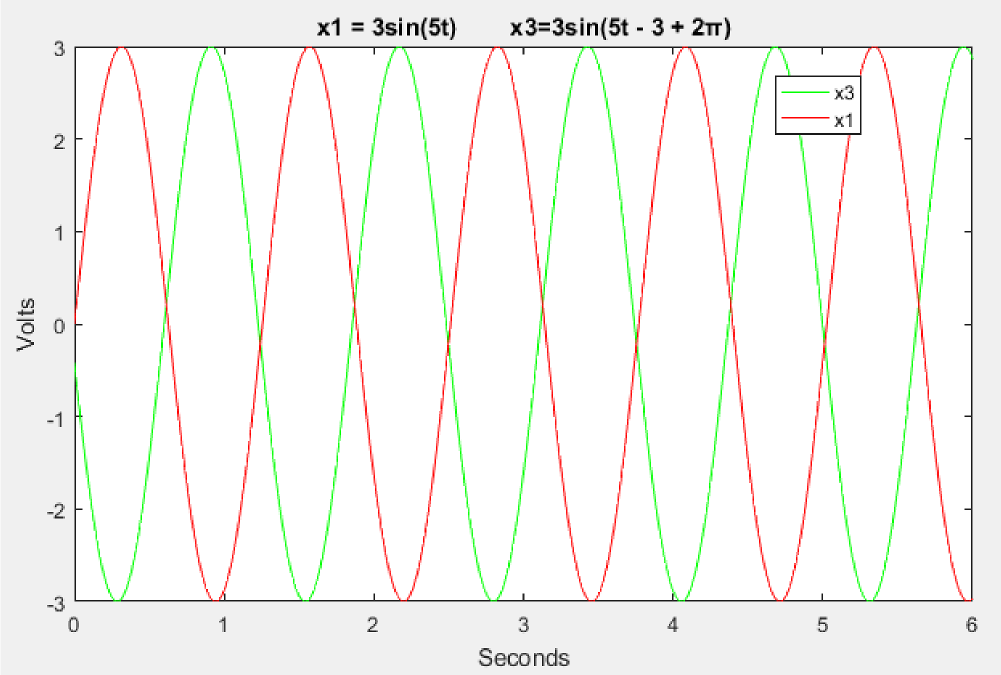


*(**figure 2.1.4** – time diff. between func. x1(t) and x2(t) in MATLAB plot)*

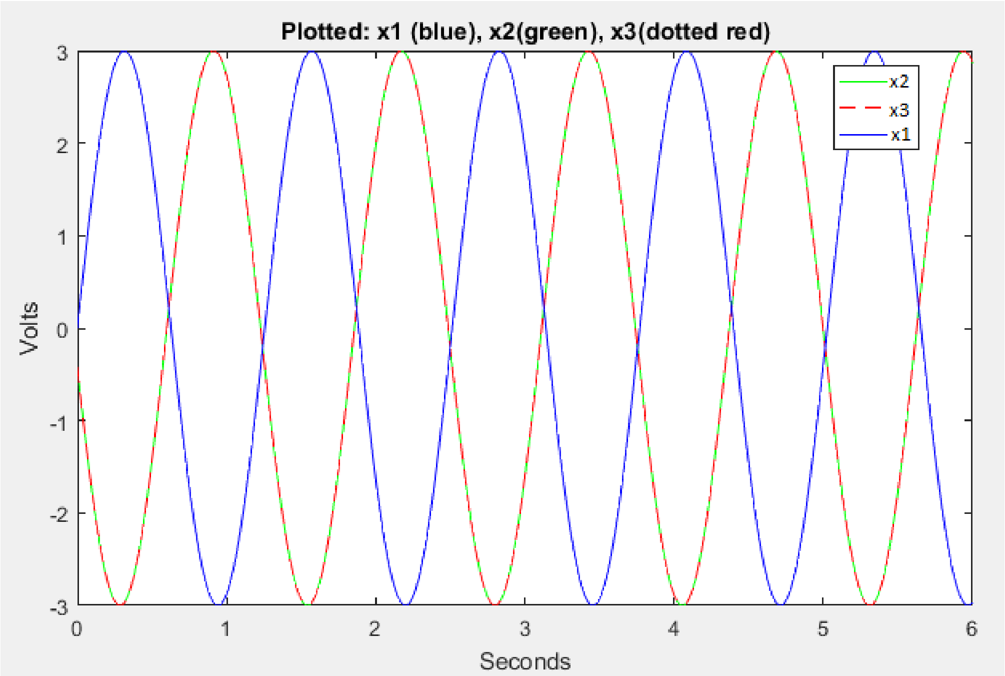
x_2 is **delayed** by **0.6 s** to x_1, calculated by: $t = ((2*\pi-\phi) / \omega_1)$

2.1.5. Use MATLAB to plot the graph of x3(t) = Asin(ω1t + ϕ) with ω1 = 5, ϕ = −3 + 2π, A = 3. Plot the function in Fig. 3 superimposed on the graph for x1 (as in previous instructions). By how much is x3 delayed (in seconds) with respect to x1? Use measurement from the displayed graph only. How does this value relate to the constant offset term in the argument for the sin function in x3? Explain any difference or similarity with x2 in terms of the properties of the sine function.

Solution:



*(**figure 2.1.5.1** –func. x1(t) and x3(t) in MATLAB plot)*



*(**figure 2.1.5.2** –func. x1(t), x2(t) and x3(t) in MATLAB plot)*

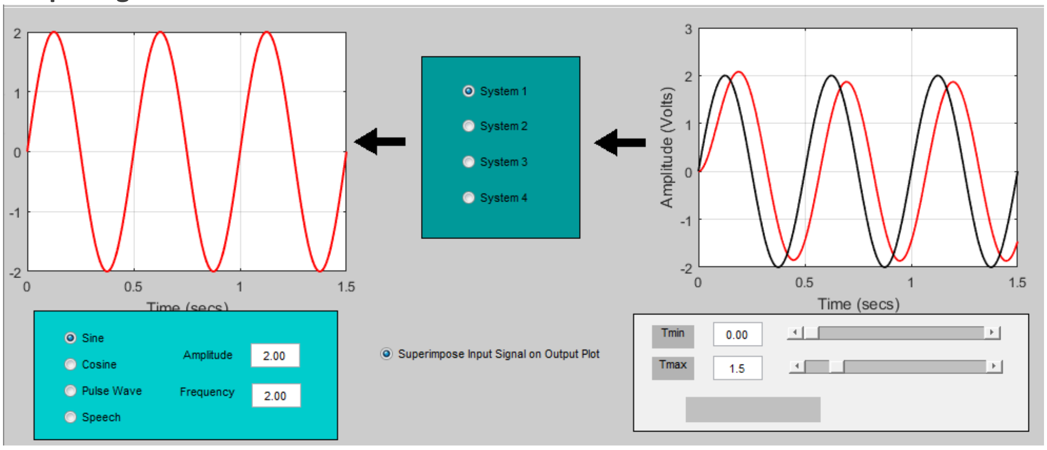
x_3 is **delayed** by **0.6 s** to x_1,
x_3 and x_2 has **phase difference of 2π**, so the resultant graph of x_3 and x_1 is same as x_2 and x_1.

Part 3: Linear Time Invariant Systems

3.1. Examine the input and output signals corresponding to the default settings of LAB1. The input signal is a Sine wave. What is the output signal? (Is it a Sine wave or a Cosine wave or something else?)

Solution:

Output signal: Sine Wave

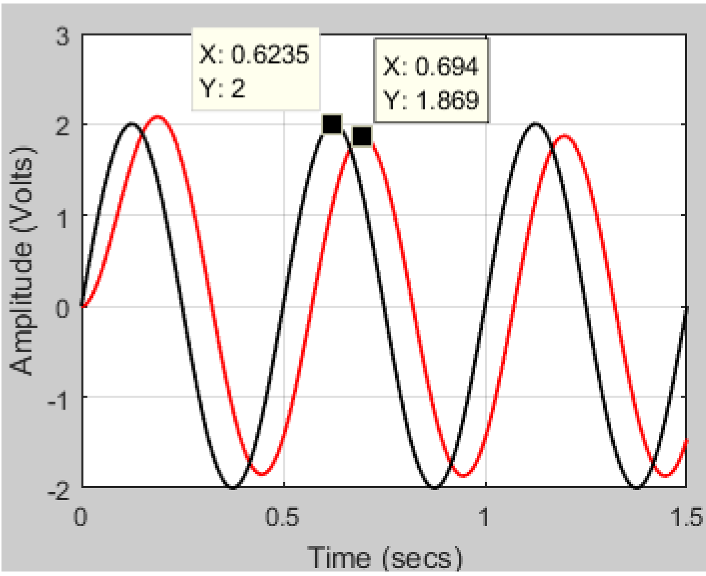


*(**figure 3.1** – output signal of LAB1)*

3.2. What is the difference between the input and output signals using system 1? What is the same? (Superimpose the input on the output to see this more clearly.)

Solution:

The input and output signals have,
Same: Amplitude & Frequency
Difference:

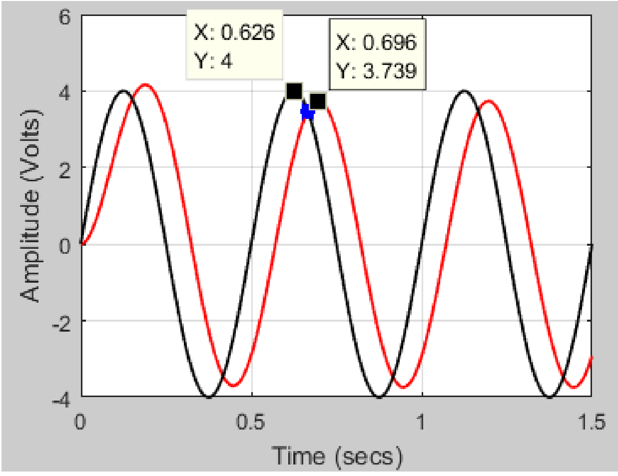


*(**figure 3.2** – super-imposed input signal on output signal of LAB1)*

3.3. Increase the amplitude of the input signal by a factor of 2. What is the corresponding increase in the amplitude of the output signal, after initial transients have decayed? How long does the system take to settle into a steady state response?

Solution:

The amplitude of input signal is doubled so does the output signal doubles. The increase is by factor of 2. The time taken by system to settle into stead state is T = 0.57125 s.



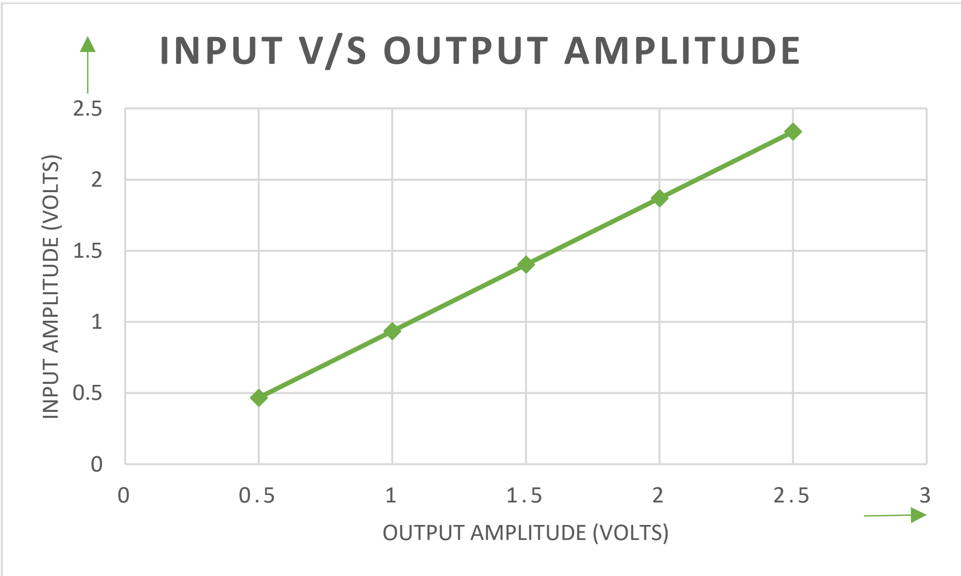
*(**figure 3.3** – super-imposed input signal on output signal of LAB1)*

3.4. For input amplitudes of 0.5, 1.0, 1.5, 2.0, 2.5, measure the corresponding output amplitudes in the output signal and plot a graph of i/p amplitude vs output. Label your axes carefully and include the plot in your write up.

Solution:

Amplitude (Volts)	
Input	Output
0.5	0.4674
1	0.9347
1.5	1.402
2	1.8693
2.5	2.3366

*(**table 3.4** – amplitude table for input v/s output)*



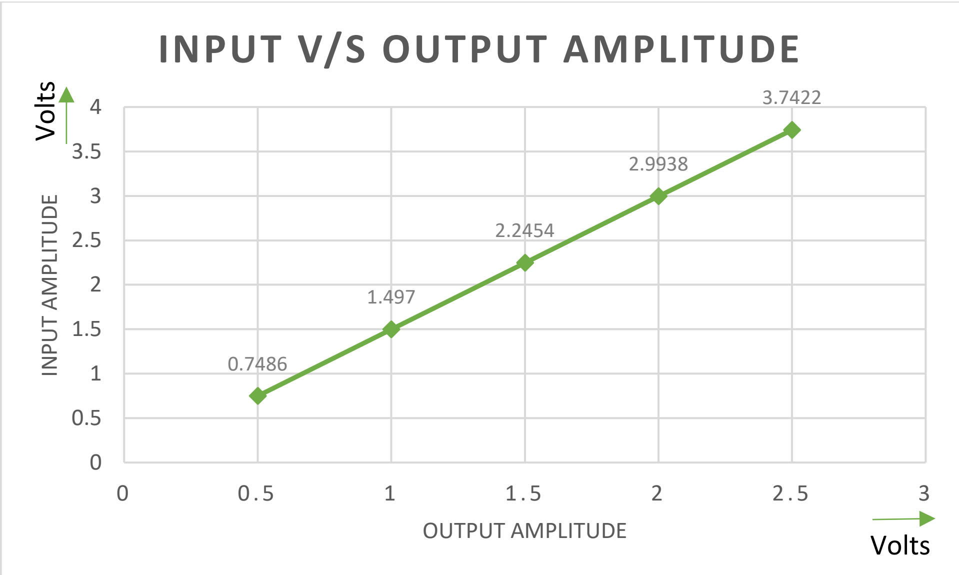
*(**figure 3.4** – graph for input v/s output amplitude)*

3.5. Switch the input signal to the Pulse Wave and set its frequency to 0.8 Hz. and repeat the above experiment. Take the output amplitude to be the maximum value of the output pulse after initial transients have decayed.

Solution:

Amplitude (Volts)	
Input	Output
0.5	0.7486
1	1.497
1.5	2.2454
2	2.9938
2.5	3.7422

*(**table 3.5** – amplitude table for input v/s output)*

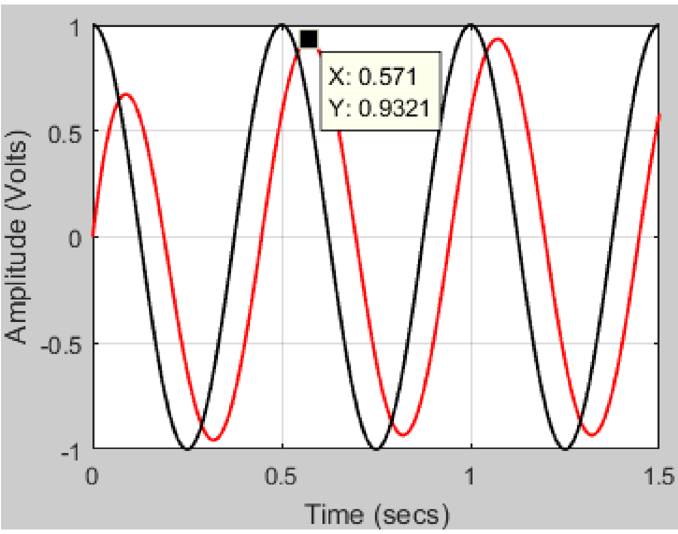


*(**figure 3.5** – graph for input v/s output amplitude)*

3.6. Select the Cosine as the input signal and set the frequency to 2Hz. What is the difference between the input and output signals using system 1? What is the same?

Solution:

The output signal formed is Sine Wave as shown in the figure.
Change in output signal amplitude and phase compared to input signal.
Time period is similar in both input and output as superimposed on output signal.



*(**figure 3.6** – graph for input cosine & output sine wave)*

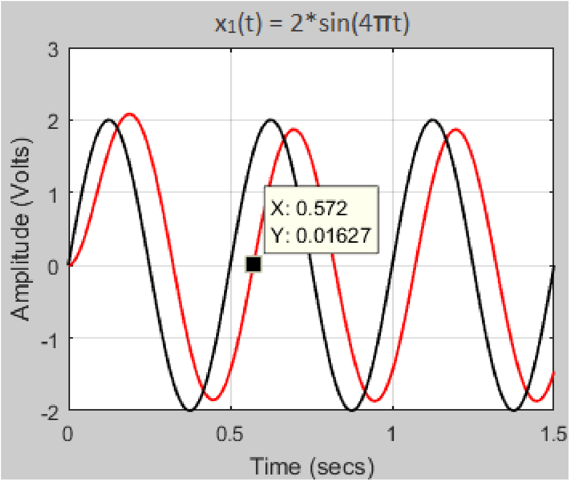
3.7. For both the Sine and Cosine input signals (at Frequency 2Hz and amplitude 2) with system 1, write mathematical expressions for the input and output signals using LAB1 to help you make measurements.

Solution:

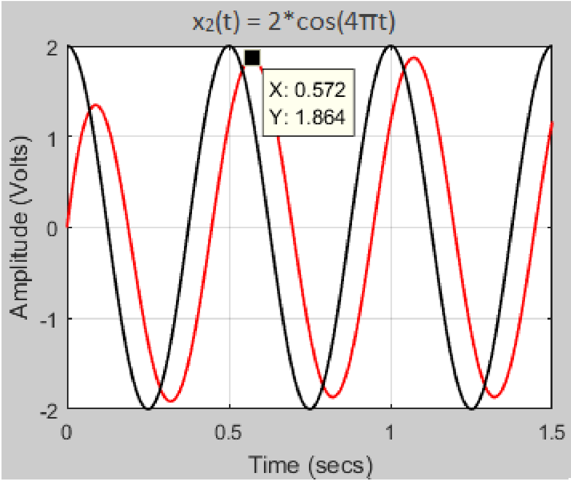
Mathematical expressions used in input signals are as follows:

$x_1(t) = 2*\sin(4\pi t)$,

$x_2(t) = 2*\cos(4\pi t)$



*(**figure 3.7.1** – input sine wave)*



*(**figure 3.7.2** – input cosine wave)*

3.8. Using what you now know about LTI systems, classify Systems 2,3,4 as LTI or non–LTI. Remember to check input amplitudes covering a wide range (e.g. 1 ↔ 6).

Solution:

LTI Systems – System 1 & System 2.

Non-LTI Systems – System 3 & System 4.

Part 3.1: Gain and Phase as a function of frequency

3.1.1. Complete the table below for the Sine input at various frequencies using system 1.

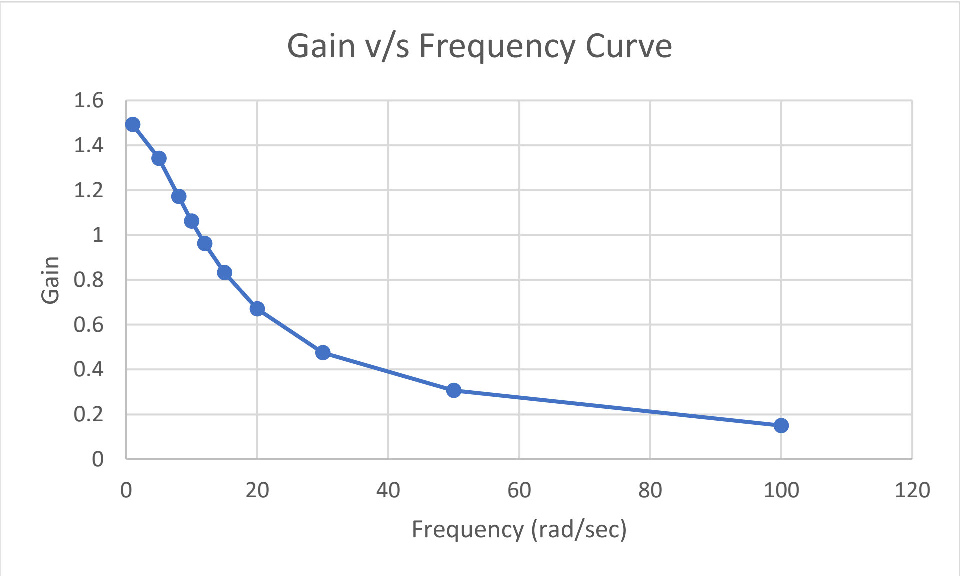
Solution:

INPUT			OUTPUT				
Frequency (rad/sec)	Frequency (Hz)	Amplitude (*x*)	Frequency (rad/sec)	Amplitude (*y*)	Phase Lag (sec)	Phase Lag 10^{-7}(rad)	Gain (*y/x*)
1	0.159	1	1	1.492	0	0	1.492
5	0.795	1	5	1.341	0.087	4.217	1.341
8	1.272	1	8	1.172	0.081	3.926	1.172
10	1.59	1	10	1.061	0.074	3.587	1.061
12	1.908	1	12	0.961	0.072	3.49	0.961
15	2.385	1	15	0.832	0.065	3.151	0.832
20	3.18	1	20	0.671	0.057	2.763	0.671
30	4.77	1	30	0.475	0.041	1.987	0.475
50	7.95	1	50	0.307	0.027	1.308	0.307
100	15.9	1	100	0.15	0.014	0.678	0.15

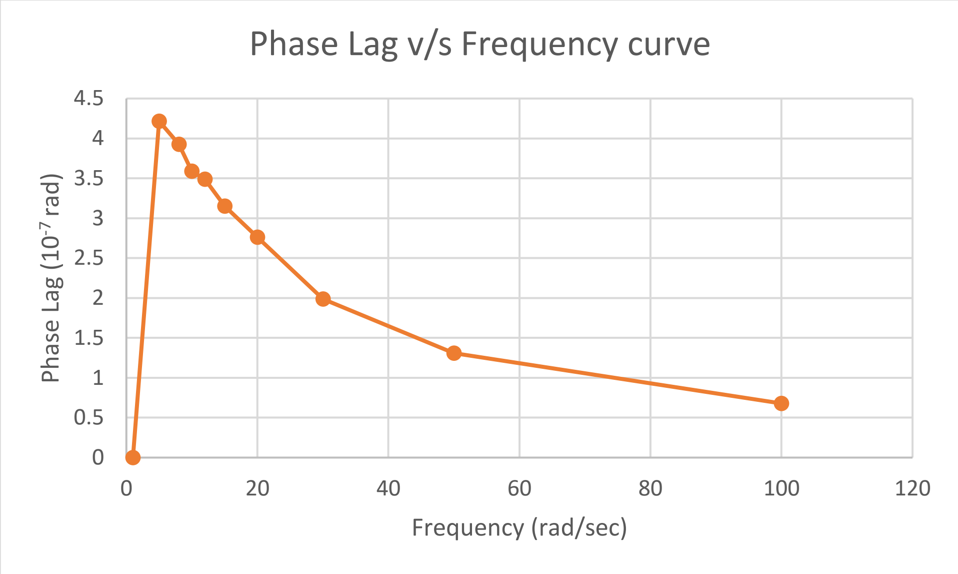
*(**table 3.1.1** – Completed with respect to Lab1 program)*

3.1.2. Plot a graph of Gain vs Frequency (rad/sec) and Phase (rad) vs Frequency (rad/sec) for system 1 using the information above. Label axes carefully.

Solution:



*(**figure 3.1.2.1** – Gain v/s Frequency curve)*



(figure 3.1.2.2 – Phase Lag v/s Frequency curve*)*

3.1.3. Is the effect of the system the same at all frequencies? How does the system behaviour change with frequency?

Solution:

The effect of the system follows same pattern as the **frequency increases** the **amplitude of the output signal decreases**.

3.1.4. Discuss the significance of this plot with respect to the effect of system 1 on the Pulse Wave and the Speech signal.

Solution:

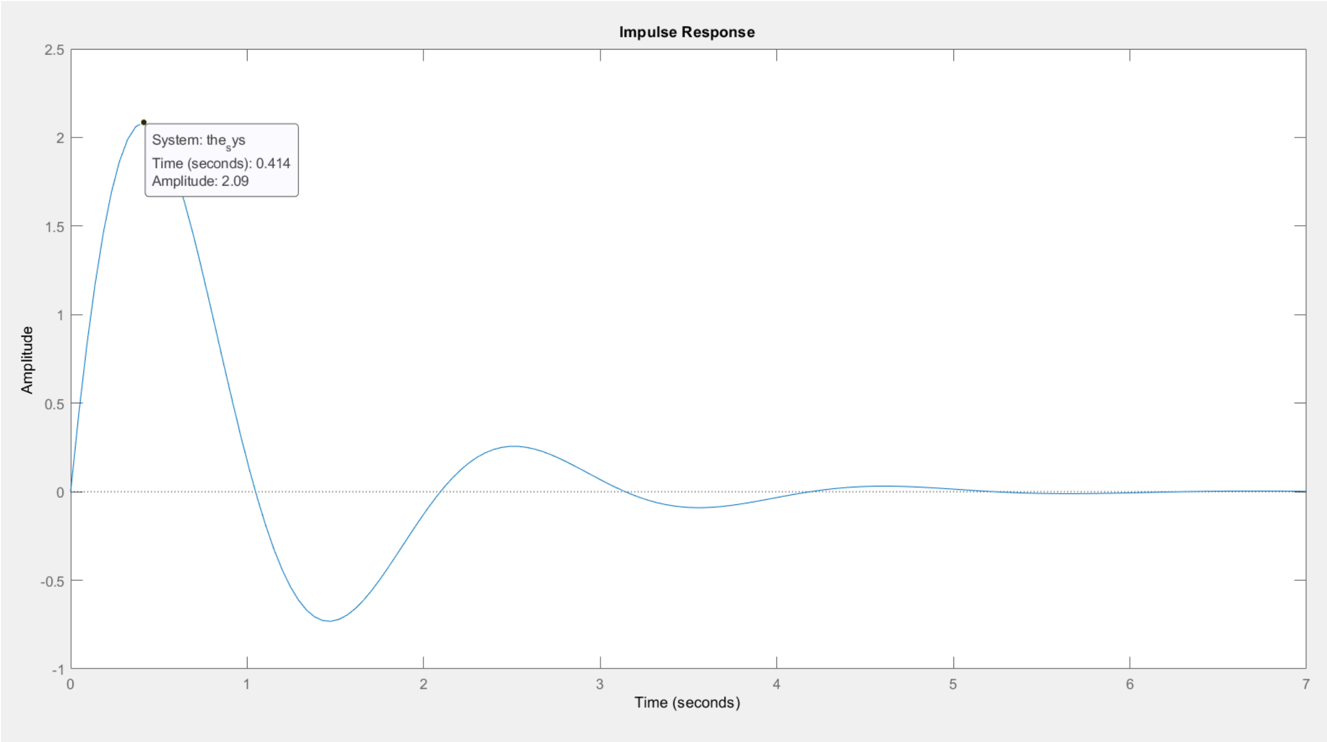
The data follow is with respect to effect of system 1:
Pulse Wave amplitude increases by 0.438 in output signal compared to the input signal.
Speech Signal remains the same.

Part 3.2: Characterising the Transient Responses of a System

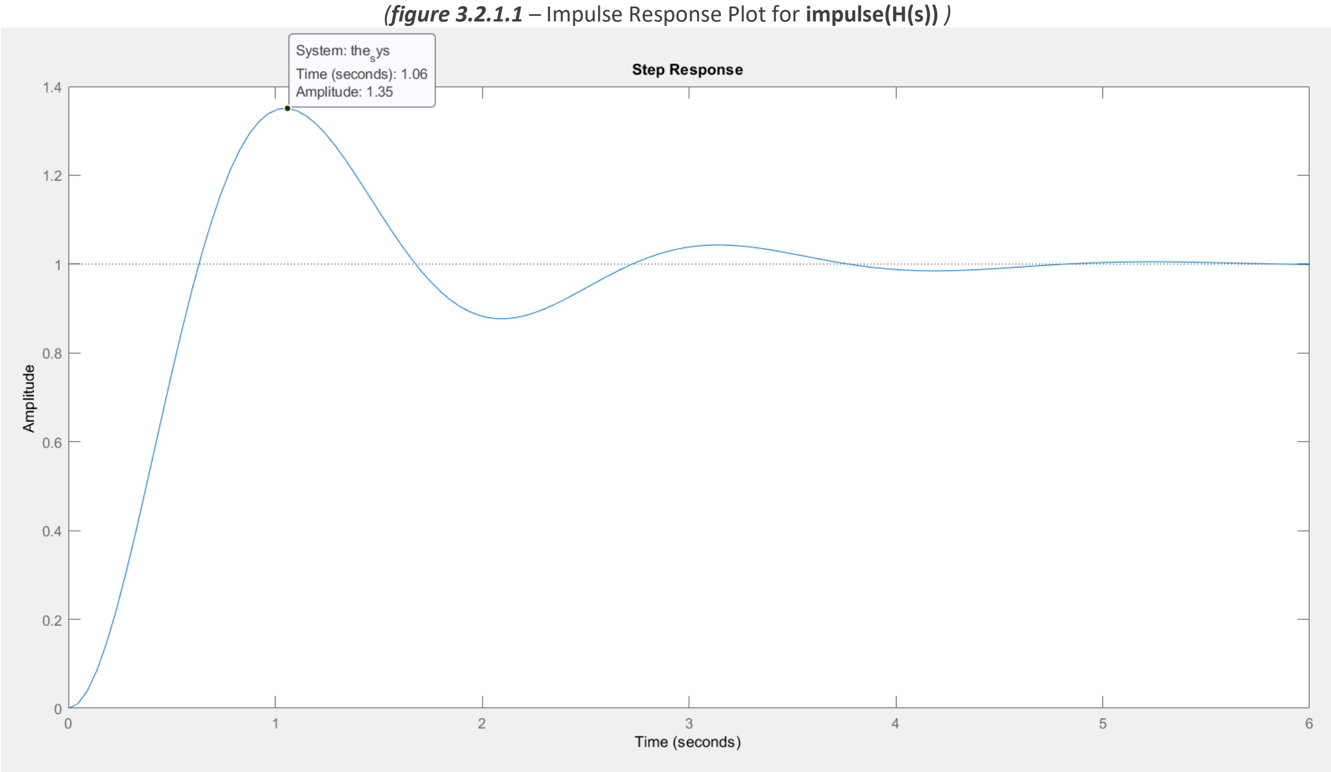
3.2.1. Is the effect of the system the same at all frequencies? How does the system behaviour change with frequency?

Solution:

$$H(s) = \frac{10}{s^2+2s+10}$$



*(**figure 3.2.1.1*** – Impulse Response Plot for **impulse(H(s))** *)*



(***figure 3.2.1.2*** – Step Response Plot for **step(H(s))**)

3.2.2. Using the plots generated previously, record values for all the parameters listed.

Solution:

$$H(s) = \frac{10}{s^2+2s+10}$$

Impulse Response

- peak time = 0.414 s
- peak value = 2.09
- settling time = 3.95 s

Step Response

- steady state value = 1
- rise time = 0.426
- % overshoot = 35.1 %
- settling time = 3.45 s

3.2.3. Determine the poles and zeros of the transfer function H(s)

Solution:

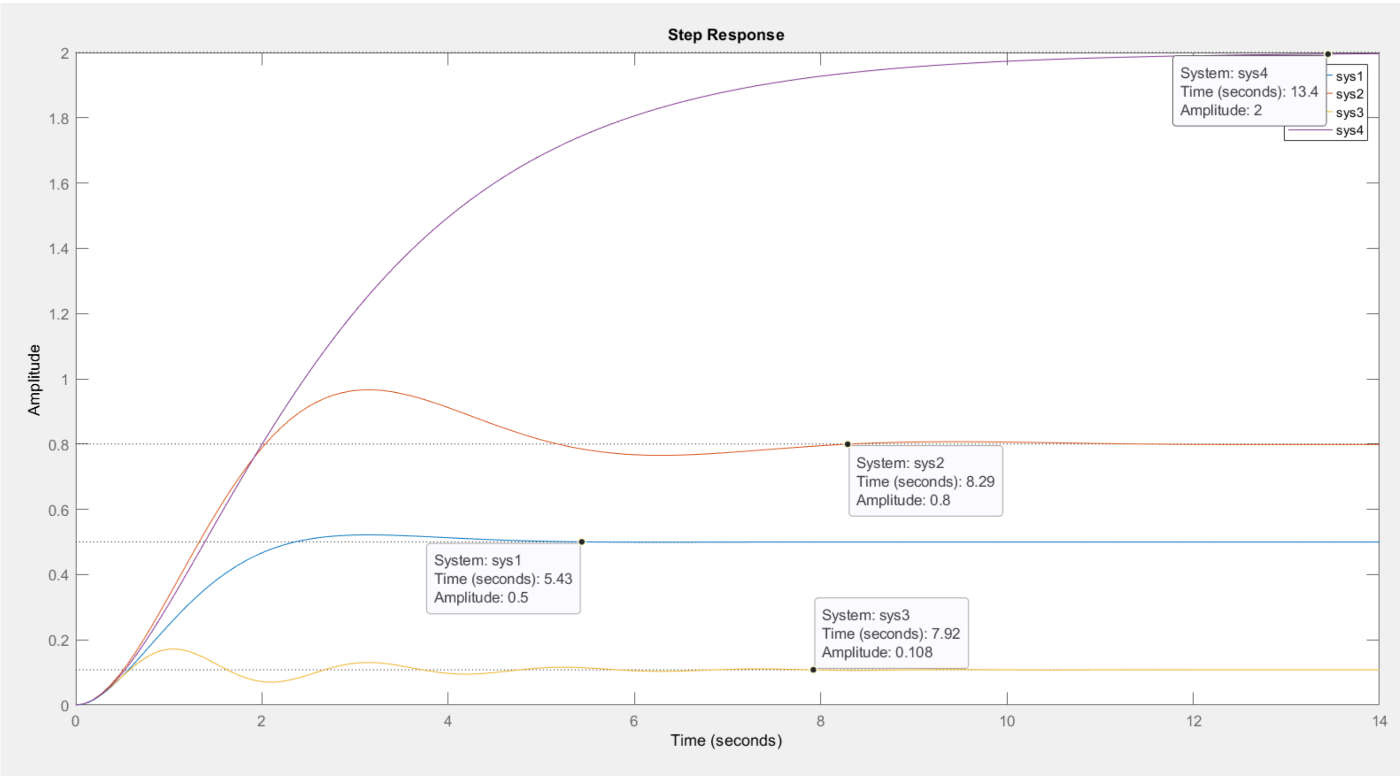
Poles: $-1+3i$
$-1-3i$

Zeros: *None*

3.2.4. Create four 2nd order systems, each having a gain of 1, no zeros in the transfer function, and with the poles specified below:

- System 1 - poles at s = −1 + j, s = −1−j
- System 2 - poles at s = −0.5−j, s = −0.5 + j
- System 3 - poles at s = −0.5 + 3j, s = −0.5−3j
- System 4 - poles at s = −1, s = −0.5

Solution:



(***figure 3.2.4*** – Step Response Plot for **all 4 systems** with given poles)

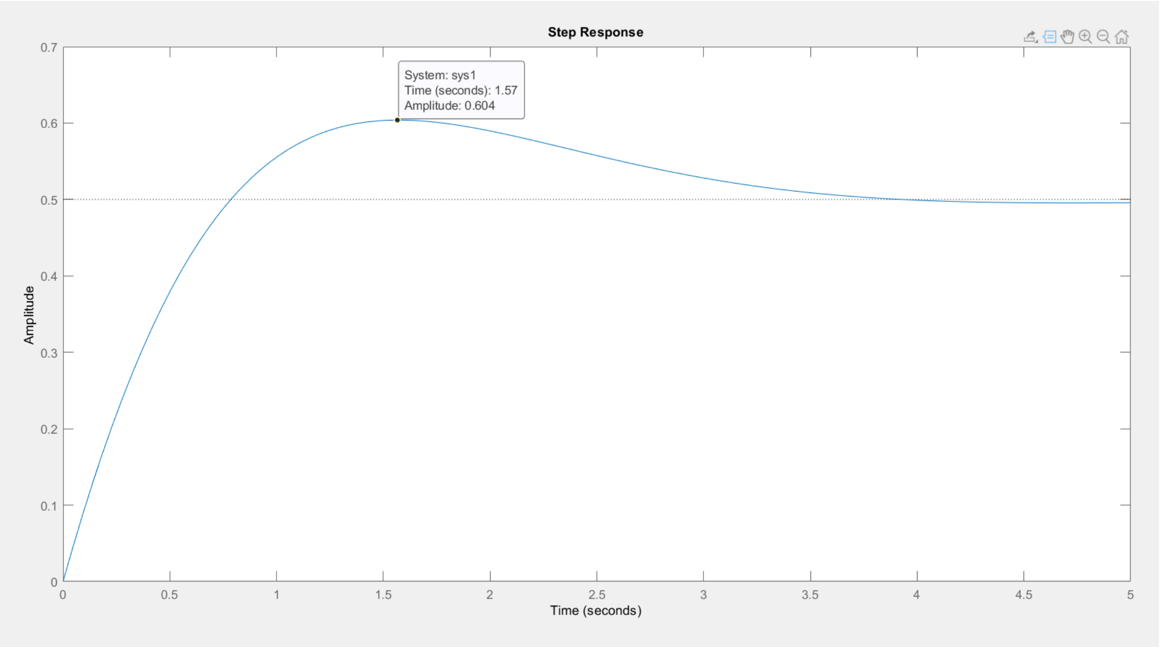
- **System 1**: slight oscillation and settling time = 5.4 s
- **System 2**: single oscillation and settling time = 8.3 s

- **System 3**: multi oscillations (3-4) and settling time = 9.8 s
- **System 4**: no oscillations and settling time = 13.4 s

3.2.5. Using System 1 above investigate what happens to the step response of a system when a zero is added at s = −1.

Solution:

System 1 - poles at s = −1 + j, s = −1−j, with zero at s = -1.



*(**figure 3.2.4** – Step Response Plot for **modified System 1** with given poles)*

Sudden **increase in peak value to 0.604 V** before introduction of zero it was around 0.52 V.
Settling **time reduced to 3.92 s** previously it was 5.2 s