Qspice Project - Fourier Analysis and Fourier Synthesis

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7-15-2025

Fourier Analysis and Fourier Synthesis

- Fourier Analysis
  - Breaking down a signal into its frequency components (calculating a_0, a_1, b_1, etc.)
- Fourier Synthesis (or Inverse Fourier Series)
  - Rebuilding the signal by summing its sinusoidal components using the coefficients
- Project Description
  - This project builds a DLL-block (.cpp) to compute Fourier series coefficients from input signal, and synthesis an output signal with these coefficients in every signal period

Calculate the Fourier series coefficients and Fourier Synthesis

- This mathematics need to know exactly the signal period (fundamental frequency), assume signal is function of $f(t)$
- Calculate a_0
  - a_0 represents the average value of the function over one period
  - $a_0 = \frac{1}{T}\int_0^T f(t)\,dt$
- Calculate a_n and b_n
  - To calculate the Fourier series coefficients a_n (cosine terms) and b_n (sine terms)
  - $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt$
  - $b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$
  - Here, n is the harmonic order, $\omega = 2\pi f = \frac{2\pi}{T}$ is the period of the signal
- Fourier Synthesis
  - $f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$

Code Explanation : Only Calculate DC and Fundamental

Qspice : \Fourier Series Coefficients in Real Time\01 DC and Fundamental\fourier.cpp

Integration (for example)

$$\int_0^T f(t)\,dt$$

$$\int_0^T f(t)\cos(\omega t)\,dt\ ,\ \int_0^T f(t)\sin(\omega t)\,dt$$

Reached a period (calculate coefficient) →

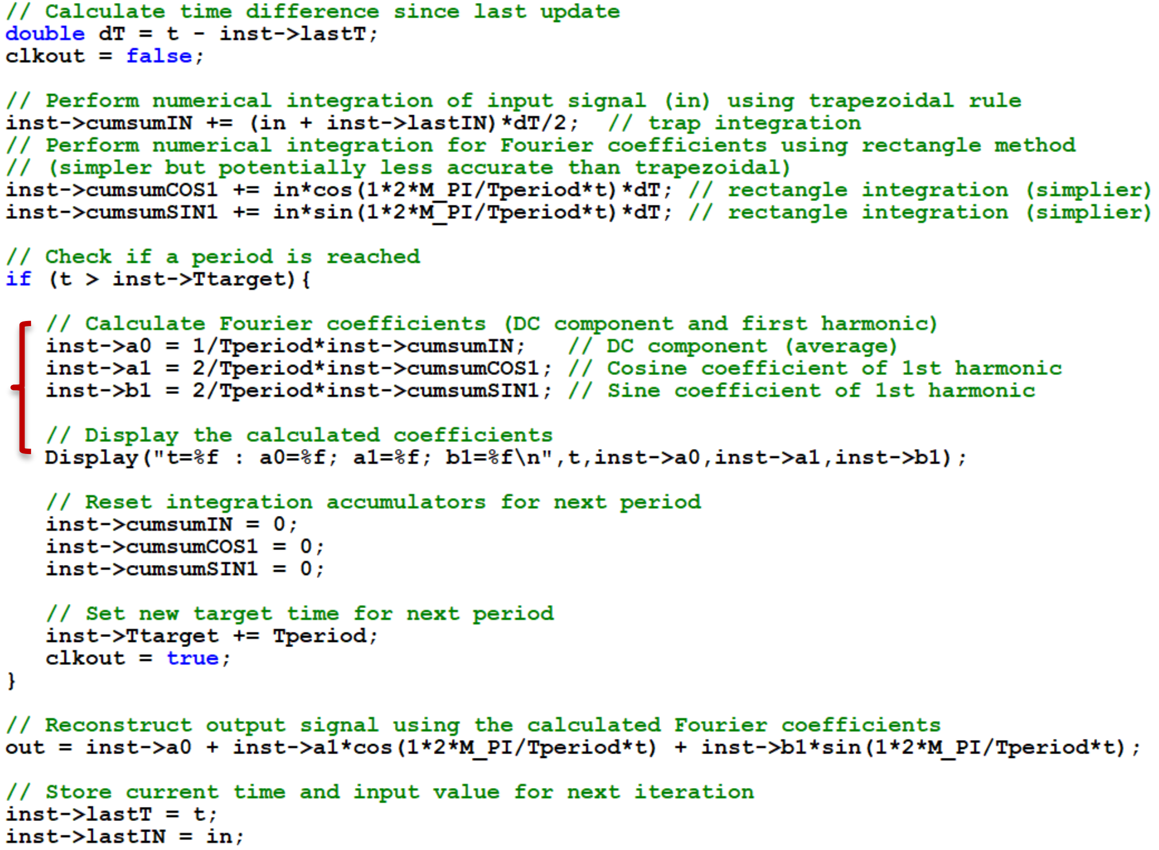
Calculate coefficient

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt\ ,\ a_1 = \frac{2}{T}\int_0^T f(t)\cos(\omega t)\,dt$$

$$b_1 = \frac{2}{T}\int_0^T f(t)\sin(\omega t)\,dt$$

Fourier Synthesis →

$$f(t) = a_0 + a_1\cos(\omega t) + b_1\sin(\omega t)$$



```cpp
// Calculate time difference since last update
double dT = t - inst->lastT;
clkout = false;

// Perform numerical integration of input signal (in) using trapezoidal rule
inst->cumsumIN += (in + inst->lastIN)*dT/2;  // trap integration
// Perform numerical integration for Fourier coefficients using rectangle method
// (simpler but potentially less accurate than trapezoidal)
inst->cumsumCOS1 += in*cos(1*2*M_PI/Tperiod*t)*dT; // rectangle integration (simplier)
inst->cumsumSIN1 += in*sin(1*2*M_PI/Tperiod*t)*dT; // rectangle integration (simplier)

// Check if a period is reached
if (t > inst->Ttarget){

   // Calculate Fourier coefficients (DC component and first harmonic)
   inst->a0 = 1/Tperiod*inst->cumsumIN;   // DC component (average)
   inst->a1 = 2/Tperiod*inst->cumsumCOS1; // Cosine coefficient of 1st harmonic
   inst->b1 = 2/Tperiod*inst->cumsumSIN1; // Sine coefficient of 1st harmonic

   // Display the calculated coefficients
   Display("t=%f : a0=%f; a1=%f; b1=%f\n",t,inst->a0,inst->a1,inst->b1);

   // Reset integration accumulators for next period
   inst->cumsumIN = 0;
   inst->cumsumCOS1 = 0;
   inst->cumsumSIN1 = 0;

   // Set new target time for next period
   inst->Ttarget += Tperiod;
   clkout = true;
}

// Reconstruct output signal using the calculated Fourier coefficients
out = inst->a0 + inst->a1*cos(1*2*M_PI/Tperiod*t) + inst->b1*sin(1*2*M_PI/Tperiod*t);

// Store current time and input value for next iteration
inst->lastT = t;
inst->lastIN = in;
```

Code Explanation : General Form

Qspice : \Fourier Series Coefficients in Real Time\02 General Form\fourier.cpp

Calculate coefficient

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

Calculate coefficient

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt\ ,\ b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

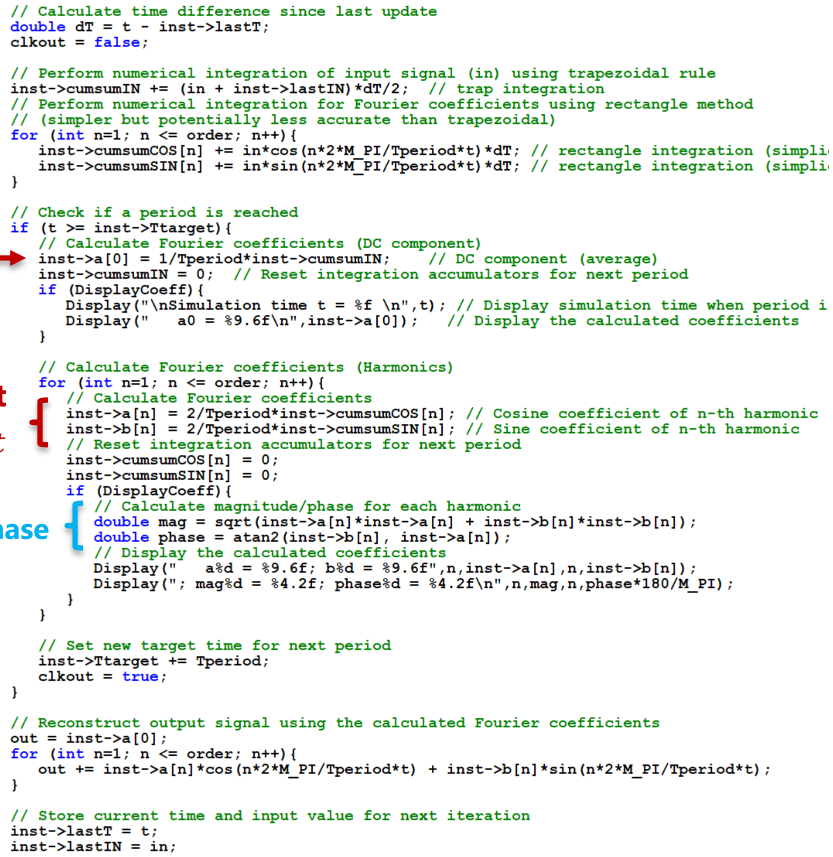
Calculate coefficient in magnitude and phase

$$MAG_n = \sqrt{a_n^2 + b_n^2}\,, PHASE_n = \tan^{-1}\frac{b_n}{a_n}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} MAG_n\ \cos(n\omega t + PHASE_n)$$

Fourier Synthesis

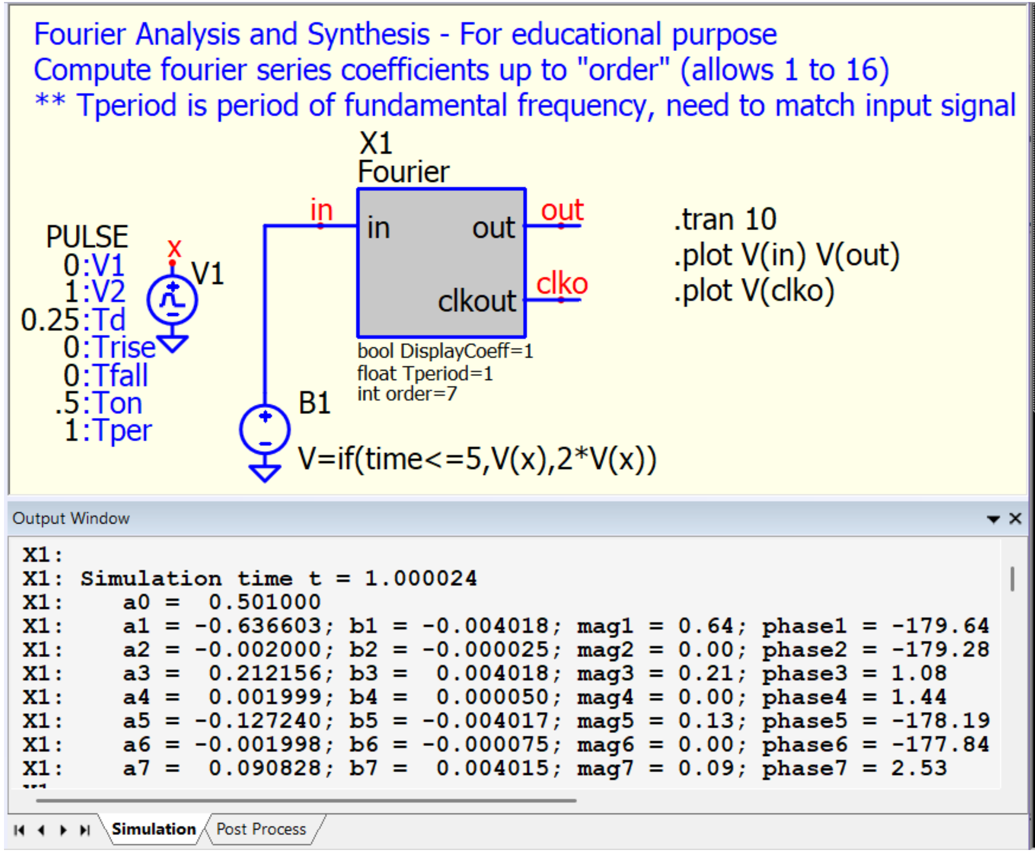
$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$$



```
// Calculate time difference since last update
double dT = t - inst->lastT;
clkout = false;

// Perform numerical integration of input signal (in) using trapezoidal rule
inst->cumsumIN += (in + inst->lastIN)*dT/2;  // trap integration
// Perform numerical integration for Fourier coefficients using rectangle method
// (simpler but potentially less accurate than trapezoidal)
for (int n=1; n <= order; n++){
   inst->cumsumCOS[n] += in*cos(n*2*M_PI/Tperiod*t)*dT; // rectangle integration (simpli
   inst->cumsumSIN[n] += in*sin(n*2*M_PI/Tperiod*t)*dT; // rectangle integration (simpli
}

// Check if a period is reached
if (t >= inst->Ttarget){
   // Calculate Fourier coefficients (DC component)
   inst->a[0] = 1/Tperiod*inst->cumsumIN;     // DC component (average)
   inst->cumsumIN = 0;  // Reset integration accumulators for next period
   if (DisplayCoeff){
      Display("\nSimulation time t = %f \n",t); // Display simulation time when period i
      Display("   a0 = %9.6f\n",inst->a[0]);   // Display the calculated coefficients
   }

   // Calculate Fourier coefficients (Harmonics)
   for (int n=1; n <= order; n++){
      // Calculate Fourier coefficients
      inst->a[n] = 2/Tperiod*inst->cumsumCOS[n]; // Cosine coefficient of n-th harmonic
      inst->b[n] = 2/Tperiod*inst->cumsumSIN[n]; // Sine coefficient of n-th harmonic
      // Reset integration accumulators for next period
      inst->cumsumCOS[n] = 0;
      inst->cumsumSIN[n] = 0;
      if (DisplayCoeff){
         // Calculate magnitude/phase for each harmonic
         double mag = sqrt(inst->a[n]*inst->a[n] + inst->b[n]*inst->b[n]);
         double phase = atan2(inst->b[n], inst->a[n]);
         // Display the calculated coefficients
         Display("    a%d = %9.6f; b%d = %9.6f",n,inst->a[n],n,inst->b[n]);
         Display("; mag%d = %4.2f; phase%d = %4.2f\n",n,mag,n,phase*180/M_PI);
      }
   }

   // Set new target time for next period
   inst->Ttarget += Tperiod;
   clkout = true;
}

// Reconstruct output signal using the calculated Fourier coefficients
out = inst->a[0];
for (int n=1; n <= order; n++){
   out += inst->a[n]*cos(n*2*M_PI/Tperiod*t) + inst->b[n]*sin(n*2*M_PI/Tperiod*t);
}

// Store current time and input value for next iteration
inst->lastT = t;
inst->lastIN = in;
```

Example of Fourier Analysis and Synthesis

Qspice : \Fourier Series Coefficients in Real Time\02 General Form\fourier.cpp



Fourier Analysis and Synthesis - For educational purpose
Compute fourier series coefficients up to "order" (allows 1 to 16)
** Tperiod is period of fundamental frequency, need to match input signal

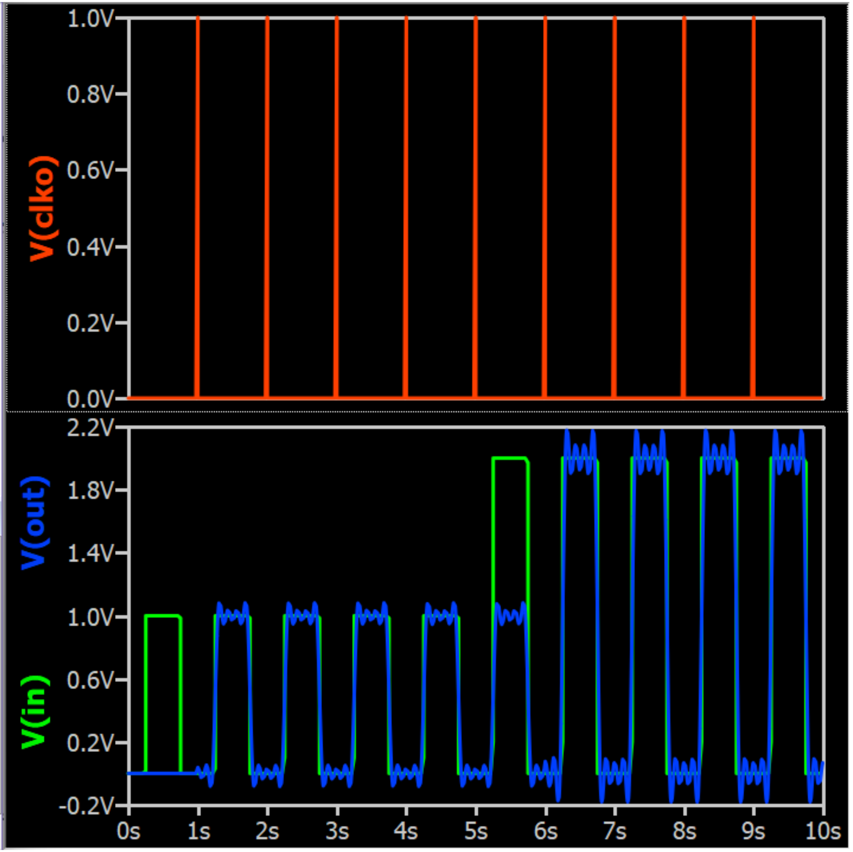
PULSE
0:V1
1:V2
0.25:Td
0:Trise
0:Tfall
.5:Ton
1:Tper

bool DisplayCoeff=1
float Tperiod=1
int order=7

V=if(time<=5,V(x),2*V(x))

```
.tran 10
.plot V(in) V(out)
.plot V(clko)
```

```
X1:
X1:  Simulation time t = 1.000024
X1:     a0 =  0.501000
X1:     a1 = -0.636603; b1 = -0.004018; mag1 = 0.64; phase1 = -179.64
X1:     a2 = -0.002000; b2 = -0.000025; mag2 = 0.00; phase2 = -179.28
X1:     a3 =  0.212156; b3 =  0.004018; mag3 = 0.21; phase3 = 1.08
X1:     a4 =  0.001999; b4 =  0.000050; mag4 = 0.00; phase4 = 1.44
X1:     a5 = -0.127240; b5 = -0.004017; mag5 = 0.13; phase5 = -178.19
X1:     a6 = -0.001998; b6 = -0.000075; mag6 = 0.00; phase6 = -177.84
X1:     a7 =  0.090828; b7 =  0.004015; mag7 = 0.09; phase7 = 2.53
```



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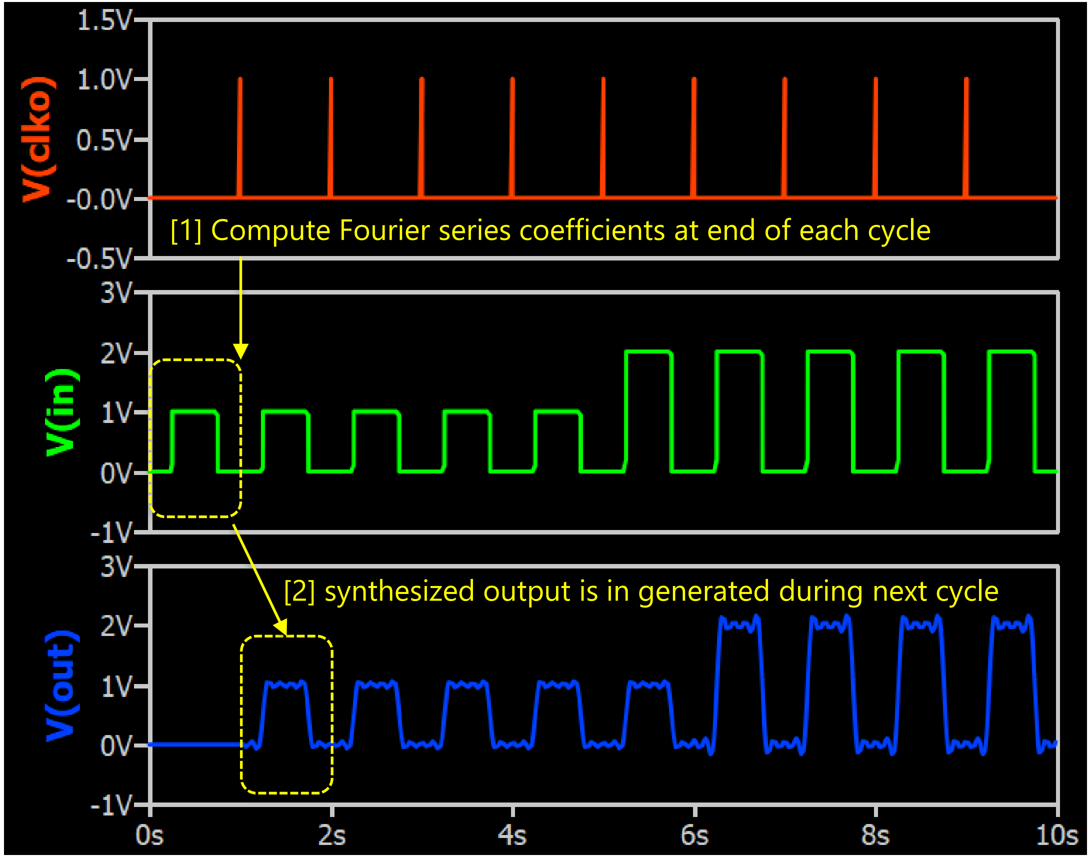

[1] Compute Fourier series coefficients at end of each cycle

[2] synthesized output is in generated during next cycle

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

a_n and b_n can only be computed after obtaining the full cycle integration results Therefore, the synthesized output is delayed by at least one full cycle.