CS 580, Fall 2022; Instructor: Simina Brânzei.
TAs: Shoaib Khan, Anuj Singh, Lu Yan, Zheng Zhong.
Due: November 14, 11:59PM EST. Submit to Gradescope

Problem Set 4

Collaboration policy : Acknowledge your collaborators on the homework. You may discuss proof strategies, but the solution must be written individually in your own words.

Submission format : The solutions must be typed in Latex and submitted via Gradescope.

Problem 1 (10 points). *Consider the language $A = \{ba^kb \mid k \in \mathbb{N}, k \geq 0\}$ over the alphabet $\Sigma = \{a, b\}$. Design a Turing machine to decide A; explain how your machine works (a) in words, using the style from the book and class (see slides with TM examples) and (b) draw the state diagram for it. State and justify the runtime of your Turing machine. [Reading: Sipser chapter 3. See also example from class slides.]*

Problem 2 (15 points). *(a) Show that the collection of Turing-recognizable languages is closed under the operation of concatenation. Reading: Sipser chapter 3.*

(b) Show that the class NP is closed under union. Reading: Sipser chapter 7.

Problem 3 (20 points). *For this problem you are given that CircuitSAT, 3SAT, and SAT are NP-complete. Reading: Sipser chapter 7.*

(a) Let $D-SAT = \{\phi \mid \phi$ is a 3CNF formula with at least two satisfying assignments$\}$. Show that $D - SAT$ is NP-complete.

(b) Let ϕ be a 3CNF formula. An $\neq$-assignment to the variables of ϕ is one where each clause contains two literals with unequal truth values.

(i) Show that the negation of any $\neq$ assignment to ϕ is also a $\neq$ assignment.

(ii) Let $\neq$ SAT be the collection of 3CNF formulas that have a $\neq$ assignment. Show that we obtain a polynomial time reduction from 3SAT to $\neq$ SAT by replacing each clause c_i of a $3SAT$ formula with a few other clauses of the form required by $\neq$ SAT.

(iii) Conclude that $\neq$ SAT is NP-complete. Justify your answers.

Problem 4 (Bonus, 10 points). *Let $A \subseteq 1^*$ be any unary language. Show that if A is NP-complete, then $P = NP$.*