Turing machines: examples

CS 580

Exercise: Describe a Turing machine that decides the language $A = \{0^{2^n} | n \geq 0\}$; i.e. the language consisting of all strings of 0s whose length is a power of 2.

Solution: The machine is:

M = “On input string w:

1. Sweep left to right across the tape, crossing off every other 0.
2. If in step 1 the tape contained a single 0, accept.
3. If in step 1 the tape contained more than a single 0 and the number of 0s was odd, reject.
4. Return the head to the left-hand end of the tape.
5. Go to step 1.”

Explanation: Each iteration of stage 1 cuts the number of 0s in half.

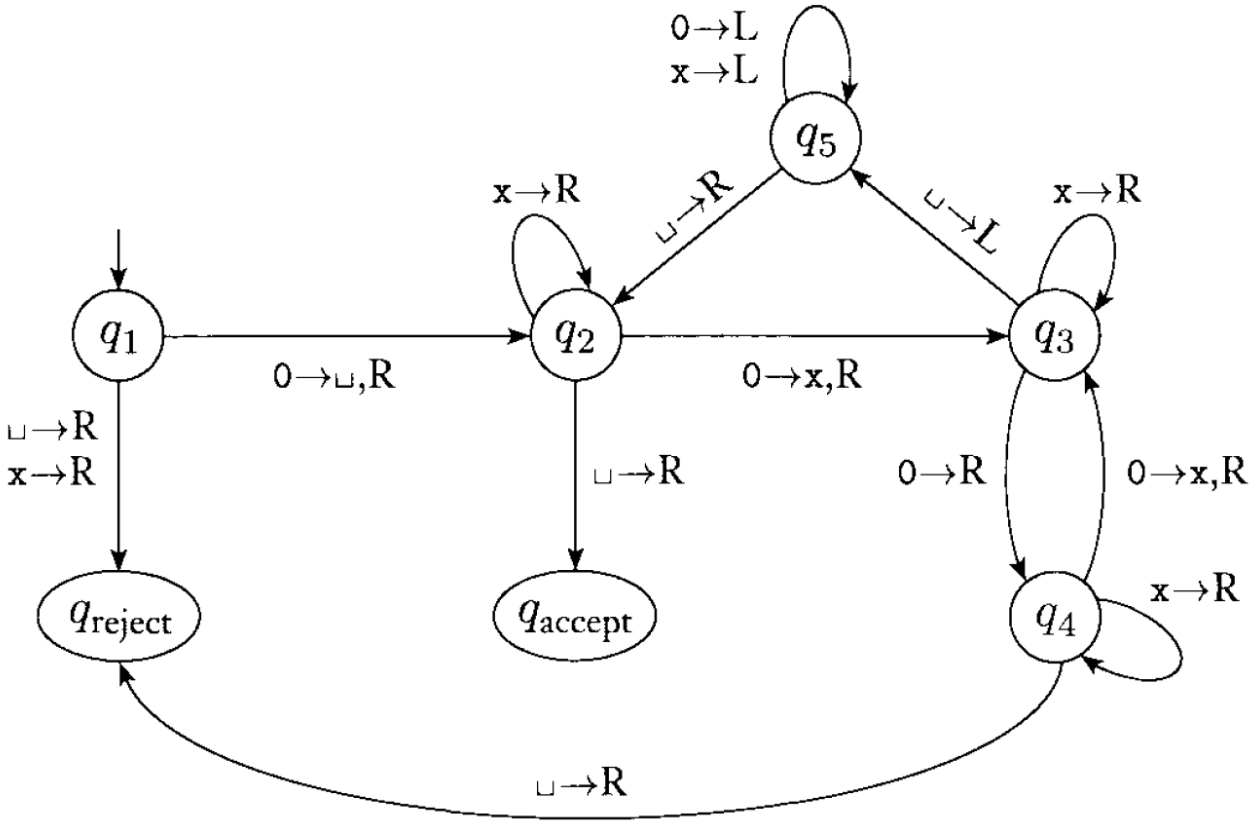
Formal description: $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{reject})$:

$$Q = \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\}$$

$$\Sigma = \{0\}$$

$$\Gamma = \{0, x, \sqcup\}$$

δ: described via state diagram. The start state is q_1, the accept state is q_{accept}, and the reject state is



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M = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, *reject*. Cross off symbols as they are checked to keep track of which symbols correspond.
2. When all symbols to the left of the # have been crossed off, check for any remaining symbols to the right of the #. If any symbols remain, *reject*; otherwise, *accept*."

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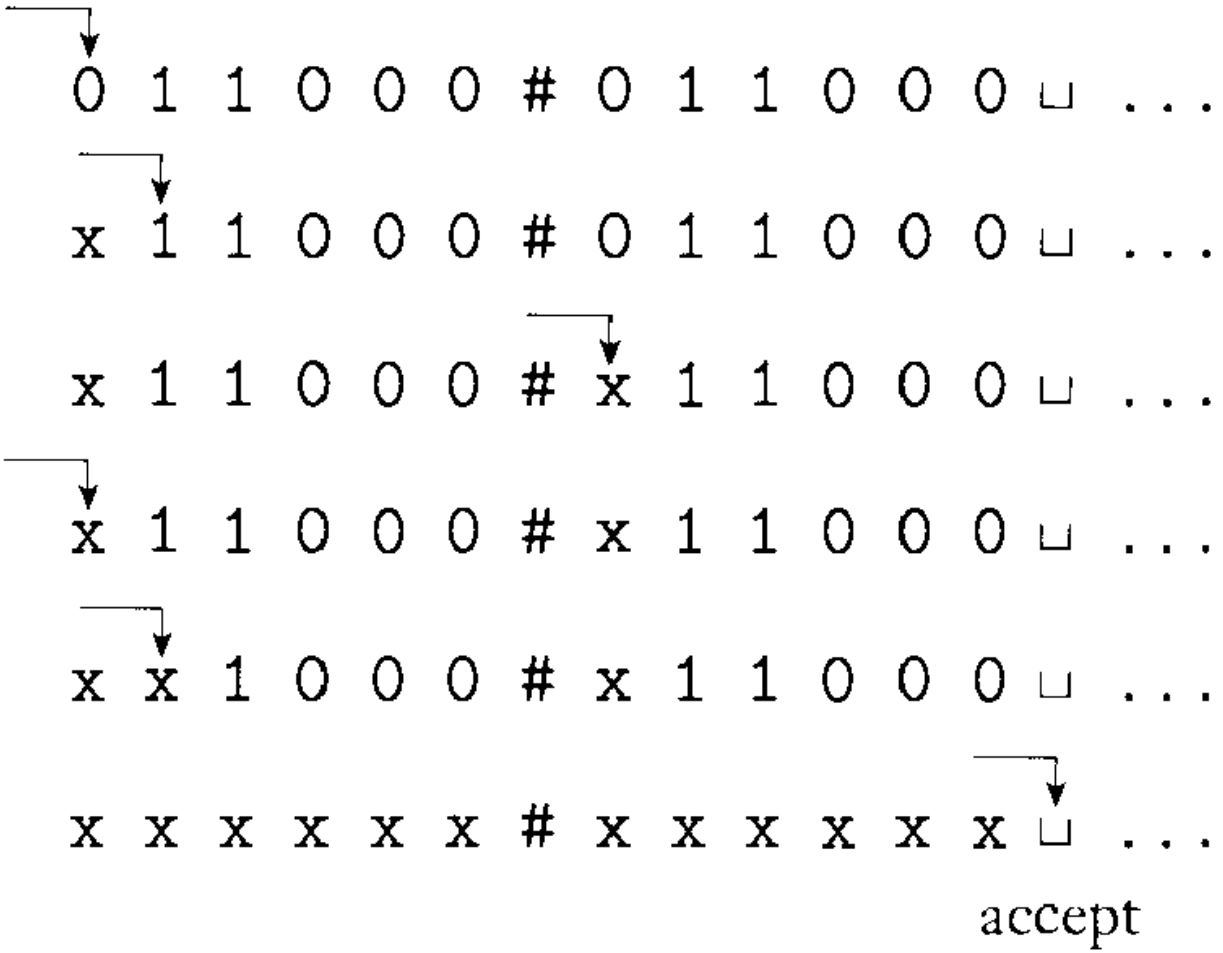
Snapshots of the machine on input 011000#011000.



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State diagram.