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We prove by induction that  $T(m) \leq \alpha \cdot m$  for all  $m \in N$ .

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**Induction step:** By the induction hypothesis we get:

$$T(n) \leq \beta n + \alpha \left( \frac{n}{5} + 1 + \frac{7n}{10} + 6 \right)$$

This is

$$T(n) \leq \beta n + \frac{9\alpha}{10}n + 7\alpha$$

$$T(n) \leq \alpha n + \beta n - \frac{\alpha}{10}n + 7\alpha$$

which is  $\leq \alpha n$  if

$$\beta n - \frac{\alpha}{10}n + 7\alpha \leq 0$$

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$$-10\beta n + (n - 70)\alpha \geq 0$$

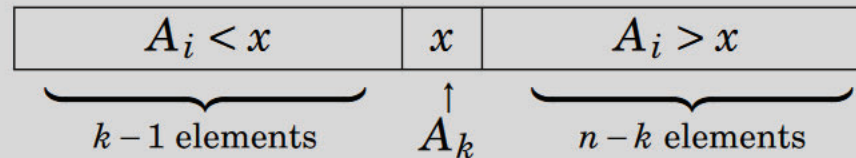
$$\alpha \geq 10\beta \frac{n}{n - 70}$$

Let  $B = 140$ , choose  $\alpha \geq 20\beta$  to show  
 $T(n) \leq \alpha n$ .

# Pseudocode

**Algorithm:** SELECT( $A, i$ )

1. Divide the  $n$  items into groups of 5 (plus any remainder).
2. Find the median of each group of 5 (by rote). (If the remainder group has an even number of elements, then break ties arbitrarily, for example by choosing the lower median.)
3. Use SELECT recursively to find the median (call it  $x$ ) of these  $\lceil n/5 \rceil$  medians.
4. Partition around  $x$ .<sup>\*</sup> Let  $k = \text{rank}(x)$ .<sup>†</sup>



5.
  - If  $i = k$ , then return  $x$ .
  - Else, if  $i < k$ , use SELECT recursively by calling SELECT( $A[1, \dots, k-1], i$ ).<sup>‡</sup>
  - Else, if  $i > k$ , use SELECT recursively by calling SELECT( $A[k+1, \dots, i], i-k$ ).