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Induction step: By the induction hypothesis we get:

$$T(n) \leq \beta n + \alpha\left(\frac{n}{5} + 1 + \frac{7n}{10} + 6\right)$$

This is

$$T(n) \leq \beta n + \frac{9\alpha}{10} n + 7\alpha$$

$$T(n) \leq \alpha n + \beta n - \frac{\alpha}{10} n + 7\alpha$$

which is $\leq \alpha n$ if

$$\beta n - \frac{\alpha}{10} n + 7\alpha \leq 0$$

$$\beta n - \frac{\alpha}{10} n + 7\alpha \le 0$$

$$-10\beta n + (n - 70)\alpha \ge 0$$

$$\alpha \ge 10\beta \frac{n}{n - 70}$$

Let $B = 140$, choose $\alpha \ge 20\beta$ to show $T(n) \le \alpha n$.

Pseudocode

Algorithm: SELECT(A, i)

1. Divide the n items into groups of 5 (plus any remainder).

2. Find the median of each group of 5 (by rote). (If the remainder group has an even number of elements, then break ties arbitrarily, for example by choosing the lower median.)

3. Use SELECT recursively to find the median (call it x) of these $\lceil n/5 \rceil$ medians.

4. Partition around x.[*] Let $k = \text{rank}(x)$.[†]

$$\underbrace{\boxed{A_i < x}}_{k-1 \text{ elements}}\underset{\substack{\uparrow \\ A_k}}{\boxed{x}}\underbrace{\boxed{A_i > x}}_{n-k \text{ elements}}$$

5.
   - If $i = k$, then return x.
   - Else, if $i < k$, use SELECT recursively by calling SELECT$(A[1, \ldots, k-1],\ i)$.[‡]
   - Else, if $i > k$, use SELECT recursively by calling SELECT$(A[k+1, \ldots, i],\ i-k)$.