CS 580, Fall 2022; Instructor: Simina Brânzei.
TAs: Shoaib Khan, Anuj Singh, Lu Yan, Zheng Zhong.
Due: October 31, 11:59PM EST. Submit to Gradescope

Problem Set 3

Reading material : Slides from class; Kleinberg-Tardos and CLRS for additional readings on divide-and-conquer and greedy algorithms.

Collaboration policy : Acknowledge your collaborators on the homework. You may discuss proof strategies, but the solution must be written individually in your own words.

Submission format : The solutions must be typed in Latex and submitted via Gradescope.

Problem 1. *(20 points) Recall the Median-of-Medians algorithm from class. Consider now a variant which works the same way, except each bin has size $2k+1$, for some constant $k \geq 1$. If the number of elements in the vector is not divisible by $2k+1$, then the last bin will have fewer than $2k+1$ elements.*

For which values of the constant k can you still show the algorithm makes $O(n)$ comparisons? Are there values of k for which you cannot obtain this bound? Justify your answers, both for yes/no cases (depending on which ones occur).

Problem 2. *(20 points) You are doing an internship at Designhill and your manager asks you to design k logos over the first k days of the internship. The logos can be designed in any order you like, however you are expected to submit one logo per day in each of the first k days.*

Logo i requires effort e_i. If you hand in logo i on day j, you get paid $e_i \cdot (k-j)$ USD. Suppose the efforts e_i are distinct.

Can you design an efficient algorithm to optimize your payment? Justify why the algorithm is correct (i.e. indeed maximizes the payment) and argue its runtime.