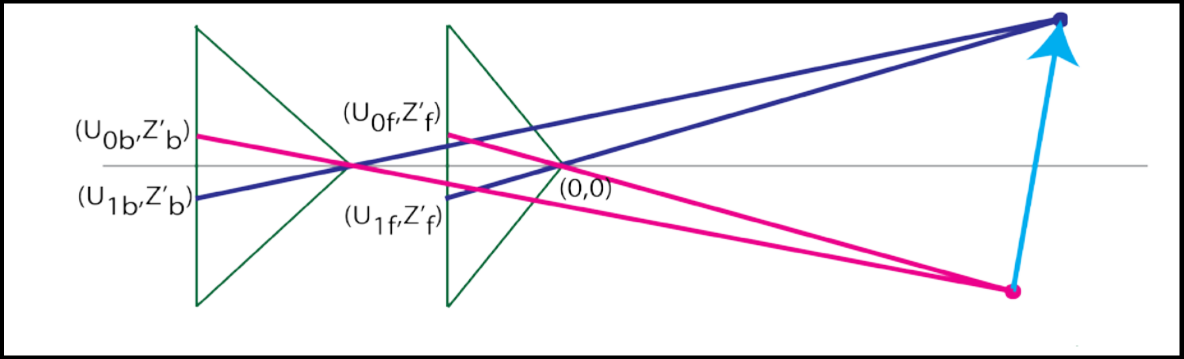
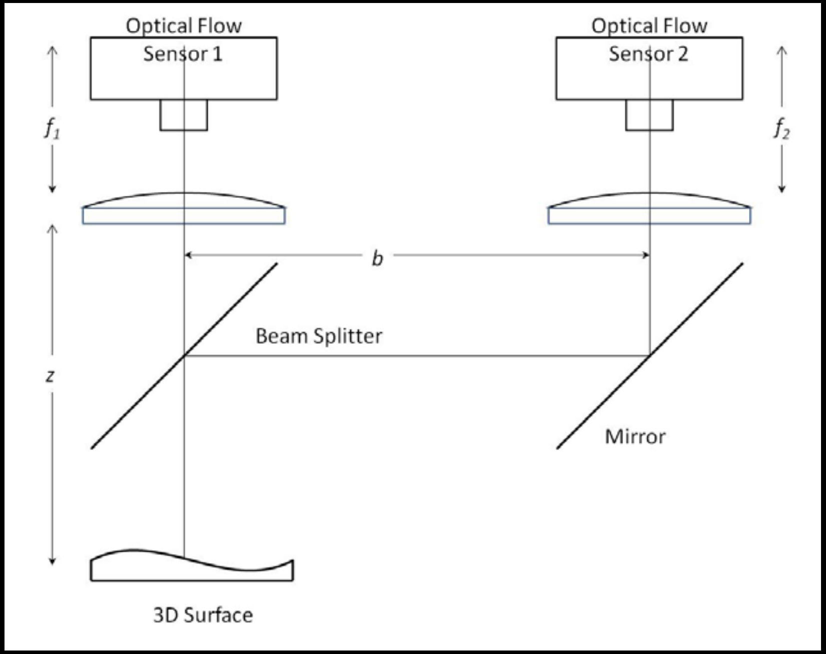
3D reconstruction from images taken with a coaxial camera rig

Richard Kirby and Ross Whitaker
University of Utah

What is a coaxial camera rig?





Why use a coaxial?

- Advantages:
  - Common center point, provides one known correspondence.
  - Baseline can be wrapped up inside the camera.
  - Substantial reduction of occlusions vs. binocular stereo
  - No minimum working distance vs. binocular stereo
- Disadvantage:
  - No pixel disparity in center region – “unrecoverable point problem”

Original Motivation

Original Motivation

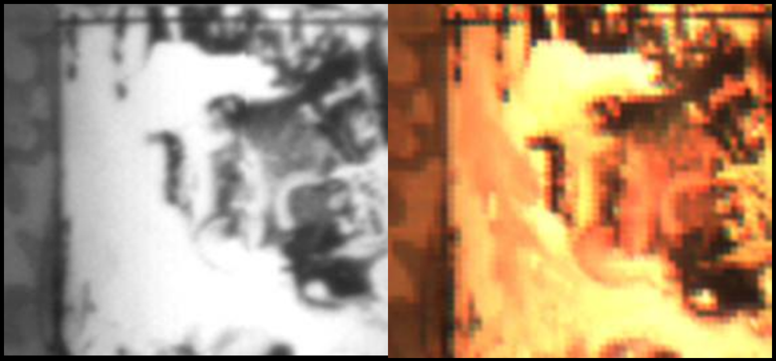
Original Motivation

Can 3D velocity be estimated directly from flow fields obtained by a coaxial camera rig?

- Direct flow field to velocity conversion e.g. bypass stereo correspondence finding.
- Resolves the unrecoverable point problem.
- Fewer occlusions
- Camera rig could be handheld and portable.
- Faster than stereo??

Applications

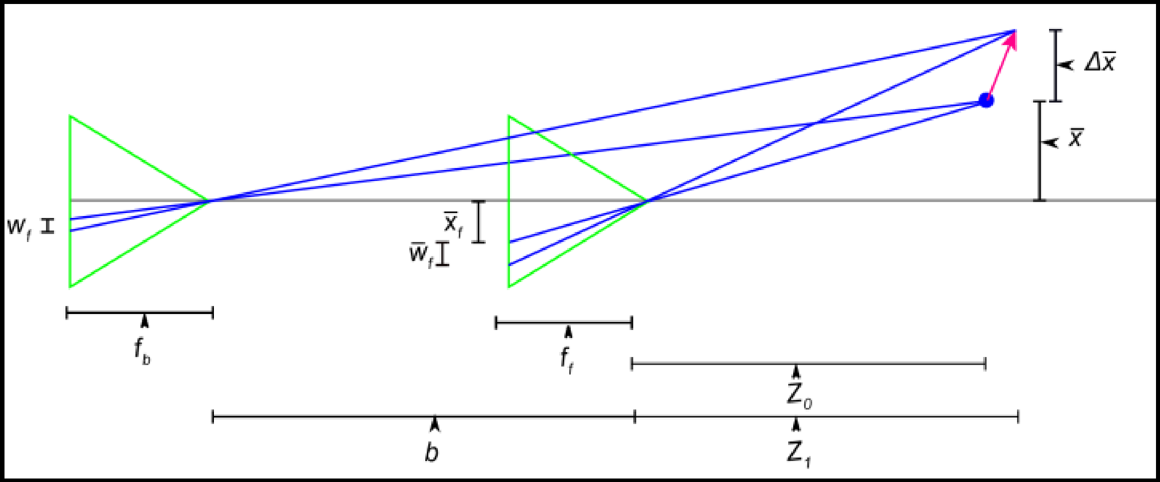
- 3D velocity measuring camera
- 3D Endoscope/borescope
- Multi-modal cameras



Derivation Assumptions

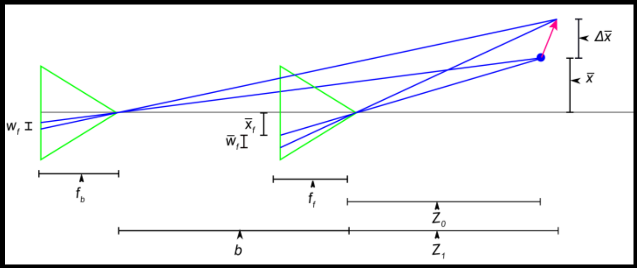
- Scene is smoothly deformable
- Optical flow need not be an accurate projection of the 3D motion field, but errors in optical flow computation should be consistent with projection equations.
- Scene must have enough visual texture to generate optical flow

Derivation Definitions



Energy derivation – Optical Flow

$$\frac{dx}{dt} = w = -f\,\frac{d}{dt}\left(\frac{X}{Z}\right) = -f\,\frac{d}{dt}\left(XZ^{-1}\right)$$



$$w = -f\,\frac{dX}{dt}Z^{-1} - fX\,\frac{dZ^{-1}}{dt}$$

$$w = -f\,\frac{U}{Z} + fX\left(Z^{-2}\right)\left(\frac{dZ}{dt}\right)$$

$$w = -f\,\frac{U}{Z} + f\,\frac{X}{Z^2}W$$

Relationship between 3D motion field
and 2D optical flow for ideal flow

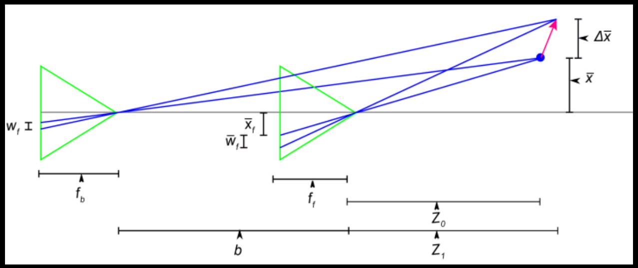
Energy derivation Optical Flow (continued)

$$X = -\frac{xZ}{f}$$

$$w = -f\frac{U}{Z} + f\frac{1}{Z^2}\left(\frac{xZ}{f}\right)W$$

$$w = -f\frac{U}{Z} + \left(\frac{x}{Z}\right)W$$

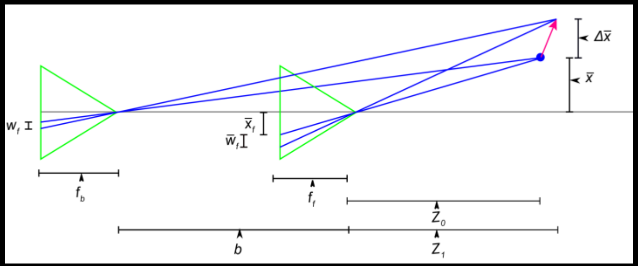
$$w = -\frac{fU - xW}{Z}$$



Energy derivation – Optical Flow
Linear algebra form

$$\bar{P} = \begin{bmatrix} 1 & 0 & x/f & 0 \\ 0 & 1 & y/f & 0 \\ 0 & 0 & 0 & -Z/f \end{bmatrix}$$

Projection matrix



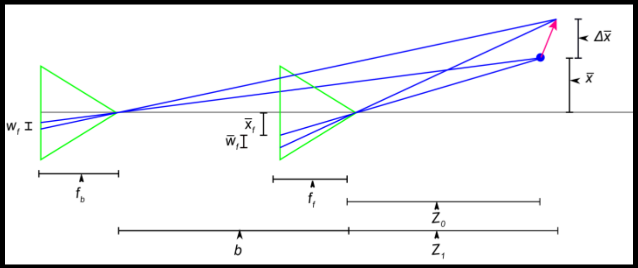
$$\bar{w} = \begin{bmatrix} 1 & 0 & -x/f & 0 \\ 0 & 1 & -y/f & 0 \\ 0 & 0 & 0 & -Z/f \end{bmatrix} \begin{bmatrix} U \\ V \\ W \\ 1 \end{bmatrix} = \begin{bmatrix} U - \dfrac{xW}{f} \\ V - \dfrac{yW}{f} \\ -\dfrac{Z}{f} \end{bmatrix} = - \begin{bmatrix} \dfrac{fU - xW}{Z} \\ \dfrac{fV - yW}{Z} \end{bmatrix}$$

$$\bar{R} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix

Energy derivation
Elimination of W

$$\bar{w} = \bar{R}\bar{P}\begin{bmatrix} U \\ V \\ W \\ 1 \end{bmatrix}$$



$$\bar{w} = \begin{bmatrix} \left(U - \frac{xW}{f}\right)\cos(\theta) + \left(V - \frac{yW}{f}\right)\sin(\theta) \\ -\left(U - \frac{xW}{f}\right)\sin(\theta) + \left(V - \frac{yW}{f}\right)\cos(\theta) \\ \frac{Z}{f} \end{bmatrix}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

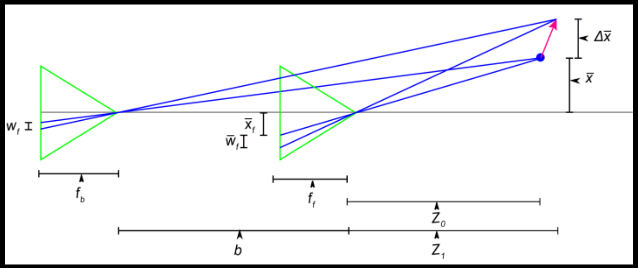
Rotation of coordinate system by theta aligns start point of flow with optical axis

Energy derivation
Elimination of W (continued)

$$\sin\left(\tan^{-1}\frac{y}{x}\right) = \frac{\frac{y}{x}}{\sqrt{1+\left(\frac{y}{x}\right)^2}}$$

$$\cos\left(\tan^{-1}\frac{y}{x}\right) = \frac{1}{\sqrt{1+\left(\frac{y}{x}\right)^2}}$$

Geometric identities



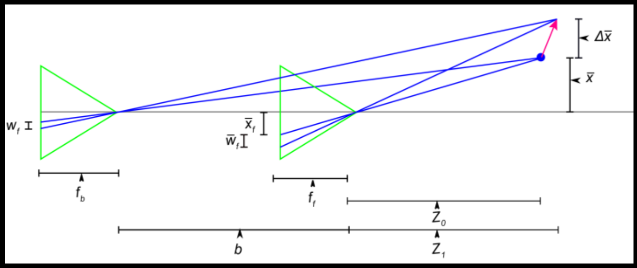
$$\frac{dy}{dt} = \left(-U\sin\theta + \left(\frac{xW}{f}\right)\left(\frac{\frac{y}{x}}{\sqrt{1+\left(\frac{y}{x}\right)^2}}\right) + V\cos\theta - \left(\frac{yW}{f}\right)\left(\frac{1}{\sqrt{1+\left(\frac{y}{x}\right)^2}}\right)\right)\frac{f}{Z}$$

$$\frac{dy}{dt} = (-U\sin\theta + V\cos\theta)\frac{f}{Z}$$

Two equations two unknowns

Energy derivation
Energy minimization form

$$\frac{dy_b(x+h)}{dt} = (-U\sin\theta + V\cos\theta)\left(\frac{f_b}{Z+b}\right)$$



$$p(x_f)\left(\frac{dy_f(x)}{dt}\right) = \frac{dy_b(x+h)}{dt}$$

$$p(x_f) = \left(\frac{Z}{Z+b}\right)\left(\frac{f_f}{f_b}\right)$$

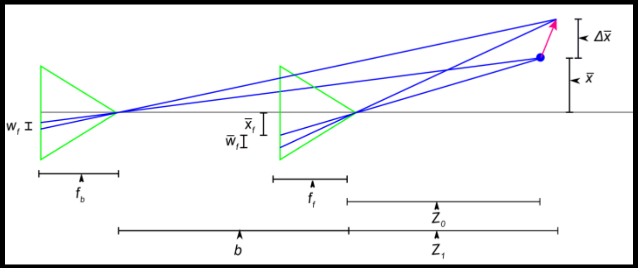
$$h(\bar{x}_f) = \frac{x_f\left(\frac{f_b}{f_f}Z - Z - b\right)}{Z+b}$$

Energy derivation (continued)

$$m(\bar{x}_f)w_f(\bar{x}_f) = c(\bar{x}_f)w_b\left(\bar{x}_f m(\bar{x}_f)\right)$$

$$m(\bar{x}_f) = \left(\frac{f_b}{f_f}\right)\left(\frac{Z(\bar{x}_f)}{(Z(\bar{x}_f) + b)}\right)$$

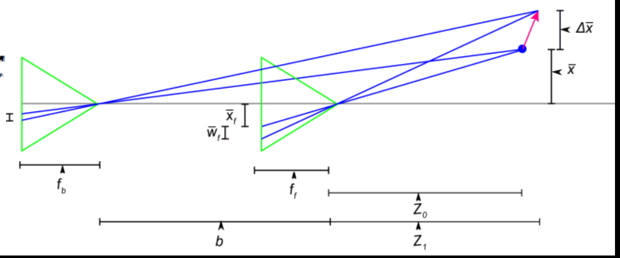
$$c(\bar{x}_f) = \left(\frac{w_f(\bar{x}_f)}{\left(\frac{Z_0(\bar{x}_f)+b}{Z_1(\bar{x}_f)+b}\right)\left(\frac{Z_1(\bar{x}_f)}{Z_0(\bar{\bar{x}}_f)}\right)(w_f(\bar{x}_f)+\bar{x}_f)-\bar{x}_f}\right)$$



Note: in a coaxial camera epipolar lines are radial lines.
This results in a 1D optimization, similar to binocular stereo using pixel correspondences.

Energy derivation (continued)

$$E_{match} = \int_a^b \frac{1}{2}\left[m(\bar{x}_f)w_f(\bar{x}_f) - c(\bar{x}_f)w_b\left(\bar{x}_f m(\bar{x}_f)\right)\right]^2 dx$$



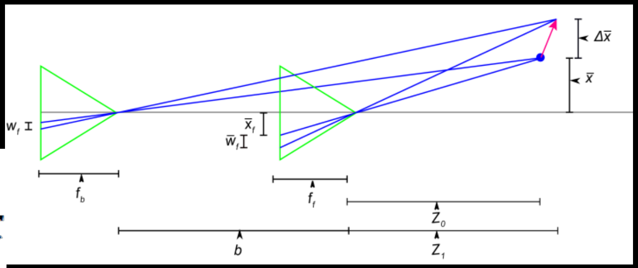
$$E_{smooth} = \frac{1}{2}\int_a^b \left\|\nabla Z(\bar{x}_f)\right\|^2 dx$$

$$E_{total} = \gamma E_{match} + \alpha E_{smooth}$$

Note: the smoothness term is 2D, along the epipolar line and between epipolar lines. We adjust alpha independently for along and perpendicular to epipolar lines.

Energy derivation (continued)

$$p(x_l)w_l(x_l) = w_l\big(x_l + h(x_l)\big)$$



$$E_{match} = \int_a^b \frac{1}{2}\left[p(\bar{x}_l)w_l(\bar{x}_l) - w_r(\bar{x}_r + h(\bar{x}_l))\right]^2 dx$$

$$E_{smooth} = \frac{1}{2}\int_a^b \|\nabla h(\bar{x}_l)\|^2\, dx$$

$$E_{total} = \gamma E_{match} + \alpha E_{smooth}$$

Numerical Solution
Gradient Decent

$$\gamma w_z(pw_l - w_r)(m'w_f + mw_f' - c'w_b(mx) -$$

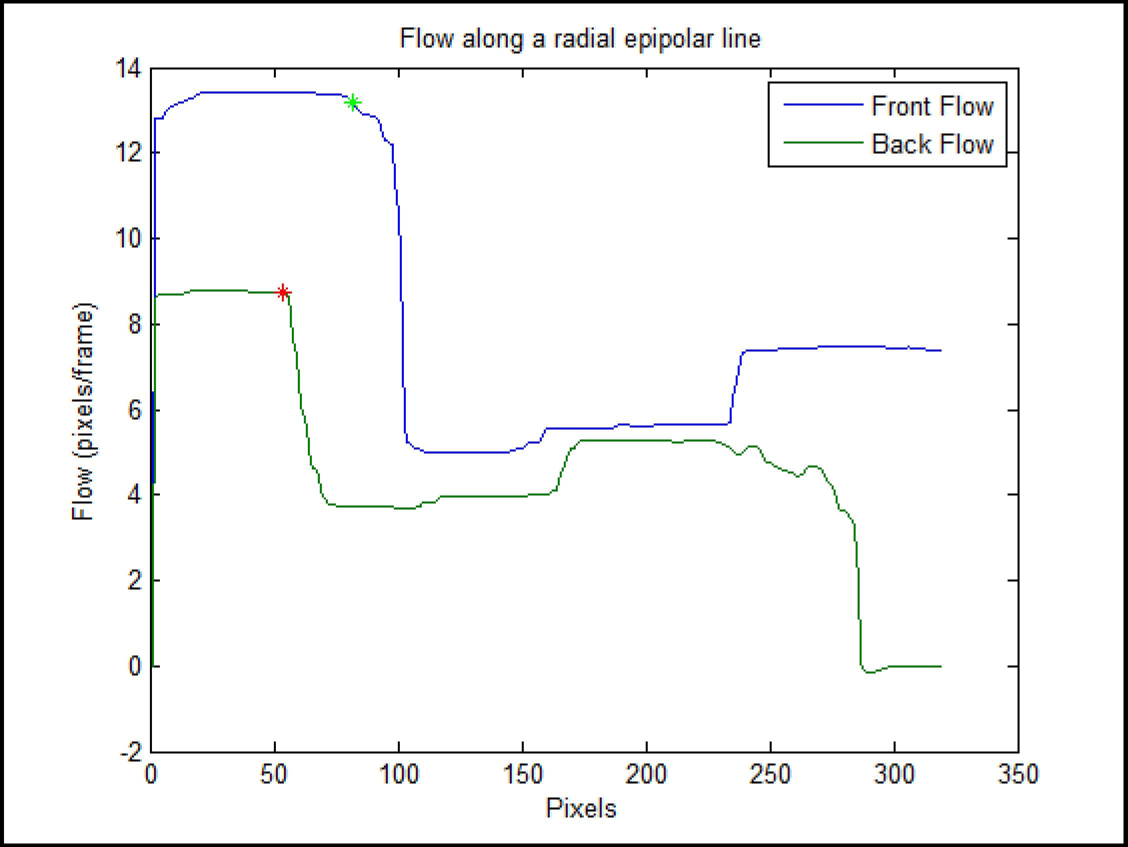
$$cw_b'm_f'(mx)m'x) - \alpha\nabla^2 Z_1 = 0$$

$$w_z = m(\bar{x}_f)w_f(\bar{x}_f) - c(\bar{x}_f)w_b\left(\bar{x}_f m(\bar{x}_f)\right)$$

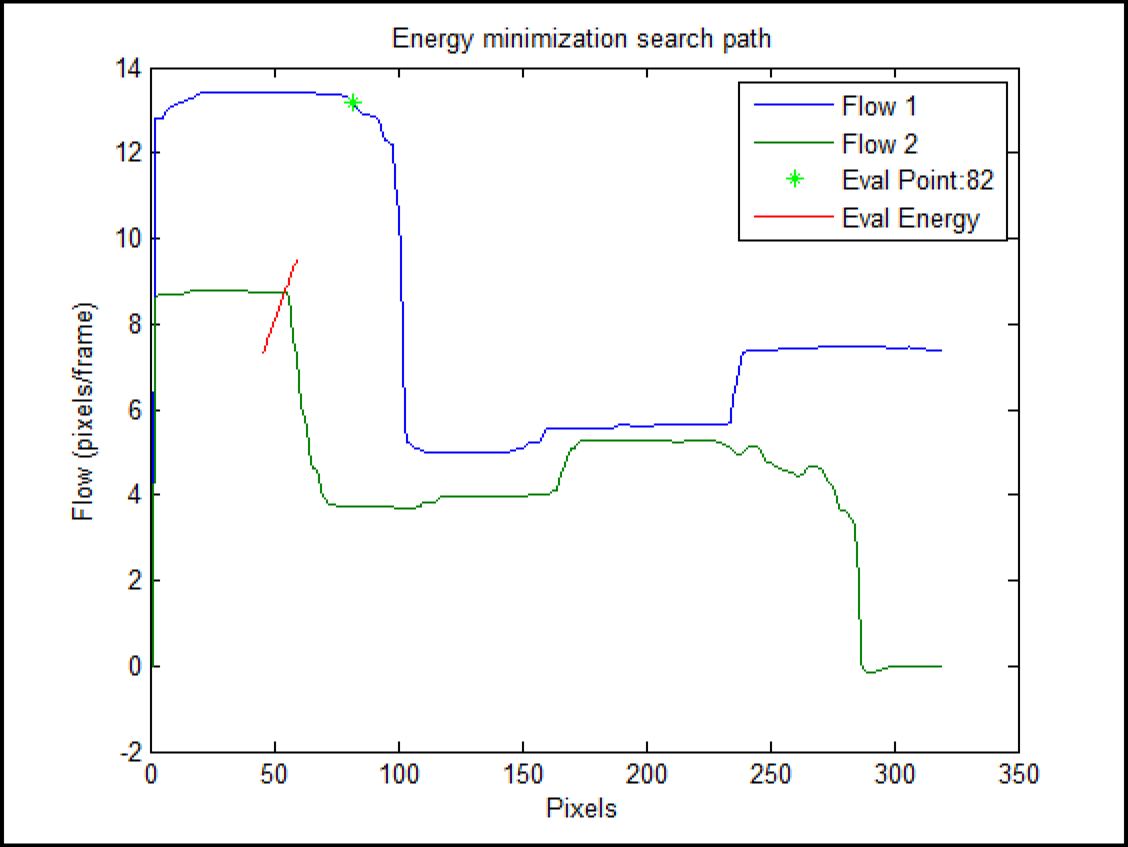
$$w_l' = \frac{\partial w_l}{\partial Z} = -\frac{w_l}{Z_1}$$

$$w_r' = \frac{\partial w_r}{\partial Z} = -\frac{w_r}{Z_1}$$

Optimization along epipolar lines

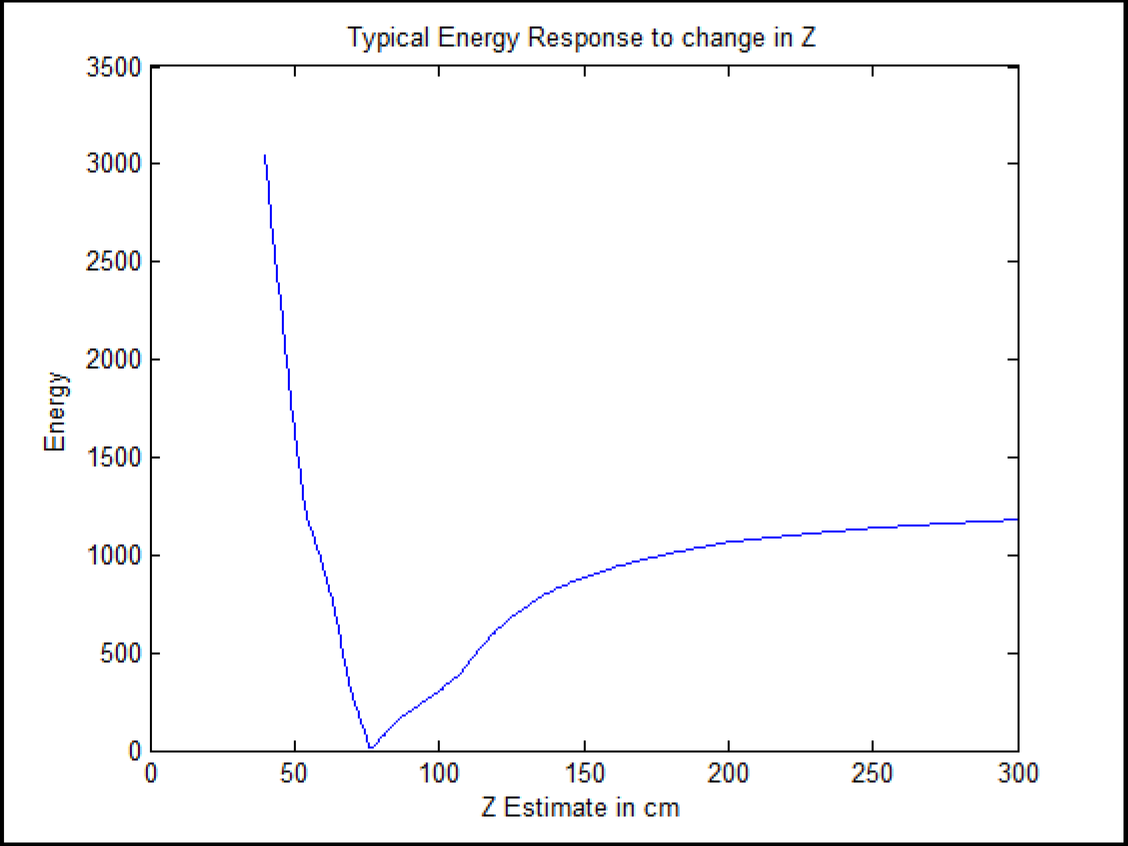


Optimization along epipolar lines



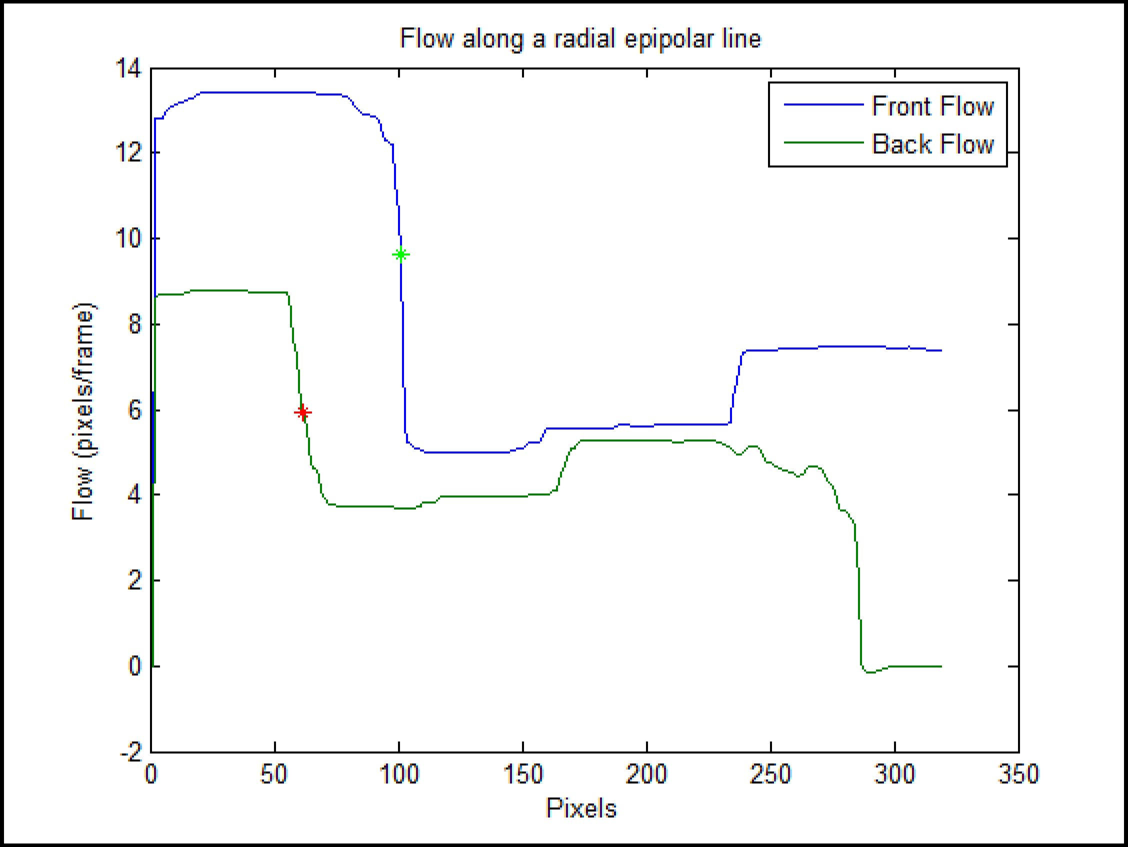
Disparity and magnitude are adjusted simultaneously for each Z estimate. Gradient decent stops when error is sufficiently small.

Energy vs. Z estimate

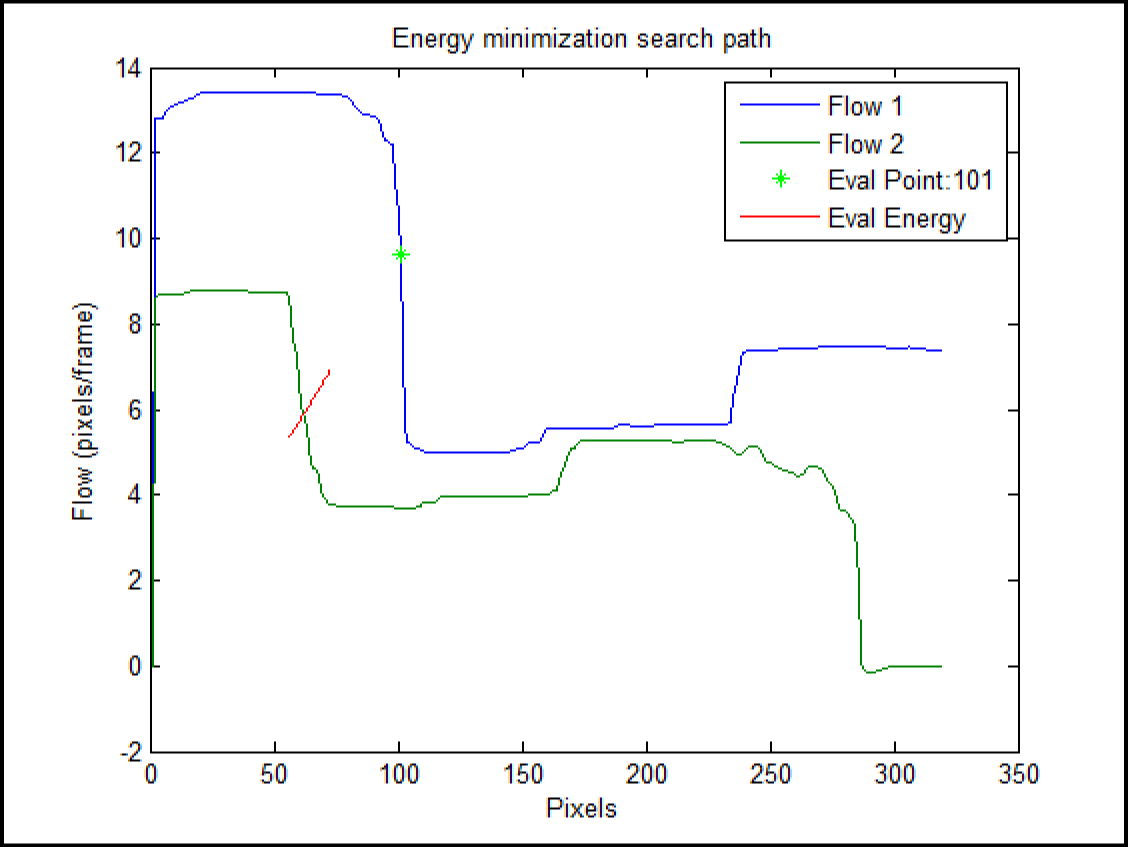


Note: gradient decent stops before error starts to increase.

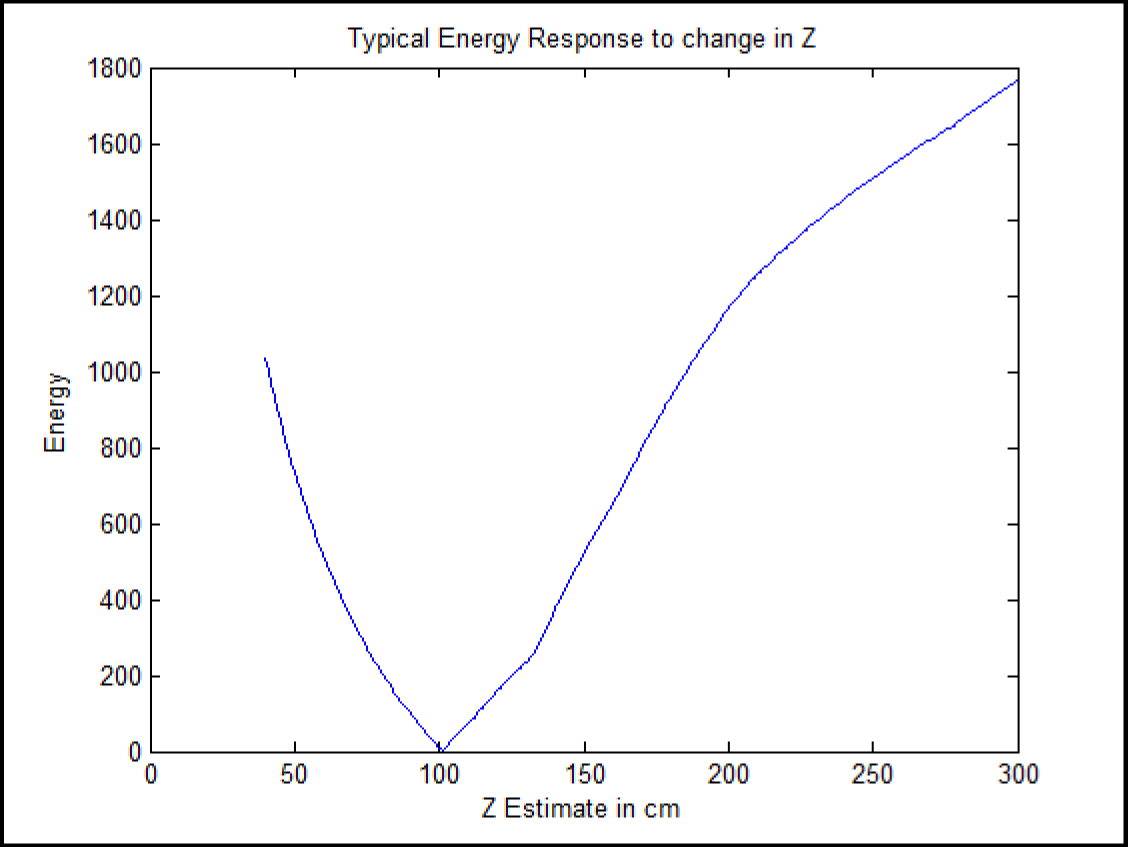
Optimization along epipolar lines



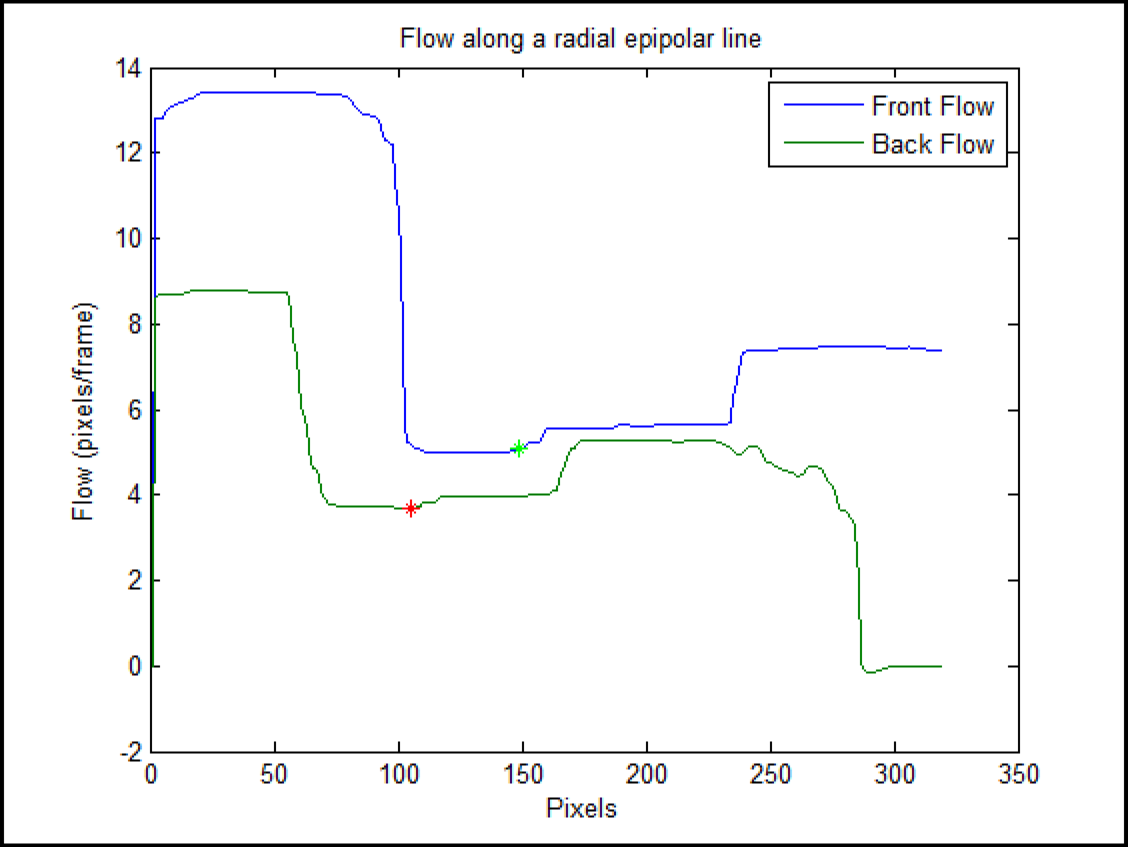
Optimization along epipolar lines



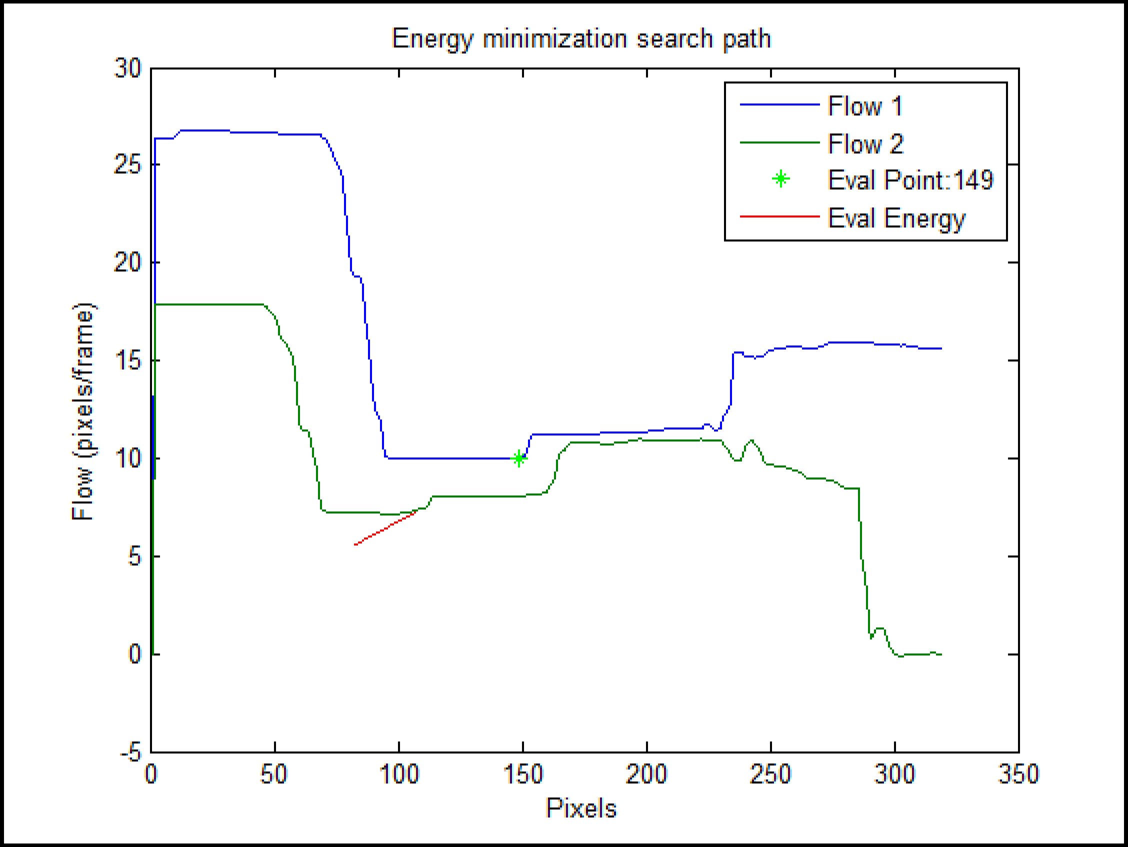
Energy vs. Z estimate



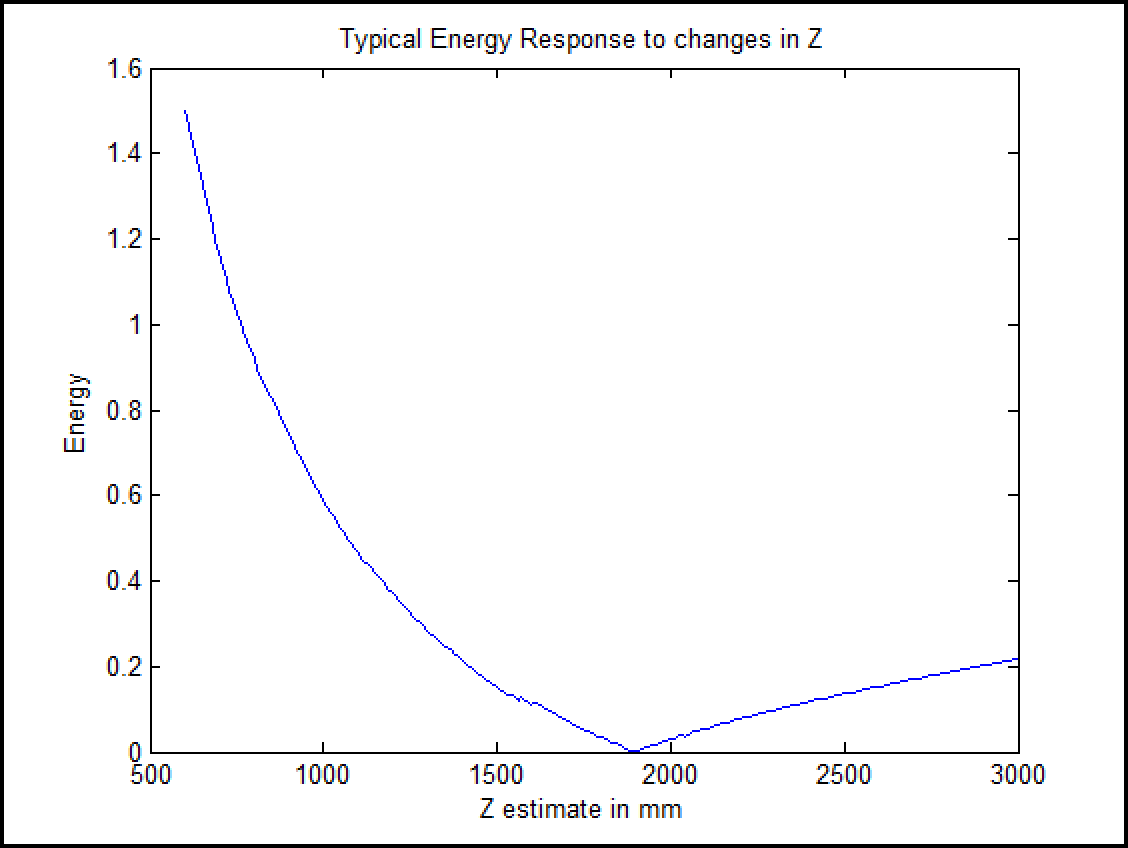
Optimization along epipolar lines



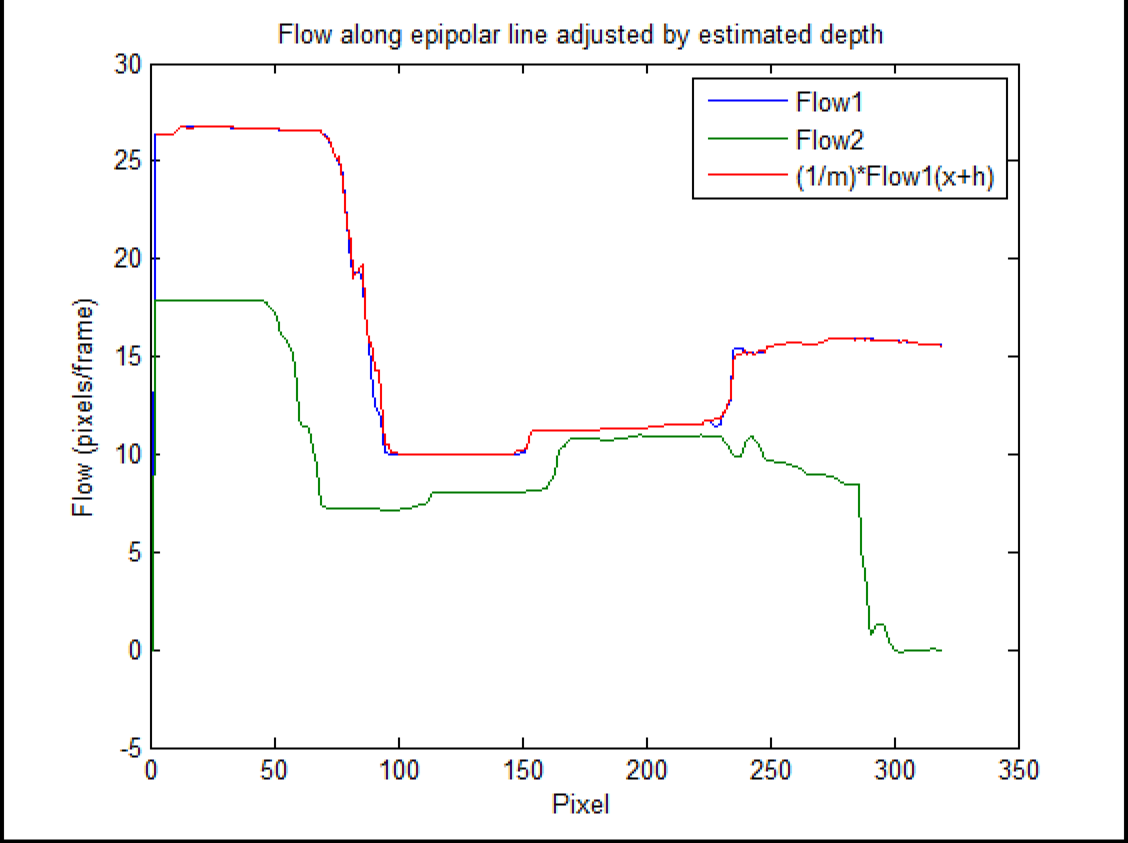
Optimization along epipolar lines



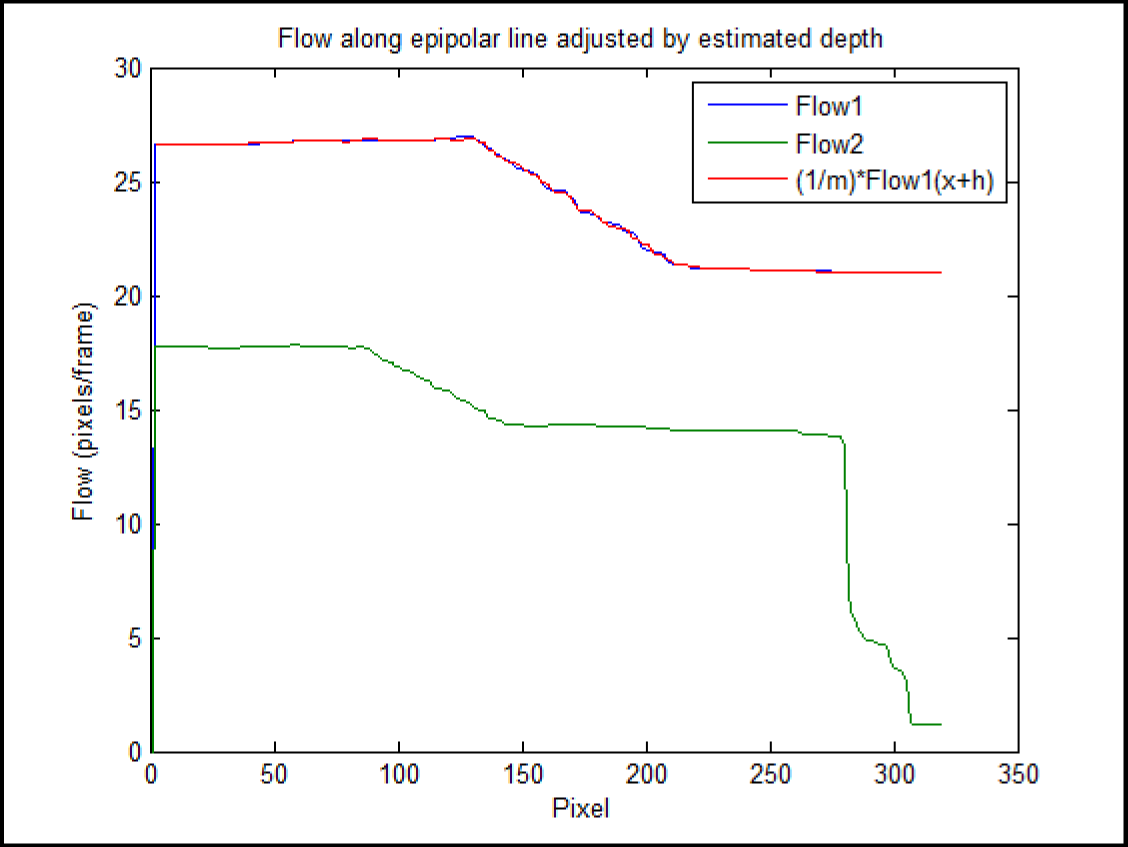
Energy vs. Z estimate

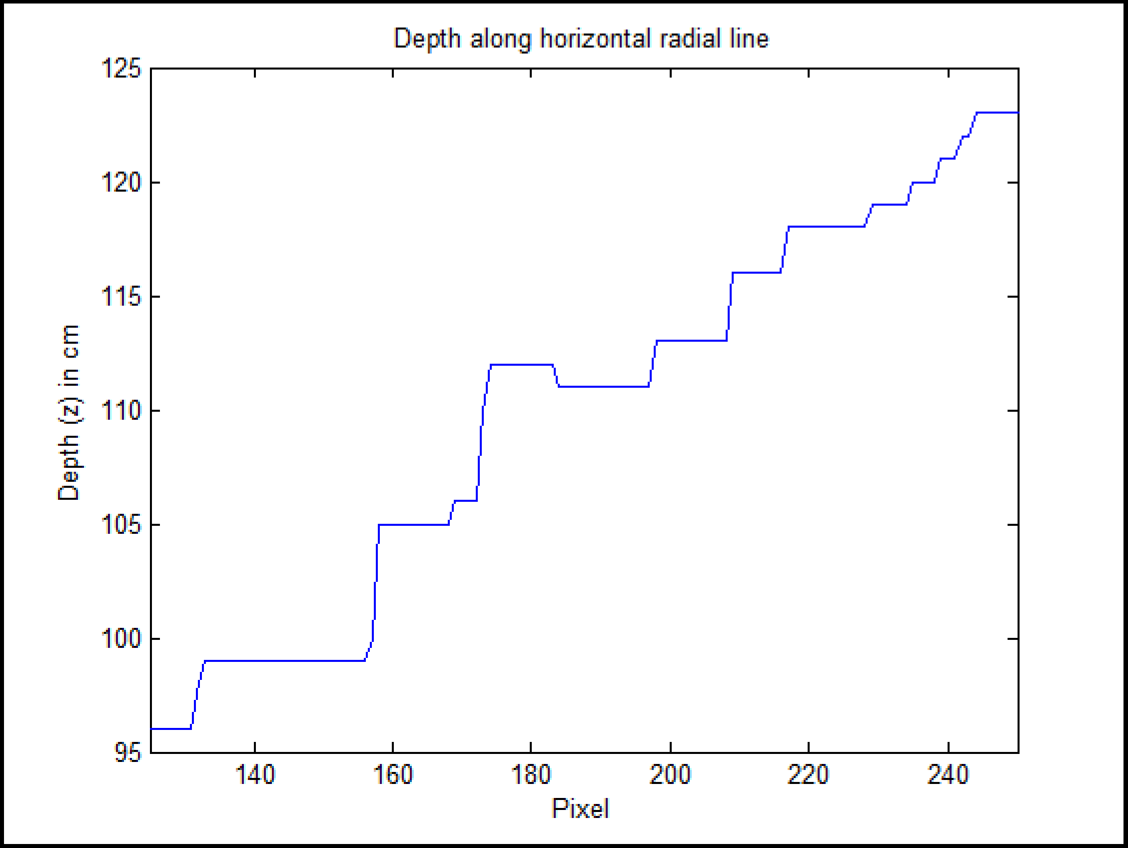


Results of optimization

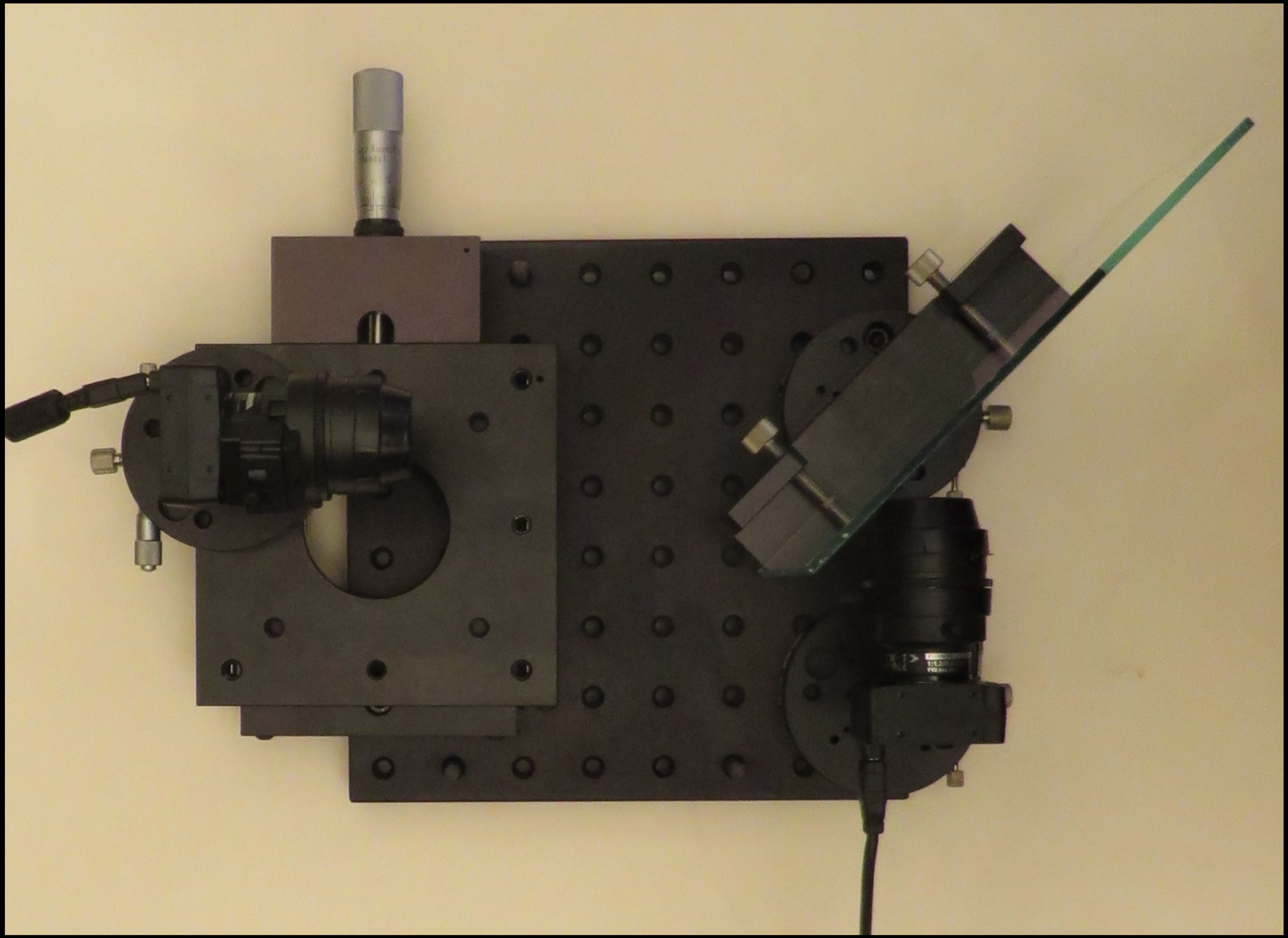


<u>RMS error = 0.078 pixels!!!</u>


Flow along epipolar line adjusted by estimated depth
Flow1
Flow2
(1/m)*Flow1(x+h)
Flow (pixels/frame)
Pixel
0
5
10
15
20
25
30
0
50
100
150
200
250
300
350


Depth along horizontal radial line
Depth (z) in cm
125
120
115
110
105
100
95
140
160
180
200
220
240
Pixel

Camera Rig



Conclusions

- Optical flow as a basis for
- Course shape
- Built-in smooth of optical flow causes
- Potentially bypassing optical flow all-together and going from four images to depth

Thank you
and
Questions