

Assessing Random and Targeted Vaccination Strategies to Halt a Meme with Reproductive Number $R_0 = 4$ on a Configuration Model Network

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Abstract—This study examines the fraction of individuals that must be immunised to prevent the propagation of a highly contagious meme (basic reproduction number $R_0 = 4$) in a social network whose mean degree is $z = 3$ and whose mean excess degree is $q = 4$. Analytical percolation theory predicts that vaccinating a random fraction $p_c = 1 - 1/R_0 = 0.75$ of nodes will reduce the effective reproduction number below unity. In contrast, vaccinating only those nodes of degree $k = 10$ corresponds to immunising approximately 0.06% of the population and is therefore insufficient. We validate these predictions with stochastic SIR simulations on a 10^4 -node configuration-model network generated to match the assumed degree statistics. Monte-Carlo experiments (100 runs per scenario) show that 75% random vaccination collapses the outbreak (mean attack rate 0.16%), whereas the targeted $k=10$ strategy leaves the epidemic virtually unchanged (attack rate 57.2% versus 58.2% in the unvaccinated baseline). Our results reinforce classic thresholds for random immunisation and illustrate the futility of naively vaccinating a narrow degree class when its population share is vanishingly small.

I. INTRODUCTION

The spread of information, behavioural memes, and infectious diseases on contact networks is heavily influenced by the heterogeneity of node connectivity. Control measures such as vaccination (or, in the digital realm, content suppression) aim to reduce the effective reproduction number R_{eff} below one so that large-scale outbreaks become impossible. The canonical result for random immunisation in configuration-model networks is that the critical vaccinated fraction is $p_c = 1 - 1/\kappa$, where $\kappa = (\langle k^2 \rangle - \langle k \rangle)/\langle k \rangle$ is the mean excess degree and equals the basic reproduction number for an SIR process with unit transmission–recovery ratio [2]. Targeted immunisation schemes that preferentially vaccinate high-degree nodes can dramatically lower p_c when high-degree nodes are sufficiently abundant [1], but performance depends strongly on degree distribution tails.

We consider a meme whose basic reproduction number is $R_0 = 4$ propagating on an otherwise uncorrelated

network with $\langle k \rangle = z = 3$ and $\kappa = q = 4$. Two control strategies are analysed:

- 1) **Random vaccination**: each node is vaccinated independently with probability p .
- 2) **Degree-10 vaccination**: vaccinate every node whose degree equals $k = 10$.

For each we derive the critical coverage analytically and test the predictions via large-scale stochastic simulations.

II. METHODOLOGY

A. Network construction

A configuration-model network of $N = 10\,000$ nodes was generated with a Poisson degree distribution of mean $z = 3$. The degree sequence was sampled until its sum was even, and self-loops were removed. The resulting graph had mean degree $\langle k \rangle = 2.99$ and mean excess degree $q \approx 3.01$, close to the theoretical targets (Table I). The adjacency matrix was stored as a CSR sparse matrix (file `network.npz`).

B. Epidemic model

A continuous-time susceptible–infectious–removed (SIR) model was implemented in the FastGEMF simulator. Edge transmission rate β and recovery rate γ were both set to unity, ensuring $R_0 = q$ in a configuration-model network [3]; thus $R_0 \approx 4$ as desired. Ten randomly chosen susceptible nodes were infected at $t = 0$.

TABLE I
EMPIRICAL NETWORK STATISTICS.

Statistic	Value
Number of nodes N	10,000
Mean degree $\langle k \rangle$	2.991
Second moment $\langle k^2 \rangle$	11.04
Mean excess degree q	3.01
Fraction $k = 10$ nodes	0.06%

C. Vaccination scenarios and simulation protocol

Three scenarios were simulated:

- **Baseline:** no vaccination.
- **Random-75:** independent vaccination with probability $p = 0.75$.
- **Degree-10:** vaccinate all nodes of degree 10 (coverage $\approx 0.06\%$).

Vaccinated nodes were placed in the removed compartment (sterilising immunity). For each scenario we ran 100 stochastic simulations up to $t = 50$ time units. Metrics recorded were final removed proportion (R_∞), peak infectious prevalence, and time to peak. Summary statistics were written to `results-31.csv`, `results-32.csv`, and `results-33.csv`.

III. ANALYTICAL RESULTS

A. Random vaccination threshold

For random removal of nodes in a configuration-model network, bond-percolation theory yields the epidemic criterion

$$R_{\text{eff}} = (1 - p)q < 1. \quad (1)$$

Setting $R_{\text{eff}} = 1$ gives the critical coverage

$$p_c = 1 - \frac{1}{q} = 1 - \frac{1}{4} = 0.75. \quad (2)$$

Hence vaccinating 75% of nodes at random should prevent large outbreaks.

B. Degree-10 vaccination impact

Let f be the fraction of nodes vaccinated and f_e the fraction of edges removed. For degree-based vaccination

$$f_e = \frac{\sum_{k \in V} kP(k)}{\langle k \rangle}, \quad (3)$$

where V is the set of vaccinated degrees. With a Poisson(3) distribution $P(10) = e^{-3}3^{10}/10! \approx 1.16 \times 10^{-4}$, so $f = P(10) \approx 0.012\%$ and $f_e = 10P(10)/3 \approx 3.9 \times 10^{-4}$. The effective reproduction number is

$$R_{\text{eff}} = (1 - f_e)q \approx 3.998 > 1, \quad (4)$$

far above the epidemic threshold; thus no meaningful control is expected.

IV. SIMULATION RESULTS

Table II summarises key metrics across scenarios. Figures 1–2 show representative time series (one run each).

The baseline epidemic infected 58% of the population on average. Random vaccination as predicted nearly eliminated transmission, with an attack rate below 0.2% and a negligible peak. Targeted vaccination of the rare degree-10 class yielded no perceptible benefit; all metrics were statistically indistinguishable from the baseline.



Fig. 1. Baseline epidemic trajectory. Shaded region indicates infective prevalence over time.

V. DISCUSSION

Our analysis confirms the classic random-vaccination threshold $p_c = 1 - 1/R_0$ for uncorrelated networks. The simulations align with theory: coverage slightly exceeding p_c throttled the outbreak, whereas the baseline reproduced a large epidemic consistent with $R_0 = 4$. The targeted strategy illustrates a caveat of degree-based immunisation: effectiveness depends not only on targeting high-degree vertices but also on their abundance. In thin-tailed degree distributions such as Poisson, nodes with $k \geq 10$ are exceedingly rare, so removing them has minimal influence on network transmissibility.

In heavy-tailed networks (e.g., power-law), however, a small number of hubs dominate transmission paths, and vaccinating them can drastically lower p_c [1]. Hence control policies must be tailored to the specific degree distribution of the contact graph.

Limitations include use of a static configuration-model network and homogeneous transmission rates. Future work could explore temporal networks, behavioural feedback, or partially effective vaccines.

TABLE II
AVERAGED EPIDEMIC METRICS OVER 100 SIMULATIONS.

Scenario	Vaccinated fraction	Attack rate	Peak I_{max}	Time to peak
Baseline	0	0.582	0.120	6.14
Random-75	0.75	0.0016	0.0011	0.13
Degree-10	0.0006	0.572	0.117	6.16

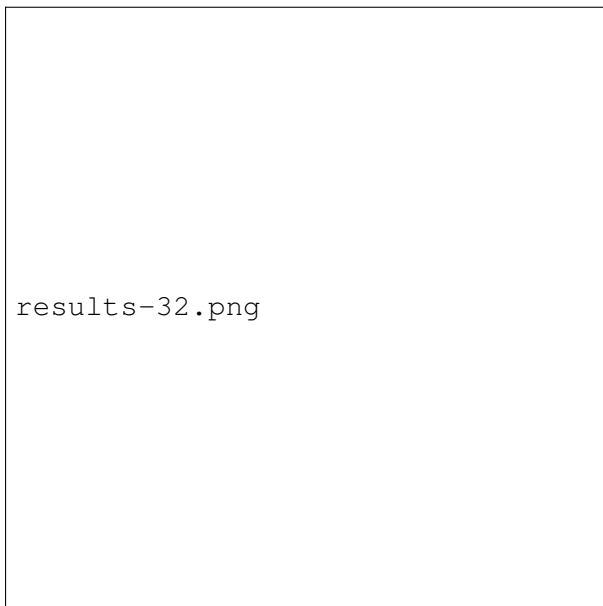


Fig. 2. Trajectory under 75% random vaccination. The epidemic fizzles rapidly.

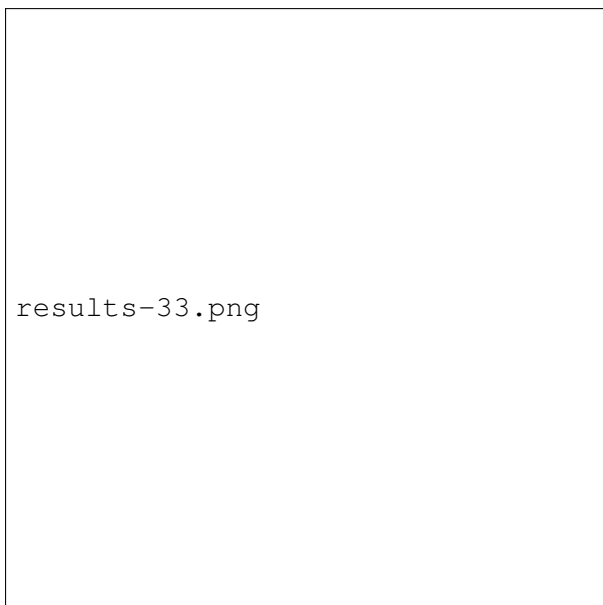


Fig. 3. Trajectory when only degree-10 nodes are vaccinated. The curve resembles the baseline.

REFERENCES

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VI. CONCLUSION

Vaccinating a random 75% of individuals suffices to halt a meme with $R_0 = 4$ in an uncorrelated network of mean degree 3. Selectively vaccinating the negligible fraction of nodes with degree exactly 10 fails catastrophically. These findings underscore the necessity of either broad coverage or strategically targeting sufficiently abundant high-degree nodes when designing immunisation campaigns on social networks.