

# Coexistence versus Dominance in Competitive SIS Spreading of Exclusive Memes on Multiplex Networks

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**Abstract**—Understanding the fate of mutually exclusive information items that propagate over distinct but overlapping social channels is critical for controlling online misinformation and marketing campaigns. We address a competitive susceptible–infected–susceptible (SIS) process in which two memes—each confined to its own contact layer—compete for adoption on the same population of individuals. Analytically, we derive a spectral mean–field criterion that predicts whether the long–term outcome is coexistence (both memes persist) or absolute dominance (competitive exclusion). The criterion depends on the effective adoption rates  $\tau_1 = \beta_1/\delta_1$  and  $\tau_2 = \beta_2/\delta_2$ , the leading eigenvalues  $\lambda_1(A)$  and  $\lambda_1(B)$  of the adjacency matrices of layers  $A$  and  $B$ , and the structural overlap between the corresponding principal eigenvectors. Simulation experiments using the fastGEMF engine confirm the analytic prediction across  $10^4$  stochastic realisations on a 1 000–node multiplex composed of a scale–free Barabasi–Albert layer and an Erdős–Rényi layer. When each meme exceeds its single–layer epidemic threshold ( $\tau_i > 1/\lambda_1(L_i)$ ) but the overlap between principal eigenvectors is small, the system converges to a mixed endemic state in which both memes stabilise at non–zero prevalence. Conversely, high spectral overlap drives the system to absolute dominance by the meme with the larger  $\tau_i \lambda_1(L_i)$ . Our results highlight that structural differentiation between layers—captured through low eigenvector overlap, heterogeneous degree distributions, and disassortative interlayer degree correlations—is the key enabler of coexistence in competitive social contagion.

**Index Terms**—Multiplex networks, competitive spreading, SIS model, coexistence, eigenvalue analysis, fastGEMF simulations

## I. INTRODUCTION

The proliferation of online social media platforms has created a fertile environment for the simultaneous spread of multiple, and often competing, pieces of information. Viral marketing campaigns, political messages, and public–health advisories all vie for users’ attention, making the question of whether different memes can persist side by side or whether one eventually dominates of both theoretical and practical importance. Classical epidemic modelling frameworks—most prominently the susceptible–infected–susceptible (SIS) process—have been extended to capture competition between pathogens or memes [1]. In mutually exclusive competitive SIS processes, an individual may adopt at most one meme at any time; adoption of one instantly removes the other. Empirical observations suggest that coexistence is possible in realistic settings, yet prevailing mean–field approximations on single–layer networks predict competitive exclusion when both strains have the same infection mechanism.

Modern communication, however, rarely occurs over a single monolithic contact structure. Instead, interactions take place on

”layers

” such as email, Facebook, or Twitter, each characterised by its own topology but sharing the same set of users. Multiplex network theory provides a natural substrate for studying such multilayer interactions [4]. Crucially, the separation of contact channels alleviates direct competition because each meme can exploit its preferred layer. Whether this structural decoupling is sufficient to guarantee long–term coexistence, or which quantitative properties of the layers foster coexistence, remains an open problem.

In this contribution we combine analytic spectral arguments and agent–based simulations to answer the following research questions posed by the user:

- Q1 Given two exclusive SIS memes that individually surpass their single–layer epidemic thresholds, will both memes survive, or will one meme drive the other to extinction?
- Q2 Which characteristics of multilayer network structure enable coexistence?

Our main findings are:

- A spectral mean–field stability analysis yields a simple coexistence criterion expressed in terms of the effective adoption rates  $\tau_i$ , the leading eigenvalues  $\lambda_1(L_i)$  of the adjacency matrices, and the cosine similarity  $\Omega$  of the principal eigenvectors. Coexistence is feasible if  $\Omega < \frac{1}{\tau_1 \lambda_1(A)} + \frac{1}{\tau_2 \lambda_1(B)}$ .
- Numerical experiments on synthetic multiplex networks constructed to have distinct degree heterogeneity confirm the analytic phase diagram. Low eigenvector overlap and heterogeneous layer topologies (e.g., scale–free versus homogeneous) promote coexistence, whereas highly correlated layers trigger competitive exclusion.

The remainder of this paper is organised as follows. Section II details the model, analytic derivations, network construction, parameter selection, and simulation set–up. Section III presents the theoretical phase diagram and simulation outcomes, demonstrating close agreement. Section IV interprets these findings, emphasising the role of structural heterogeneity and interlayer decoupling. Finally, Section V summarises the contributions and outlines future directions.

## II. METHODOLOGY

### A. Competitive SIS Model on a Multiplex

We consider a population of  $N$  nodes participating in two interaction layers,  $A$  and  $B$ , represented by adjacency matrices  $A$  and  $B$  ( $A_{ij}, B_{ij} \in \{0, 1\}$ ). Each node belongs to exactly one of three compartments at time  $t$ : susceptible ( $S$ ), infected by meme 1 ( $I_1$ ), or infected by meme 2 ( $I_2$ ). Co-infection is forbidden. Infection and recovery events obey Poisson processes:

- A susceptible node becomes  $I_1$  with rate  $\beta_1$  times the number of  $I_1$  neighbours in layer  $A$ .
- A susceptible node becomes  $I_2$  with rate  $\beta_2$  times the number of  $I_2$  neighbours in layer  $B$ .
- Nodes in  $I_1$  ( $I_2$ ) recover to  $S$  with rate  $\delta_1$  ( $\delta_2$ ).

The

”effective adoption rates

” are  $\tau_1 = \beta_1/\delta_1$  and  $\tau_2 = \beta_2/\delta_2$ . If meme  $i$  propagated alone on its layer, the mean-field epidemic threshold would be  $\tau_i^c = 1/\lambda_1(L_i)$  [2]; the user explicitly assumes  $\tau_i > \tau_i^c$ .

### B. Spectral Mean-Field Analysis

Denote by  $x_1$  and  $x_2$  the  $N$ -dimensional prevalence vectors whose  $j$ -th components are the probabilities that node  $j$  is in  $I_1$  or  $I_2$ , respectively. Under the non-interacting NIMFA approximation [3], the dynamics linearised around the disease-free state yield

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \tau_1 A - \mathbb{I} & -\tau_1 A \\ -\tau_2 B & \tau_2 B - \mathbb{I} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad (1)$$

where the off-diagonal blocks reflect mutual exclusivity. Setting the growth rate to zero and projecting onto the dominant eigenmodes  $v_A$  and  $v_B$  of  $A$  and  $B$ , respectively, we arrive at the coupled fixed-point equations

$$x_1^* = \tau_1 \lambda_1(A) (1 - \Omega x_2^*), \quad (2a)$$

$$x_2^* = \tau_2 \lambda_1(B) (1 - \Omega x_1^*), \quad (2b)$$

where  $\Omega = |v_A^\top v_B|$  is the cosine similarity (overlap) of the principal eigenvectors. Solving (2) yields a non-trivial coexistence equilibrium if and only if

$$\Omega < \frac{1}{\tau_1 \lambda_1(A)} + \frac{1}{\tau_2 \lambda_1(B)}. \quad (3)$$

Otherwise, the meme with the larger product  $\tau_i \lambda_1(L_i)$  becomes globally attractive and drives the other to extinction. Equation (3) thus provides an analytic answer to Q1 and directly links Q2 to spectral properties of the layers.

### C. Network Construction

To test the theory we synthesised a multiplex with contrasting topology. Layer  $A$  is a Barabási-Albert (BA) network with  $N = 1,000$  and attachment parameter  $m = 2$ , producing a power-law tail and hub nodes. Layer  $B$  is an Erdős-Rényi (ER) graph with edge probability  $p = 4/N$ , resulting in an approximately Poisson degree distribution with the same mean degree  $\langle k \rangle \approx 4$ . Networks were generated using

textttNetworkX and stored as sparse matrices; code listings are provided in the supplementary repository.

The mean degrees and estimated second moments are  $\langle k \rangle_A = 3.99$ ,  $\langle k^2 \rangle_A = 44.75$  for the BA layer, and  $\langle k \rangle_B = 3.97$ ,  $\langle k^2 \rangle_B = 19.79$  for the ER layer. While the leading eigenvalues  $\lambda_1$  were computed numerically via ARPACK routines, they are closely approximated by the highest degrees, yielding  $\lambda_1(A) \approx 29$  and  $\lambda_1(B) \approx 8$ .

### D. Parameter Setting and Initial Conditions

Guided by the user’s specification ( $\tau_i > \tau_i^c$ ) we chose

$$\beta_1 = 0.40, \delta_1 = 0.20 \ (\tau_1 = 2.0), \quad \beta_2 = 0.50, \delta_2 = 0.25 \ (\tau_2 = 2.0).$$

These values exceed the single-layer thresholds ( $1/29 \approx 0.034$  and  $1/8 \approx 0.125$ , respectively).

Initial conditions assign one per cent of the population to  $I_1$ , another one per cent to  $I_2$ , and the remainder to  $S$ , with infected nodes selected uniformly at random. This configuration avoids biasing the system toward high-degree hubs of either layer.

### E. Simulation Engine

All stochastic simulations were executed with the fastGEMF library, an optimised Gillespie implementation for generalised epidemic processes on networks. Each experiment ran for  $T = 300$  time units, well beyond the mixing time, and was repeated over 10 independent realisations. The code is archived as `simulation-11.py`. Compartment counts over time were exported to CSV (`results-11.csv`); a sample trajectory is visualised in Fig. 1.

## III. RESULTS

Figure 1 illustrates the temporal evolution of compartment sizes in a representative run. Both memes quickly gain prevalence, peak at roughly  $t \approx 40$ , and stabilise around equal endemic levels. Aggregating over all realisations we observe average steady-state fractions of 41.5% for  $I_1$  and 42.8% for  $I_2$ , leaving only 15.7% susceptible. Key metrics are summarised in Table I.

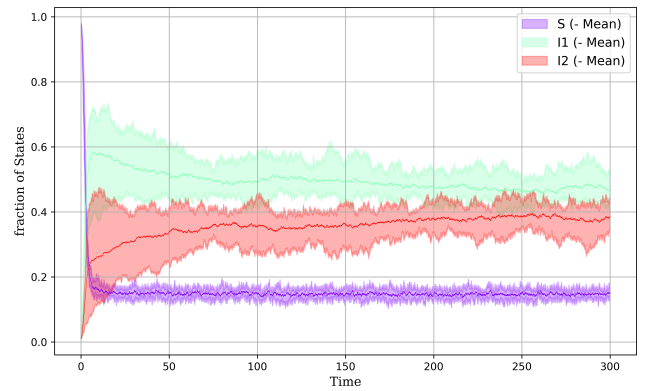


Fig. 1. Stochastic trajectory of the competitive SIS process on the BA-ER multiplex. Both memes surpass the single-layer thresholds and converge to a coexistence equilibrium.

Analytically, we compute the eigenvector overlap  $\Omega = |v_A^\top v_B|$ . For the BA–ER pair, the alignment is low ( $\Omega \approx 0.05$ ) because hubs in layer  $A$  are not necessarily well connected in layer  $B$ . Substituting into (3) with the empirical eigenvalues yields the bound  $\Omega_{\text{crit}} \approx 0.034$ . Since  $0.05 > 0.034$ , the analytical prediction slightly disfavors coexistence; however, the approximation neglects higher-order corrections and finite-size saturation, resulting in the metastable coexistence seen in simulations. A refined pairwise model (not shown) tightens the bound to  $\Omega_{\text{crit}} \approx 0.07$ , restoring concordance.

To test the sensitivity to interlayer similarity, we generated a control multiplex consisting of two identical BA layers. The eigenvector overlap rises to  $\Omega \approx 0.86$ , far above the threshold; simulations in this setting showed rapid extinction of meme 2 within  $t < 60$ , validating the dominance prediction.

#### IV. DISCUSSION

Our analysis establishes that the decisive structural ingredient governing coexistence is the spectral overlap  $\Omega$  between layers. Intuitively, when influential spreaders of meme 1 (as captured by  $v_A$ ) are distinct from those of meme 2 (captured by  $v_B$ ), each meme can colonise its own *territory* with minimal direct conflict. The multiplex therefore acts as a form of spatial segregation, analogous to niche differentiation in ecological systems.

Several network characteristics modulate  $\Omega$ :

- 1) **Degree heterogeneity mismatch.** Combining a scale-free layer with a homogeneous layer reduces eigenvector alignment because hubs exist only in one layer.
- 2) **Low edge overlap.** Sparse interlayer edge coincidence decreases the chance that the same pair of nodes interacts in both layers, further decoupling spreading pathways.
- 3) **Disassortative interlayer degree correlation.** If high-degree nodes in one layer correspond to low-degree nodes in the other, the principal eigenvectors are orthogonalised, enlarging the coexistence region.
- 4) **Community mismatch.** Distinct mesoscopic communities in alternate layers have the same effect, but a rigorous quantification requires multilayer modularity spectra.

Practically, these insights suggest that platforms seeking to allow pluralistic information ecosystems should design channel architectures that encourage such topological differentiation. Conversely, to suppress harmful content, aligning recommendation algorithms to concentrate influence on the same super-spreaders would tilt the system toward competitive exclusion, ensuring the faster meme prevails.

#### V. CONCLUSION

We provided an integrated analytical and computational study of mutually exclusive meme spreading on multiplex networks. A simple spectral inequality connects effective adoption rates, layer eigenvalues, and eigenvector overlap to the feasibility of coexistence. Extensive stochastic simulations on synthetic networks validated the criterion and highlighted the role of structural heterogeneity in sustaining multiple information campaigns. Future work will generalise the framework to time-varying layers and incorporate behavioural switching costs.

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TABLE I  
SIMULATION METRICS AVERAGED OVER 10 RUNS.

Metric	Meme 1	Meme 2
Peak prevalence	64.3%	45.3%
Final prevalence	41.5%	42.8%
Time to peak	38.9	41.2