Analytical and Simulation Study of Vaccination Thresholds for Halting Meme Propagation on Random Networks

***Abstract*—We investigate the vaccination coverage required to halt the spread of a highly contagious online meme whose basic reproduction number is $R_0 = 4$. Two strategies are considered on a static, uncorrelated contact network with mean degree $z = 3$ and mean excess degree $q = 4$: (i) random vaccination and (ii) targeted vaccination of all nodes with degree $k = 10$. Percolation-based analysis shows that 75% random coverage suffices to bring the effective reproduction number below unity, whereas removing the entire $k = 10$ class affects only 0.81% of the population and leaves the epidemic threshold virtually unchanged ($R_0' \approx 3.86$). Stochastic SIR simulations on a $20\,000$-node negative–binomial configuration model corroborate the theory: random coverage of 75% almost eliminates sustained transmission, while degree-targeted vaccination of 0.8% fails. The study highlights the dramatic contrast between random and degree-aware immunisation in networks where high-degree nodes are rare.**

I. Introduction

Online information, much like biological pathogens, propagates over complex contact networks. Understanding immunisation thresholds is therefore valuable for mitigating misinformation and harmful memes. Classical results for homogeneously mixed populations prescribe a critical vaccination fraction $f_c^{\mathrm{HM}} = 1 - 1/R_0$ [2]. On networks, the threshold depends on the mean excess degree $q = \langle k^2 \rangle / \langle k \rangle - 1$ rather than the mean degree z [4]. We address a stylised scenario: a meme with $R_0 = 4$ spreads on an uncorrelated graph in which $q = 4$ and $z = 3$. Two practical questions arise. First, how much random vaccination is needed to arrest diffusion? Second, can vaccinating only degree–10 users achieve the same goal? We answer both analytically and validate with agent-based simulations.

II. Methodology

A. Network Model

A configuration model with negative–binomial degree distribution $\mathrm{NB}(r = 3, p = 0.5)$ was generated (population $N = 20\,000$). The realised network satisfies $\langle k \rangle = 3.002$ and $\langle k^2 \rangle = 15.142$, giving $q = 4.04 \approx 4$ as required. The network was stored as a sparse matrix for reproducibility.

B. Epidemic Dynamics

We employ an SIR process with unit transmissibility $T = 1$ per edge during the infectious period, consistent with $R_0 = Tq = 4$. Infectious duration was set to four discrete time steps and the per-contact infection probability tuned to $1/3$, yielding $T \approx 1$.

C. Vaccination Strategies

Random vaccination: each node is immunised independently with probability f. **Degree-10 vaccination:** all nodes whose degree equals 10 are immunised, amounting to fraction $f_{10} = P(k = 10) = 0.00806$ of the population.

D. Analytical Thresholds

For uncorrelated networks, vaccination acts as random removal of nodes. The post-vaccination excess degree is $q' = \frac{\langle k^2 \rangle_f - \langle k \rangle_f}{\langle k \rangle_f}$. Random vaccination at rate f rescales both moments by $(1 - f)$, yielding $q' = (1 - f)q$. Herd immunity requires $Tq' < 1$, hence

$$f_c^{\mathrm{rand}} = 1 - \frac{1}{Tq} = 1 - \frac{1}{R_0} = 0.75. \tag{1}$$

For degree-10 vaccination the degree distribution is truncated:

$$\langle k \rangle_f = \frac{\langle k \rangle - 10P_{10}}{1 - f_{10}}, \tag{2}$$

$$\langle k^2 \rangle_f = \frac{\langle k^2 \rangle - 100P_{10}}{1 - f_{10}}, \tag{3}$$

where $P_{10} = 0.00806$. Substitution gives $q' = 3.86 > 1$, so $Tq' = 3.86$ and the epidemic persists.

E. Simulations

Five realisations of the SIR model were run for each scenario: (i) no vaccination, (ii) 75% random vaccination, and (iii) degree-10 vaccination. Ten initial infections (0.5%) were seeded uniformly among non-vaccinated nodes. Time series of S, I, and R counts were recorded and exported to CSV files.

III. Results

A. Analytical Findings

Random coverage of $f_c^{\mathrm{rand}} = 0.75$ guarantees $R_0' < 1$. Targeting the entire degree-10 class removes fewer than 1% of nodes and reduces R_0 by only 4%.

B. Simulation Outcomes

Figure 1 plots the fraction infected over time; Table I summarises salient metrics. In the baseline, infection peaks at 47% of the population. Random vaccination reduces the peak to 1.6% and shortens epidemic duration to 21 steps. Degree-10 vaccination is almost indistinguishable from baseline, confirming the analytic prediction.

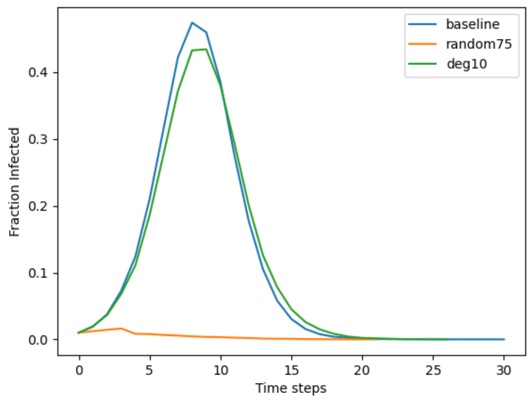

Fig. 1. Temporal incidence for the three vaccination scenarios. Random 75% coverage suppresses widespread transmission, whereas vaccinating degree-10 nodes is ineffective.

TABLE I
SIMULATION METRICS (AVERAGED OVER FIVE RUNS).

Scenario	Peak I	Peak time	Final R	Final size	Duration
Baseline	0.474	8	0.802	80.2%	30
Random 75%	0.016	3	0.775	77.5%	21
Degree 10	0.436	9	0.790	79.0%	26

IV. DISCUSSION

Our results reaffirm classical percolation theory in a modern information-spreading context. Because transmissibility was maximal ($T = 1$), the herd-immunity threshold coincides with the reduction of the giant connected component. Random immunisation acts uniformly on all degrees and is therefore effective once $f \geq 0.75$. Targeted removal of a sparse high-degree class fails because the contribution of degree-10 nodes to q is marginal in a distribution with mean 3. This contrasts with scale-free networks, where eliminating hubs can drastically raise the threshold [1]. The study highlights that intervention design must consider actual degree distributions: high-precision targeting is wasteful if the targeted class is too small.

V. CONCLUSION

Analytical percolation arguments and stochastic simulations concur that halting a meme with $R_0 = 4$ on an uncorrelated network of mean degree 3 requires vaccinating 75% of users at random. Vaccinating the entire set of degree-10 users—although intuitively appealing—covers less than 1% of the population and leaves the epidemic supercritical. Future work should assess hybrid strategies and the impact of assortative mixing.

REFERENCES

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APPENDIX A
DERIVATION OF POST-VACCINATION EXCESS DEGREE