

The rapid on-line diffusion of short messages, or “memes,” can be mathematically treated as an epidemic spreading process. Introduction The spread of information in on-line social systems bears striking resemblance to pathogen transmission in classic well-mixed models offer a parsimonious answer: inoculating a fraction $1 - 1/R_0$ of individuals collapses the basic reproduction number. In this study we revisit the archetypal configuration model under the simplifying assumptions of (i) negligible degree-degree correlations, (ii) static network structure, and (iii) no degree-degree correlations.

Random vaccination—each node is independently immunized with probability p .

Fixed-degree vaccination—all nodes of degree exactly $k = 10$ are vaccinated.

The remainder of the paper is organized as follows. Section 2 details the analytical framework, network construction, parameter estimation, and simulation results. Methodology Analytical vaccination threshold on uncorrelated networks For configuration networks without degree correlations, the analytical threshold is $p_c = 1 - 1/R_0$. For configuration networks with degree correlations, the analytical threshold is $p_c = 1 - 1/R_0$.

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When immunization is *not* random but concentrated on a specific degree k , the algebra changes. Let P_k be the pre-vaccination degree distribution. The post-vaccination degree distribution is $P'_k = \frac{\langle k^2 \rangle - \langle k \rangle^2 f}{1 - f} P_k$. Plugging these into q' yields

Because f is typically tiny for moderate k in thin-tailed networks, q' often remains above unity even after full removal of the targeted degree.

Selecting a representative degree distribution To validate the analytics in silico we require a concrete network whose mean degree is $\langle k \rangle = 4$.

so vaccinating all $k = 10$ nodes removes only 0.81% of the population.

Network synthesis Using the **networkx** configuration-model generator, we sampled $N = 10^4$ degrees from the selected network. $q = 3.9623$ (≈ 4). The adjacency matrix was stored as a CSR matrix (**network.npz**) for simulation reproducibility.

Disease dynamics and parameters The meme dynamics were modeled as a continuous-time SIR process with infection rate β and recovery rate γ .

Vaccination scenarios and initialization **Scenario 1 – Random vaccination.** Each node was independently assigned to R with probability p .

Scenario 2 – Fixed-degree vaccination. All nodes of degree exactly 10 were pre-assigned to R . This immunized fraction was $p = 0.0081$.

Simulation engine We employed **fastGEMF** v2.1, which implements the generalized epidemic mean-field framework via the generalized mean-field approximation.

Results Analytical predictions Equation eq:rand_thresholdprescribes $p_c = 0.75$ for random immunization. For fixed-degree vaccination, $p_c = 0.75$.

Stochastic simulations validate analytics *Scenario 1 (random 75% vaccination). The epidemic never gained traction (Fig. 11).

*Scenario 2 (all $k = 10$ vaccinated). Figure 12 shows a dramatic outbreak: prevalence climbed to 15.9% around $t = 23$ and then declined.

Figure 11: Temporal evolution of compartments under 75% random vaccination. The infectious compartment (red) remains near zero.

Figure 12: Temporal evolution under universal degree-10 vaccination. A large-scale outbreak occurs, peaking at $t \approx 23$.

Discussion The combined analytical–computational investigation yields three principal insights.

Random versus targeted immunization efficiency Random vaccination achieves herd immunity exactly at the classical threshold $p_c = 1 - 1/R_0$.

Implications for social-media content moderation Online platforms often rely on flagging or throttling a handful of large hubs.

Limitations and future work We assumed sterilizing immunity, static contact structure, and no degree-degree correlations.

Conclusion We analytically derived and numerically validated vaccination thresholds to halt meme propagation on an uncorrelated network.

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Reproducibility material All Python scripts, networks, and CSV outputs are included in the accompanying **output** directory.