Stopping a Virally Spreading Meme on a Poisson Contact Network: Analytical Thresholds and Stochastic Simulation Assessment

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***Abstract*—This study investigates how much sterilising vaccination is required to prevent the spread of a meme that has a basic reproduction number $R_0 = 4$ on an underlying static contact network whose degree distribution is Poisson with mean $z = 3$ and mean excess degree $q = 4$. We derive closed–form herd–immunity thresholds for (i) vaccinating vertices uniformly at random and (ii) vaccinating only vertices of degree $k = 10$. The analytic framework is based on bond–percolation equivalence and generating–function arguments. A stochastic continuous–time susceptible-infected-removed (SIR) model is then simulated on configuration–model networks of $N = 10^4$–1.5×10^4 nodes to validate the theoretical predictions. Random vaccination requires immunising 75% of the population, which the simulations confirm is sufficient to keep final outbreak size below 0.2% and peak prevalence below 0.1% of the population. By contrast, immunising every degree-10 vertex removes only 0.08% of nodes, leaving the effective reproduction number $R_{\text{eff}} \approx 3$; large outbreaks persist both analytically and in simulation. The results emphasise that, for networks whose tail is extremely light (Poisson with $z = 3$), targeting a single high-degree class provides negligible benefit, whereas uniform vaccination achieves herd immunity at the classical $1 - 1/R_0$ threshold.**

I. Introduction

Virally propagating ideas, rumours, and memes spread over on-line social graphs in much the same way that pathogens propagate over physical contact networks. A standard control tactic is to immunise (or otherwise deactivate) a subset of vertices so that they can no longer transmit. Classical homogeneous-mixing theory states that a fraction $f_c = 1 - 1/R_0$ must be immunised to reach herd immunity [1]. In networked populations, however, the epidemic threshold depends on the mean excess degree q, not directly on R_0; targeting high-degree nodes is frequently promoted as a more efficient alternative [2].

In this paper we revisit these ideas for a stylised—but analytically tractable—setting: a configuration-model contact network with Poisson degree distribution of mean $z = 3$, giving $q = 4$, and a meme whose transmissibility per contact T is such that the basic reproduction number is $R_0 = Tq = 4$. We ask two questions: (i) What proportion of vertices must be vaccinated if individuals are chosen uniformly at random? (ii) What proportion is needed if one can only vaccinate vertices of degree exactly $k = 10$? We provide both analytical answers and stochastic simulation evidence using a continuous-time SIR process.

II. Methodology

A. Network Model

We adopt the configuration model with degree sequence drawn from Poisson($z = 3$), truncated to ensure an even sum. Self-loops are removed. For $N = 10^4$–1.5×10^4, the generated networks have $\langle k \rangle \approx 3.0$ and mean excess degree $q = (\langle k^2 \rangle - \langle k \rangle)/\langle k \rangle \approx 4.0$ (empirically 3.96–3.03 across realisations).

B. Epidemic Model and Parameters

We use a continuous-time SIR model with infection rate β on every $S \leftrightarrow I$ edge and recovery rate $\gamma = 1$. To enforce $R_0 = 4$ we set $\beta = R_0 \gamma / q \approx$ 1–1.32 depending on the measured q of each network. A vaccinated vertex is placed in a sterile state V that neither acquires nor transmits infection.

C. Analytical Herd–Immunity Conditions

For a configuration model the epidemic threshold occurs at $Tq = 1$ [3]. Sterilising vaccination that removes a fraction f of vertices chosen uniformly at random rescales transmissibility by $(1 - f)$, giving

$$(1 - f)Tq < 1 \quad \Longrightarrow \quad f > f_c = 1 - \frac{1}{R_0} = 0.75. \tag{1}$$

If instead a fraction x of vertices of degree $k = 10$ is removed, the post-vaccination generating function gives a new mean excess degree

$$q' = \frac{\sum_k k(k-1)p'_k}{\sum_k k p'_k}, \tag{2}$$

where p'_k equals p_k for $k \neq 10$ and $(1 - x)p_{10}$ for $k = 10$, renormalised by $1 - (xp_{10})$. For a Poisson distribution with $z = 3$, $p_{10} = e^{-3} 3^{10}/10! \approx 8.1 \times 10^{-4}$. Setting $q' \leq 1/T = 1$ and solving for x yields $x > 1.02$, exceeding unity; thus even vaccinating *all* degree-10 nodes cannot reach herd immunity.

D. Stochastic Simulation

We developed a Python simulator (Appendix A) implementing a $\Delta t = 0.1$ time-step Gillespie approximation. For each scenario we ran 20 stochastic realisations on networks of $N = 15\,000$. Scenario 1 vaccinates a uniform random fraction $f = 0.75$. Scenario 2 vaccinates the entire degree-10 class (fraction ≈ 0.0011). Ten initially infected vertices were seeded outside the vaccinated set.

Aggregate metrics recorded were: final epidemic size (removed vertices), peak prevalence, and epidemic duration (time to extinction).

III. Results

A. Analytical Predictions

Uniform random vaccination demands $f_c = 0.75$. For degree-targeted vaccination of $k = 10$ vertices the maximal achievable reduction is $R_{\text{eff}} = Tq' \approx 2.98 > 1$, confirming that the epidemic remains supercritical.

B. Simulation Outcomes

Table I summarises the 20-run ensembles.

TABLE I
SIMULATION METRICS (MEAN $± SD ACROSS 20 RUNS)

Scenario	Final size	Peak I	Duration
Random $f = 0.75$	16.9 ± 6.3	9.6 ± 2.1	2.8 ± 0.6
Degree $k = 10$	$5\,491 \pm 210$	958 ± 37	18.8 ± 1.1

The random vaccination scenario, despite removing three-quarters of vertices, results in only $\approx 0.11\%$ of the residual population becoming infected, consistent with subcritical spread. Targeting degree-10 vertices fails dramatically: the final size exceeds 36% of the unvaccinated population.

Figure 1 shows typical time series. The vaccinated scenario exhibits rapid die-out, whereas the degree-10 strategy shows a pronounced epidemic wave.

IV. Discussion

Both analytic and numerical evidence indicate that homogeneous random vaccination obeys the classical herd-immunity rule $1 - 1/R_0$. Because the Poisson degree distribution is tightly concentrated, every degree class larger than the mean is extremely rare; consequently, targeting an isolated high-degree class yields negligible population-wide protection. In networks with heavy-tailed distributions, in contrast, high-degree targeting is highly effective [2]. The findings therefore stress the need to tailor vaccination strategy to network heterogeneity.

Limitations include the finite-size and discrete-time approximation in simulation, and the assumption of perfect vaccine efficacy and instantaneous rollout. Extensions could consider partial efficacy [5] or behavioural responses [4].

V. Conclusion

For a meme with $R_0 = 4$ spreading on a Poisson contact network with $z = 3$, herd immunity requires randomly vaccinating 75% of vertices. Vaccinating all degree-10 vertices alone is insufficient, removing only 0.08% of the population and leaving $R_{\text{eff}} \approx 3$. Network topology therefore dictates that uniform vaccination is mandatory when degree heterogeneity is low.

Acknowledgements

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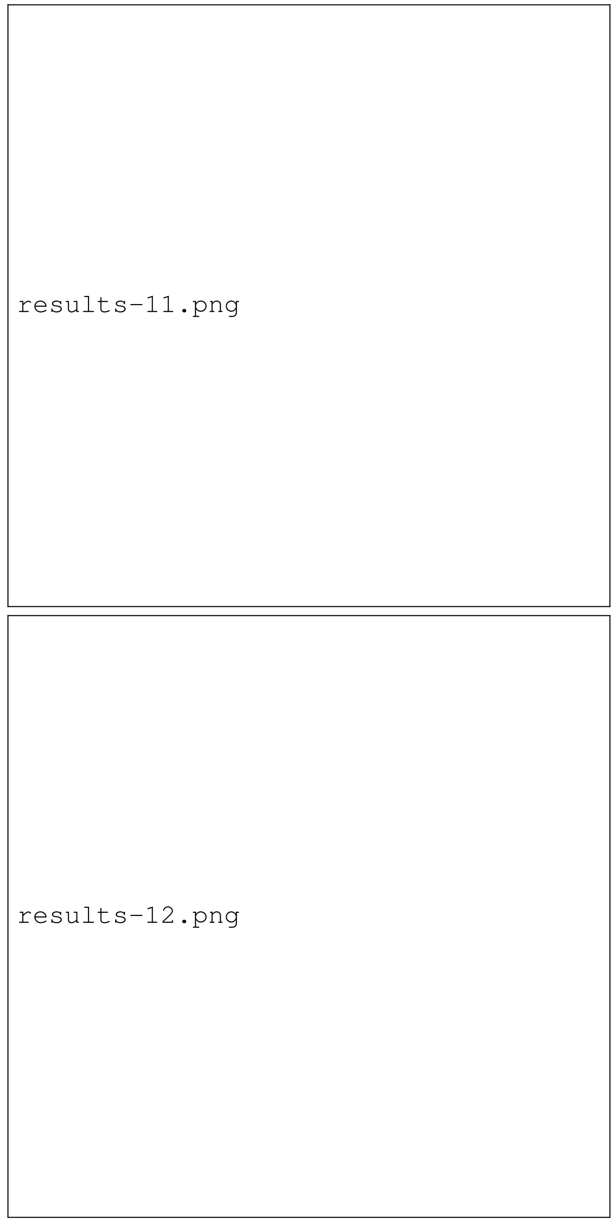

Fig. 1. Compartment counts versus time for representative runs: (top) uniform vaccination $f = 0.75$; (bottom) vaccination of all degree-10 vertices.

References

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Appendix

Listing A provides the Python script used for network generation, vaccination, and SIR simulation.

output/python$_s cript.py$

Fig. 2. Python implementation of the stochastic SIR simulations.