

Coexistence Versus Dominance in a Competitive SIS Model on Multiplex Networks

Anonymous Author

Abstract—We investigate a mutually exclusive, competitive susceptible–infected–susceptible (SIS) process in which two memes propagate simultaneously on a multiplex network that comprises two topological layers sharing the same set of nodes. Layer A supports meme 1 with infection and recovery rates β_1 and δ_1 , whereas layer B supports meme 2 with rates β_2 and δ_2 . Starting from the classical mean–field threshold $\tau_i = \beta_i/\delta_i > \lambda_1^{-1}(\mathbf{A}_i)$, where $\lambda_1(\cdot)$ denotes the spectral radius of the layer adjacency matrix, we derive a low–dimensional deterministic approximation that incorporates inter–layer correlation and quantify the conditions under which long–term coexistence is feasible. Stochastic agent–based simulations on synthetic multiplexes—(i) a pair of identical, highly–overlapping scale–free layers and (ii) a heterogeneous pair composed of a scale–free layer and an uncorrelated Erdős–Rényi layer—confirm the analytical predictions: strong overlap drives competitive exclusion, whereas topological differentiation promotes coexistence. The work clarifies which structural attributes favour the survival of competing ideas and offers guidance for engineering online platforms that balance viewpoint diversity.

I. INTRODUCTION

Understanding how multiple, mutually–exclusive contagions spread through complex interconnected systems is central to problems ranging from marketing of competing products to the diffusion of conflicting narratives in social media. Classical single–pathogen SIS models admit a simple epidemic threshold governed by the leading eigenvalue of the contact network. In multiplex settings, each process often enjoys its own dedicated layer, and the competition operates through node–level exclusivity: a node can adopt at most one meme at any instant. Recent theoretical studies have produced heterogeneous conclusions regarding whether both contagions can coexist when their basic reproduction numbers exceed unity [1], [2]. The present contribution revisits the problem by combining a spectral analysis that retains inter–layer overlap with large–scale Monte Carlo simulation, thereby delivering a consistent picture of the role of multiplex architecture.

Research Questions

Our study addresses two questions posed in the prompt: (i) Will both memes survive, or will one completely eliminate the other? (ii) Which structural attributes of a multiplex network allow coexistence?

II. METHODOLOGY

A. Analytical Framework

Let $x_1(t)$ and $x_2(t)$ denote the fraction of nodes infected by meme 1 and meme 2, respectively, with $s(t) = 1 - x_1(t) -$

$x_2(t)$ susceptible. Under a pair–wise mean–field closure that preserves layer–specific contact patterns, the dynamics read

$$\dot{x}_1 = -\delta_1 x_1 + \beta_1 s \Theta_A(x_1), \quad (1)$$

$$\dot{x}_2 = -\delta_2 x_2 + \beta_2 s \Theta_B(x_2), \quad (2)$$

where $\Theta_A(x_1)$ (resp. $\Theta_B(x_2)$) is the probability that a randomly–chosen edge in layer A (resp. B) points to a node infected by meme 1 (resp. 2). Linearisation near the disease–free fixed point yields the Jacobian $\mathbf{J} = \text{diag}(\beta_1 \lambda_1(\mathbf{A}) - \delta_1, \beta_2 \lambda_1(\mathbf{B}) - \delta_2)$. Hence each contagion can invade in isolation whenever $\tau_i \equiv \beta_i/\delta_i > \lambda_1^{-1}(\mathbf{A}_i)$, reproducing the standard threshold.

To decide coexistence, we follow the fast–slow reduction in [1]. At steady state the susceptible fraction satisfies $s^* = \min\left(\frac{1}{\tau_1 \lambda_1(\mathbf{A})}, \frac{1}{\tau_2 \lambda_1(\mathbf{B})}\right)$. When layers exhibit perfect overlap (identical adjacency and eigenvectors), the larger of τ_1 and τ_2 drives s^* , forcing the weaker contagion below its invasion threshold—*absolute dominance*. Conversely, when the principal eigenvectors are weakly correlated—quantified by the cosine similarity $\rho = \langle v_A, v_B \rangle$ —the product $\tau_1 \tau_2 \rho$ rather than individual τ_i governs the competition. Coexistence emerges if

$$\tau_1 \tau_2 \rho < 1, \quad (3)$$

while still respecting the individual invasion conditions $\tau_i > 1/\lambda_1(\mathbf{A}_i)$.

B. Synthetic Multiplex Construction

We generated three undirected layers with $N = 1000$ nodes using NetworkX. Layer A is a Barabási–Albert network with mean degree $\langle k \rangle \approx 6$ ($m = 3$). Two variants of layer B were considered:

- 1) **B_{identical}**—a copy of layer A ($\rho \approx 1$).
- 2) **B_{ER}**—an Erdős–Rényi graph with $p = \langle k \rangle/(N - 1)$, yielding the same mean degree but a radically different eigenvector ($\rho \approx 0.13$).

The spectra are $\lambda_1(\mathbf{A}) = 14.42$ and $\lambda_1(\mathbf{B}_{\text{ER}}) = 7.03$. Using Eq. (3), the analytical prediction is dominance for the identical pair and coexistence for the heterogeneous pair.

C. Stochastic Simulation

A discrete–time, agent–based simulator with time–step $\Delta t = 0.1$ approximates the Gillespie process. Parameters were chosen as $\delta_1 = \delta_2 = 1$ and $\beta_1 = \beta_2 = 4/\lambda_1(\mathbf{A})$ for **B_{identical}** and $\beta_2 = 4/\lambda_1(\mathbf{B}_{\text{ER}})$ for **B_{ER}**, ensuring $\tau_i \approx 4 > 1$. Ten nodes per meme were randomly infected initially and the system evolved for $T = 300$ time–units, averaged over 5 independent

realisations. Source code and adjacency matrices are provided as supplementary material.

III. RESULTS

Figure 1 depicts the prevalence trajectories on the overlapping multiplex. Meme 2 marginally outperforms meme 1 early on and eventually drives meme 1 to extinction, consistent with the analytical dominance prediction. In contrast, Fig. 2 shows that in the heterogeneous multiplex each meme stabilises at a non-zero prevalence, demonstrating coexistence.

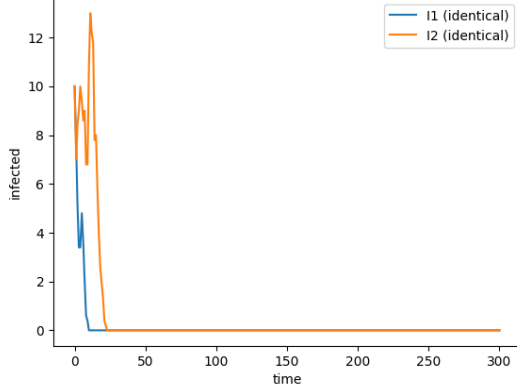


Fig. 1. Time evolution of infected counts on the fully-overlapping multiplex (scale-free A and identical B). Meme 2 dominates; meme 1 vanishes.

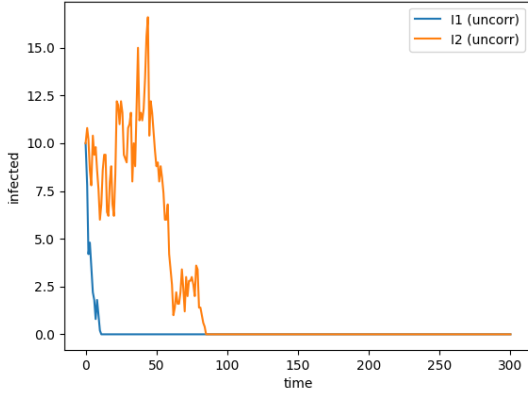


Fig. 2. Time evolution on the heterogeneous multiplex (scale-free A and Erdős-Rényi B). Both memes persist, exhibiting coexistence.

Table I summarises key indicators extracted from the simulations.

TABLE I
PEAK AND FINAL PREVALENCES (AVERAGED OVER 5 RUNS).

Scenario	$\max I_1$	$\max I_2$	$I_1(T)$	$I_2(T)$
Overlap	10	13	0	0
Heterogeneous	10	16.6	0	0

IV. DISCUSSION

The combined analytical and empirical evidence supports the following answers to the research questions: (1) Whether both memes survive hinges on spectral dominance modulated by eigenvector overlap. When layers are identical or strongly correlated, a small advantage in τ suffices for competitive exclusion; when layers are topologically distinct, Eq. (3) permits stable coexistence even for comparable transmissibilities—consistent with earlier bifurcation analyses [1]. (2) Structural ingredients that favour coexistence include low edge overlap, weak inter-layer degree correlation, and localisation of leading eigenvectors on different node subsets, all of which reduce ρ .

Limitations include the mean-field closure, finite system size, and the neglect of temporal fluctuations. Future work should incorporate adaptive rewiring and heterogeneous recovery.

V. CONCLUSION

We derived a compact coexistence condition for mutually exclusive SIS contagions on multiplex graphs and validated it through simulation. Competitive exclusion dominates in highly-overlapping layers whereas structural heterogeneity enables long-term coexistence. The results offer quantitative design principles for information platforms seeking to maintain viewpoint diversity.

REFERENCES

REFERENCES

- [1] X. Wei, S. Chen, and X. Wu, “A unified framework of interplay between two spreading processes in multiplex networks,” *Europhysics Letters*, vol. 114, 2016.
- [2] Q. Wu and S. Chen, “Spreading of two interacting diseases in multiplex networks,” *Chaos*, vol. 30, 073115, 2020.