Cooperative Game Theory

Kanaparthy S V Samhita
Machine Learning Lab,
International Institute of Information Technology,
s.v.samhita@research.iiit.ac.in

Sujit P Gujar
Machine Learning Lab,
International Institute of Information Technology,
sujit.gujar@iiit.ac.in

Abstract

As we extend game theory from two-player zero-sum games to n player games for $n \geq 3$, there is a huge difference from a non-cooperative game-theoretic point of view. The existence of Nash equilibrium still holds and the challenges observed in two-player games follow here as well, But there is a new phenomenon that must be taken into account. That is the formation of a coalition among the subsets of players and act together to gain more utility than they could have gained by acting independently. This forms an essential aspect of game theory and is widely studied as part of Cooperative Game Theory. This report includes a concise study of Cooperative Games referred from the book "Game Theory and Mechanism Design" [1].

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Games with Contracts

The motivation to look at games with cooperation is the fact that in many games, Nash equilibrium yields non-optimal payoff compared to certain non-equilibrium outcomes.

	x2	**y2**
x1	4,4	0,7
y1	7,0	2,2

Table 1: **Pay-off Matrix**

Consider the payoff matrix shown above, this has $(y1, y2)$ as equilibrium which yields payoff (2,2). However, the non-equilibrium strategy $(x1, x2)$ yield higher payoffs (4,4) for both the players. In these cases, players may communicate themselves to coordinate their strategies or form contracts.

1.1 Correlated Strategy and Equilibrium

In games with contracts, a player who signs the contract is required to play according the designated strategy which is called *correlated strategy*.

Definition 1.1 (Correlated Strategy)**.** *Let $\Gamma = \langle N, (S_i), (u_i) \rangle$ be a strategic form game. A correlated strategy for a non-empty subset C (coalition) of the players is any probability distribution over the set of possible combinations of pure strategies that the player can choose. A correlated strategy τ_C for given coalition C belongs to $\Delta(S_C)$ where $S_C = \times_{i \in C} S_i$*

In the above definition a strategic form game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ where $N = 1, 2, \ldots, n$ is a finite set of players; $S_1, S_2, \ldots, S_n$ are the strategy sets of the players; and $u_i : S_1 \times S_2 \times \ldots \times S_n \to \mathbb{R}$ for $i = 1, 2, \ldots, n$ are utility functions.

Coalition with all the players (N) is called *grand coalition*. A coalition strategy for grand coalition is denoted by τ_N. Readers should note that there is a difference between a correlated strategy and a mixed strategy profile. A correlated strategy for grand coalition is a member of $\Delta(\times(S_i))$ and a mixed strategy is of $\times(\Delta(S_i))$.

In the games with contracts, contract is defined as,

Definition 1.2 (Contract). *Consider the vector* $\tau = (\tau_C)_{C\subseteq N}$.

$$\tau \in \times_{C\subseteq N}(\Delta(\times_{i\in C} S_i))$$

The vector τ of correlated strategies of all coalitions is a contract. Contract defines an extended game which is called *contract signing game*.

Definition 1.3 (Correlated Equilibrium). *Given a finite strategic form game* $\Gamma = \langle N, (S_i), (u_i)\rangle$, *a correlated strategy* $\alpha \in \Delta(S)$ *is called correlated equilibrium if*

$$u_i \geq \sum_{(s_i,s_{-i})\in S} \alpha(s_i, s_{-i}) \forall \delta_i : S_i \to S_i \forall i \in N$$

2

Nash Bargaining Theory

According to Nash, cooperative actions can be considered as the culmination of a certain process of bargaining among the cooperating players and consequently, cooperation between players can be studied using core concepts of non-cooperative game theory. This will transform a strategic form game into another strategic form game that has an extended strategy space for each player that captures bargaining with the other players to jointly plan cooperative strategies.

The *bargaining* refers to a situation in which

- two or more individuals have a possibility of concluding a mutually beneficial agreement
- there is a conflict of interest about which agreement to conclude
- no agreement may be imposed on any player without that player's approval

2.1 Two Player Bargaining Problem

The two person bargaining problem consists of a pair (F, v) where F is called the feasible set and v is called the disagreement point.

- F, the feasible set of allocations, is a closed, convex subset of $\mathbb{R}^2$
- The disagreement point $v = (v_1, v_2) \in \mathbb{R}^2$ represents the disagreement payoff allocation for the players. Also called *status-quo-point*, *default point*, *de facto point*. These are payoffs given in case the negotiations fail.
- The set $F \cap \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \geq v_1; x_2 \geq v_2\}$ is assumed to be non-empty and bounded.

2.2 Axioms

Any ideal bargaining solution is expected to satisfy the five following axioms:

1. Strong Efficiency
2. Individual Rationality
3. Scale Covariance
4. Independence of Irrelevant Alternatives
5. Symmetry

Strong Efficiency: Given a feasible set F, an allocation $x = (x_1, x_2) \in F$ is strongly efficient (Strongly Pareto Efficient) iff there do not exist another allocation $y = (y_1, y_2) \in F$ such that $y_1 \geq x_1; y_2 \geq y_1$ with at least one player having strict inequality.

An allocation $x = (x_1, x_2) \in F$ is weakly Pareto efficient or weakly efficient iff there exists no other point $y = (y_1, y_2) \in F$ such that $y_1 > x_1; y_2 > x_2$.

Let $f(F, v) = (f_1(F, v), f_2(F, v))$ denote the Nash bargaining solution for the bargaining problem (F, v).

Individual Rationality: States that no player should get less in bargaining solution than he/she could get in disagreement solution, i.e., $f_1(F, v) \geq v_1; f_2(F, v) \geq v_2$.

Scale Covariance: For numbers $\lambda_1, \lambda_2, \mu_1, \mu_2$ with $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ define the set $G = \{(\lambda_1 x_1 + \mu_1, \lambda_2 x_2 + \mu_2 : (x_1, x_2) \in F)\}$ and point $w = (\lambda_1 v_1 + \mu_1, \lambda_2 v_2 + \mu_2)$. Then the solution $f(G, w)$ is given by:

$$f(G, w) = (\lambda_1 f_1(F, w) + \mu_1, \lambda_2 f_2(F, w) + \mu_2)$$

The axiom says that applying affine utility transformations on (F, v) will not effect any relevant properties of the utility functions. The solution for transformed problem $(G, v))$ can be derived from that of (F, v) by the same transformations.

Independence of Irrelevant Alternatives: For any closed, convex set G,

$$G \subseteq F \text{ and } f(F, v) \in G \implies f(G, v) = f(F, v)$$

The axiom asserts that eliminating irrelevant alternatives should not effect the solution.

Symmetry: This axiom asserts that if the positions of two players are completely symmetric in the bargaining problem, then the solution should treat them symmetrically. That is,

$$v_1 = v_2 \text{ and } \{(x_2, x_1) : (x_1, x_2) \in F\} = F \implies f_1(F, v) = f_2(F, v)$$

There exist only unique solution that satisfies all the five axioms. The following theorem presents the solution:

Theorem 2.1. *Given a two person bargaining problem (F, v), there exists a unique solution function $f(.,.)$ that satisfies Axioms (1) - (5). The solution function satisfies, for every two person bargaining problem (F, v),*

$$\begin{aligned} f(F, v) \in &\operatorname{argmax}((x_1 - v_1)(x_2 - v_2)) \\ &(x_1, x_2) \in F \\ &x_1 \geq v_1; x_2 \geq v_2 \end{aligned}$$

2.3 Solutions - Bargaining Problem

There are two well-known approaches to the bargaining problem, Egalitarian and Utilitarian solutions.

Egalitarian Solution is a unique point $(x_1, x_2) \in F$ that is weakly efficient in F and satisfies the equal gain condition $(x_1 - v_1 = x_2 - v_2)$

Utilitarian Solution is any solution function that selects, for every two players bargaining problem (F, v), an allocation $(x_1, x_2) \in F$ such that $x_1 + x_2 = \max_{(y_1, y_2) \in F}(y_1 + y_2)$

Essential Bargaining Problem: Let (F, v) be an essential two player bargaining problem. Suppose x is an allocation vector such that $x \in F$ and $x \geq v$. Then x is the Nash bargaining solution for (F, v) iff there exist strictly positive numbers λ_1, λ_2 such that

$$\begin{aligned} \lambda_1 x_1 - \lambda_1 v_1 &= \lambda_2 x_2 - \lambda_2 v_2 \\ \lambda_1 x_1 + \lambda_2 x_2 &= \max_{y \in F}(\lambda_1 y_1 + \lambda_2 y_2) \end{aligned}$$

3

MultiPlayer Cooperative Games

We have seen two-player bargaining problem and their solutions. However, in $n > 2$ player games the Nash bargaining solutions may ignore the possibility of cooperation among proper subsets of the players. If the grand coalition negotiate effectively the n-player Nash bargaining solution is relevant. In cases where other coalitions negotiate effectively it is no longer relevant. There are $2^n - 1$ such coalitions possible for negotiation to be conducted and we need to understand what should outcome be as the result of balance of power among various coalitions.

3.1 TU Games

The assumption of *Transferable Utility* makes the cooperative games easier to analyze. The TU assumption implies that there is a commodity called money that the players can freely transfer among themselves such that the payoff of any player increases by one unit for every unit of money that it gets. With this assumption, cooperative possibilities of a game can be described as follows:

Definition 3.1 (Tranferable Utility Game (TU Game))**.** *A cooperative game with transferable utility is defined as the pair (N, v) where $N = \{1, 2, \ldots, n\}$ is a set of players and $v : 2^N \to \mathbb{R}$ is a characteristic function, with $v(\emptyset) = 0$.*

$v(C)$ is called the worth or value of the coalition C and it captures the total amount of transferable utility that the members of C could earn without any help from the players outside of C.

In a TU game *imputation* is an allocation which satisfies individual rationality and distributes the total value of the grand coalition among players. Formally, Imputation is given as follows

Definition 3.2 (Imputation)**.** *Given a TU game (N, v), an imputation is an allocation $x = \{x_1, \ldots, x_n\} \in \mathbb{R}$ that satisfies*

$$\begin{gathered} x_i \geq v(\{i\}) \quad \forall i \in N \\ \sum_{i \in N} x_i = v(N) \end{gathered}$$

3.1.1 TU Games with Special Structure

In this section we look at special classes of TU games.

Monotonic Games: A TU game (N, v) is called monotonic if

$$v(C) \leq v(D) \quad \forall C \subseteq D \subseteq N$$

In a monotonic TU game as we add players to a coalition, the worth of the coalition does not decrease.

Superadditive Games: A TU game (N, v) is said to be superadditive if

$$v(c \cup D) \leq v(C) + v(D) \quad \forall C, D \subseteq N \text{ such that } C \cap D$$

In Superadditive TU games in which two disjoint coalitions, on coming together, produce a non-negative additional value beyond the sum of the individual values.

Essential Superadditive Games: A superadditive game (N, v) is said to be inessential if

$$\sum_{i \in N} v(i) = v(N)$$

and essential otherwise.

3.1.2 Strategic Equivalence of TU Games

Two TU games (N, v) and (N, w) are said to be strategically equivalent if there exist constants $c_1, c_2, \ldots, c_n$ and $b > 0$ such that

$$w(C) = b(v(C) + \sum_{i \in C} c_i) \quad \forall C \subseteq N$$

3.2 Convex Games

Definition 3.3 (Convex Game)**.** *A TU game is said to be convex if*

$$v(C \cup D) + v(C \cap D) \geq v(C) + v(D) \quad \forall C, D \subseteq N$$

In the above definition, if C and D are disjoint ($C \cap D = \emptyset$). We have,

$$v(C \cup D) \geq v(C) + v(D) \quad \forall C, D \subseteq N$$

The above equation is definition of superaddivity. Every convex game is superadditive. The converse need not be true.

The following propositions are equivalent definitions for convex games

Proposition 1. *A TU game (N, v) is convex iff*

$$v(C \cup \{i\}) - v(C) \leq v(D \cup \{i\}) - v(D) \forall C \subseteq D \subseteq N \forall i \in N \backslash D$$

Proposition 2. *A TU game (N, v) is convex iff*

$$v(C \cup E) - v(C) \leq v(D \cup E) - v(D) \forall C \subseteq D \subseteq N \forall E \subseteq N \backslash D$$

3.3 Cost Games

The TU games seen so far are profit games since the worth $v(C)$ of a coalition C represents the total value that could be earned together by the members of the coalition. Instead if we consider the worth of a coalition C as the minimum total price the members of the coalition have to pay if the coalition were to form, then the resulting TU game is called a *cost game.*

4

Core of Coalition Games

Given a TU game, the key questions that are of our interest are: Which coalition will form? How should the value be divided among the members of coalition?

For this we will first look at concept *core* and its prerequisites.

Given a TU game (N, v), core is a feasible set of payoff allocations possible. A payoff allocation $x = (x_1, x_2, \ldots, x_n)$ is any vector in $\mathbb{R}^n$ where x_i is the payoff of player i.

Definition 4.1 (Feasible Allocation)**.** *An allocation $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$ is said to be feasible for a coalition C if $\sum_{i \in C} x_i \leq v(C)$*

Definition 4.2 (Rational Allocation)**.** *A payoff allocation $x = (x_1, x_2, \ldots, x_n)$ is said to be individually rational if*

$$x_i \geq v(\{i\}) \forall i \in N$$

It is collectively rational if

$$\sum_{i \in N} x_i = v(N)$$

It is coalitionally rational if

$$\sum_{i \in C} x_i \geq v(C) \quad \forall C \subseteq N$$

Collectively rational is same as (Pareto) efficiency. And coalitionally rational implies individual rationality. An Imputation is a payoff allocation that is individually rational and collectively rational.

Definition 4.3 (Core)**.** *The core of a TU game (N, v) denoted by $\mathbb{C}(N, v)$ is the set of all payoff allocations that are coalitionally rational and collectively rational.*

$$\mathbb{C}(N, v) = \Big\{ (x_1, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = v(N); \sum_{i \in C} x_i \geq v(C) \forall C \subseteq N \Big\}$$

In other words, the core is the set of all imputations that are coalitionally rational.

4.1 Key Notes

- If an allocation x that is feasible for N is not in the core, then there would exist some coalition C such that the players in C could all do strictly better than in x by cooperating together and dividing the worth $v(C)$ among themselves.
- If an allocation x belongs to the core, then it implies that for each player, a unilateral deviation will not make the player strictly better off. This means x is a Nash equilibrium of an underlying contract signing game.
- Each allocation in the core could be viewed as resulting out of effective negotiations.
- The core of a TU game can be empty. If the core is empty, then we are unable to draw any conclusions about the game.
- If the core consists of a large number of elements, then we have difficulty in preferring any particular allocation in the core.

- Convexity and compactness are two desirable properties satisfied by the core.

The necessary and sufficient condition for non-emptiness of core is called *balancedness*.

Definition 4.4 (Balanced TU games)**.** *A TU game (N, v) is said to be balanced if*

$$\sum_{C \supseteq \{i\}} \alpha(C) = 1 \forall i \in N \implies \sum_{C \subseteq N} \alpha(C) v(C) \leq v(N)$$

Observe that, any convex game has non-empty core.

Let $\Pi(N)$ denote the set of all permutations of $N = \{1, 2, \ldots, n\}$. For any permutation $\pi \in \Pi(N)$, $P(\pi, i)$ denotes the set of players who are predecessors of i in the permutation π, i.e.,

$$P(\pi, i) = \{j \in N : \pi(j) < \pi(i)\}$$

The marginal contribution of a player i to his predecessors in π by

$$m(P, i) = v(P(\pi, i) \cup \{i\}) - v(P(\pi, i))$$

Proposition 3. *Suppose (N, v) is a convex game and $\in \Pi(N)$ is any permutation. Then the allocation $y^{\Pi} = (m(\Pi, 1), m(\Pi, 2), \ldots, m(\Pi, n))$ belongs to the core of (N, v). That is, $y^{\Pi} \in C(N, v)$.*

5

Shapley Value

Given a coalitional game, the core may be empty or may be very large or even uncountably infinite. These certainly cause difficulties in getting sharp predictions for the game. The Shapley value is a solution concept which predicts a unique expected payoff allocation for every given coalitional game.

Given a game in coalitional form (N, v), we denote the Shapley value by $\phi(N, v) = (\phi_1(N, v), \ldots, \phi_n(N, v))$ where $\phi_i(N, v)$ is the expected payoff to player i.

5.1 Axioms

Three axioms were proposed to describe the desirable properties of a good solution concept: (1) Symmetry; (2) Linearity; (3) Carrier.

5.1.1 Symmetry

For any $v \in \mathbb{R}^{2^n - 1}$, any permutation π on N , and any player $i \in N$, the symmetry axiom states that

$$\phi_{\pi(i)}(\pi v) = \phi_i(v)$$

5.1.2 Linearity

Let (N, v) and (N, w) be any two coalition games. Suppose $p \in [0, 1]$, the coalition game $(N, pv + (1-p)w)$ has

$$pv + (1-p)w(C) = pv(C) + (1-p)w(C) \quad \forall C \subseteq N$$

5.1.3 Carrier

A coalition D is said to be a carrier of a coalitional game (N, v) if

$$v(C \cap D) = v(C) \quad \forall C \subseteq N$$

If D is a carrier and $i \notin D$, then

$$v(\{i\}) = v(\{i\} \cap D) = v(\phi) = 0$$

If D is a carrier of (N, v) then all players $j \in N \setminus D$ are called *Dummies*. A carrier contains all influential players, might also include non-influential players.

This axiom asserts that the dummy players are allocated nothing form the worth obtained.

Theorem 5.1. *There is exactly one mapping $\phi : \mathbb{R}^{2^n - 1} \to \mathbb{R}$ that satisfies Symmetry, Linearity, Carrier axioms. This mapping satisfies $\forall i \in N, \forall v \in \mathbb{R}^{2^n - 1}$*

$$\phi_i(v) = \sum_{C \subseteq N \setminus \{i\}} \frac{|C|!(n - |C| - 1)!}{n!} \{v(C \cup \{i\}) - v(C)\}$$

In the above the fraction $\frac{|C|!(n-|C|-1)!}{n!}$ is the probability that in any permutation, the members of C are ahead of distinguished player i. $v(C \cup \{i\}) - v(C)$ gives the marginal contribution of player i to the worth of the coalition C.

We have seen that core of any convex game is non-empty. Now, we see that the Shapley value of the convex game belongs to the core.

Theorem 5.2. *Let (N, v) be a convex game. Then $\phi(v) \in C(N, v)$.*

If we are able to model a physical situation as a convex game, then the Shapley value has the added characteristic that it is also a stable allocation since it belongs to the core. Thus for convex games, the Shapley value provides an excellent solution concept.

6

More Solution Concepts

So far we have seen set solution concept core and single point solution concept Shapley. There are many other solution concepts studied in cooperative game theory. Here we briefly look at few of them.

6.1 Stable Sets

Definition 6.1 (Excess). *Given a TU game (N, v), a coalition C, and an allocation $x = (x_1, \ldots, x_n)$, the excess of C at x is defined as*

$$e(C, x) = v(C) - \sum_{i \in C} x_i$$

The excess $e(C, x)$ is the net transferable utility that coalition C would be left with after allocating value to every player in C.

Definition 6.2 (Domination). *An imputation $x = (x_1, \ldots, x_n)$ is said to dominate another imputation $y = (y_1, \ldots, y_n)$ if there is some coalition $C \subseteq N$ such that*

$$e(C, x) \geq 0$$
$$x_i > y_i \forall i \in C$$

An imputation is said to be undominated if no other imputation that dominates it.

Definition 6.3 (Internally Stable). *Given a TU game (N, v), a set of imputations Z is said to be internally stable if*

$$\forall x, y \in Z, \forall C \subseteq N, x_i > y_i \forall i \in C \implies e(C, x) < 0$$

Definition 6.4 (Externally Stable). *Given a TU game (N, v), a set of imputations Z is said to be externally stable if every imputation not in Z is dominated by some imputation in Z*

Definition 6.5 (Stable Set). *A stable set of a TU game (N, v) is a set of imputations Z satisfying internal stability as well as external stability.*

6.2 Bargaining Sets

Definition 6.6 (Objection). *An objection by a player i against another player j and a payoff allocation x is a pair (y, C) where y is another payoff allocation and C is a coalition such that*

$$i \in C; j \notin C$$
$$e(C, y) = 0$$
$$y_k > x_k \forall k \in C$$

Definition 6.7 (Counter Objection). *Given a player i's objection (y, C) against player j and a payoff allocation x, a counterobjection by player j is any pair (z, D) where z is another payoff allocation and D is a coalition such that*

$$j \in D$$
$$i \notin D$$
$$C \cap D \neq \emptyset$$
$$e(D, z) = 0$$
$$z_k \geq x_k \forall k \in D$$
$$z_k \geq y_k \forall k \in C \cap D$$

Let (N, v) be a TU game and Q is a partition of N. We define

$$I(Q) = \left\{ x \in \mathbb{R}^n : x_i \geq v(\{i\}) \forall i \in N, \sum_{i \in C} x_i = v(C) \forall C \in Q \right\}$$

Definition 6.8 (Bargaining Sets). *Given a TU game (N, v) and a partition Q of N, a bargaining set is a collection of payoff allocations $x \in \mathbb{R}^n$ such that*

1. *$x \in I(Q)$*
2. *For any coalition D in Q and for any two players $i, j \in D$, there exists a counterobjection to any objection by i against j and x.*

6.2.1 Key Notes

- The core is a subset of the bargaining set of (N, v) relative to the partition $Q = \{N\}$.
- Given a partition Q, if $I(Q)$ is non-empty, then the bargaining set relative to Q is non-empty.
- If the game (N, v) is superadditive, then for any partition Q, the set $I(Q)$ is non-empty and hence the bargaining set with respect to Q is also non-empty.
- The allocations suggested by a bargaining set need not be fair in the sense of Shapley value.

6.3 Kernel

The kernel of a TU game (N, v) is defined with respect to a partition Q of N. Kernel is also a subset of $I(Q)$. The intuition behind this is that if two players i and j belong to the same coalition in Q, then the highest excess that i can make in a coalition without j should be the same as the highest excess that j can make in a coalition without i.

Definition 6.9 (Kernel). *Given a TU game (N, v) and a partition Q of N, the kernel is a set of allocations $x \in \mathbb{R}^n$ such that*

1. *$x \in I(Q)$*
2. *For every coalition $C \in Q$ and every pair of players $i, j \in C$,*

$$\max_{D \subseteq N \setminus \{j\} i \in D} e(D, x) = \max_{E \subseteq N \setminus \{i\} j \in E} e(E, x)$$

6.4 Nucleolus

We have seen that the excess of a coalition C with respect to an allocation x is a measure of unhappiness of C with allocation x. The nucleolus is based on the idea of minimizing the level of unhappiness of the unhappy coalitions.

Definition 6.10 (Nucleolus)**.** *Consider any allocation* $x = (x_1, \dots, x_n)$ *and let* $e_k(x)$ *be the kth largest excess generated by any coalition with allocation* x*. This means the cardinalities of the stated sets satisfy:*

$$|\{C \subseteq N : e(C, x) \geq e_k(x)\}| \geq k$$
$$|\{C \subseteq N : e(C, x) > e_k(x)\}| < k$$

6.4.1 Key Notes

- The nucleolus always exists and is unique.
- If the core is non-empty, the nucleolus belongs to the core.

6.5 Gately Point

Suppose we have a TU game (N, v) with $N = \{1, 2, \dots, n\}$. Let a player i breaks away from the grand coalition, it might result in a loss (or gain) for the players. If $x = (x_1, \dots, x_n)$ is the original allocation for the grand coalition. Then the loss to the player i is $x_i - v(i)$. The joint loss to the rest of the players is

$$\sum_{j \neq i} x_j - v(N \setminus \{i\})$$

The disruption caused by player i is called *propensity to disrupt* which is defined as

$$d_i(x) = \frac{\sum_{j \neq i} x_j - v(N \setminus \{i\})}{x_i - v(\{i\})}$$

The Gately point is defined as an imputation that minimizes the maximum propensity to disrupt.

7

Our Work

In crowdsourcing to incentivize players to exert efforts and report truth rewards are introduced. But these rewards are often not fair. This unfair rewarding may discourage players to participate. And most of these tasks need players to submit reports early. Motivated with this, we worked on designing a fair reward mechanism using reputation scores of the players, which incentivizes early reporting. We also aimed for the mechanism to be non-collusive. The complete work on this is provided in the appendix. The manuscript will be submitted to IJCAI '21.

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