Peer Prediction with Heterogeneous Tasks

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1 Problem Addressed

The problem of peer prediction with heterogeneous tasks, where each task is associated with different distribution on responses is addressed. We extend *correlated algorithm (CA)* mechanism to this setting, aligning incentives for investing effort without creating opportunities for coordinated manipulations.

2 Previous Work

- Jurca and Faltings 2009 proposed a three-peer mechanism to eliminate uninformative, pure strategy equilibrium.
- Kong et al. provided a method to design robust, single task, binary signal mechanism.
- Dasgupta and Ghosh came up with DG mechanism which reward agents if they provide same signal on same task and punish if one agent's report on one task is same as another agent's on another task.
- Shnayder generalise DG mechanism and handle multiple signals and show how the required knowledge can be estimated from the reports without compromising incentives.
- Agarwal generalised the CA mechanism when users are heterogeneous and derive sample complexity bounds for learning the reward matrices.

3 Basic Assumptions

- We do not assume the tasks are *ex ante* identical.

- Signals for different tasks are assumed to be drawn independently.
- Agents are exchangeable in their roles in the distribution, with same marginal distributions and joint distributions for any pair of agents.
- It is assumed that agent adopts same strategy across all the tasks.

4 Heterogeneous Multi-Task Peer Prediction

Consider agents 1 and 2, each is assigned to a set of M = {1,2,..m} tasks. Let ${S_k}^1$ and ${S_k}^2$ are the signals of agents on task k respectively. $P_k(i,j) = \Pr({S_k}^1 = i,\ {S_k}^2 = j)$ be joint probability for a pair of signals (i,j) on task k. $P_k(i)$ and $P_k(j)$ be corresponding marginal probabilities.

- *Definition 1*:
  Strong Truthful: A peer prediction mechanism is strong truthful if and only if for all strategies F,G we have $E(I,I) \geq E(F,G)$, where equality may hold only when F and G are both the same permutation strategy.
- *Definition 2*:
  Informed Truthful: A peer prediction mechanism is informed truthful if and only if for all strategies F,G we have $E(I,I) \geq E(F,G)$, where equality may hold only when F and G are informed strategies.
- These two Properties imply that truthful reporting is strict and weak correlated equilibrium. t
- CA mechanism fails to be informed truthful for some cases.

5 Correlated Agreement Heterogeneous (CAH) Mechanism

Algorithm: CAH mechanism

Require: Joint probability distribution $P_b(.,.)$, marginal probability distributions $\{P_l(.)\}_{l \neq b}$ and reports $\{r_k^1, r_k^2\}_{k=1}^m$
1 : $b \leftarrow$ uniformly at random from $\{1, 2, ..m\}$ (bonus task)
2 : $l^{'} \leftarrow$ uniformly at random from $\{1, 2, ..m\} \backslash \{b\}$ (penalty task assigned to agent 1)
3 : $l^{''} \leftarrow$ uniformly at random from $\{1, 2, ..m\} \backslash \{b, l^{'}\}$ (penalty task assigned to agent 2)
4 : Define $\Delta_b(i, j)$ as

$$P_b(i, j) - \frac{1}{(m-1)(m-2)} \sum_{t^{'}, t^{''} \in [m] \backslash \{b\} \& t^{'} \neq t^{''}} P_{t^{'}}(i) P_{t^{''}}(j)$$

5 : Let $S_b(i, j)$ be the corresponding score matrix i.e.

$$S_b(i, j) = \begin{cases} 1, & \text{if } \Delta_b(i, j) > 0 \\ 0, & \text{otherwise.} \end{cases}$$

6 : Make payment $S_b(r_b^1, r_b^2) - S_b({r_{l^{'}}}^1, r_{l^{''}}^2)$ to each agent.

- *Expected Score*: $E(F, G) = \frac{1}{m} \sum_{b=1}^m \sum_{i,j} \Delta_b(i, j) S_b(F_i, G_j)$

- *Lemma1*: For each task b, we have $\sum_{i,j} \Delta_b(i, j) = 0$.

- **Theorem1**: If for each task b, Δ_b is symmetric and each entry of Δ_b is non-zero, then the CAH mechanism is informed truthful.

- *Condition1*:
  - $\Delta_b(i, i) > 0, \forall b \forall a$
  - $\sum_{b=1}^m \Delta_b(i, j) < 0, \forall i \neq j$

- **Theorem2**: If $\{\Delta_b\}_{b=1}^m$ satisfy Condition 1, then the CAH mechanism is strongly truthful.

- CAH mechanism remains approximately informed truthful as the score matrix that corresponds to correct statistics is best possible score matrix for agents.

6 CAHR (CAH Recomputed)

Theorem: If there are at least $q = \Omega(\frac{n}{\epsilon^2} log(\frac{m}{\delta}))$ agents reviewing each task, for m tasks and n possible signals, then with probability at least $1 - \delta$, then CAHR satisfies $E[I, I] \geq E[F, G] - \epsilon \; \forall F, G$

Algorithm: CAHR mechanism

Require: Agent p of a population of q agents provides reviews $(r_1^p, ..., r_m^p)$ on each of the m tasks.
1 : $T_k(i, j) \leftarrow$ observed frequency of signal pair i, j on task k.
2 : Pair up the agents uniformly at random, and run CAH for each pair with the estimated distribution $\{T_k(., .)\}_{k=1}^m$

7 Cross Correlated Agreement

- Responses of two users to two different tasks may be correlated.
- Let $P_{l',l''}(i, j)$ denote the probability that the user observe signal i on task $l^{'}$ and another user observes signal j on task $l^{''}$.
- When there is no correlation among signals for different questions, then $P_{l',l''}(i, j) = P_{l'}(i)P_{l''}(j)$
- CCAH is same as CAH expect it defines $\Delta_b(i, j)$ as:

$$P_b(i, j) - \frac{1}{(m-1)(m-2)} \sum_{t',t'' \in [m] \setminus \{b\} \& t' \neq t''} P_{t',t''}(i, j)$$

- CCAH is strong truthful and informed truthful under similar conditions as stated for CAH.

8 Conclusion

CAH mechanism which is informed-truthful under mild conditions was introduced. The simulation results suggest that the mechanism provides better incentives for being truthful and is more resistant to coordinate misreports than the RPTS and Kamble mechanisms.