Honours Report 1

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Abstract

In this report, we look into the different papers focusing on problems related to game theory which mainly extend over the concepts of Peer Prediction, Prediction Markets and their incentive compatible mechanisms. We see how peer prediction mechanisms are designed for various settings and are being used in several applications. Peer Prediction plays a vital role in eliciting information as its mechanisms are designed with scoring rules which incentivize users to provide honest reports. Thus, they are extremely useful in applications of peer prediction extending from peer grading to rating.

Contents

1 Introduction

In many practical settings, an effective way of evaluating products/services of interest is to collect evaluations from dispersed individuals and aggregate these evaluations together. For example, many millions of users rely on feedback from Yelp, Amazon to choose among competing movies, restaurants and products. Crowd-sourcing platforms help us obtain services or content by soliciting contributions from a large group of people and especially from the online community rather than from traditional employees. This has many advantages such as reaching a wider range of reviewers with huge diversity in opinions. This is effective in information aggregation, but the greatest difficulty here is to detect poor quality entries or misleading contributions. Some participants may not know much about the content nor have taken time to learn it but share their reviews. Participants who are aware may not be truthful or may not be willing to share the truth. Thus, information elicitation is very important. Therefore, we have peer prediction mechanisms that involve game-theoretic aspects to capture these issues. Many new mechanisms are being proposed for different settings that are less restrictive compared to classical mechanisms which make them more realistic. Here, we study different papers on real-world applications of peer prediction and also a few settings where new mechanisms are proposed as the classical mechanism can't be applied.

2 Preliminaries

In this section, we shall review some basic concepts and mechanisms from the game theory which are essential for the study of research papers mentioned in the report.

Definition 1 (Dominant Strategy Equilibrium) *A strategy profile $(s_1^*, .., s_n^*)$ is called dominant strategy equilibrium of the game $\Gamma = (N, (S_i), (u_i))$ if, $\forall i = 1, 2, .., n$, the strategy s_i^*, $u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i})$, $\forall s_i \neq s_i^*$ and $\forall s_{-i} \in S_{-i}$.*

Here, $N = \{1, 2, .., n\}$ is a finite set of players, $S_1, S_2, .., S_n$ are strategy sets of players, and $u_i : S_1 \times S_2 \times ...S_n \to \mathbf{R}$ for $i = 1, 2, ..n$ are utility functions.

Definition 2 (Individual Rationality) *Individual rationality of a social choice function essentially means that each agent gains a utility that is no less than he would get without participating in a mechanism that implements the social choice function.*

Definition 3 (Minimal Mechanism) *A minimal peer prediction mechanism is a function $M : [m] \times [m] \to R$, where $M(x_i, x_j)$ specifies the payment to agent i when she reports signal x_i and her peer agent j reports signal x_j .*

Definition 4 (Truthful Mechanism) *Mechanism M is truthful if we have $s_i = \underset{x_i}{a}rgmax \underset{S_j}{E}[M(x_i, S_j)|S_i = s_i]$, $\forall i \in [n]$ and $\forall s_i \in [m]$, where s_i is signal reported by agent i and S_j signal reported by agent j.*

Definition 5 (Universal Peer Prediction Mechanism) *A peer-prediction mechanism is universal if it can be operated without knowledge of the joint distribution of the high-quality signals s^h_{ij} (i.e., it is "detail free" [14]) and well defined for any number of agents $n \geq 3$, where s^h_{ij} is high quality signal for object j, agent i has access.*

Definition 6 (Output Agreement) *Output agreement mechanisms only collect signal reports from agents and reward an agent i for evaluating object j based on agents' signal reports for the object, where agent i is only rewarded when his signal report matches that of another randomly chosen agent j. Agent i's reward is given by, $z_i(r) = \mathbf{1}_{\{r_{ij}=r_{i'j}\}}$.*

Definition 7 (Peer Truth Serum) *It is an incentive mechanism which utilizes the distribution of reported data from similar tasks as the prior probability of possible answers, and proportionally scale the reward given for agreement between agents with this distribution.*

Bayesian truth Serum It is a mechanism that doesn't assume any prior. Take both information report and prediction report. Rewards responses that are surprisingly common.

Formally defining, If there are r indexed respondents, with m responses and let $t^r = (t^r_1, .., t^r_m)$ be truthful opinion, $w = (w_1, .., w_m)$ be distribution of frequencies over population, $x^r = (x^r_1, .., x^r_m)$ be reported frequencies, $y^r = (y^r_1, .., y^r_m)$ be predicted distribution of frequencies. The scoring rule is defined as sum of information score and prediction score:
$u^r = \sum_k x^r_k log\frac{\overline{x_k}}{\overline{y_k}} + \alpha. \sum_k \overline{x_k} log\frac{y^r_k}{\overline{x_k}}$

Definition 8 (Second Price Auction) *In the second price auction, the one who bids highest will be allocated the item but the winning bidder will pay an amount equal to the second-highest bid.*

Definition 9 (Stochastic Relevance) *Random variable S_i is stochastically relevant for random variable S_j if and only if the distribution of S_j conditional on S_i is different for all possible values of S_i.*

Dollar Partition It is Strategyproof peer selection mechanism, which works as follows:

- All the participants are partitioned into l clusters of equal sizes.
- Every participant i evaluates m other peers from different clusters.
- Participant i assigns value v^j_i to each agent j among the m agents reviewed by i.
- Based on the values given by the participants outside the cluster, the normalized weight χ_j is assigned to each cluster C_j.

Definition 10 (Robust Bayesian Truth Serum) *This mechanism rewards agent i for how well his belief report b_i and shadowed belief report $b_i^{'}$ predict the signal reports of another randomly chosen agent k whose signal is r_k. Agent i's reward is $z_i(r, b) = R(b_i^{'}, r_k) + R(b_i, r_k)$. Agent i's shadowed belief report is calculated based on his signal report and another random agent j's belief report: $b_i^{'} = b_j + \lambda$ if $r_i = 1$ and $b_i^{'} = b_j - \lambda$ if $r_i = 0$ where $\lambda = min(b_j, 1 - b_j)$.*

Definition 11 (Correlated Agreement Mechanism) *It is informed truthful and proper multi-task mechanism with score matrix = Sign(Δ).*

- Δ is an $n \times n$ matrix with $\Delta_{ij} = P(i, j) - P(i)P(j)$.
- $P(i, j)$ is joint probability distribution, $P(i)$ and $P(j)$ are marginal probabilities on signals i, j of agents 1 and 2 respectively.

Definition 12 (Positive affine transformation) *A mechanism $M^{'}$ is a positive affine transformation of mechanism M if there exists $f : [m] \to \mathbf{R}$ and $\alpha > 0$ such that for all $x_i, x_j \in [m]$, $M^{'}(x_i, x_j) = \alpha M(x_i, x_j) + f(x_j)$. Here, x_i is the signal reported by agent i.*

Definition 13 (Power Diagram) *A power diagram is a partitioning of Δ_m into sets called cells, defined by collection of points $\{v^s \in \mathbf{R}^m : s \in [m]\}$ called sites with associated weights $w(s) \in \mathbf{R}$, given by $cell(v^s) = \{u \in \mathbf{R^m} : s = \underset{x \in [m]}{argmin}\{||u - v^x||^2 - w(x)\}\}$*

- For probability distributions over only 3 signals, there is a convenient graphical representation of the probability simplex Δ_m as an equilateral triangle, where the three corners represent the signals.

Paper	Appeared in	Problem Addressed	Solution Proposed
Applications of Peer Prediction			
TSP: Truthful Grading Based Strategy Proof Peer Selection for MOOCs	IEEE-18	Selection of best k students in MOOC (massive open online courses) settings using peer grading.	Proposed a truthful grading-based strategy-proof peer selection scheme.
Identifying Vulnerabilities in Trust and Reputation Systems	IJCAI-19	To evaluate trust and reputation systems against known attacks avoiding heavy reliant on expert analysts.	A novel method for automatically identifying vulnerabilities in such systems is proposed by formulating the problem as a derivative-free optimization problem.
Designing Payment Mechanisms			
Contingent Payment Mechanism for Resource Utilization	AAMAS-19	Assigning resources to improve their utilization for a setting where agents have uncertainty.	Introduces a Contingent Payment (CP) mechanism which optimizes social welfare for assigning resources.
Mechanism Design in Different Settings			
Peer Prediction with Heterogeneous Tasks	ArXiv-16	Introduces peer prediction mechanism for a setting where tasks are heterogeneous.	Extends correlated agreement (CA) mechanism to this setting, aligning incentives for investing effort without creating opportunities for coordinated manipulations.
Peer Prediction with Heterogeneous Users	ACM-EC 18	Introduces peer prediction mechanism for a setting where users differ in their taste, judgement and reliability.	Proposed algorithms for learning the clustering of agents based on their reporting behaviour and a mechanism that works with clusters of agents.
Personalized Peer Truth Serum for Eliciting Multi-Attribute Personal Data	UIA-19	Considers the problem of eliciting personal attributes of the agents.	Extends the peer consistency incentive mechanism to propose a Personalized Peer Truth Serum (PPTS).
Analysis of Peer Prediction Mechanism			
A Geometric Method to Construct Minimal Peer Prediction Mechanisms	AAAI-16	Proves that minimal peer prediction mechanisms are equivalent to power diagrams using geometric perspective.	Proposed a belief model constraint on which mechanisms are truthful, proved that minimal peer prediction mechanism is equivalent to a power diagram.
Incentivizing Evaluation via Limited Access to Ground Truth: Peer-Prediction Makes Things Worse	AIJ-19	Analysis of peer prediction mechanisms when given limited access to the ground truth.	Shows that the simpler mechanism achieves stronger incentive guarantees given less access to the ground truth than a large set of peer-prediction mechanisms.
Prediction Markets			
Crowdsourced Outcome Determination in Prediction Markets	AAAI-16	Considers the challenges in outcome determination while implementing and scaling decentralised prediction market.	Proposed a specific prediction market mechanism with crowdsourced outcome determination.

Table 1: **Research Papers Index**

3 Classification of Papers

Table [1] gives the classification and overview of all research papers.
The papers mentioned in this report can broadly be classified into the following:

3.A Applications of Peer Prediction

Truthful Grading-Based Strategyproof Peer Selection for MOOCs

- Selection of best k students in MOOC (massive open online courses) settings using peer prediction scheme.

Identifying Vulnerabilities in Trust and Reputation Systems

- Identify vulnerabilities in TRSs by formulating the problem as a derivative free optimisation problem.

3.B Designing Payment Mechanisms

Contingent Payment Mechanisms for Resource Utilization

- Improving resource utilization by introducing contingent payments mechanisms.

3.C Mechanism Design in Different Settings

Peer Prediction with Heterogeneous Tasks

- Designing peer prediction mechanism where each task is associated with different distribution on responses.

Peer Prediction with Heterogeneous Users

- Designing peer prediction mechanism where users differ in their taste, judgement and reliability.

Personalized Peer Truth Serum for Eliciting Multi-Attribute Personal Data

- Designing a peer consistency incentive mechanism in a setting for collecting personal attributes.

3.D Analysis of Peer Prediction Mechanism

A Geometric Method to Construct Minimal Peer Prediction Mechanisms

- Proving that minimal peer prediction mechanisms are equivalent to power diagrams using geometric perspective.

Incentivizing Evaluation via Limited Access to Ground Truth: Peer-Prediction Makes Things Worse

- Given limited access to ground truth, shows that simpler mechanism is better than large set of peer prediction mechanisms.

3.E Prediction Markets

Crowdsourced Outcome Determination in Prediction Markets

- This paper proposes and analyzes a mechanism where the outcome of an MSR (Market Scoring Rule) prediction market is determined via a peer prediction mechanism among a set of arbiters.

4 Research Paper Summaries

4.A Research Paper - 1

Title: TSP: Truthful Grading-Based Strategyproof Peer Selection for MOOCs [P1]
Author: Yufeng Wang, Hui Fang, Chongu Cheng, Qun Jin

4.A.1 Problem Addressed

Analyzing the fundamental challenges in using peer prediction to select best-k students in MOOC (massive open online courses) settings and designing a strategy-proof, truthful grading based peer prediction scheme.

4.A.2 Previous Work

- A strategy-proof selection mechanism called *Dollar Partition*[1] was proposed. However, it didn't take into account that the agents instead of exerting effort and time would report arbitrary. (For brief description of Dollar Partition refer [2]).
- There are mechanisms to motivate agents to put the effort in a setting that assumes the "gold standard" questions[2], but this paper has no such assumptions.
- Some of the mechanisms leverage correlation in peers reports eliciting truthful reports under a scenario of non-ground truth (i.e. Output agreement), which is not incentive compatible.
- Peer Truth Serum (PTS) mechanism was proposed, which utilizes the distribution of the reported data from similar tasks as prior probability and proportionally scale the reward. (For brief description of PTS refer [7]).

4.A.3 Basic Assumptions

- All participants enjoy the same situation and infer the final grading of all the students.
- No ground truth available and assumes no gold standard questions in the setting.

4.A.4 TSP Mechanism Framework

TSP scheme is composed of three components.

- **Component 1**: Partitioning students uniformly into clusters and peer assessment.
- **Component 2**: Reward calculation according to the rule of surprisingly common.
- **Component 3**: Best k-students selection into winning set W.

Student Partitioning and Peer Assessment

- Population of $N = [n]$ students are partitioned into a set of clusters $C = C_1, ..., C_l$ such that the sizes of all the clusters are almost same.
- Every student w evaluates $min \leq m \leq n-1$ of other students from different clusters. M_w is set of students reviewed by w. $|M_w| = m$.
- Each student $w \in N$ reports an evaluation v_w^p to each student $p \in M_w$.
- We assume that the signal values are extracted from $X = [x]$ and if there exist no evaluation between students w and p, set $v_w^p = 0$

Reward Function
Parameters: R(x) is prior probability distribution, x_p, x_w are reports of agents p & w, $\alpha > 0$

Reward: $\tau(x_w, x_p) = \alpha.(\tau(x_w, x_p) - 1)$, where

$$\tau_0(x_w, x_p) = \begin{cases} \frac{1}{R(x)}, & \text{if } x_w = x_p \\ 0, & \text{if } x_w \neq x_p. \end{cases}$$

Final Reward is the avg of rewards over all peers in $S(w)$, where $S(w)$ denotes the set of peers p who grade the same assignment as w.

$\tau(w) = \alpha.(\frac{1}{|S(w)|}.\sum_{p\in S(w)}(\tau_0(x_w, x_p) - 1))$

Selection of Best k Students

- Each cluster is assigned a normalized weight $x_j = \frac{1}{n}.\sum_{w\in C_j, p\in C_j} v_w^p$, $\forall w, p \in N$.
- From each cluster C_j, t_j peers should be selected, $t_j = min(flooring(s_j, |C_j|)$ where $s_j = x_j.k$.
- For each student $p \in C(p)$ grading level is calculated as $g^p = \sum_{w\in C(p)} V_w^p$.
- t_j students with highest scores from each cluster C_j into the winning set W.
- The size of winning set may be less than k. The other $(k - |W|)$ students are selected in lottery way: For each cluster C_j let $l_j = \frac{s_j - t_j}{\sum_{i\in[l]}(s_i - t_i)}$.
- Randomly select a student with highest score from left students in cluster C_j with probability l_j. Repeat this for $(k - |W|)$ times until $|W| = k$.

4.A.5 Novelty

- This recognizes the strategic aspect of agents and motivate them to report truthfully by making a proper effort.

- The principle of rewarding 'surprisingly common' answers is realized. Common because reports are consistent. Surprising because less likely answers lead to higher reward.
- Agents can't increase his/her chances of getting selected by manipulating their valuation for other agents outside the cluster because he/she only contributes to the probability weight of $\frac{1}{n}$.

4.A.6 Comparison with Previous Mechanisms

For traditional strategy-proof peer selection mechanisms, students may report random evaluation on other peers instead of exerting efforts to evaluate, especially for the case that those evaluations can't be verified (because of extreme difficulty or no ground truth existed).

4.A.7 Conclusion

A novel truthful grading based strategy-proof peer selection for MOOCs, TSP is proposed. The theoretical analysis proves that TSP can motivate students to put effort into peer assessment and also report their grading honestly. Even the simulation results show that the accuracy of the selection of best-k students in a strategy-proof way is better than that of the traditional non-incentive scheme.

4.B Research Paper - 2

Title: Contingent Payment Mechanism for Resource Utilization [P2]
Author: Hongyao Ma, Reshef Meir, David C. Parkes, James Zoa

4.B.1 Problem Addressed

Considering the problem of assigning resources to improve their utilization, for settings where agents have uncertainty about their values for using a resource, and where it is in the interest of the society or the planner that resources be used and not wasted, and designing mechanisms, presenting a simulation study of the welfare properties of these mechanisms.

4.B.2 Previous Work

- Previously arisen contingent payments are conditional on observable world states rather than agent's downstream actions.
- In past applications, the principal role of contingent payment is to improve revenue and to limit risk.
- Atakan and Ekmecki [4] studied auctions where the value of taking each action depends on the collective actions by others, but the timing of information is quite different than in our model.
- Courty and Li [5] studied the problem of selling airline tickets. However, they consider distributions that satisfy either mean-preserving spread or stochastic dominance and reduce the type space to one-dimensional.
- A mechanism was designed to incentivize reliability in demand-side response in electric power systems where any number of agents can be selected. In our model, only one agent can be assigned to a resource.

4.B.3 Basic Assumptions

- All agents are risk-neutral, expected utility maximizers with quasi-linear utility functions.
- The cumulative distribution function (CDF) F_i of V_i of agent $i's$ private information at period 0, corresponds to her *type*. The following are the assumptions about F_i for each $i \in N$:
  - $(A1) : \mathrm{E}[V_i^+] > 0$, which means that V_i takes the positive value with non-zero probability, thus the option to use the resource as one wishes has a positive value.
  - $(A2) : \mathrm{E}[V_i^+] < \infty$, which means that agents do not get infinite expected utility from the option to use the resource.

4.B.4 Contingent Payment Mechanism

Definition 14 *The contingent payment mechanism with maximum penalty Z collects two-part bids $b = (b_1...b_n)$. For each $i \in N$, $b_i = (b_i^{(0)}, b_i^{(1)}) \in R$, where $R = \{(z, y) \in R^2 | 0 \le z \le Z, y = 0\} \cup \{(z, y) \in R^2 | z = Z, y \ge 0\}$*

- *Allocation rule*: $x_{i^*}(b) = 1$ for $i^* \in \underset{i \in N}{argmax}\{b_i^{(0)} + b_i^{(1)}\}$ (breaking ties at random); $x_i(b) = 0$ for all $i \neq i^*$
- *Payment rule*: let $i' \in \underset{i \neq i^*}{argmax}\{b_i^{(0)} + b_i^{(1)}\}$. $t_{i^*}^{(0)}(b) = b_{i'}{}^{(0)}$; $t_{i^*}^{(1)}(b) = b_{i'}{}^{(1)}$; $t_i^{(0)}(b) = t_i^{(1)}(b) = 0$. for all $i \neq i^*$

Theorem 1 *(Dominant Strategy in CP): Given $(A1) - (A2)$, under the contingent payment mechanism with maximum penalty Z, it is a dominant strategy for each agent $i \in N$ to bid $b_{i^*,CP} = (Z, u_i(Z))$ if $u_i(Z) \ge 0$. Otherwise, it is a dominant strategy to bid $b_{i^*,CP} = (z_i^0, 0)$, where z_i^0 is the unique zero-crossing of $u_i(z)$.*

Theorem 2 *For any set of agent types satisfying $(A1) - (A2)$, under the dominant strategy equilibria, the CP(W) mechanism Pareto-dominates the second price auction in the utilization and social welfare.*

Characterization and Optimality:

- $P1$. Dominant-strategy equilibrium (refer [1])
- $P2$. Individually rational (refer [2])
- $P3$. No deficit
- $P4$. Anonymous
- $P5$. Deterministic
- $P6$. No subsidy

4.B.5 Optimality of CP(W) Mechanism

Theorem 3 *Assume the type space is the set of all value distributions satisfying* $(A1)$ *and* $(A2)$. *Assuming generic input, no two-period mechanism under* $(P1)-(P6)$ *achieves weakly higher social welfare than the CP(W) mechanism for all type profiles, and a strictly higher social welfare than the CP(W) mechanism for at least one type profile.*

Theorem 4 *Assume the type space is the set of all value distributions satisfying* $(A1)$ *and* $(A2)$. *With the generic input assumption, the CP(W) mechanism is welfare-optimal type profile by type profile, among all two-period mechanisms that satisfy* $(P1)-(P6)$, *always allocate the resource, and use an ordered payment space.*

4.B.6 Uniqueness & Optimality of CSP

Theorem 5 *Assume the type space is the set of all value distributions satisfying* $(A1)-(A3)$, *assume generic input, and consider two-period mechanisms that satisfy* $(P1)-(P6)$*:*

i. The CSP mechanism is the unique mechanism that always allocates the resource, and does not charge the allocated agent if the resource is utilized.

ii. For the (w_i, p_i) type space, the CSP mechanism is optimal for utilization, type profile by type profile.

iii. The CSP mechanism is not dominated for utilization.

iv. The CSP mechanism is utilization optimal type profile by type profile, among all mechanisms that always allocate the resource and use an ordered payment space.

4.B.7 Novelty

- Introduced, a family of two-period mechanisms that make use of payments which are contingent on whether or not a resource is used.
- The contingent payment mechanism has a simple dominant strategy.
- CP(W) is not dominated for expected welfare by any other mechanism.
- Amongst mechanisms that always allocate the resource and support a simple indirect structure, CP(W) is ex-post optimal, i.e. maximizes social welfare profile by profile.

4.B.8 Conclusion

The CP(W) mechanism which was given in the paper optimizes social welfare for assigning a single resource and can be generalized to multiple heterogeneous resources. Simulations also demonstrate the effectiveness and robustness of the mechanism.

4.B.9 Future Work

- Generalizing the model to allow more than two periods
- Repeated assignments of resources using points.
- Folding in considerations of behavioral economics, understanding the impact of present bias on resource utilization and designing commitment devices through the CP mechanism.

4.C Research Paper - 3

Title: Identifying Vulnerabilities in Trust and Reputation Systems [P3]
Author: Taha D. Gunes, Long Tran-Thanh, Timothy J. Norman

4.C.1 Problem Addressed

To evaluate trust and reputation systems against known attacks, By presenting a method to automatically identify vulnerabilities in existing trust models. To provide reliable and objective means to assess how these systems are towards different kinds of attacks.

4.C.2 Previous Work

- Numerous kind of attacks and defence strategies have been explored [Hoffman et al., 2009][6], but considered relatively simple attack profiles.
- The BRS (Beta Reputation System) with filtering [7], focuses on excluding attackers who provide unfair feedback by badmouthing or ballot-stuffing.
- The TRAVOS [8] discounts outlying ratings in making trust assessments.
- The HABIT [9]ite model uses a hierarchical Bayesian model to identify participants with various profiles of reliability, and factor into aggregated ratings.
- Bidgoly & Ladani [10] considered injecting false evidence and whitewashing, which are modelled as primitive actions in a planning mechanism (POMDP).

4.C.3 Contributions

The contributions made here are three-fold.

- **First**: Model coordinated, strategic attacks with a specific objective as a derivative-free optimization problem.

- **Second**: Two search methods are proposed for efficiently identifying coordinated attacks in complex attack spaces through sampling-based optimization.

- **Third**: This method is used to analyze a selection of existing trust models, providing evidence for the kinds of complex attacks they are vulnerable to.

4.C.4 Basic Assumptions

- Prediction of the future behaviour of an agent (i.e. a trust assessment) at time t is, $\varepsilon = \{O^{0:t}_{c_i->p_i} | c_i \in C, p_i \in P\}$

- We investigate cases in which an attacker is limited by:

  - *Power*, the number of observations that it can add through the attack ($\rho = |\varepsilon^{'}|$)
  - *Control* over the witnesses ($W^{'} \subseteq W$).

4.C.5 Attack Space

- The space of possible attacks is χ,

$$|\chi| = \begin{cases} \rho + k.|\{O^{0:t}_{w_i->p_j} | w_i \in W^{'}, p_i \in P\}| - 1 \\ k.|\{O^{0:t}_{w_i->p_j} | w_i \in W^{'}, p_i \in P\}|. \end{cases}$$

- The space of attacks is defined in terms of:

  - The number of witnesses to be used, s.
  - The distribution of the attack power, ρ across these selected witnesses, considering those they can report on:

    i. All restricted partitions of ρ into s ($D = RP_s(\rho)$) and their permutations without repetition: ${P_s}^D$
    ii. The distribution of these permutations to each witness-provider pair, such that the number of possible distributions is $(|P|.k)^s$

- The number of attacks in reduced space is, $|\chi| = \binom{|W^{'}|}{s} D.{P_s}^D.(|P|.k)^s$

- To solve attackers optimisation problem, 'Monte Carlo Sampling' or 'Hierarchical Sampling' based techniques are used.

4.C.6 Conclusion

A novel method for identifying vulnerabilities in trust and reputation systems is introduced and its practical value is demonstrated. Model when employed to search for effective strategies through derivative-free optimization methods, output a set of attack profiles and an estimate of the vulnerability of the TRS to an attack of that kind.

4.D Research Paper - 4

Title: Peer Prediction with Heterogeneous Tasks [P4]
Author: Debmalya Mandal, Matthew Leifer, David C. Parkes, Galen Pickard, Victor Shnayder

4.D.1 Problem Addressed

The problem of peer prediction with heterogeneous tasks, where each task is associated with different distribution on responses is addressed. We extend *correlated algorithm* (CA) [3] mechanism to this setting, aligning incentives for investing effort without creating opportunities for coordinated manipulations.

4.D.2 Previous Work

- Jurca and Faltings 2009 proposed a three-peer mechanism to eliminate uninformative, pure strategy equilibrium.

- Kong et al. provided a method to design robust, single task, binary signal mechanism.

- Dasgupta and Ghosh [16]came up with DG mechanism which reward agents if they provide same signal on same task and punish if one agent's report on one task is same as another agent's on another task.

- Shnayder [17] generalise DG mechanism and handle multiple signals and show how the required knowledge can be estimated from the reports without compromising incentives.

- Agarwal generalised the CA mechanism (refer [11]) when users are heterogeneous and derive sample complexity bounds for learning the reward matrices.

4.D.3 Basic Assumptions

- We do not assume the tasks are *ex ante* identical.

- Signals for different tasks are assumed to be drawn independently.

- Agents are exchangeable in their roles in the distribution, with same marginal distributions and joint distributions for any pair of agents.

- It is assumed that agent adopts same strategy across all the tasks.

4.D.4 Heterogeneous Multi-Task Peer Prediction

Consider agents 1 and 2, each is assigned to a set of $M = \{1, 2, ..m\}$ tasks. Let ${S_k}^1$ and ${S_k}^2$ are the signals of agents on task k respectively. $P_k(i, j) = Pr({S_k}^1 = i, {S_k}^2 = j$ be joint probability for a pair of signals (i, j) on task k. $P_k(i)$ and $P_k(j)$ be corresponding marginal probabilities. Let $\mathbf{I}$ denote the truthful strategy $\mathbf{I}(j) = j$.

Definition 15 (Strong Truthful) *A peer prediction mechanism is strong truthful if and only if for all strategies F,G we have $E(\mathbf{I},\mathbf{I}) \geq E(F,G)$, where equality may hold only when F and G are both the same permutation strategy.*

Definition 16 (Informed Truthful) *A peer prediction mechanism is informed truthful if and only if for all strategies F,G we have $E(\mathbf{I},\mathbf{I}) \geq E(F,G)$, where equality may hold only when F and G are informed strategies.*

- These two properties imply that truthful reporting is strict and weak correlated equilibrium.
- CA mechanism fails to be informed truthful for some cases.

4.D.5 Correlated Agreement Heterogeneous (CAH) Mechanism

> *Algorithm:* CAH mechanism
>
> *Require*: Joint probability distribution $P_b(.,.)$, marginal probability distributions $\{P_l(.)\}_{l\neq b}$ and reports $\{r_k^1, r_k^2\}_{k=1}^m$
> 1: $b \leftarrow$ uniformly at random from $\{1, 2, ..m\}$ (bonus task)
> 2: $l^{'} \leftarrow$ uniformly at random from $\{1, 2, ..m\}\backslash\{b\}$ (penalty task assigned to agent 1)
> 3: $l^{''} \leftarrow$ uniformly at random from $\{1, 2, ..m\}\backslash\{b, l^{'}\}$ (penalty task assigned to agent 2)
> 4: Define $\Delta_b(i,j)$ as
>
> $$P_b(i,j) - \frac{\sum_{t^{'},t^{''}\in[m]\backslash\{b\}\&t^{'}\neq t^{''}} P_{t^{'}}(i)P_{t^{''}}(j)}{(m-1)(m-2)}$$
>
> 5: Let $S_b(i,j)$ be the corresponding score matrix i.e.
>
> $$S_b(i,j) = \begin{cases} 1, & \text{if } \Delta_b(i,j) > 0 \\ 0, & \text{otherwise.} \end{cases}$$
>
> 6 : Make payment $S_b(r_b^1, r_b^2) - S_b({r_{l^{'}}}^1, r_{l^{''}}^2)$ to each agent.

- *Expected Score*: $E(F,G) = \frac{1}{m}\sum_{b=1}^m \sum_{i,j} \Delta_b(i,j) S_b(F_i, G_j)$
- **Lemma 1** *For each task b, we have $\sum_{i,j} \Delta_b(i,j) = 0$.*
- **Theorem 6** *If for each task b, Δ_b is symmetric and each entry of Δ_b is non-zero, then the CAH mechanism is informed truthful.*
- *Condition1*:
  - $\Delta_b(i,i) > 0,\ \forall b \forall a$
  - $\sum_{b=1}^m \Delta_b(i,j) < 0,\ \forall i \neq j$
- **Theorem 7** *If $\{\Delta_b\}_{b=1}^m$ satisfy Condition 1, then the CAH mechanism is strongly truthful.*
- CAH mechanism remains approximately informed truthful as the score matrix that corresponds to correct statistics is best possible score matrix for agents.

4.D.6 CAHR (CAH Recomputed)

Theorem 8 *If there are at least $q = \Omega(\frac{n}{\epsilon^2} log(\frac{m}{\delta}))$ agents reviewing each task, for m tasks and n possible signals, then with probability at least $1-\delta$, then CAHR satisfies $E[I,I] \geq E[F,G] - \epsilon\ \forall F,G$.*

The above theorem shows that CAHR is (ϵ, δ)- informed truthful.

> *Algorithm:* CAHR mechanism
>
> *Require*: Agent p of a population of q agents provides reviews $(r_1^p, ..., r_m^p)$ on each of the m tasks.
> 1 : $T_k(i,j) \leftarrow$ observed frequency of signal pair i,j on task k.
> 2 : Pair up the agents uniformly at random, and run CAH for each pair with the estimated distribution $\{T_k(.,.)\}_{k=1}^m$

4.D.7 Cross Correlated Agreement

- Responses of two users to two different tasks may be correlated.
- Let $P_{l^{'},l^{''}}(i,j)$ denote the probability that the user observe signal i on task $l^{'}$ and another user observes signal j on task $l^{''}$.
- When there is no correlation among signals for different questions, then $P_{l^{'},l^{''}}(i,j) = P_{l^{'}}(i)P_{l^{''}}(j)$
- CCAH is same as CAH expect it defines $\Delta_b(i,j)$ as:

$$P_b(i,j) - \frac{1}{(m-1)(m-2)} \sum_{t^{'},t^{''}\in[m]\backslash\{b\}\&t^{'}\neq t^{''}} P_{t^{'},t^{''}}(i,j)$$

- CCAH is strong truthful and informed truthful under similar conditions as stated for CAH.

4.D.8 Conclusion

CAH mechanism which is informed-truthful under mild conditions was introduced. The simulation results suggest that the mechanism provides better incentives for being truthful and is more resistant to coordinate misreports than the RPTS and Kamble mechanisms.

4.E Research Paper - 5

Title: Peer Prediction with Heterogeneous Users [P5]
Author: Arpit Agarwal, Debmalya Mandal, David C. Parkes, Nisarg Shah

4.E.1 Problem Addressed

The problem of peer prediction with heterogeneous users, where users differ in their taste, judgement and reliability is addressed. We solve this problem by clustering agents based on their reporting behavior, propose a mechanism that works with clusters of agents and design algorithms that learn such a clustering.

4.E.2 Previous Work

- Jurca and Faltings proposed a three-peer mechanism to eliminate uninformative, pure strategy equilibrium.
- Kong et al. provided a method to design robust, single task, binary signal mechanism.
- Witwoski and Parkes introduced the combination of learning and peer prediction, coupling the estimation of signal prior with shadowing mechanism.
- Dasgupta and Ghosh showed that robustness to coordinated misreports can be achieved for binary signals in small population by using multi-task mechanism.
- CA mechanism [Shnayder] generalised DG mechanism to handle multiple signals and shows how the required knowledge can be estimated from the reports without compromising incentives.
- For a binary setting, Kamble designed a mechanism with strict incentive compatibility for a large number of heterogeneous agents, when the number of tasks grow without bound.

4.E.3 Model Description

- Let $[t]$ denote $\{1,..,t\}$ for $t \in \mathbf{N}$. Consider population of agents $P = [l]$. The set of tasks be $M = [m]$. When an agent performs a task, the signal is received from $N = [n]$.
- We assume that the effort of an agent is binary. And the tasks are *ex ante* identical, i.e. signal of an agent for different tasks are sampled i.i.d
- Let S_p be random variable for signal of agent p for a task. Use $D_{p,q}(i,j)$ to denote joint probability for a pair of signals (i,j) received by agents p,q respectively on a random task. And $D_p(i)$ and $D_q(j)$ for the marginal probabilities.
- *Delta* matrix $\Delta_{p,q}$ between agents p and q is defined as:

$$D_{p,q}(i,j) - D_p(i)D_q(j)$$

- It is assumed that agent's strategy is uniform across different tasks.
- Strategy of agent p is denoted by F^p, defines distribution for each possible signal i, $F^p_{i,r} = Pr(R_p = r|S_p = i)$. $\{F^p\}_{p\in P}$ is the strategy profile for agent p.
- A strategy is *informed* if there exist distinct $i,j \in [n]$ and $r \in [n]$ such that $F^p_{i,r} \neq F^p_{j,r}$. Otherwise it is *uninformed.*

4.E.4 Multi-Task Peer Prediction

- For every pair of agents $p,q \in P$, we define *scoring matrix* $S_{p,q} : [n] \times [n] \rightarrow \mathbf{F}$ as a means of scoring agents reports.
- The set of tasks performed by each agent p are divided into nonempty sets of *bonus tasks* and *penalty tasks*, Denoted by M^p_1 and M^p_2 respectively.
- To calculate payment to an agent p for a *bonus task* $t \in M^p_2$
  1. Randomly select agent $q \in P\backslash\{p\}$ such that $t \in M^p_1$.
  2. Pick penalty tasks $t^{'} \in M^p_2$ and $t^{''} \in M^q_2$ at random such that $t^{'} \neq t^{''}$.
  3. Let the reports of agent p be r^t_p and $r^{t^{'}}_p$ and of agent q be r^t_q and $r^{t^{''}}_q$.
  4. Payment of agent p for task t is then $S_{p,q}(r^t_p, r^t_q) - S_{p,q}(r^{t^{'}}_p, r^{t^{''}}_q)$.
  5. The total payment to the agent is the sum of payments for the agent's bonus tasks.
  6. The expected payment to an agent p is given by: $u_p(F^p, \{F^q\}_{q\neq p}) =$

     $\frac{1}{(l-1)} \sum_{q\neq p} \sum_{i,j} \Delta_{p,q}(i,j).S_{p,q}(F^p_i, F^q_j)$
- **Lemma 2** *For every agent p, and any strategy others, there always exists optimal strategy F^p maximizing u_p that is deterministic.*
- **ϵ- Informed Truthful:** A multi-task peer prediction mechanism is ϵ- informed truthful for some $\epsilon \geq 0$, if and only if for every strategy profile $\{F^p\}_{q\in p}$ and every agent p, we have $u_p(\mathbf{I}, \{\mathbf{I}\}_{q\neq p}) \geq u_p(F^p, \{F^q\}_{q\neq p}) - \epsilon$, where $\mathbf{I}$ is the truthful strategy, and $u_p(\mathbf{I}, \{\mathbf{I}\}_{q\neq p}) > u_p(F^p_0, \{F^q\}_{q\neq p})$ where F^p_0 is an uninformed strategy.
- **Agent Clustering:**
  - we assume agents are clustered into K clusters, denoted by $G_1,..,G_K$. Let $G(p)$ denote the cluster to which the agent p belongs.
  - *Definition:* Clustering is ϵ_1 - accurate, for some $\epsilon_1 \geq 0$, if for every pair of agents $p,q \in P$, $||\Delta_{p,q} - \Delta_{G(p),G(q)}||_1 \leq \epsilon_1$
  - $\Delta_{G(p),G(q)}$ is cluster Delta matrix between clusters $G(p), G(q)$, defined as

$$\Delta_{G_s,G_t} = \frac{1}{|G_s|\times|G_t|} \sum_{p\in G_s, q\in G_t} \Delta_{p,q}$$

4.E.5 Correlated Agreement for Clustered, Heterogeneous Agents (CAHU)

Definition: Clustering $\{G_s\}_{s\in[K]}$ and estimates $\{\overline{\Delta}_{G_s,G-t}\}_{s,t\in[K]}$ are (ϵ_1, ϵ_2) - accurate if:

- $||\Delta_{p,q} - \Delta_{G(p),G(q)}||_1 \leq \epsilon_1$ for all agents $p, q \in P$, i.e. the clustering is ϵ_1 accurate,

- $||\Delta_{G(s),G(t)} - \Delta_{G(s),G(t)}||_1 \leq \epsilon_2$ for all clusters $s, t \in [K]$, i.e. the cluster Delta matrix estimates are ϵ_2 - accurate.

Algorithm: CAHU mechanism
Input: Clustering according to the above definition; for each agent $p \in P$, bonus task M_1^p, penalty tasks M_1^p and responses $\{r_p^b\}_{b\in M_1^p\cup M_2^p}$

Method:
1 : **for** every agent $p \in P$ **do**
2 : **for** every task $b \in M_1^p$ **do**
3 : $q \leftarrow$ uniformly at random conditioned on $b \in M_1^q \cup M_2^q$ ($M_2^p \neq M_2^q$ and $|M_2^p| \geq, 2|M_2^q| \geq 2$)
4 : Pick tasks $b^{'} \in M_2^p$ and $b^{''} \in M_2^q$ randomly such that $b^{'} \neq b^{''}$
5 : $S_{G(p),G(q)} \leftarrow \text{Sign}(\overline{\Delta}_{G(p),G(q)})$.
6 : Reward to agent p for task b is $S_{G(p),G(q)}(r_b^p, r_b^q) - S_{G(p),G(q)}(r_{b'}{}^p, r_{b''}^q)$.
7 : **end for**
8 : **end for**

Lemma 3 *For a strategy profile $\{F^p\}_{q\neq p}$ and an agent $p \in P$, define*

$$u_p(F^p, \{F^q\}_{q\neq p}) = \frac{1}{l-1}\sum_{q\in P\setminus\{p\}}\sum_{i,j}\Delta_{p,q}(i,j).S_{G(p),G(q)}(F_i^p, F_j^q)$$

Theorem 9 *With $(\epsilon_1$,$\epsilon_2)$-accurate clustering and learning, mechanism CAHU is $(\epsilon_1 + \epsilon_2)$ informed truthful if $min_p u_p{}^*(\mathbf{I}, \{\mathbf{I}\}_{q\neq p}) > \epsilon_1$. In particular,*

1. *For every profile $\{F^q\}_{q\in P}$ and agent $P \in P$, we have $u_p(F^p, \{F^q\}_{q\neq p}) - \epsilon_1 - \epsilon_2$*

2. *For any uninformed strategy $F_0{}^p$, $u_p(F_0^p, \{F^q\}_{q\neq p}) < u_p(\mathbf{I}, \{\mathbf{I}\}_{q\neq p})$*

4.E.6 Clustering

Definition 17 *A clustering $G_1, ..., G_k$ is ϵ-good if for some $\gamma > 0$*
$G(q) = G(r) \Rightarrow ||\delta_{pq} - \delta_{pr}||_1 \leq \epsilon - 4\gamma\ \forall p \in [l]\backslash\{q, r\}$
$G(q) \neq G(r) \Rightarrow ||\delta_{pq} - \delta_{pr}||_1 > \epsilon - 4\gamma\ \forall p \in [l]\backslash\{q, r\}$

Theorem 10 *Suppose there exist two clustering $\{G_j\}_{j\in[K]}$ and $\{T_i\}_{i\in[K']}$ that are ϵ-good. Then $K' = K$ and $G_j = T_{\pi(j)}$ for some permutation π over $[K]$*

Algorithm: Learning-Δ-No-Assumption
1 : **for** $t = 1, ...K$ **do**.
2 : Choose agent $q_t \in G_t$ arbitrarily.
3 : **end for**
4 : **for** each pair of clusters G_s, G_t **do**
5 : Let q_s and q_t be the chosen agents for G_s and G_t respectively
6 : Let $\overline{D}_{q_s,q_t}$ be the empirical estimate of D_{q_s,q_t} such that $||\overline{D}_{q_s,q_t} - D_{q_s,q_t}||_1 \leq \epsilon^{'}$ with probability at least $1 - \frac{\delta}{K^2}$
7 : Let $\overline{\Delta}_{q_s,q_t}$ be the empirical Delta matrix computed using $\overline{D}_{q_s,q_t}$
8 : Set $\overline{\Delta}_{G_s,G_t} = \overline{\Delta}_{q_s,q_t}$
9 : **end for**

4.E.7 Conclusion

A solution for the problem of peer prediction with heterogeneous agents is provided. There are interesting directions for ongoing research. It is possible to solve this problem with similar complexity but without a clustering approach and to couple methods of peer prediction with optimal methods for inference in crowd-sourced classification.

4.F Research Paper - 6

Title: A Geometric Method to Construct Minimal Peer Prediction Mechanisms [P6]
Author: Rafael Frongillo, Jens Witkowski

4.F.1 Problem Addressed

This paper proves that minimal peer prediction mechanisms are equivalent to power diagrams, a type of weighted Voronoi diagram using geometric perspective. Using computational geometry, it shows that many mechanisms are unique up to affine transformations and also introduces a general method to construct new truthful mechanisms.

4.F.2 Existing Mechanisms

The classical peer prediction mechanism compares the information reported by two participants and compute a payment rule which ensures that truth revelation is a strategic equilibrium. But this requires too much common knowledge.

Bayesian Truth Serum relaxes common knowledge assumptions but require participants to report both information and prediction report. The mechanism is not minimal (refer [2] for BTS).

The shadowing method and $\frac{1}{p}$ mechanism are minimal (refer [3]) with less assumption on common knowledge.

4.F.3 Model and Assumptions

There is group of $n \geq 2$ rational, risk neutral and self interested agents. Each agent i can observe a signal S_i from $[m] := \{1, .., m\}$ and $m \geq 2$. $p_i(s_i|s_j)$ denotes agent i's posterior belief that agent j receives signal s_j given agent i's signal s_i.

It is assumed that every agents belief model satisfies *stochastic relevance* (refer [9]). The equilibrium is *subjective* as it allows each agent to have distinct belief model, and *ex-post* because it doesn't require knowledge of other agents.

Lemma 4 *Let $M^{'}$ be a positive affine transformation (refer [12]) of M. Then $M^{'}$ is truthful if and only if M is truthful (refer [4]).*

Definition 18 (Effort Incentives) *Given an agent j invests effort and reports truthfully. The effort incentive for agent i by peer prediction mechanism M is difference in expected utility of investing effort followed by truthful reporting and not investing effort.*
$e_i(M) = \underset{S_i S_j}{\mathbf{E}}[M[S_i, S_j] - \underset{x_i \in [m] S_i}{max\,\mathbf{E}}[M(x_i, S_j)]$

Lemma 5 *For any mechanism M, and any positive affine transformation $M^{'} = \alpha.M + f$, we have $e_i(M^{'}) = \alpha.e_i(M)$*

4.F.4 Mechanisms and Power Diagrams

Definition 19 (Belief Model Constraint) *A belief model constraint is a collection $D = \{D_s \subseteq \Delta_m : s \in [m]\}$ of disjoint sets D_s of distributions.*
If we have $cl(\cup_s D_s) = \Delta_m$, i.e. if D partitions the simplex, D is maximal.

- A mechanism $M(.,.)$ is truthful with respect to belief model constraint D. M is truthful whenever $p_i(.|s) \in D_s$ for all agents $i \in [n]$ and all signals $s \in [m]$.
- Every truthful minimal peer prediction mechanism requires a belief model constraint.
- For an arbitrary mechanism M, if we restrict it to only those belief models under which M is truthful. It turns out that the described by a belief model constraint, which we call the constraint *induced* by M.

Lemma 6 *Let $M : [m] \times [m] \to \mathbf{R}$ be an arbitrary mechanism, and let D^M be the belief model constraint given by $D^M = \{p_i(.|s) : s = \underset{x_i}{argmax} \underset{S_j \sim p_i(.|s)}{\mathbf{E}} M(x_i, S_j)\}$, M is truthful with respect to D^M, but not truthful for belief models not satisfying D^M.*

4.F.5 Equivalence to Power Diagram

- Given a mechanism $M : [m] \times [m] \to \mathbf{R}$, we construct sites and weights as follows:
  $v^s = M(s, .)$, $w(s) = ||v^s||^2 = ||M(s, .)||^2$
- Given a power diagram (refer [13]) with sites and weights, we can construct a mechanism M as follows: $M(x_i, x_j) = v^{x_i}(x_j) - \frac{1}{2}||v^{x_i}||^2 + \frac{1}{2}w(x_i)$, where $v^{x_i}(x_j)$ is the x_jth entry of v^{x_i}.

Theorem 11 *Given any mechanism $M : [m] \times [m] \to \mathbf{R}$, the induced belief constraint D^M is a power diagram. Conversely, for every power diagram given by sites v^s and weights $w(s)$, there is a mechanism M whose induced belief model constraint D^M satisfies $D^M_s = cell(v^s)$ for all s.*

- Let D be a maximal belief model constraint. Then there exists a mechanism which is truthful with respect to D if and only if D is a power diagram.
- Conversion from mechanism to power diagram and back are inverse operations. Thus, it shows that mechanisms are in one-to-one correspondence with power diagrams on Δ_m.
- Mechanisms are unique for their respective conditions on posteriors, up to positive affine transformations.

Theorem 12 *If there exists a mechanism M that is truthful for some maximal belief model constraint D, and there is some $y \in \Delta_m$ with $y(s) > 0 \forall s$ such that $\cap_s cl(D_s) = y$, then M is the unique truthful mechanism for D up to positive affine transformations.*

4.F.6 Conclusion

new geometric perspective on minimal peer prediction mechanisms proved that it is without loss of generality, a minimal peer prediction mechanism as a power diagram. This allowed to prove uniqueness of several well-known mechanisms up to positive-affine transformations, to construct novel peer prediction mechanisms for new conditions, and to compute the maximally-robust mechanism with respect to agents subjective belief models deviating from the mechanism's.

4.G Research Paper - 7

Title: Incentivizing Evaluation via Limited Access to Ground Truth: Peer-Prediction Makes Things Worse[P7]
Author: Xi Alice Gao, James R. Wright, Kevin Leyton-Brown

4.G.1 Problem Addressed

peer-prediction mechanisms can only motivate agents to behave in a certain way as a group. An agent has a strong incentive to be truthful if all other agents are truthful; conversely, when all other agents coordinate on investing no effort, the agent again has a strong incentive to coordinate with the group.peer-prediction mechanisms thus need to provide a strong enough incentive for agents to deviate from the most attractive uninformative equilibrium in the worst case. This paper shows that the simpler mechanism achieves stronger incentive guarantees given less access to ground truth than a large set of peer-prediction mechanisms.

4.G.2 Previous Work

- Many peer grading systems are proposed with incentives for truthful grading using peer prediction scheme.
- Experimental evaluations of peer-prediction methods have mixed results. Some studies showed that agents reported truthfully [12]; another study found that agents colluded on uninformative equilibria [13].
- With a fixed probability, the mechanism obtains a trusted report and rewards the agent based on the agreement between the agent's report and the trusted report [15].
- Recent progress on peer-prediction mechanisms has focused on making the truthful equilibrium Pareto dominant, i.e., (weakly) more rewarding to every agent than any other equilibrium.

4.G.3 Peer Prediction Mechanisms

- *Universal peer-prediction mechanisms* [5] can be divided into three categories: output agreement mechanisms, multi-object mechanisms, and belief based mechanisms.
- **Output Agreement** is peer prediction mechanism in which two randomly paired users are rewarded if their reports agree with each other.(For formal definition refer [6])
- **Multi-Object Mechanism** reward each agent based on his reports for multiple objects. There are many multi-object mechanisms which differ in their scoring rule.
  - Shnayder [18] proposed a mechanisms which reward agents for agreement, as in output agreement mechanisms. The mechanism adds an additive scaling term to the reward for agreement.
  - Kamble [19], In contrast proposed a mechanism adds a multiplicative scaling term to the reward for agreement.
  - Radanovic and Faltings [20] proposed a mechanism that rewards the agents for report agreement using a reward function inspired by the quadratic scoring rule.
- **Belief Based Mechanism** collect both signal and belief reports from agents and reward each agent based on all agents' signal and belief reports for each object.
  - The robust Bayesian Truth Serum (BTS) rewards agent i for how well his belief report b_i and shadowed belief report $b_i^{'}$ predict the signal reports of another randomly chosen agent k.(For formal definition refer [10])
  - The multi-valued robust BTS[21] rewards agent i if his signal report matches that of another random agent j and his belief report accurately predicts agent j's signal report.
  - The divergence-based BTS [22] rewards agent i if his belief report accurately predicts another random agent j's signal report.
  - The Riley[23] mechanism rewards agent i for how well his belief report predicts other agents' signal reports.

4.G.4 Problem Setting

Consider a game theoretic setting in which we will study the elicitation problem. A mechanism designer wishes to elicit information about a set O of objects from n risk-neutral agents. Each object j has a latent quality $q_j \in Q$, where Q is a finite set.

- For each object j, agent i has access to two pieces of private information: *high-quality signal* $s_{ij}^h \in Q$ and a *low quality signal* s_i^l.
- The mechanism designer's aim is to incentivize each agent $i \in 1, ..., n$ to gather and truthfully report information about every object in $j \in O$.
- Let r_{ij} and b_{ij} denote agent i's signal and belief reports for object j respectively. s_j^t is a trusted report on object j.
- A mechanism is defined by a reward function, which maps a profile of agent reports to a reward for each agent.

Theorem 13 *For any universal elicitation mechanism, there exists a multi-signal environment in which the truthful equilibrium is not Pareto dominant.*

Theorem 14 *For any universal elicitation mechanism, there exists a costly-observation multi-signal environment in which the truthful equilibrium is Pareto dominated.*

4.G.5 Contribution

Definition 20 (spot-checking mechanism) *A spot-checking mechanism is a tuple $M = (p, y, z)$, where p is the spot check probability; y is a vector of functions $y_{ij}(r_{ij}, s_j^t)$ called the spot check mechanism; and z is a vector of functions $z_{ij}(b, r)$ called the unchecked mechanism.*

Let z be a peer-prediction mechanism. Then any spot-checking mechanism that uses z as its unchecked mechanism is a *spot-checking peer-prediction mechanism.*

Definition 21 (peer-insensitive mechanism) *A peer-insensitive mechanism is a spot-checking mechanism in which the unchecked mechanism is a constant function. i.e.,* $z_{ij} = W$ *for some constant* $W > 0$.

Lemma 7 *The minimum spot check probability* p_{ds} *at which the truthful strategy is dominant for the peer-insensitive mechanism satisfies the following equation.*

$$p_{ds}\mathbf{E}[y(s^h, s^t)] - c^E = p_{ds}\mathbf{E}[y(g^l(s^l, s^t))]$$

Here, $g_l = argmax_{g \in G}\mathbf{E}[y(g_l(s_l, s_t)]$ be an agent's best strategy when a spot check is performed and the agent invests no effort. Let the g_l equilibrium be the equilibrium where every agent uses the g_l strategy.

Lemma 8 *For any spot-checking peer-prediction mechanism, if the* g_l *equilibrium exists when* $c^E = 0$ *and* $p = 0$, *then* $p_{el} \geq p_{ds}$ *for all* $c^E \geq 0$.

Lemma 9 *For any spot-checking peer-prediction mechanism, if the* g_l *equilibrium exists and Pareto dominates the truthful equilibrium when* $c^E = 0$ *and* $p = 0$, *then* $p_{ex} \geq p_{ds}$ *for all* $c^E \geq 0$.

Theorem 15 *For any spot-checking peer-prediction mechanism, if the* g_l *equilibrium exists and Pareto dominates the truthful equilibrium when* $c^E = 0$ *and* $p = 0$, *then* $p_{Pareto} \geq p_{ds}$ *for all* $c^E \geq 0$.

Lemma 10 *For the spot check reward function in* $y_{ij}(r_i, s^t) = \mathbf{1}_{r_{ij}=s^t} - \mathbf{1}_{r_{ij'}=s^t_{j''}}$, *an agent's best strategy conditional on not investing effort is always to report the low-quality signal* s^l .

Corollary 1 *For spot-checking peer-prediction mechanisms, the minimum spot check probability* p_{Pareto} *for the Pareto dominance of the truthful equilibrium is greater than or equal to the minimum spot check probability* p_{ds} *at which the truthful strategy is a dominant strategy for the peer-insensitive mechanism.*

Corollary 2 *For spot-checking peer-prediction mechanisms, if the peer-prediction mechanism uses a symmetric proper scoring rule, then the minimum spot check probability p Pareto for the Pareto dominance of the truthful equilibrium is greater than or equal to the minimum spot check probability* p_{ds} *at which the truthful strategy is a dominant strategy for the peer-insensitive mechanism.*

4.G.6 Conclusion

Some mechanisms in the literature have been carefully designed to ensure that the truthful equilibrium is the most attractive equilibrium to the agents. However, these mechanisms rely crucially on the unrealistic assumption that agents' only means of correlating are via the signals that the mechanism aims to elicit. This paper shows that under the more realistic assumption that agents have access to more than one signal, no universal peer-prediction mechanism has a Pareto dominant truthful equilibrium in all elicitable settings. And also presents a simpler peer-insensitive mechanism that provides incentives for effort and honesty only by checking the agents' reports against ground truth.

4.H Research Paper - 8

Title: Crowdsourced Outcome Determination in Prediction Markets[P8]
Author: Rupert Freeman, Sébastien Lahaie, David M. Pennock

4.H.1 Problem Addressed

Motivated by the recent introduction of decentralized prediction markets, this paper introduces a mechanism that allows for the outcome to be determined by the votes of a group of arbiters who may themselves hold stakes in the market and also derive conditions under which we can incentivize arbiters to vote truthfully by using funds raised from market fees to implement a peer prediction mechanism.

4.H.2 Related Work

- Prediction markets have proven effective at forecasting events in a variety of domains.
- A decentralized platform removes the requirement for a highly trusted center, but it also allows each arbiter to directly influence the outcome of the market, this is known as *output manipulation.*
- Clark [24] discussed outcome determination in crypto-based prediction markets, among several other implementation aspects.
- Chakraborty and Das [25], considered a model where two agents participate in a prediction market whose outcome is determined by a vote among the two agents.

4.H.3 Model

- Let N be a set of agents and let $A \subset N$ be a small set of distinct and verifiable *arbiters*. Let $m = |A|$ denote the no.of arbiters. The agents are anonymous. Let X be an binary event (outcome in {0,1}).

- The prediction market is implemented via a *Market Scoring Rule (MSR)*, where the underlying scoring rule is strictly proper. This can be implemented as market maker based on convex cost function.
- Let μ be the prior probability that agent receives a positive signal. Let μ_1 be the probability that given agent i receives a positive signal,another randomly chosen agent also receives a positive signal. we calculate μ_1, μ_0 given μ and signal beliefs $P(x_i = 1|X = 1)$ and $P(x_i = 1|X = 0)$.
- $\delta = \mu_1 - \mu_0$ is the update strength.
- 1/prior is the peer prediction mechanism used in the model.

4.H.4 Mechanism

Market Stage

(a) A prediction market is set up for an event X using a market scoring rule.

(b) Agents trade in the market. For a security bought at price p, a trading fee of fp is charged, and for a security sold at price p, a fee of $f(1-p)$ is charged.

(c) The market closes, trading stops.

Arbitration Stage

(a) Each arbiter i receives a signal $x_i \in \{0, 1\}$ and reports an outcome $\widehat{x}_i \in \{0, 1\}$.

(b) Each arbiter i is assigned a peer arbiter $j \neq i$ and paid according to the 1/prior with midpoint mechanism.

(c) The outcome of the market, and the payoff of each share sold, is set to the fraction of arbiters that report $\widehat{x}_i = 1$.

Lemma 11 *Let n_i be the number of securities held by arbiter i. Then truthfully reporting $\widehat{x}_i = x_i$ is a best response for arbiter i, given that all other arbiters report truthfully, if and only if $k \geq \frac{2|n_i|}{m\delta}$.*

Let q^- and q^+ be the number of outstanding securities such that the market prices are $p(q^-) = \frac{f}{(1+f)}$ and $p(q^+) = \frac{f}{(1+f)}$ respectively.

Lemma 12 *For fixed percentage fee f , the number of outstanding securities lies in the interval $[q^-, q^+]$.*

Let $\phi_b^-(B) = C_b^{-1}(B + C_b(q^+)) - q^+$ and $\phi_b^+(B) = C_b^{-1}(B + C_b(q^-)) - q^-$. where the budget is B.

Corollary 3 *At the end of the market stage, an agent i with budget B holds $n_i \in [\phi_b^-(B), \phi_b^+(B)]$ securities.*

Theorem 16 *Given that all other arbiters report truthfully, truthful reporting is a best response for arbiter i if, $k \geq \frac{2max\{|\phi_b^-(B)|,|\phi_b^+(B)|\}}{m\delta}$*
In the case that agents may participate in the market many times, truthful reporting requires that $k \geq \frac{2B(1+f)}{fm\delta}$

Theorem 17 *The mechanism generates enough fee revenue to pay the arbiters without requiring any outside subsidy if, $fM \geq \frac{2max\{|\phi_b^-(B)|,|\phi_b^+(B)|\}}{\delta}$*
If agents may participate in the market many times, then we require that $fM \geq \frac{2B(1+f)}{f\delta}$

4.H.5 Conclusion

This paper proposed and analyzed a mechanism where the outcome of an MSR prediction market is determined via a peer prediction mechanism among a set of arbiters. To incentivize truthful arbitration the mechanism relies on two key adaptions. Market shares pay out according to the proportion of arbiters who vote affirmatively and peer prediction payments are based on the midpoint of the two possible posteriors,

4.I Research Paper - 9

Title: Personalized Peer Truth Serum for Eliciting Multi-Attribute Personal Data [P9]
Author: Naman Goel, Boi Faltings

4.I.1 Problem Addressed

Considering the problem of eliciting the personal attributes of the agents where the tasks cannot be shared between two agents and designing a Personalized Peer Truth Serum (PPTS) which incentivize the peer consistency.

4.I.2 Previous Work

- Radanovic and Faltings introduced a mechanism, *Logarithmic Peer Truth Serum (LPTS)* [26].
- Miller [27] proposed original peer prediction mechanism for information elicitation without verification.
- Peer Truth Serum (PTSC) mechanism for incetivizing efforts for crowdsourcing was proposed [28].
- The Deep Bayesian Trust mechanism [29] ensures dominant strategy incentive compatibility and also computes fair rewards in large scale crowdsourcing by using both peer answers and some gold standard answers.

4.I.3 Contribution

- **Step1:** Define which agents can act as peers for one another in settings when agents can't share tasks.
- **Step2:** Show that even if such peers are estimated from the reports submitted by the agents, the incentive compatibility is not affected.
- **Step3:** Extend the mechanism to handle continuous data values instead of only discrete answers.

4.I.4 Model

- In this model the data is collected from large no.of agents $W(|W| = n \to \infty)$.
- The elicited data consists of set of attributes $A(|A| = d \geq 2)$.
- Let $P(X_{ij})$ be agent i's prior belief about measurement of attribute j.
- Random variable G_j models the global factors that affect the attribute j of any random agent. $P(G_i)$ is prior belief and $P(G_i|X_{ij})$ is posterior belief agent's have about global factors.
- For every agent i, set of other agents $N_i \in W$ called cluster of agent i which share only some personal factors.
- Let L_{ij} denote random variable for personal factors where k being the cluster to which agent i belongs. $P(L_{ij})$ is prior belief and $P(L_{ij}|X_{ij})$ is posterior belief agent's have about personal factors.
- The global distribution $P(X_{ij}|G_j)$ is modeled as $P(X_{ij}|G_j) = \sum_{k=1}^{K} \alpha.P(X_{ij}|L_{kj})$ where $K(<< N)$ is the number of clusters and α_k is the mixing probability of kth cluster.

4.I.5 Mechanism

- The center collects reports from all agents for all their attributes. Then each agent is assigned to a cluster corresponding to agent i's belief.
- jth attribute score of agent i for reporting $X_{ij} = y$ is $r_{ij} = log\frac{f(y|\widehat{\mu}_{L_{ij}}, \widehat{\sigma}^2_{L_{ij}})}{\sum_{k=1}^{K} \widehat{\alpha}_k \cdot f(y|\widehat{\mu}_{L_{kj}}, \widehat{\sigma}^2_{L_{kj}})}$ where f is a Gaussian function given by $f(x|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ where $\widehat{\mu}_{L_{kj}}, \widehat{\sigma}^2_{L_{kj}}$ are mean and variance of values of reported for attribute j by agents in the cluster N_i. $\widehat{\alpha}_k$ is the empirical relative mixing frequency of cluster k.
- Agent i finally gets a cumulative reward (CR) equal to the average of attribute score r_{ij} for all attributes $j \in \{1, 2.., d\}$. More formally, $CR(i) = \frac{\sum_{j=1}^{d} r_{ij}}{d}$

4.I.6 Analysis

- PPTS mechanism rewards 'surprisingly common' reports. The PPTS mechanism is Bayes-Nash incentive compatible, with strictly positive expected payoffs in the truthful reporting equilibrium.
- Heuristic reporting equilibria result in zero expected payoff in the mechanism.
- In the PPTS mechanism, an equilibrium strategy profile defined by a function $g(x) = ax + b$ is not in expectation more profitable than the truthful strategy.

Theorem 18 *The ex-ante expected score of a truthful agent is equal to the conditional mutual information (CMI) of the attribute measurements and the personal factors given the global factors.*

Definition 22 (ϵ- Correct Clustering Algorithm) *A clustering algorithm is called ϵ - correct, if given true reports, it assigns a true report to a wrong cluster with probability at most ϵ and ϵ is such that as $|N_k| \to \infty$, the MLE estimates $\{\widehat{\mu}_{L_{kj}}, \widehat{\sigma}^2_{L_{kj}}\}$ converge to $\{\mu_{L_{kj}}, \sigma^2_{L_{kj}}\}$ and $\widehat{\alpha}_k$ converge to α_k, $\forall k$.*

Theorem 19 *Given an ϵ - correct clustering algorithm, the PPTS is Bayes-Nash incentive compatible even if the clusters are approximated from the reports.*

4.I.7 Conclusion

A novel incentive mechanism to elicit continuous valued, multi-attribute and personal data from crowd was proposed and presents that the mechanism ensures truthful equilibrium is profitable compared to any other undesired equilibria.

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