# Monte Carlo Simulation
# &
# Black Scholes delta-hedged Portfolio

**AIM : To simulate multiple monte carlo step-wise probabilistic evolutions of asset prices over time**

$$S(t + dt) = S(t) * \{ 1 + \sigma\sqrt{dt} * \epsilon(t) \}$$

Where $\sigma$ is volatility, $\epsilon$ **(t)** is normally distributed noise

## Delta Hedged Portfolio

Delta hedging is a technique used to reduce or eliminate the portfolio value changing risk from price movements in the underlying asset. The key idea is to adjust the portfolio continuously by buying or selling the underlying asset in proportion to the delta of the portfolio.

**Initial Portfolio with $\Delta_t$ being delta of Call Option at time 't'**

$\Pi_0$ = { 1 Call Option of strike $K=S_0$ (long) , $-\Delta_0$ of S (short) }

Constantly delta hedging portfolio at each time interval $t_1$ $t_{2\,....}$

$\Pi_t$ = { 1 Call Option of strike $K=S_0$ (long) , $-\Delta_t$ of S (short) }

$\Pi_{t+dt}$ = { 1 Call Option of strike $K=S_0$ (long) , $-\Delta_{t+dt}$ of S (short) }

**Thus at time t+dt, need to short more $\Delta_{t+dt} - \Delta_t$ shares of S**

**Cash Flow Calculation**: Throughout each simulation:

- The portfolio is adjusted at each time step based on the change in delta of the option, ensuring that the portfolio remains delta-neutral.
- At the end of each simulation, the portfolio is unwound by closing the option position and adjusting the underlying shares.

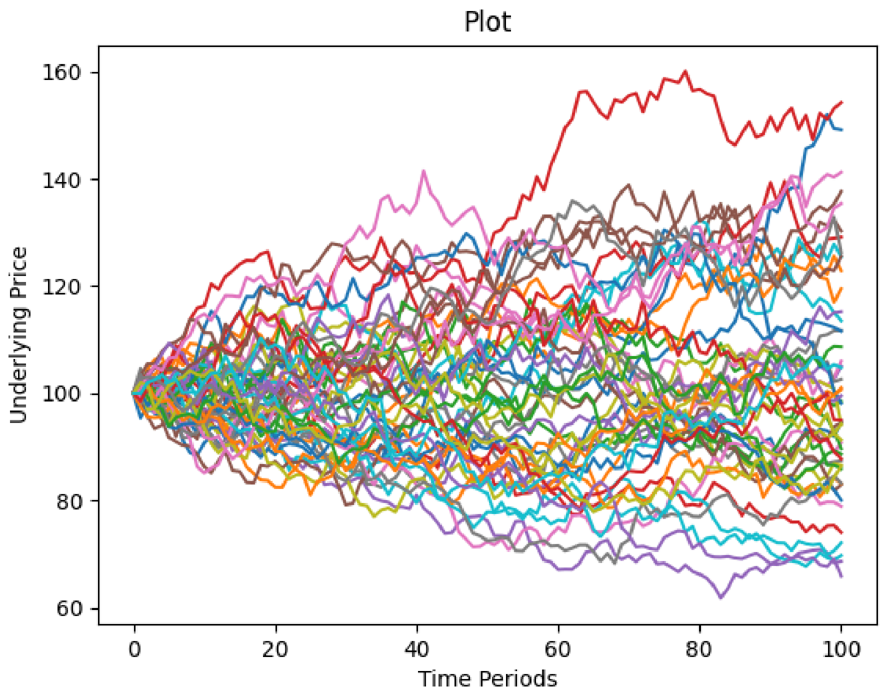
The main objective of this simulation is to demonstrate that **the expected value of the hedged portfolio at expiry is zero, assuming arbitrage free market and no risk-free interest rate**. This aligns with the fundamental premise of the Black-Scholes model, which assumes:

- The underlying asset follows geometric Brownian motion.
- There are no arbitrage opportunities.
- Delta hedging can effectively eliminate risk.

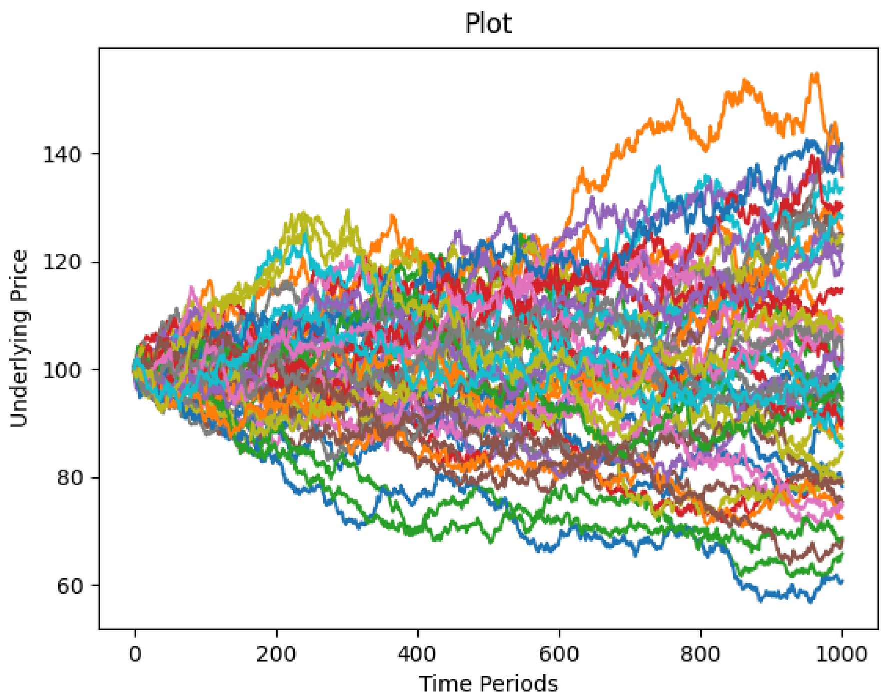
The average cash flow across all simulations is approx zero, confirming the validity of the Black-Scholes model under the assumptions made.

# Results - 1

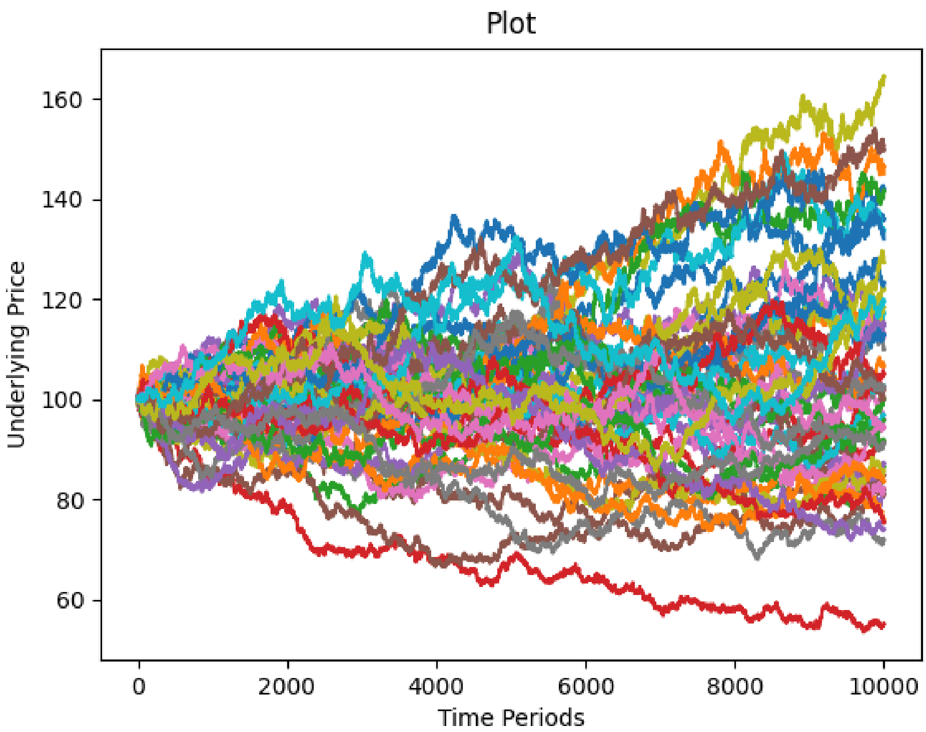
No. of Time Periods = 100     Avg Cash Flow = 0.193678




No. of Time Periods = 1000     Avg Cash Flow = 0.0221157



No. of Time Periods = 10000        Avg Cash Flow = -0.00151821




**Observation:** As the frequency of delta hedging the portfolio increases, the average cash value gets more and more closer to 0, demonstrating the correctness of Black Scholes model which assumes a perfectly hedged portfolio.

# Results - 2

Here for the Number of Time Periods = 1000 case, for different values of realised volatility in comparison to the implied volatility value, we can see the changes in the average cash flow value of the portfolio at time of expiry.

1000_IV_20_RealVol_5
Average Cash Flow across simulations: -5.84445

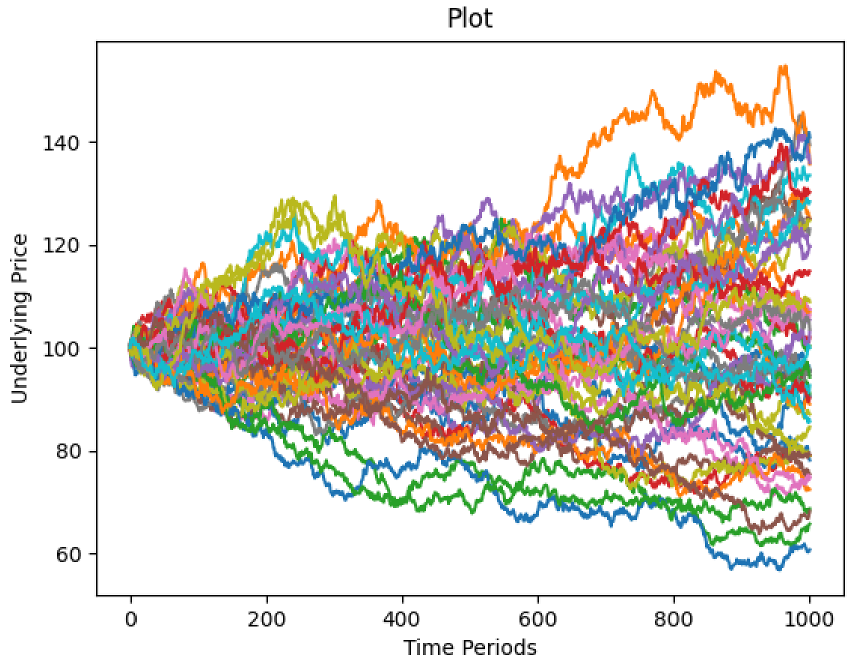
1000_IV_20_RealVol_20
Average Cash Flow across simulations: 0.0221157

1000_IV_20_RealVol_40
Average Cash Flow across simulations: 7.77404

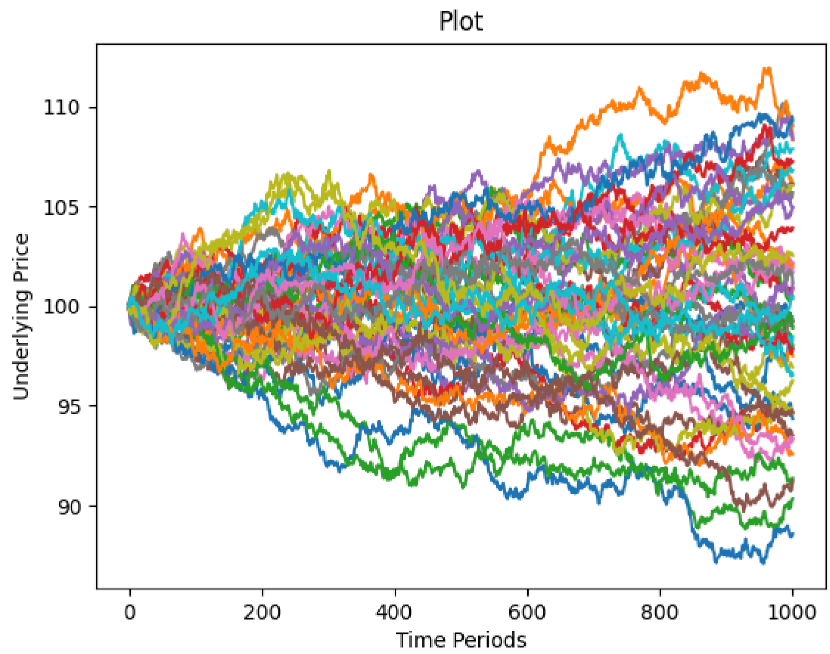
```
satyam@satyam:~/Desktop/work$ ./run.sh
100_IV_20_RealVol_20
Average Cash Flow across simulations: 0.193678
1000_IV_20_RealVol_5
Average Cash Flow across simulations: -5.84445
1000_IV_20_RealVol_20
Average Cash Flow across simulations: 0.0221157
1000_IV_20_RealVol_40
Average Cash Flow across simulations: 7.77404
10000_IV_20_RealVol_20
Average Cash Flow across simulations: -0.00151821
satyam@satyam:~/Desktop/work$
```

When **realised volatility exceeds the implied volatility**, the **market movements become greater than anticipated by the black scholes model using implied volatility,** leading to **larger adjustments in hedge positions** and **higher than expected option price**, which translate into **gain**. And similarly the opposite happens for the case of lower realised volatility.

Implied Volatility = 20%   Realised Volatility = 20%

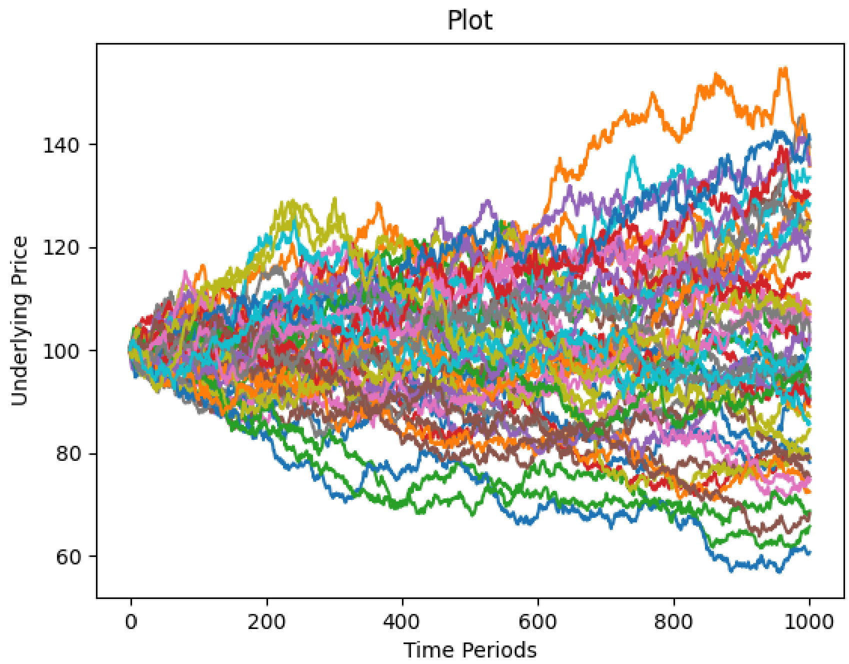



Implied Volatility = 20%   Realised Volatility = 5%

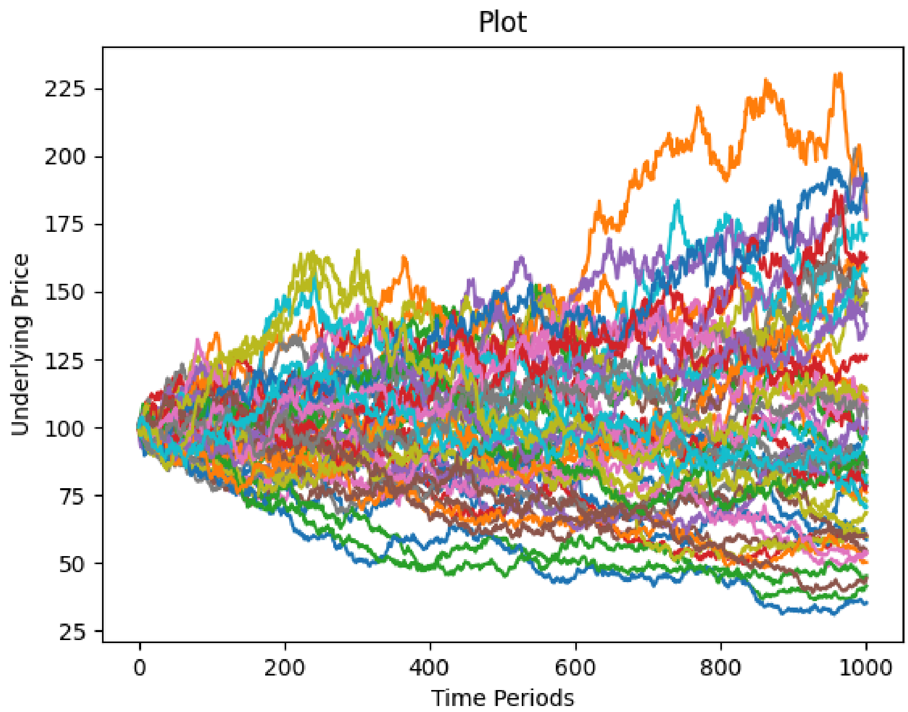



Observation The spread of Simulations has decreased in the 5% realised Volatility case as the stochastic term in the price equation is directly proportional to Volatility (60-140) to (90-110).

Implied Volatility = 20%   Realised Volatility = 20%




Implied Volatility = 20%   Realised Volatility = 40%




Similarly here the spread of Simulations has increased in the 40% realised Volatility case from (60-140) to (25-225).

~~~
~~~