MACHINE LEARNING

WEEK04

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اَللّٰهُمَّ ارْزُقْنِی عِلْمًا نَافِعًا وَاسِعًا عَمِیْقًا

اَللّٰهُمَّ اُرْزُقْنِی رِزْقًا وَاسِعًا حَلَالًا طَیِّبًا
مُبَارَكًا مِنْ عِنْدِكَ

WEEK 04

TYPE OF MACHINE LEARNING

- Supervised Learning.
- Unsupervised Learning.
- Reinforcement Learning.

SUPERVISED LEARNING

Living area (feet^2)	Price (1000\$s)
2104	400
1600	330
2400	369
1416	232
3000	540
⋮	⋮

SUPERVISED LEARNING: REGRESSION

- When we try to predict a number from historical data this type of supervised learning problem is called Regression Problem

FIND THE RELATION

- Can You predict upcoming value based on the following data.

x	y
0	1
2	5
3	7
4	9
5	11
6	13
7	?

FIND THE RELATION

- Can you write Generic Formula (function) for following ?

x	**y**
0	1
2	5
3	7
4	9
5	11
6	13
7	?

FIND THE RELATION

- $Y=2x+1$ (this is called model)
- We call y is function of x.

x	y
2	5
3	7
4	9
5	11
6	13
7	?

FIND THE RELATION

- We call y is function of x. $f(x) = 2x+1$
- More Formally, we write it $f\ x:R \longrightarrow y:R$

x	y
0	1
2	5
3	7
4	9
5	11
6	13
7	?

LINEAR FUNCTION

- Function on Graph

x	y
0	1
2	5
3	7
4	9
5	11
6	13
7	?

LINEAR FUNCTION

- This Function is called
  Linear Function

x	y
0	1
2	5
3	7
4	9
5	11
6	13
7	?

FIND THE RELATION

- Can you predict value of y when x=7 based on the following data ?

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

FIND THE RELATION

- What could be the general relation ?

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

FIND THE RELATION

- What could be the general relation ?
- $Y=x^2 + 2$

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

GRAPH THE RELATION

- Draw relation $Y=x^2+2$

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

GRAPH THE RELATION

- This kind of relation is called Quadratic Relation

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

FIND THE RELATION

- The problem of finding the relation between x and y is called Regression

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

FIND THE RELATION

- And if we assume the relation shall be a linear function, we call it linear regression

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

FEATURES: INDEPENDENT VARIABLE

- In this relation x is independent variable and y is dependent variable.

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

UNIVERIATE LINEAR REGRESSION

- If there is only singe independent variable or feature that use to predict the y, then this problem is called Univeriate Linear Regression

REGRESSION

- As an Human, we identified the function that could be $f(x)= 2x+1$ or $f(x)=x^2+2$ where $f(x)$ is y.

FEATURES: INDEPENDENT VARIABLE

- X is also called as features and y is also called target variable or label variable.

x	y
0	2
2	6
3	11
4	18
5	27
6	38
7	?

LINEAR REGRESSION

given independent variables, we need to find a linear function that can predict the value of y while value of x is given.

$$f(x) = \theta_1 x + \theta_0$$

LINEAR REGRESSION

- In previous example θ_1 =2, θ_0=1 that make it f(x)= 2x+1

$$f(x) = \theta_1 x + \theta_0$$

LINEAR REGRESSION

- This identified function is called model or hypothesis on data.

$$f(x) = \theta_1 x + \theta_0$$

LINEAR REGRESSION

The θ_1 and θ_0 are called parameters, the value of these parameters could be different for different type of data.

$$f(x) = \theta_1 x + \theta_0$$

MORE FORMALLY

Linear Regression

Given: Training data: $(x_1, y_1), \ldots, (x_n, y_n)$ / $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$

example $x_1 \rightarrow$	x_{11}	x_{12}	$\ldots$	x_{1d}	$y_1 \leftarrow$ label
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
example $x_i \rightarrow$	x_{i1}	x_{i2}	$\ldots$	x_{id}	$y_i \leftarrow$ label
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
example $x_n \rightarrow$	x_{n1}	x_{n2}	$\ldots$	x_{nd}	$y_n \leftarrow$ label

Task: Learn a regression function:

$$f : \mathbb{R}^d \longrightarrow \mathbb{R}$$
$$f(x) = y$$

Linear Regression: A regression model is said to be linear if it is represented by a linear function.

LINEAR REGRESSION

x	y
0	1
2	5
3	7
4	9
5	11
6	13
7	?

So finally, we have following problem in hand for any given data we need to find a linear function that map x to y.

$$f(x) = \theta_1 x + \theta_0$$

LINEAR REGRESSION

$$f(x) = \theta_1 x + \theta_0$$

In other terms, we need to find the parameter value of θ_0 and θ_1 that best suitable according to the given data.

LINEAR REGRESSION

$$f(x) = \theta_1 x + \theta_0$$

This linear function also represent as a line graphically.

LINEAR REGRESSION

$$f(x) = \theta_1 x + \theta_0$$

Therefore, sometime the univariate linear regression problem also seen as to find equation of line.

LINEAR REGRESSION

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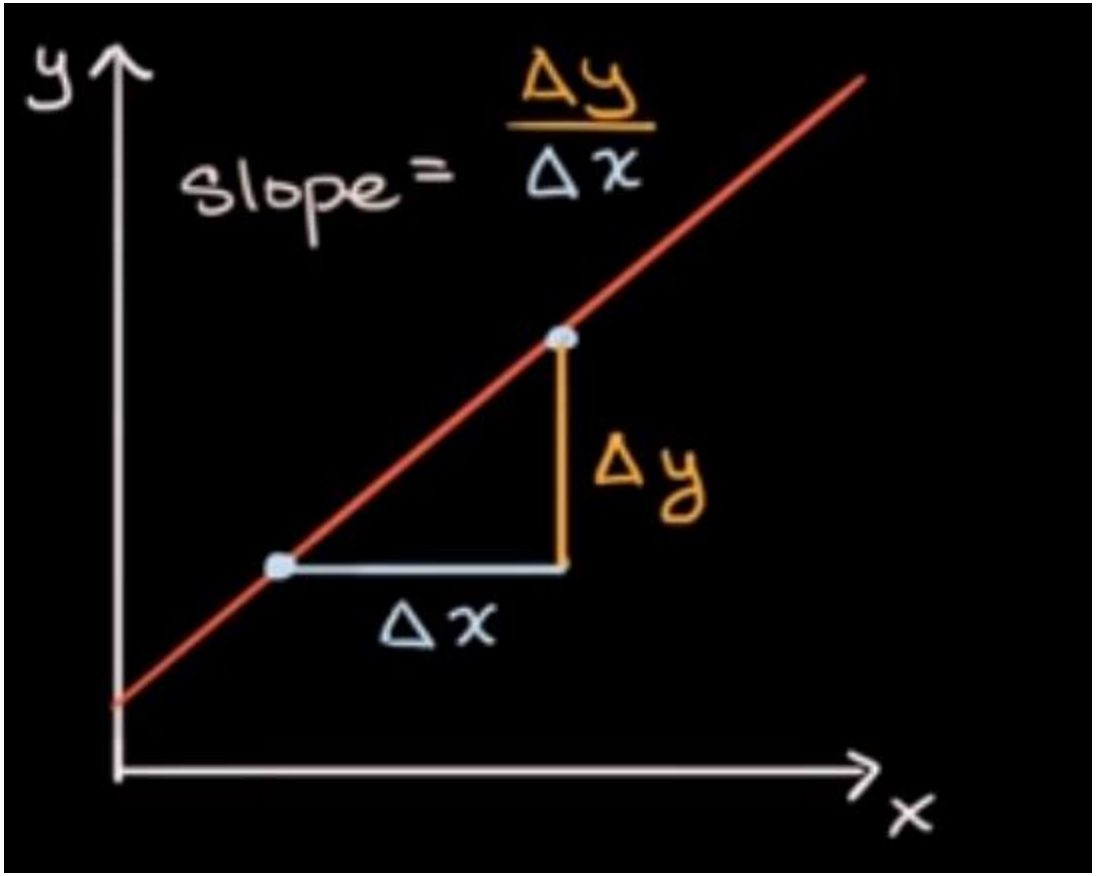
LINES REVIEW

SLOPE OF LINE

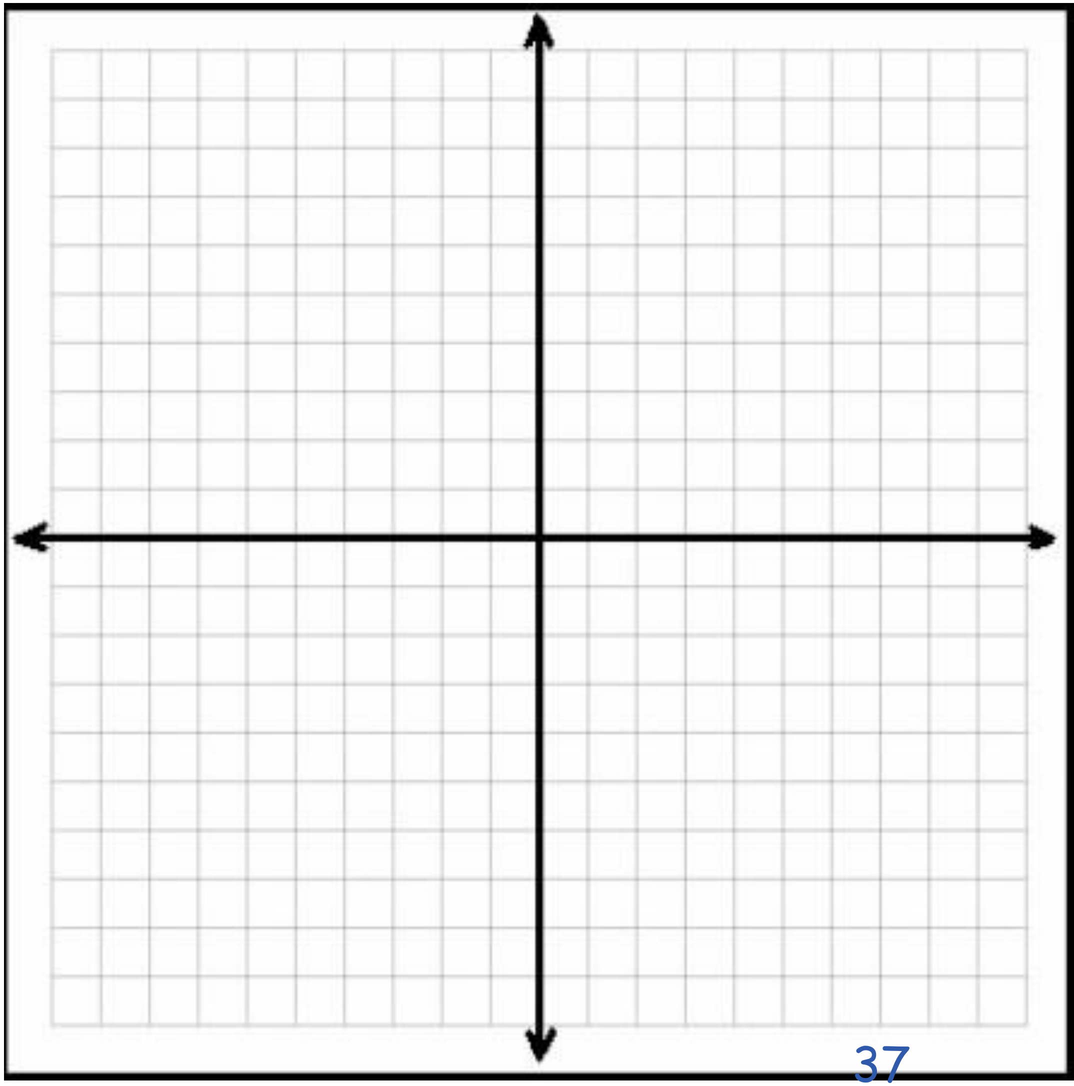


SLOPE INTERCEPT FORM

$$y = mx + c$$

Draw Line for following Equation

$$y = \frac{5}{3}x - 2$$



SLOPE INTERCEPT FORM

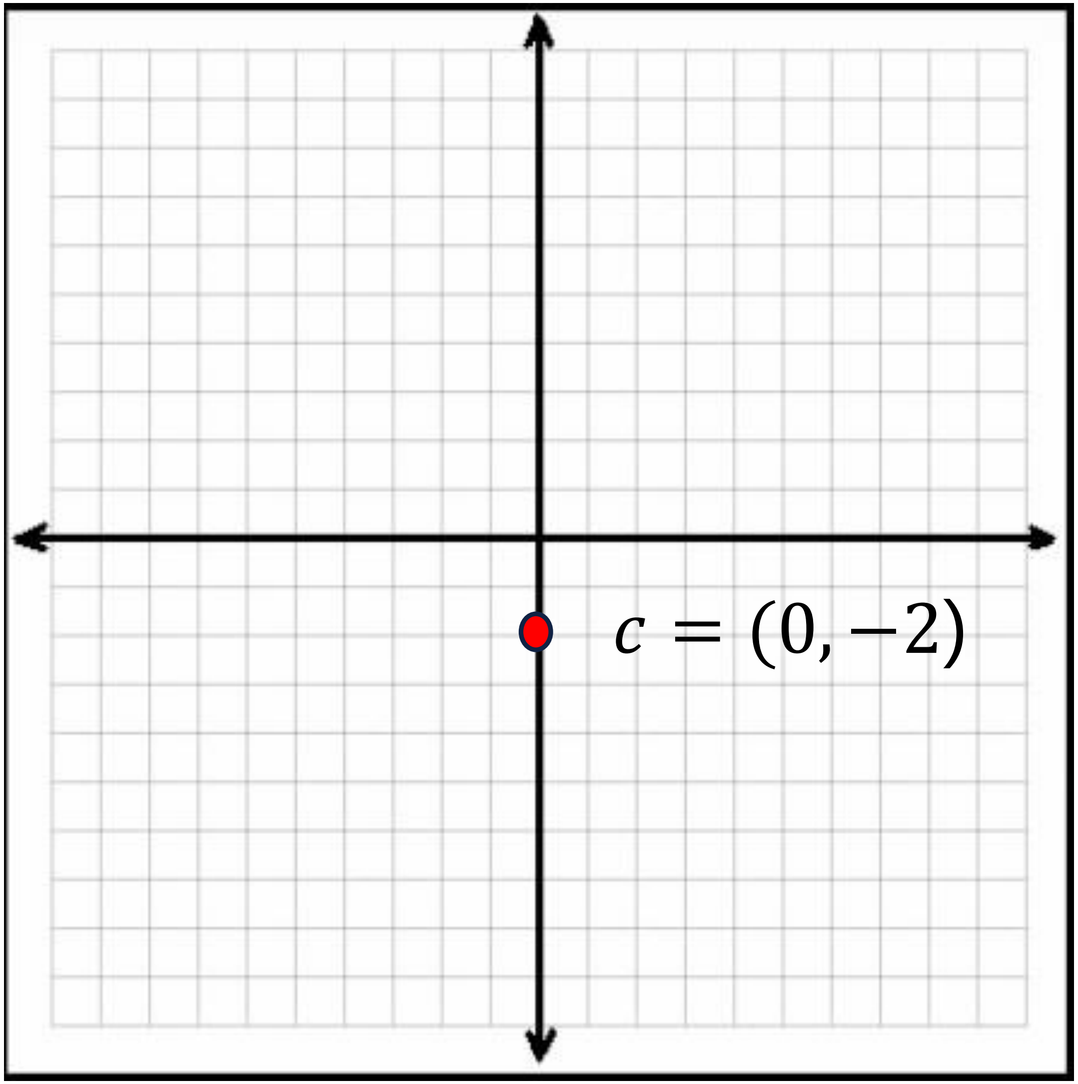
$$y = mx + c$$

$$m = \frac{\Delta Y}{\Delta X}$$

$$y = \frac{5}{3}x - 2$$

$$c = -2$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{5}{3}$$



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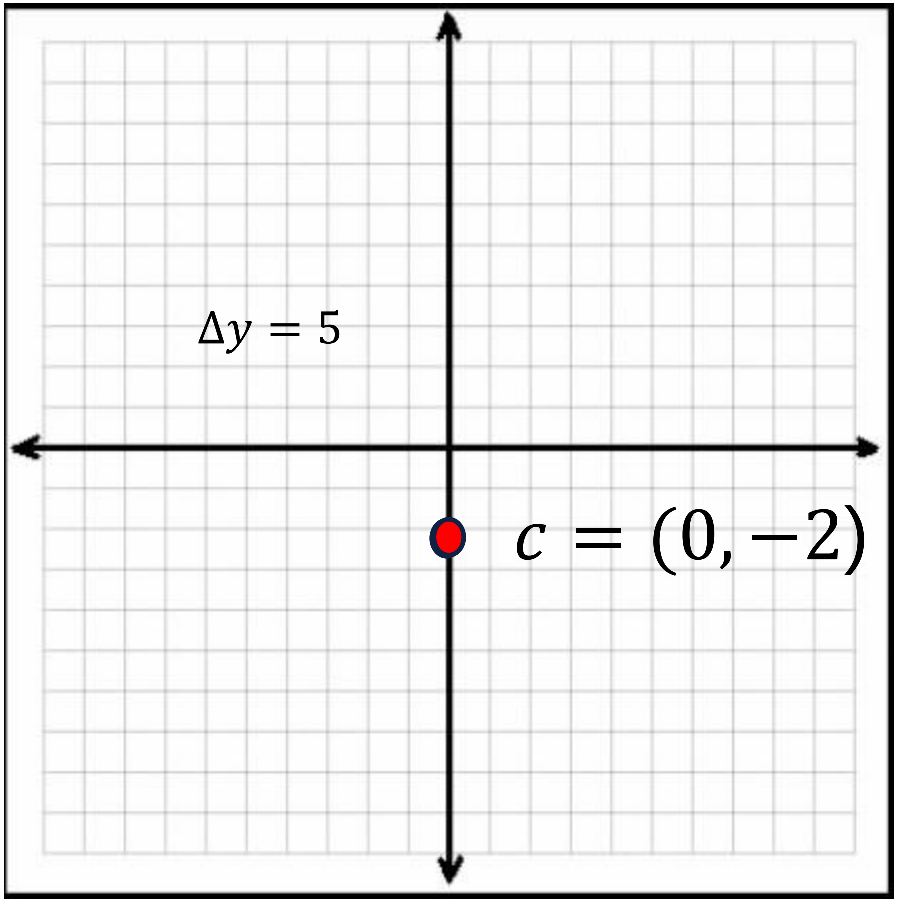

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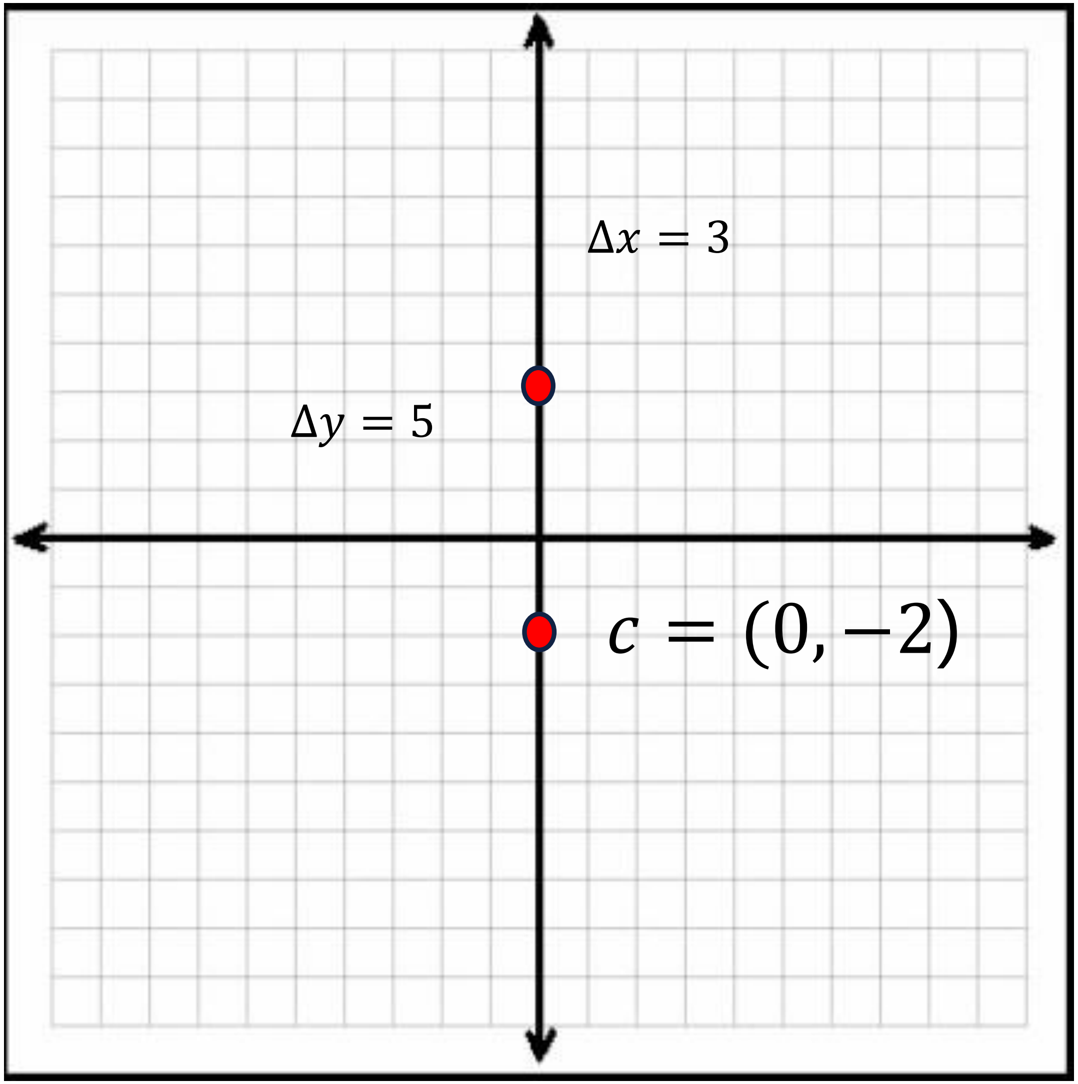


SLOPE INTERCEPT FORM

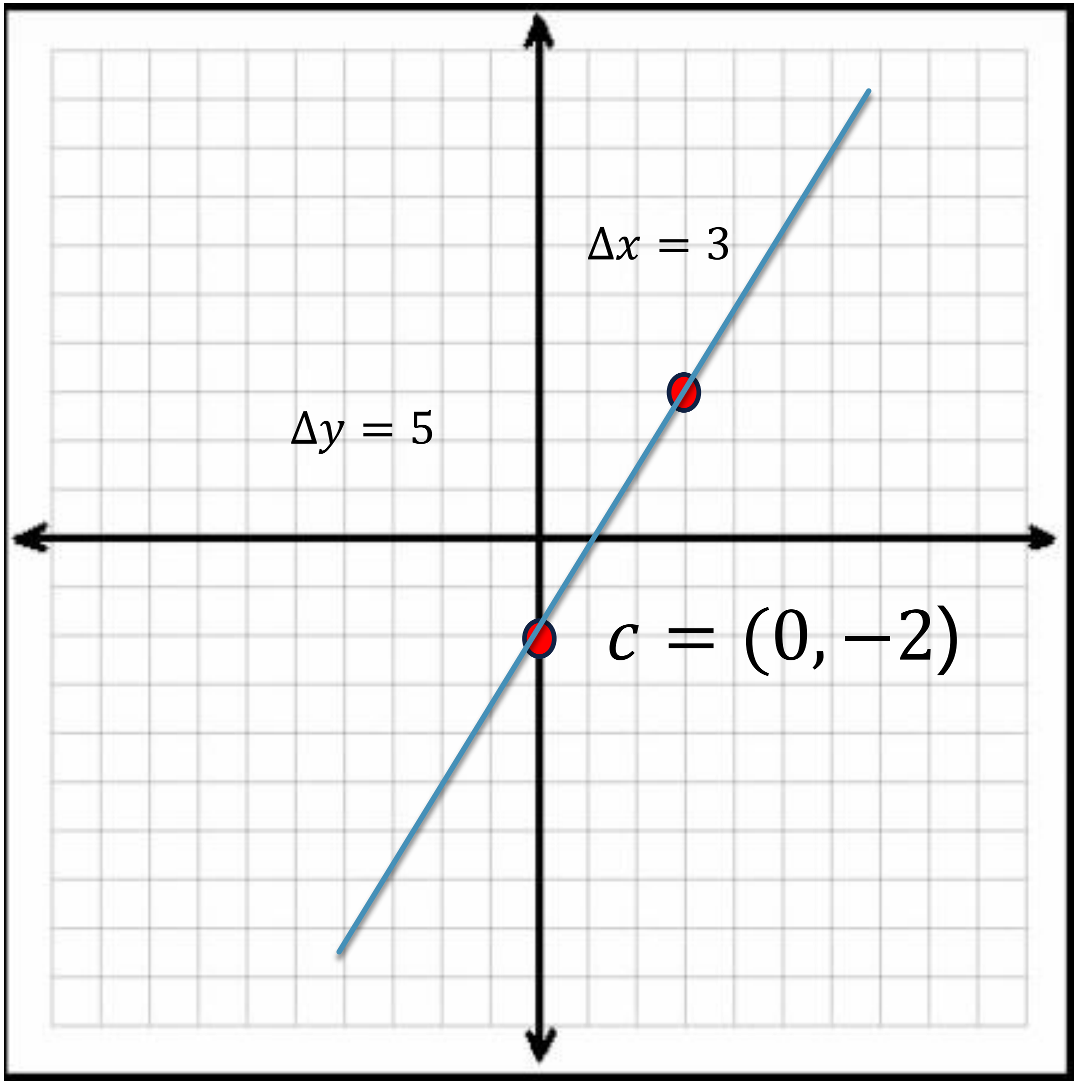
$$y = mx + c$$

$$m = \frac{\Delta Y}{\Delta X}$$

$$y = \frac{5}{3}x - 2$$

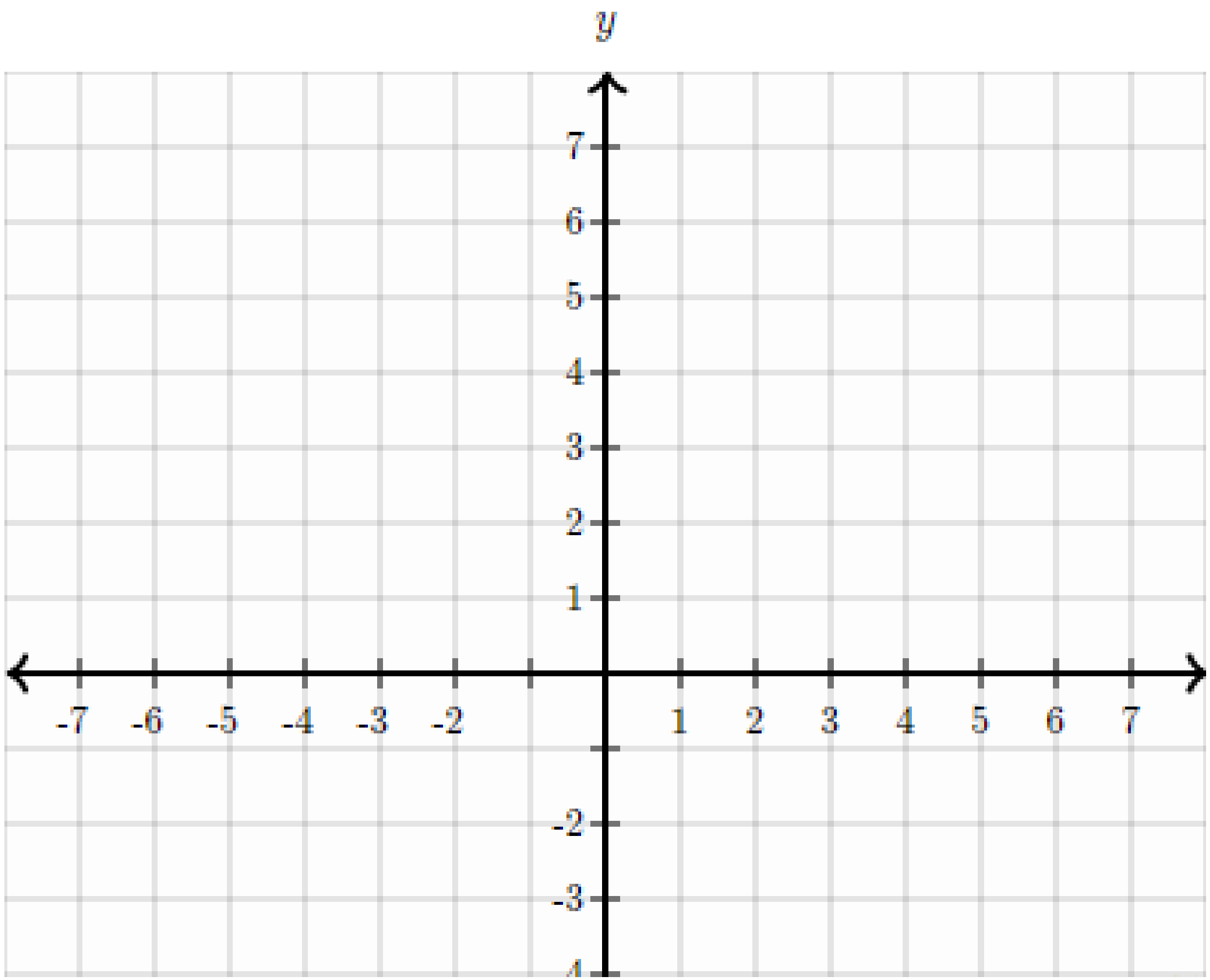
$$c = -2$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{5}{3}$$

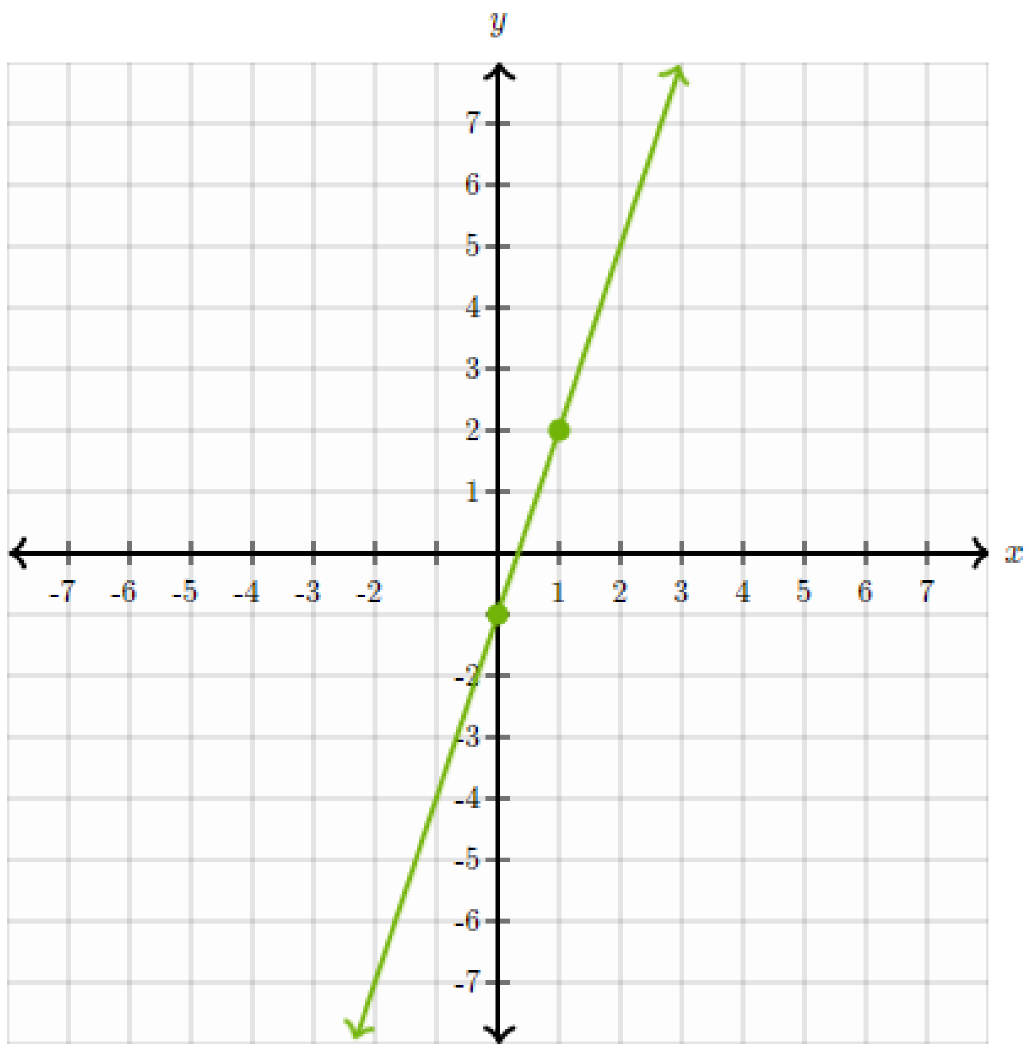


PRACTICE: DRAW LINE

$$y = 3x - 1$$

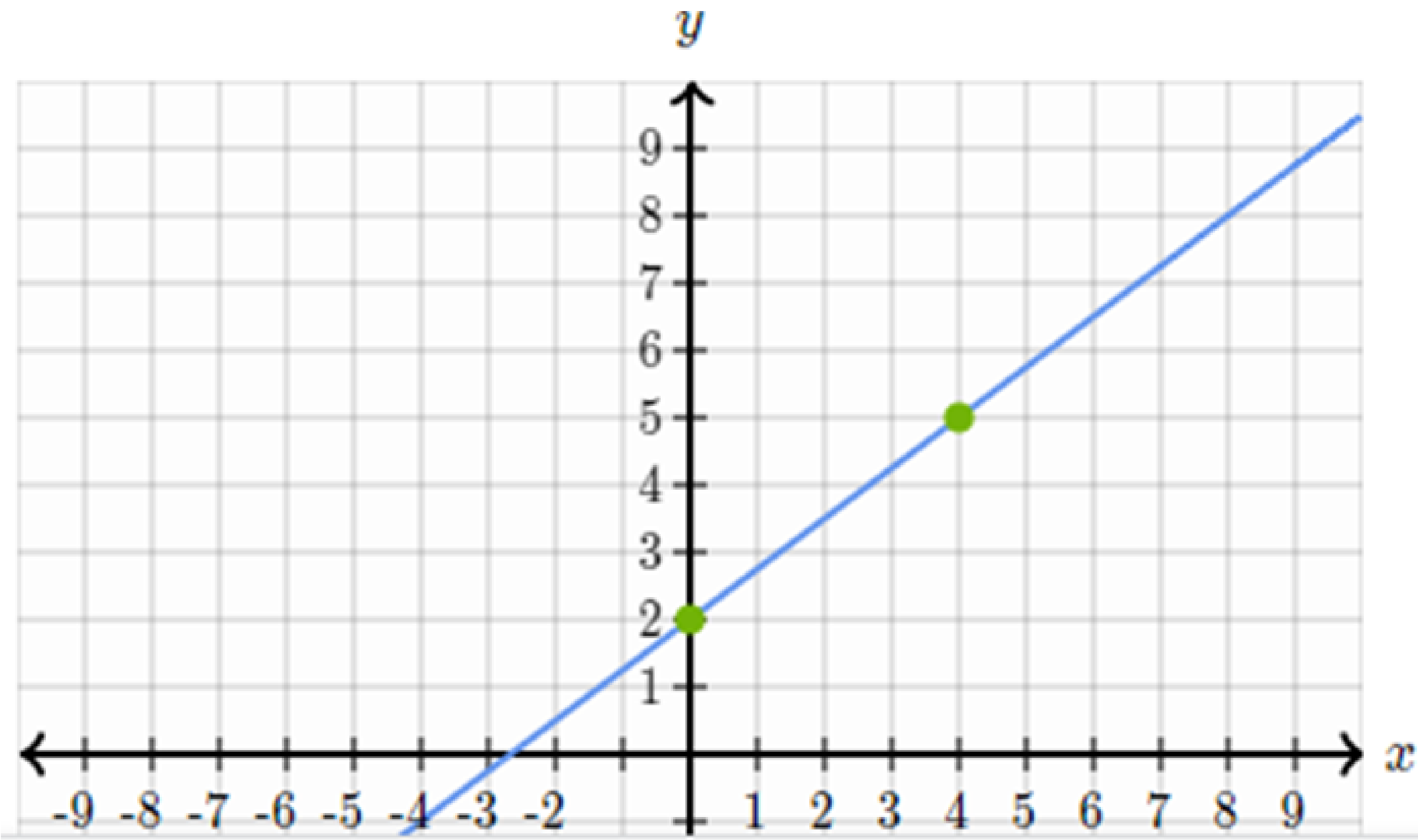


PRACTICE: DRAW LINE Y=3X-1



ONE MORE EXAMPLE

Graph $y = \frac{3}{4}x + 2.$



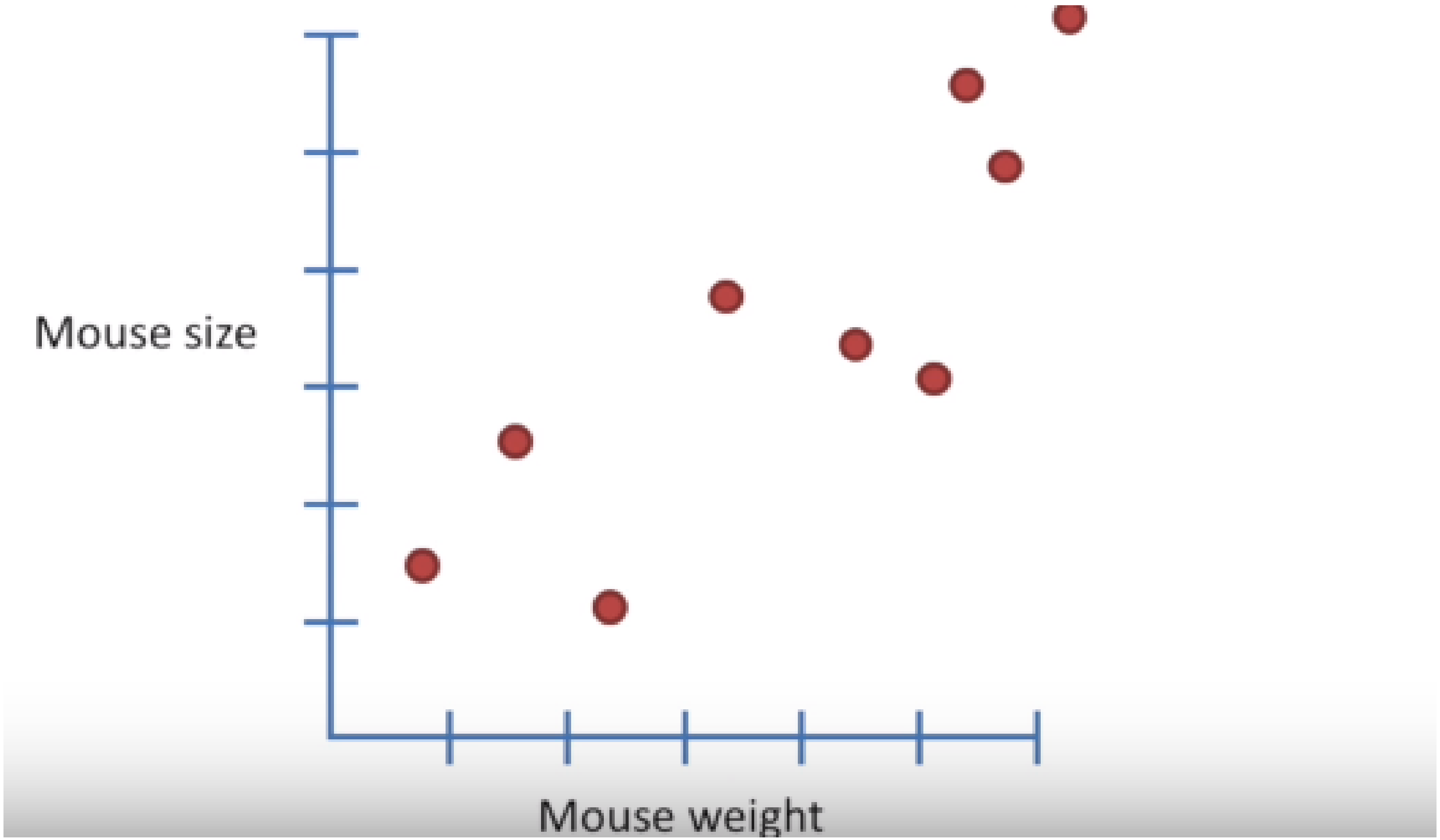
RATE OF CHANGE

- For Linear Functions, Slope represents the rate of change of function and it is constant.

WHAT IS BEST SUITABLE

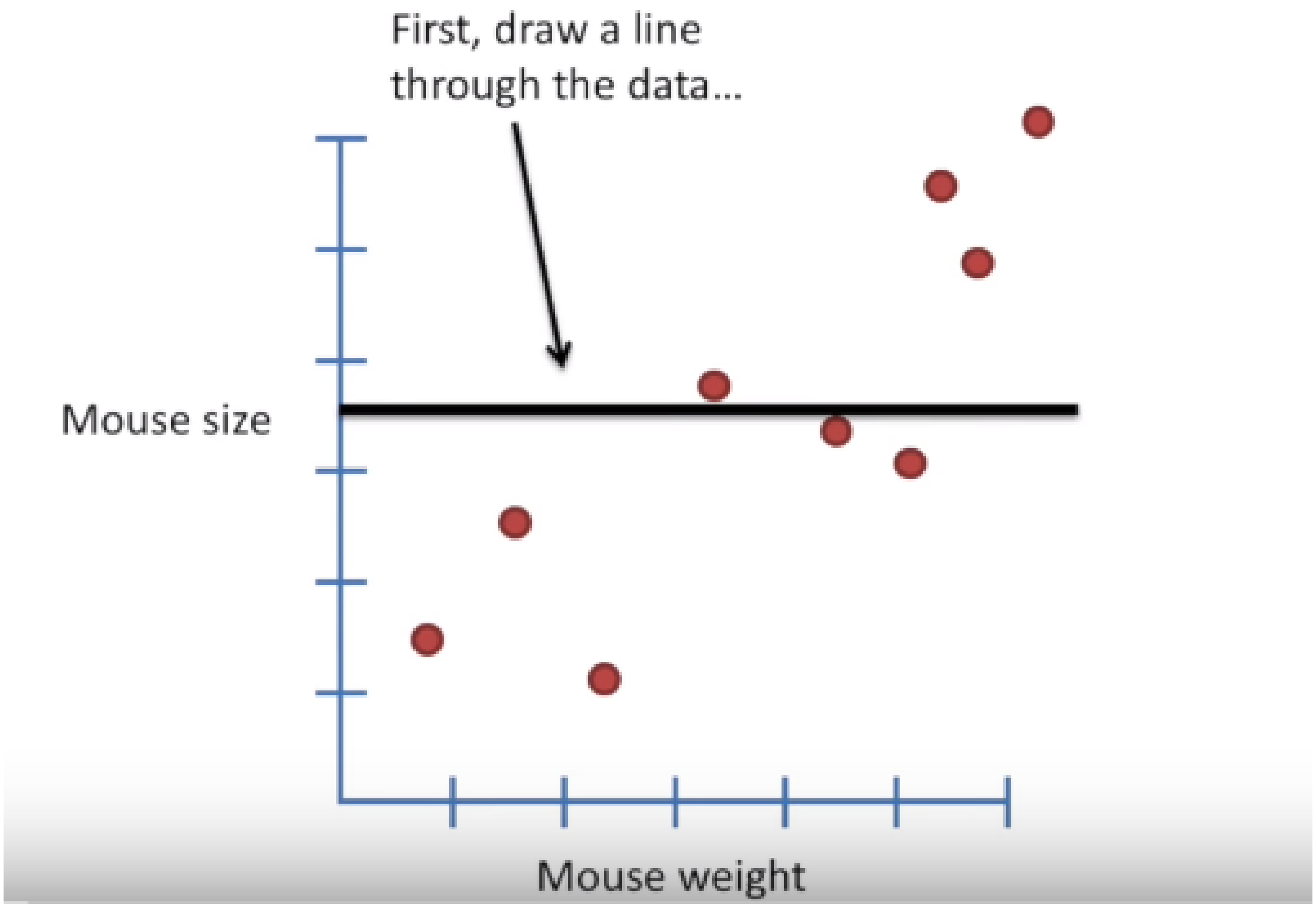
- What is best suitable parameters ? Let's visualize the problem

DATA

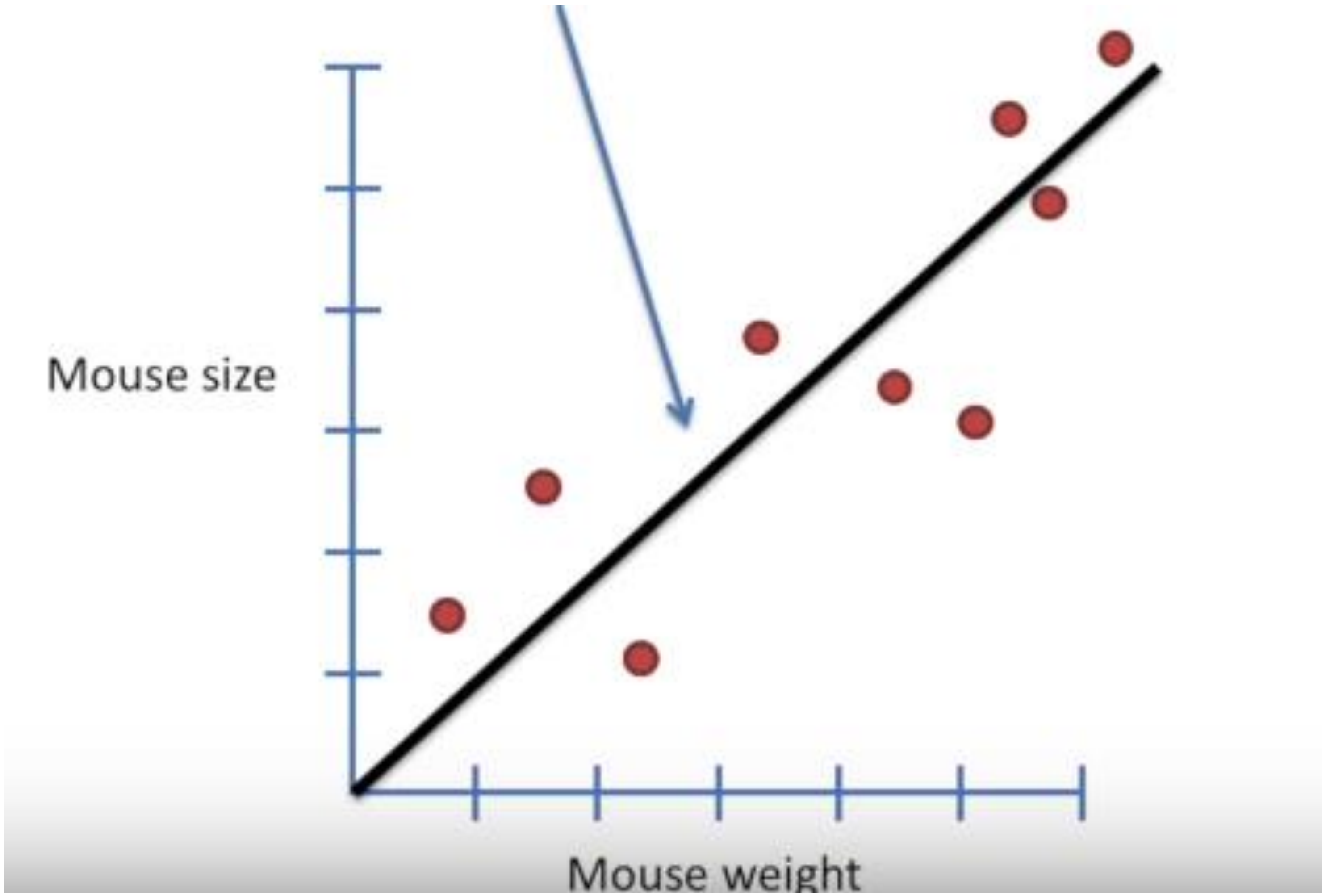


Sourcce: https://www.youtube.com/watch?v=nk2CQITm_eo

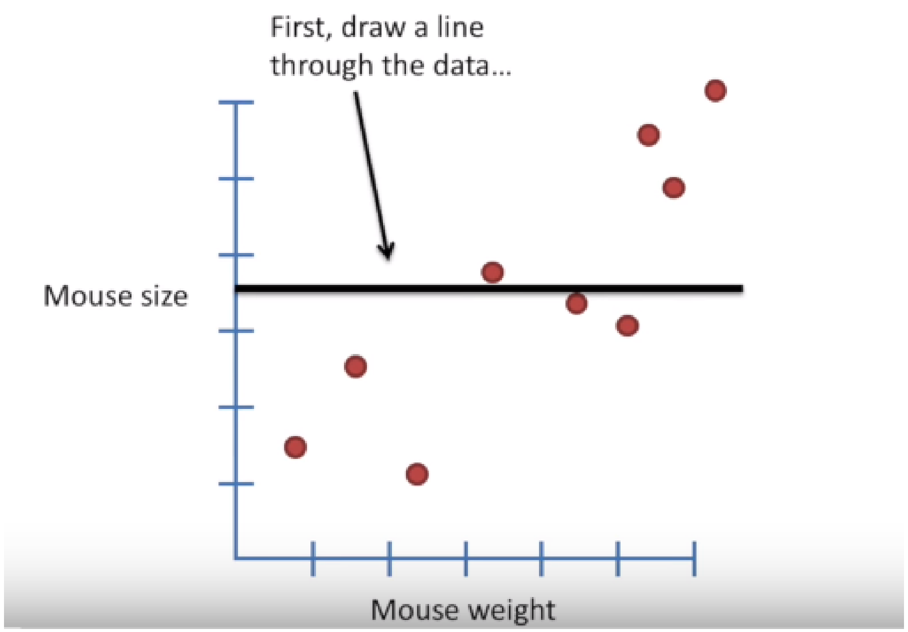
ONE POSSIBLE FUNCTION

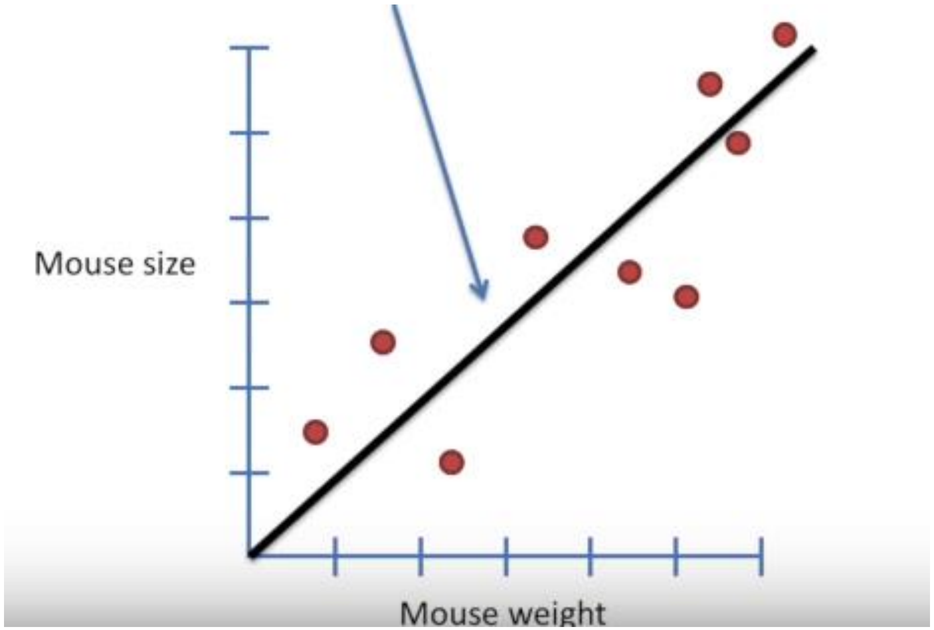


ANOTHER POSSIBLE FUNCTION

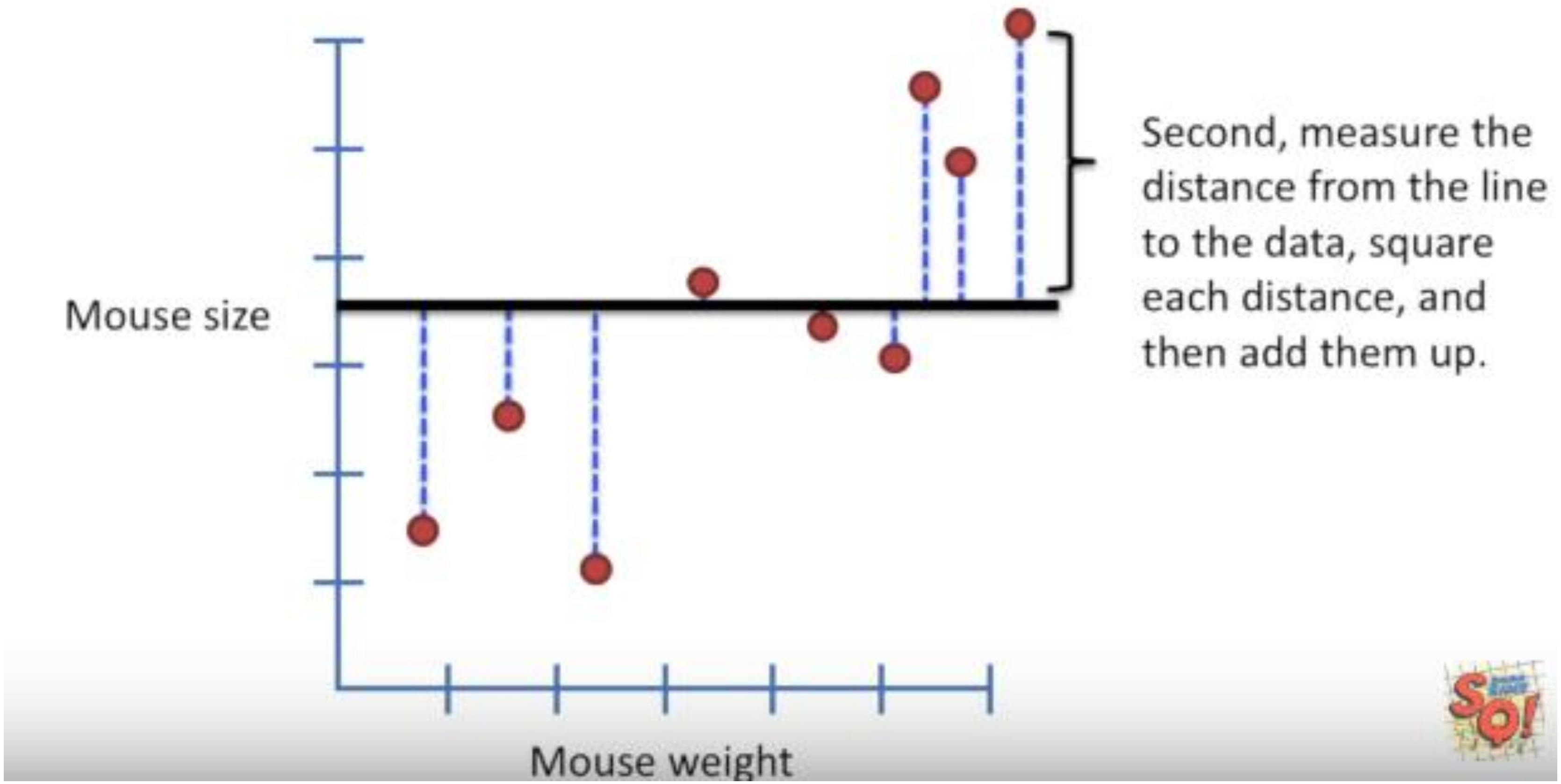


WHAT IS BEST LINE (LINEAR FUNCTION) ?

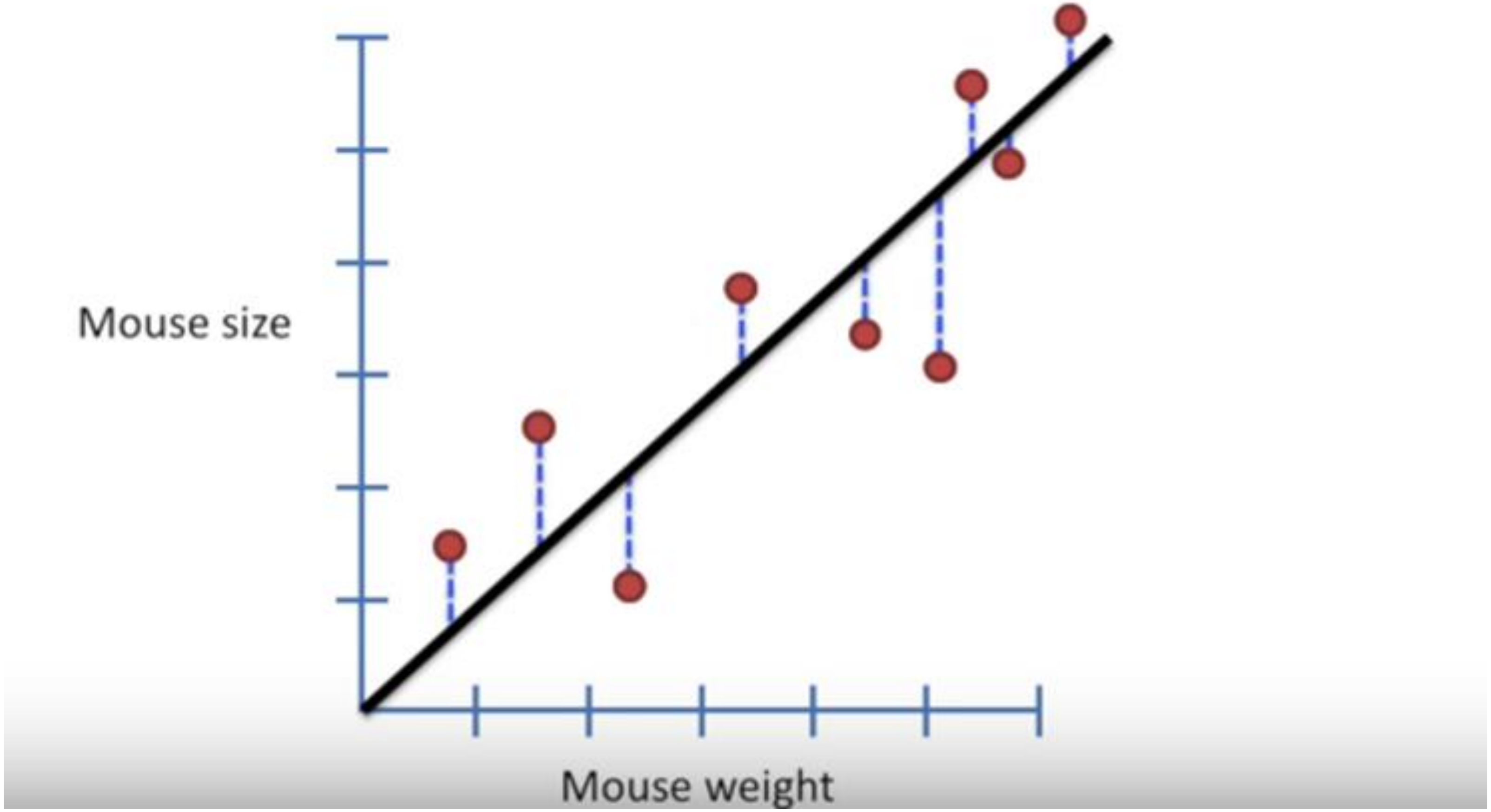




CALCULATE THE ERROR:

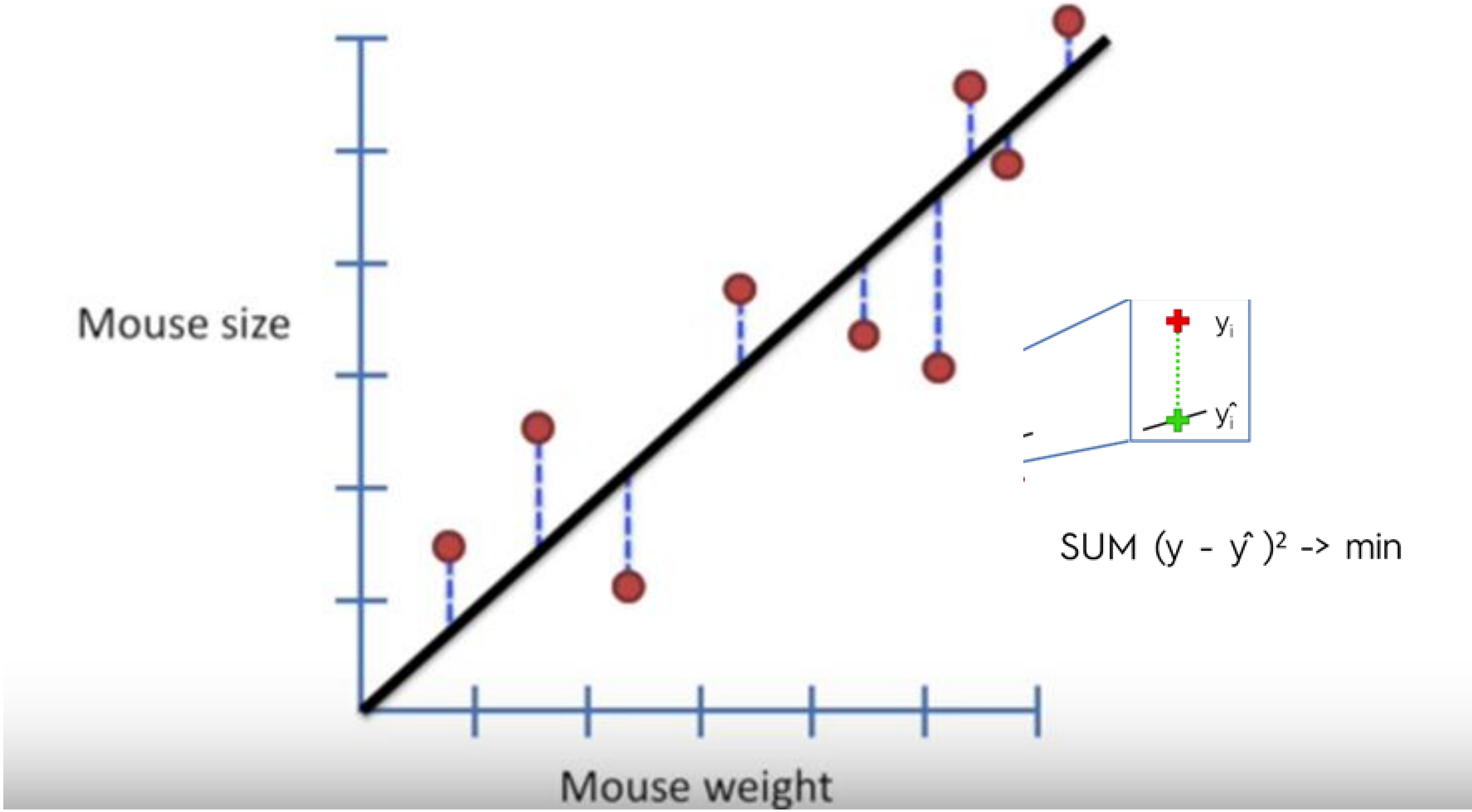


CALCULATE THE ERROR



- How to calculate Error ?

LEAST SQUARE: ONE WAY TO FIND ERROR



LOSS FUNCTION

- The function that find the error is called the LOSS function. This calculate the Loss of prediction for single example.

- $J(\theta) = (y^i - f(x^i))^2$

COST FUNCTION

- Over all cost for all examples, this function is called cost function.

- $J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (y^i - f(x^i))^2$

COST FUNCTION

- Over all cost for all examples, this function is called cost function.

- $J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (y^i - f(x^i))^2$

Remember here $f(x^i) = \theta_0 + \theta_1 x^i$

OUR OBJECTIVE

- So finally, our objective is to find the value of θ_0 and θ_1 such that the value of $J(\theta)$ is minimized.

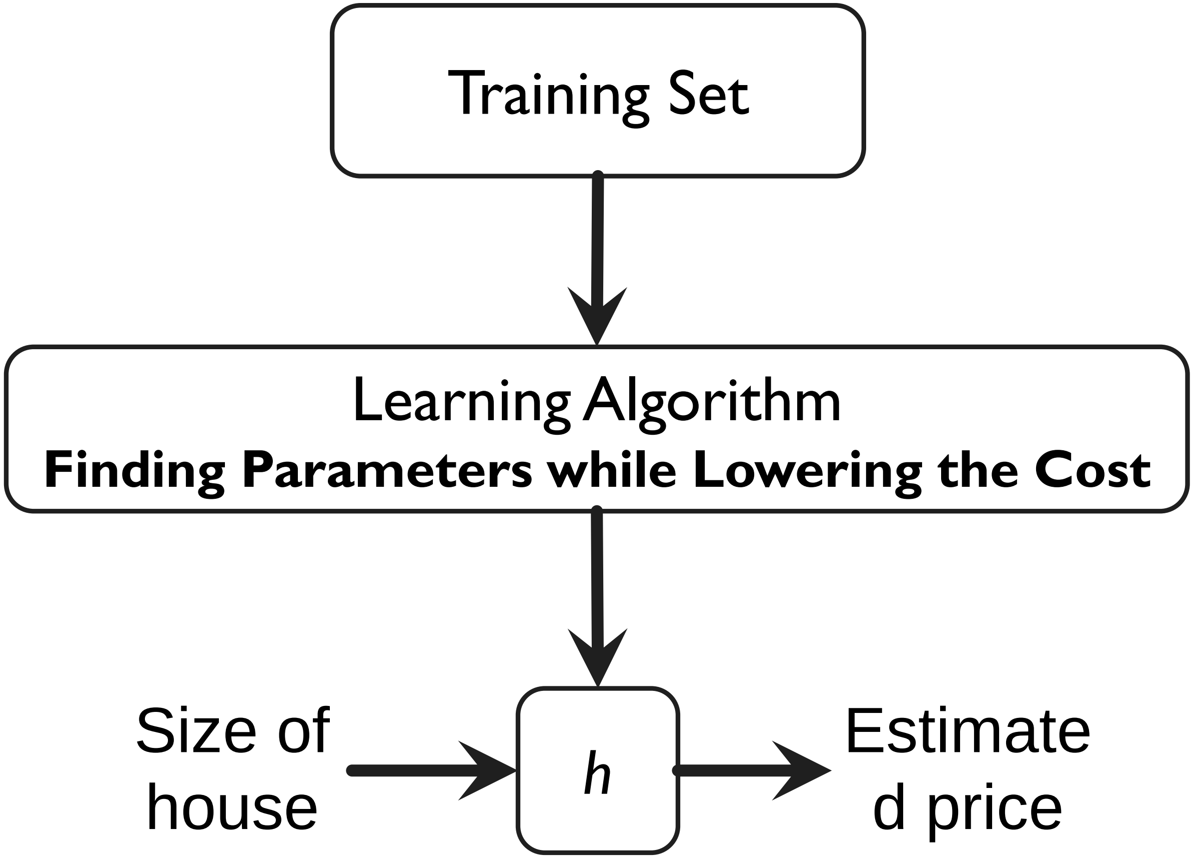
- $J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (y^i - f(x^i))^2$

Remember here $f(x^i) = \theta_0 + \theta_1 x^i$

OUR OBJECTIVE

- So finally, our objective is to find the value of θ_0 and θ_1 such that the value of $J(\theta)$ is minimized.

- $\mathrm{J}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (y^i - (\theta_0 + \theta_1 x^i))^2$



Linear regression with one variable.Univariate linear regression.

REIVEW

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

FIND THE MODEL

- With help of cost function find the θ_0 and θ_1.

x	y
0	1
1	4
2	6
3	11
4	14
5	18
6	17

HINT: FIND THE MODEL

- With help of cost function find the θ_0 and θ_1.
- Assume θ_0 and θ_1 values ranges from 1 to 5 (integers) only, now for each combination calculate the $J(\theta)$
- Select the value of θ_0 and θ_1 where $J(\theta)$ has minimum value.

x	y
0	1
1	4
2	6
3	11
4	14
5	18
6	17

VIEW THE $J(\theta)$

- Draw graph of $J(\theta)$ (as dependent variable) and θ_0 (as independent variable)

- Draw graph of $J(\theta)$ (as dependent variable) and θ_1 (as independent variable)

POSSIBLE EXAM QUESTION

- For the given two model and data set, select best model for Univariate Linear Regression

PROGRAMMING ASSIGNMENT 01

- Add comma separated Dataset in data.csv file.
- Python program take input two parameter θ_0 and θ_1 of a hypothesis function and display total cost based on the given data.
- (write a python function that take two input and return the cost value)

PROGRAMMING ASSIGNMENT 02

- Extend the previous assignment 01, now it should take parameters of two models and return the parameters of best model.

PROGRAMMING ASSIGNMENT 03

- Extend the previous assignments, write a program that read the data and proposed the linear regression parameters.
- (You can add range constrain on parameters).

PROGRAMMING ASSIGNMENT 03

- Extend the previous assignments, write a program that read the data and proposed the linear regression parameters.
- (You can add range constrain on parameters between 1 to 10).
- Hint you need to change the first parameter value and keep the second parameter constant

PROGRAMMING ASSIGNMENT 04

- Calculate the runtime of algorithm

PROGRAMMING ASSIGNMENT 05

- Repeat the Experiment for following ranges of parameters and fill the table.

Range	Number of Iterations	Time in Seconds
1 to 10		
1 to 100		
1 to 1000		
1 to 10000		