慣性運動(inertial motion) – 考慮 $a = 0$

- 慣性(inertia)—物體具有抗拒運動狀態被改變的特性。
- 任何物體都傾向維持其固有的運動狀態。如果靜止，則永遠靜止，如果運動，則永遠保持等速直線運動(相當於等速度運動)，加速度 a 為零，即牛頓第一定律的描述。
- 慣性運動可能受力，但淨力(net force)=0，詳見動力學。

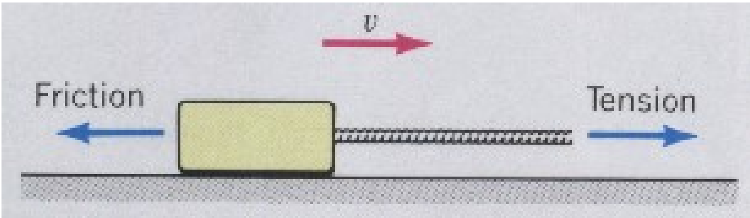


Fig.4.1

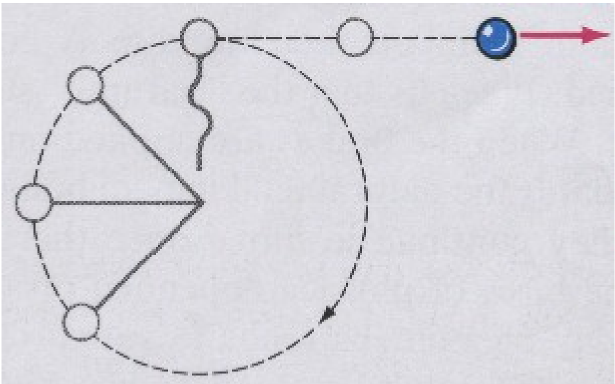


Fig.4.2

二維或三維運動(two- or three- dimensional motion)

✦利用位置向量(position vector)表示三維運動。

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \Rightarrow \vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

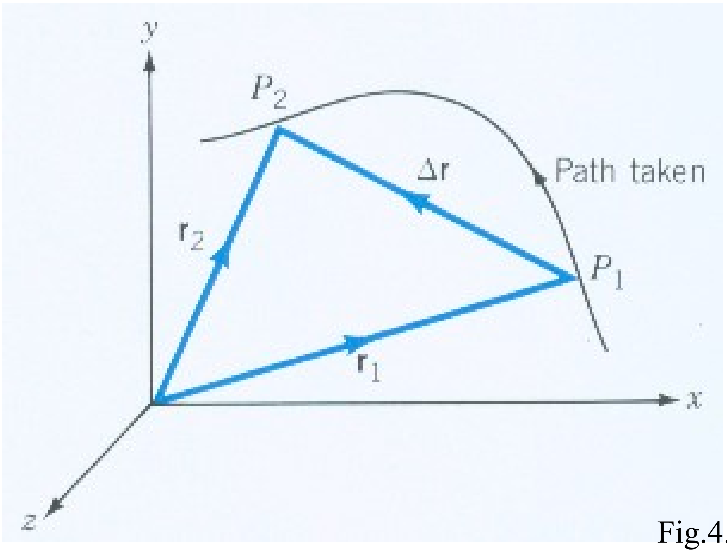


Fig.4.3

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$a = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

等加速度運動方程式

$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a}t \\ \vec{r} = \vec{r}_0 + \frac{1}{2}(\vec{v}_0 + \vec{v})t \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

➢二維等加速度運動方程式(the equations for two- dimensional motion)

x 分量	y 分量
$\mathrm{v}_x = \mathrm{v}_{0x} + a_x t$	$\mathrm{v}_y = \mathrm{v}_{0y} + a_y t$
$x = x_0 + \frac{1}{2}(\mathrm{v}_{0x} + \mathrm{v}_x)t$	$y = y_0 + \frac{1}{2}(\mathrm{v}_{0y} + \mathrm{v}_y)t$
$x = x_0 + \mathrm{v}_{0x}t + \frac{1}{2}a_x t^2$	$y = y_0 + \mathrm{v}_{0y}t + \frac{1}{2}a_y t^2$
$\mathrm{v}_x{}^2 = \mathrm{v}_{0x}{}^2 + 2a_x(x - x_0)$	$\mathrm{v}_y{}^2 = \mathrm{v}_{0y}{}^2 + 2a_y(y - y_0)$

拋體運動(projectile motion)

- 水平運動(horizontal motion) (x分量) $\Rightarrow a_x = 0$ (慣性運動)
- 垂直運動(vertical motion) (y分量) $\Rightarrow a_y = -g$ (等加速度運動)
- 水平與垂直運動相互獨立(independent)。

x 分量

- $x = x_0 + v_{0x} t$
- $v_x = v_{0x} =$ 定值

y 分量

- $y = y_0 + v_{0y} t - \frac{1}{2} g t^2$
- $v_y = v_{0y} - gt$
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

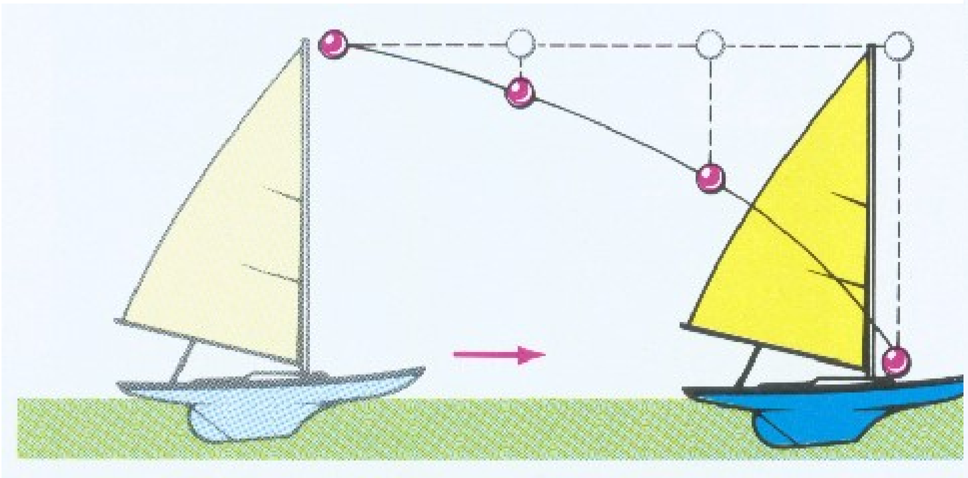
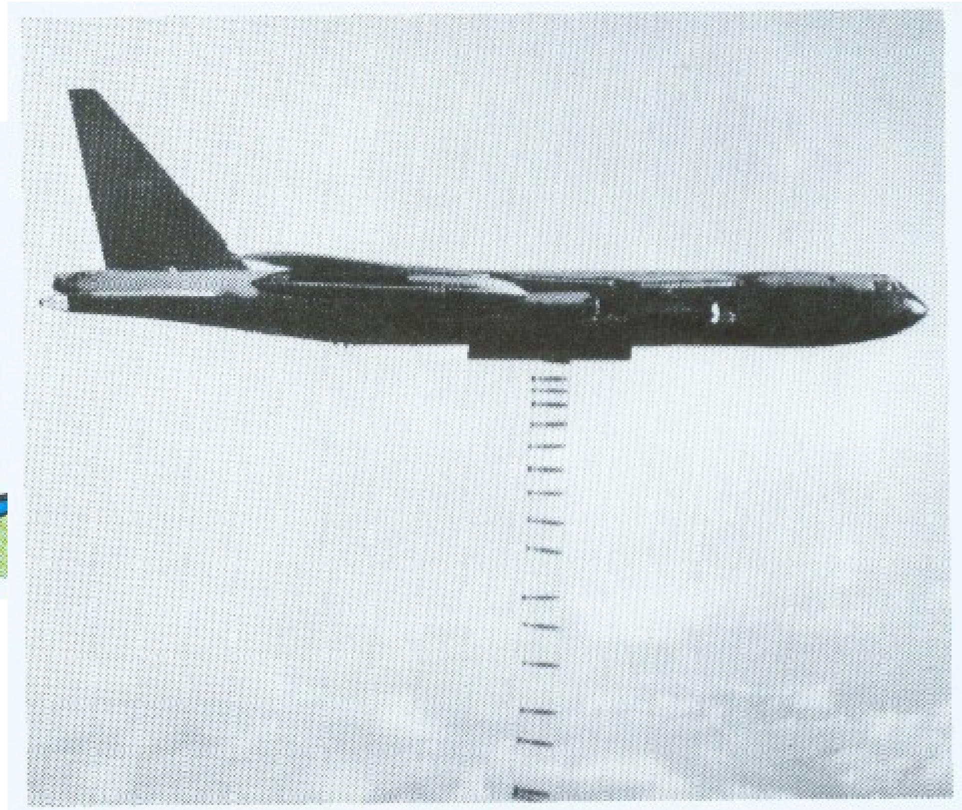


Fig.4.7



Example 4.2

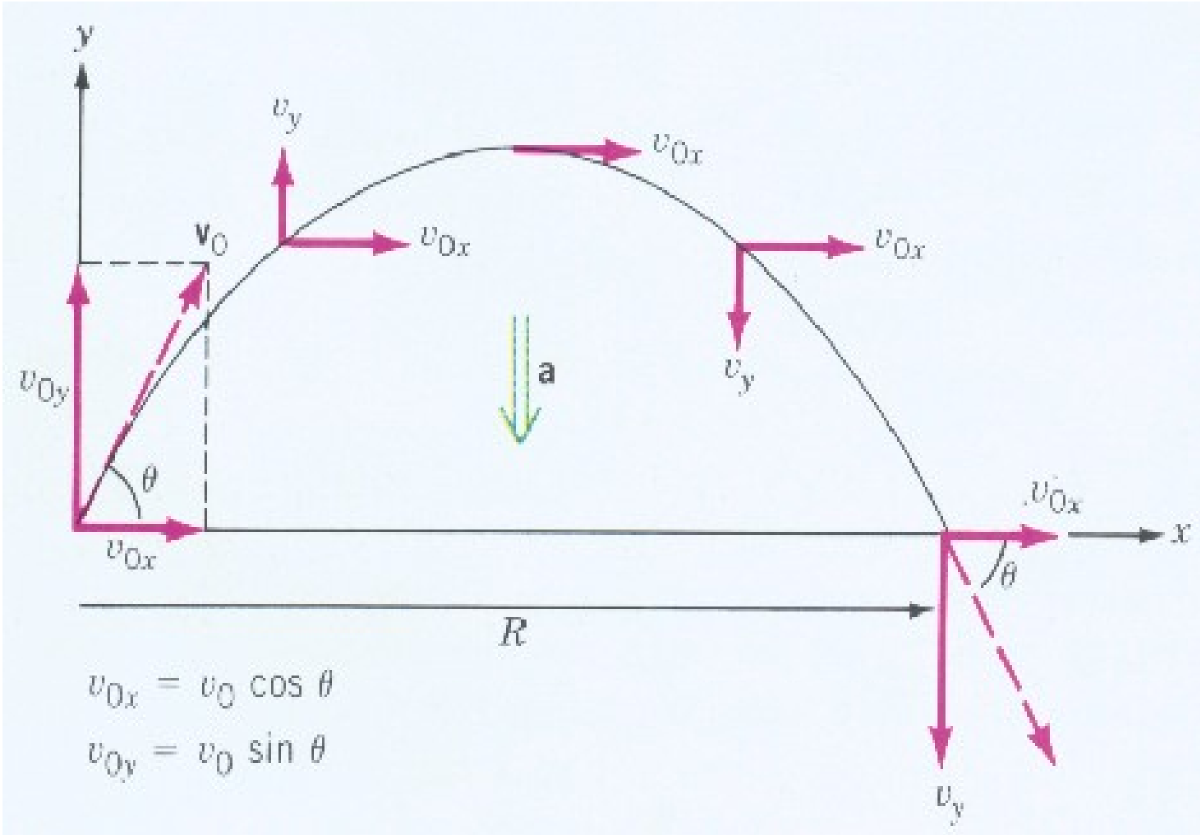


Fig.4.9

$$x = \mathrm{v}_0 \cos\theta t \qquad y = \mathrm{v}_0 \sin\theta t - \frac{1}{2} g t^2$$

飛行時間(The time of flight) $\Rightarrow y = 0, \quad t = \dfrac{2\mathrm{v}_0 \sin\theta}{g}$

最大水平範圍(the Maximum horizontal range)

$$\Rightarrow R = \mathrm{v}_0 \cos\theta t = \frac{\mathrm{v}_0 \cos\theta \cdot 2\mathrm{v}_0 \sin\theta}{g} = \frac{\mathrm{v}_0{}^2 \sin 2\theta}{g}$$

最大高度(the maximum height)

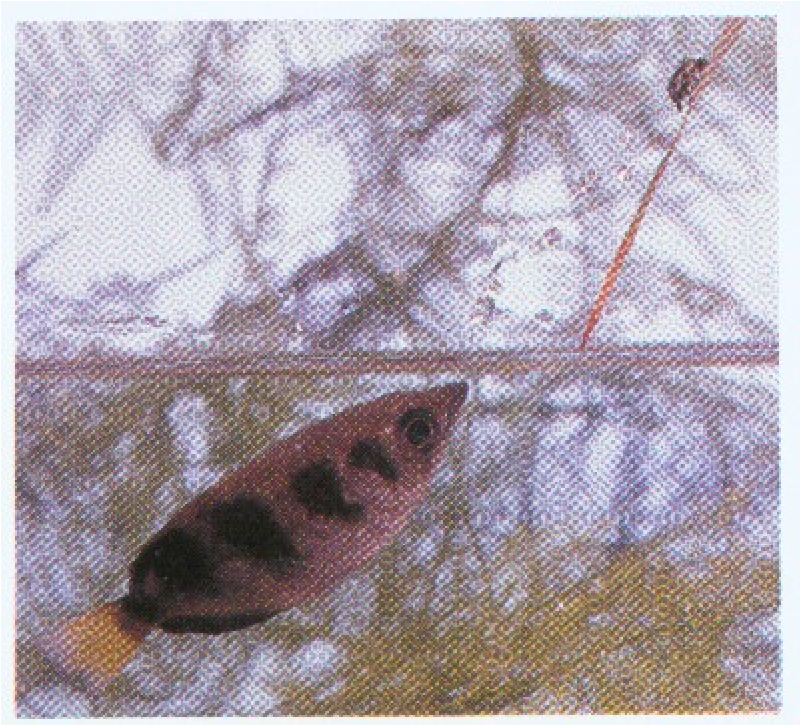
$$\Rightarrow H = \mathrm{v}_0 \sin\theta t - \frac{1}{2} g t^2 = \mathrm{v}_0 \sin\theta \cdot \frac{\mathrm{v}_0 \sin\theta}{g} - \frac{1}{2} g \left(\frac{\mathrm{v}_0 \sin\theta}{g}\right)^2$$

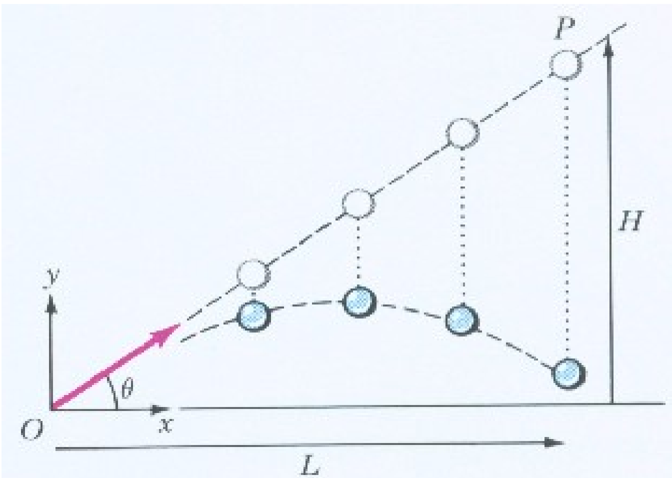
$$= \frac{\mathrm{v}_0{}^2 \sin^2\theta}{2g}$$ (因 $0 = \mathrm{v}_0 \sin\theta - gt \Rightarrow t = \mathrm{v}_0 \sin\theta / g$)

軌跡形狀(The shape of the path)

將 $t = \dfrac{x}{\mathrm{v}_0 \cos\theta}$ 代入y式$\Rightarrow y = (\tan\theta)x - \dfrac{g}{2(v_0 \cos\theta)^2} x^2$

Example 4.4：





$$y_B = L\tan\theta - \frac{1}{2}gt^2$$

$$y_D = \mathrm{v}_0 \sin\theta t - \frac{1}{2}gt^2$$

$$x_D = \mathrm{v}_0 \cos\theta t$$

若水滴軌跡可抵達甲蟲垂直下落處，

即： $x_D = L \;\Rightarrow\; t = \dfrac{L}{v_0\cos\theta}$ 代入 y_D

則 $y_D = L\tan\theta - \frac{1}{2}gt^2 = y_B$

表示水滴與甲蟲必會相撞。

等速率圓周運動(uniform circular motion)

–具有向心加速度，可使用平面極座標描述。

•向心加速度指向圓心，速率相等，方向不同(表示速度不同)。

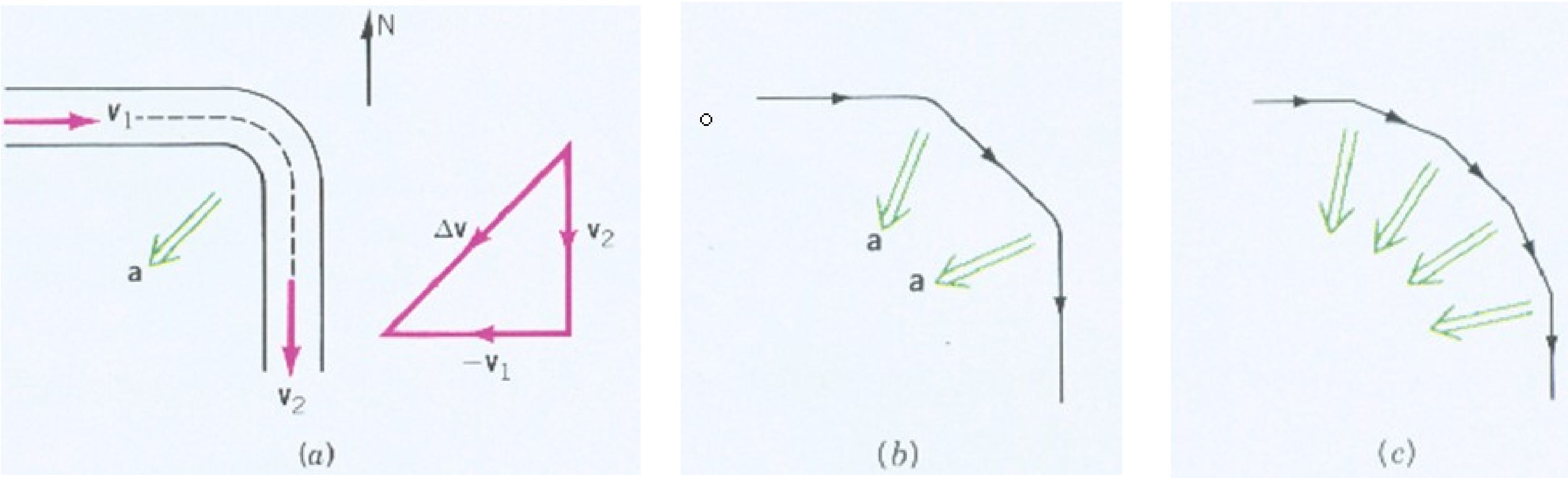
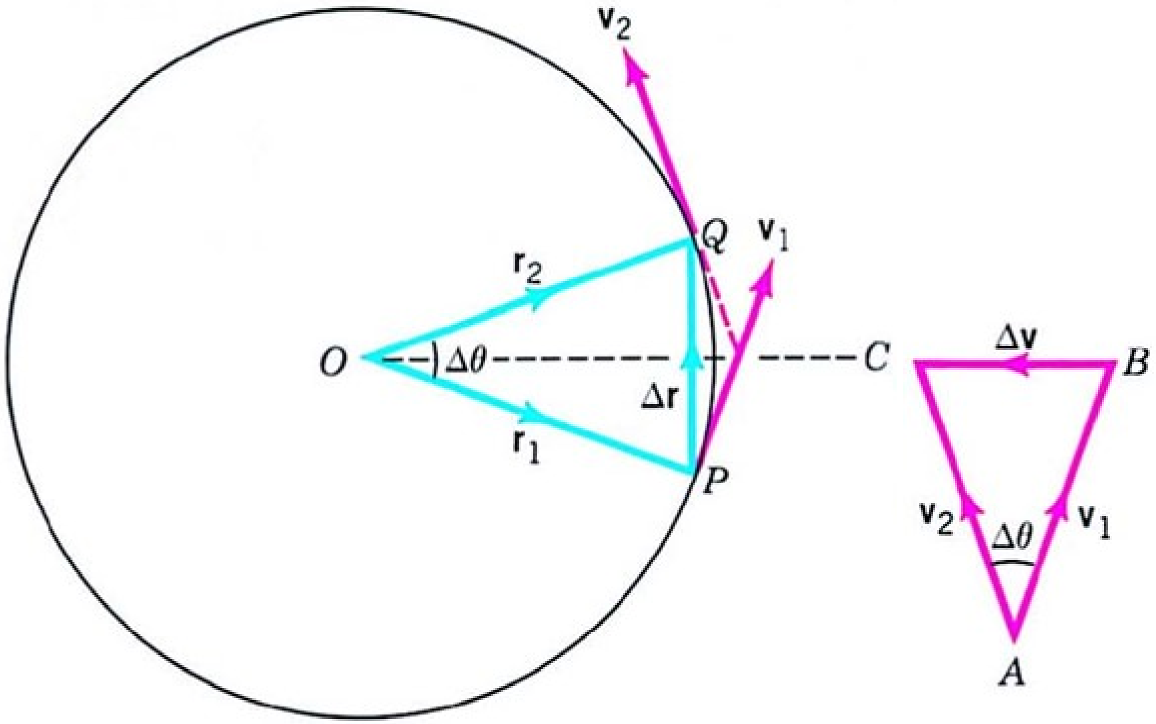


Fig.4.13

•向心加速度 $\vec{a}_r = -\dfrac{v^2}{r}\hat{r}$ ； $a_r = \dfrac{v^2}{r} = \dfrac{4\pi^2 r}{T^2} = r\omega^2$

僅改變方向，不影響速率大小。

推導：



$$\because \Delta OPQ \sim \Delta ABC$$

$$\therefore \quad \frac{|\Delta\vec{r}|}{r} = \frac{|\Delta\vec{v}|}{v}$$

$$\Rightarrow |\Delta\vec{v}| = \left(\frac{v}{r}\right)|\Delta\vec{r}|$$

$$\because \quad |\Delta\vec{r}| \approx v\Delta t$$

$$\therefore \quad |\Delta\vec{v}| = \left(\frac{v}{r}\right)(v\Delta t)$$

$$\Rightarrow \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v^2}{r} \Rightarrow \quad a = \lim_{\Delta t \to 0}\left(\frac{|\Delta\vec{v}|}{\Delta t}\right) = \frac{v^2}{r} = a_r$$

又 $v=\dfrac{2\pi r}{T}$代入a_r $\Rightarrow a_r = \dfrac{4\pi^2 r}{T^2}$，故 $a_r = \dfrac{v^2}{r} = \dfrac{4\pi^2 r}{T^2}$

非等速率圓周運動(nonuniform circular motion)

- $\vec{a} = \vec{a}_r + \vec{a}_t = -\dfrac{v^2}{r}\hat{r} + \dfrac{dv}{dt}\hat{\theta}$，其中 $a_r = \dfrac{v^2}{r}$，$a_t = \dfrac{dv}{dt}$

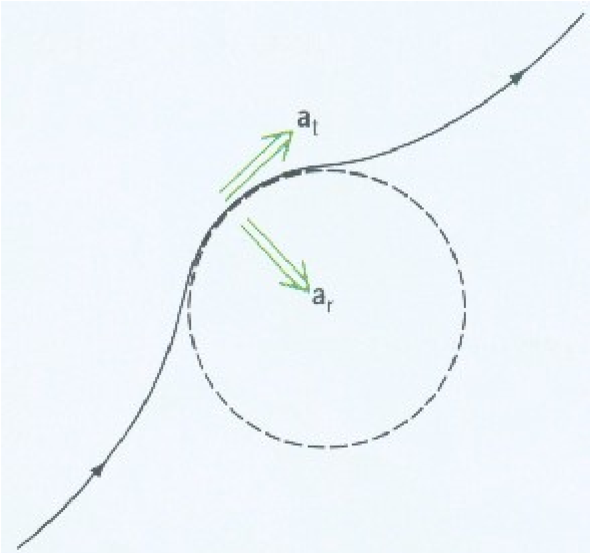


Fig.4.25

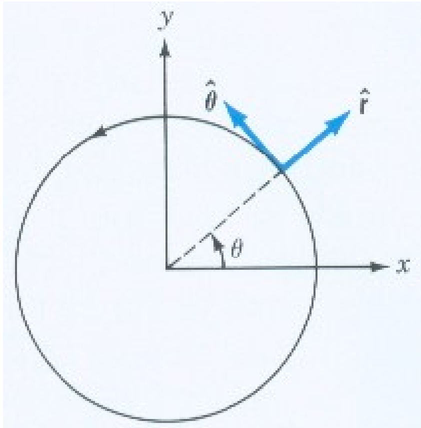
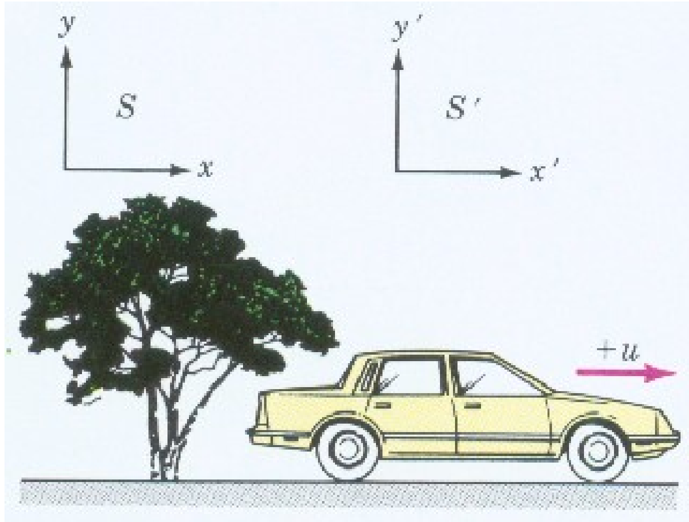


Fig.4.26

Fig.4.16



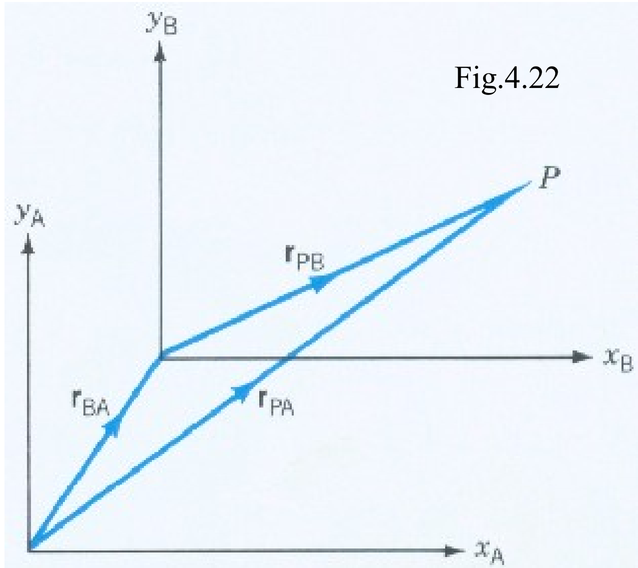
慣性座標系的相對運動(relative motion in inertial reference frame)

- 參考座標系 (reference frame) — 定義質點位置或速度方向的座標系，即觀察者位於此此座標原點上。

- 慣性參考座標系 (inertial reference frame) — 係指靜止或等速運動的參考座標系，如：地球可近似，但實際為非慣性參考座標系。

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

P相對於A的位置　　P相對於B的位置　　B相對於A的位置



- 相對速度(relative velocity)

$$\because \bar{v} = d\bar{r} / dt$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

P相對於A的速度　　P相對於B的速度　　B相對於A的速度

➢ $\vec{v}_{BA} = \vec{v}_{PA} - \vec{v}_{PB} = -(\vec{v}_{PB} - \vec{v}_{PA}) = -\vec{v}_{AB}$

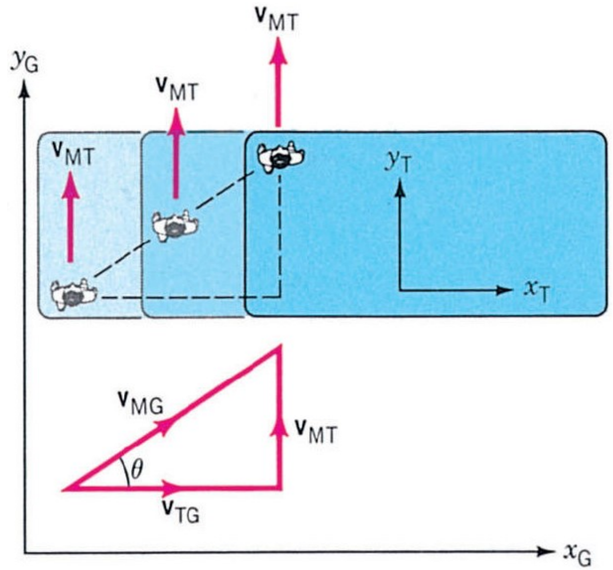
➢ Example:

$\vec{V}_{MT}$：人相對於火車的速度

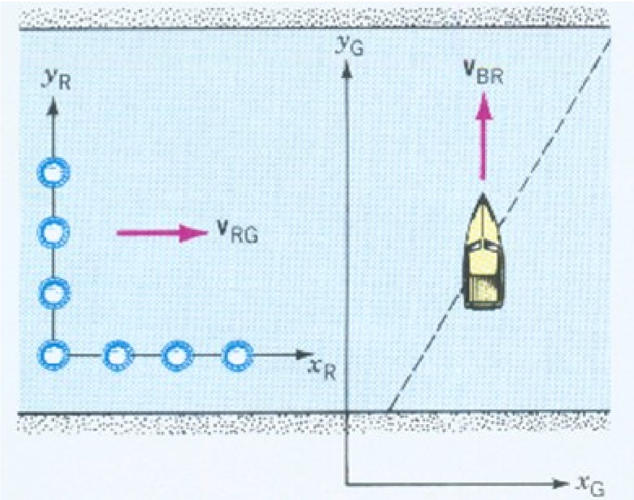
$\vec{V}_{TG}$：火車相對於地面的速度 $\Rightarrow$ $\vec{V}_{MG} = \vec{V}_{MT} + \vec{V}_{TG}$

$\vec{V}_{MG}$：人相對於地面的速度

$$\tan\theta = V_{MT} / V_{TG}$$



Example 4.8 A motor boat can travel at 10 m/s relative to the water. It starts at one bank of a river that is 100 m wide and flows eastward at 5 m/s. If the boat is pointed directly across, find: (a) its velocity relative to the bank; (b) how far downstream it travels.

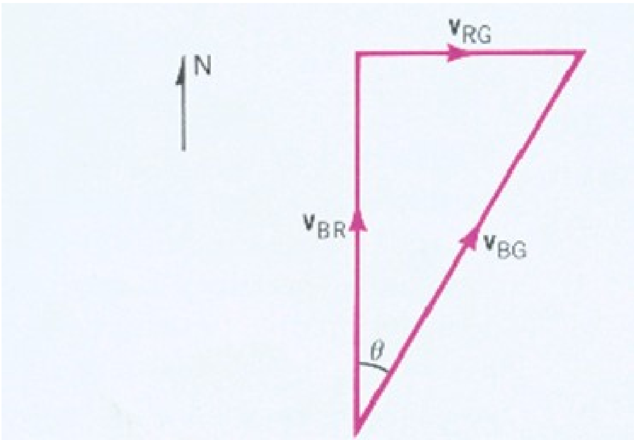


$\vec{V}_{BR}$：船(boat)相對於河水(river) =10 m/s；向北

$\vec{V}_{RG}$：河水相對於河岸 (bank)=5 m/s；向東

$\vec{V}_{BG}$：船相對於河岸(河岸為靜止座標)？; 方向？

$$\vec{V}_{BG}=\vec{V}_{BR}+\vec{V}_{RG}$$

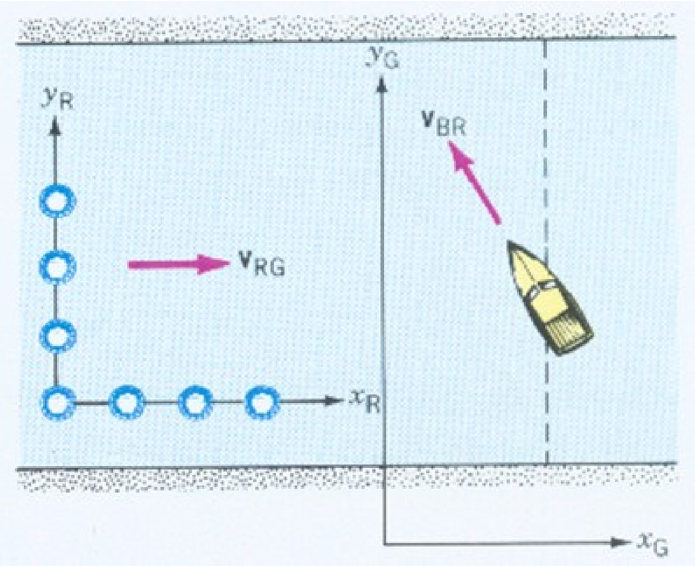


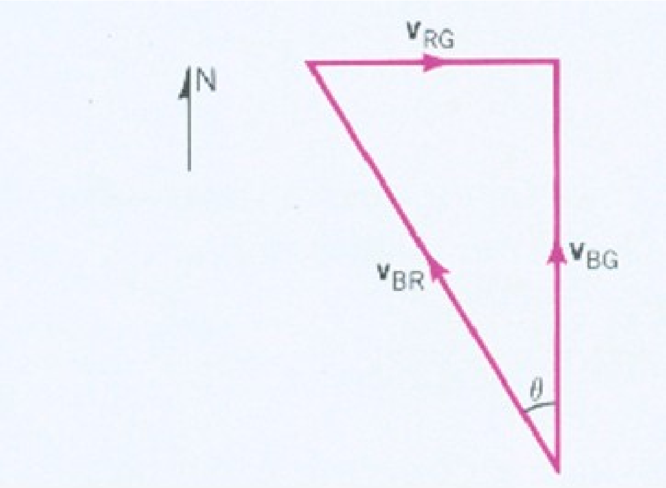
$$v_{BG}=\sqrt{10^2+5^2}=11.2\ \text{m/s}$$

$$\tan\theta=\frac{5}{10}=0.5\text{ ，}\quad \theta=26.5^0\,\text{E of N}\quad \text{(a)}$$

(100 m)/(10 m/s)=10 s ，5 m/s ×10 s=50 m (b)

Example 4.9 The captain of the boat in Example 4.8 realizes his mistake. (a)In which direction must he point the boat to get directly across? (b) How long does this take?





$$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG}$$

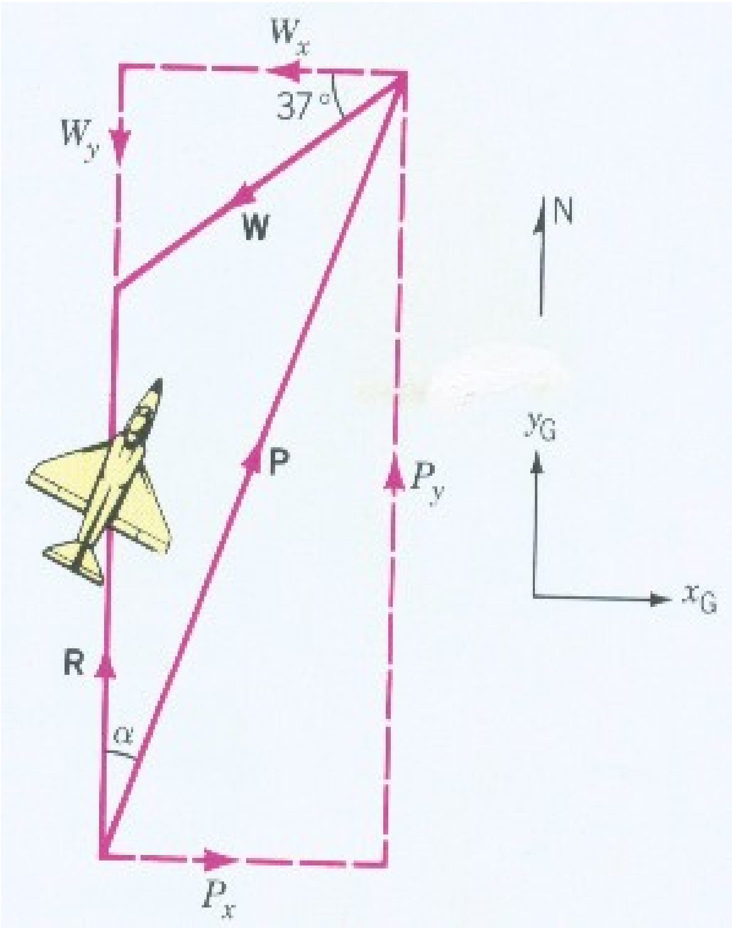
$$v_{BG} = \sqrt{10^2 - 5^2} = 8.7 \text{ m/s}$$

$$\sin\theta = \frac{5}{10} = 0.5 \text{，} \quad \theta = 30^0 \text{W of N} \quad \text{(a)}$$

(100 m)/(8.7 m/s) = 11.5 s (b)

雖然橫越至對岸的位置不會偏離，
但橫越的時間卻變長 。

Example 4.10　The pilot of an aircraft has to get to a point 320 km due north in 1 h. Ground control reports that there is a crosswind of 80 km/h toward 37^0 S of W. What is the required heading of the plane?



$\vec{v}_{PG}$：飛機相對於地面=320 km/h ；向北

$\vec{v}_{AG}$：風相對於地面=80 km/h；朝向西偏南37^0

$\vec{v}_{PA}$：飛機(plane)相對於風(或空氣)？; 方向?

$$\vec{v}_{PA} = \vec{v}_{PG} + \vec{v}_{GA} = \vec{v}_{PG} - \vec{v}_{AG} \quad \Rightarrow$$

$\vec{P} = \vec{R} - \vec{W}$ (其中$\vec{P}=\vec{v}_{PA}$, $\vec{W}=\vec{v}_{AG}$, $\vec{R}=\vec{v}_{PG}$)

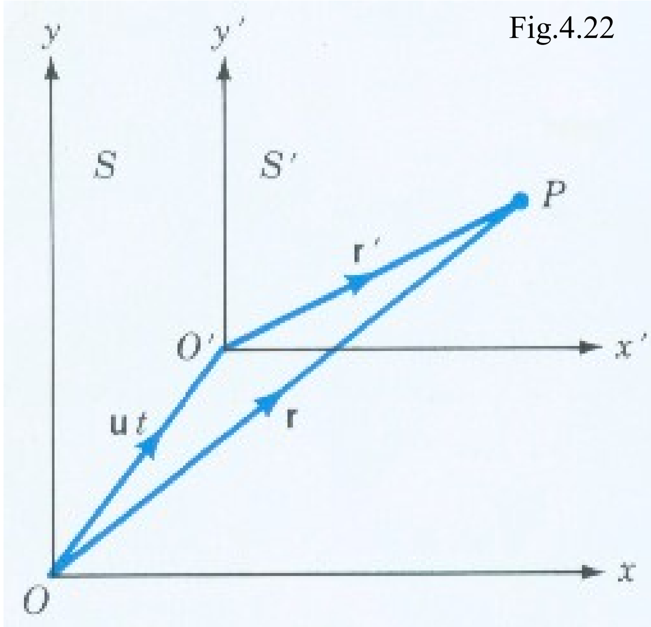
$$P_x = R_x - W_x = 0 - (-80\cos 37^0) = +64 \text{ km/h}$$

$$P_y = R_y - W_y = 320 - (-80\sin 37^0) = +368 \text{ km/h}$$

$$\vec{P} = 64\hat{i} + 368\hat{j} \quad , \quad \alpha = \tan^{-1}\left(\frac{P_x}{P_y}\right) = \tan^{-1}(0.174) = 9.9^0 \text{ E of N}$$

- 伽利略轉換(The Galilean Transformation)

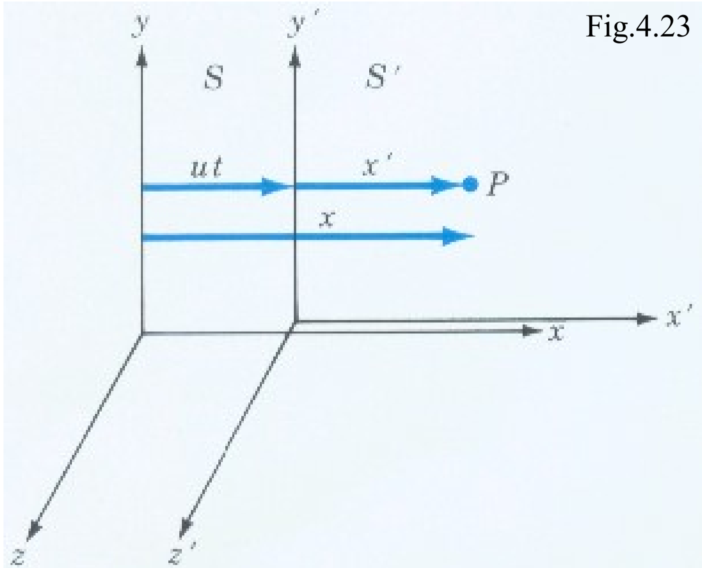
$$\vec{r}' = \vec{r} - \vec{u}t$$



假設 S' 以速度 u 沿 S 的 x 軸運動，則

$$x' = x - ut, \quad y' = y, \quad z' = z, \quad t' = t$$

此即為座標的伽利略轉換。

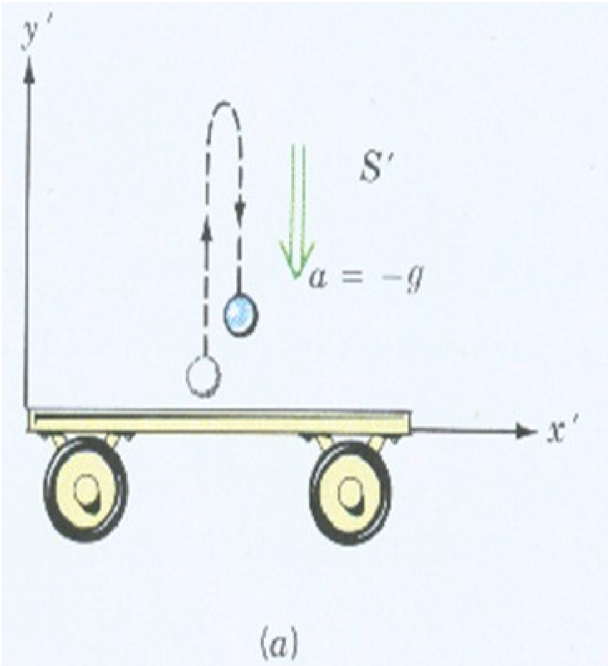


而取時間微分 ⇨ $\vec{v}' = \vec{v} - \vec{u}$

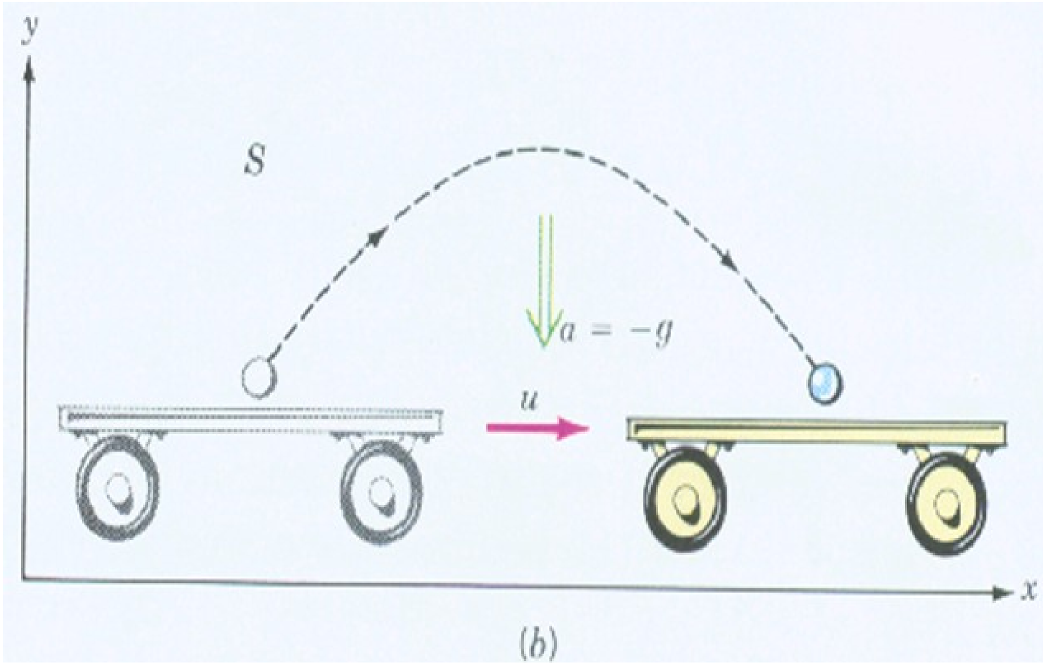
再取時間微分 ⇨ $\vec{a}' = \vec{a}$

即所有慣性參考座標系所觀察到的質點加速度皆相同。

Fig.4.24

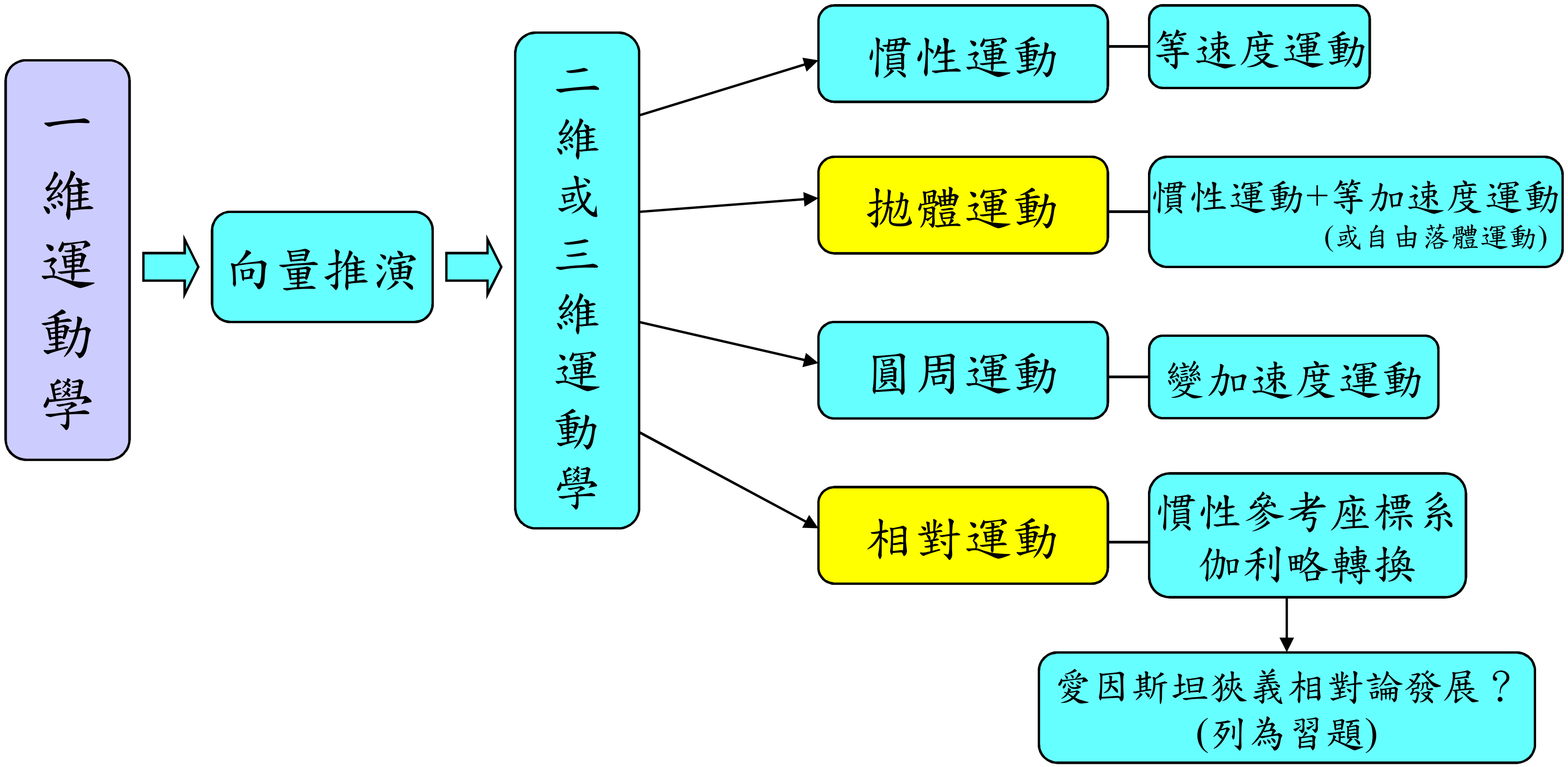


（以車為參考座標系）



（以地面為參考座標系）

本章重要觀念發展脈絡彙整



習 題

- 教科書習題(p.74~p.79)

  Exercise: 1,9,19,21,29,41,43,53,55,57,76,81

  Problem: 1,3,5,16

- 基本觀念習題：

  1.請推導理想二維拋體運動的飛行時間 $t=\dfrac{2v_0\sin\theta}{g}$；最大水平距離 $R=\dfrac{v_0^2\sin 2\theta_0}{g}$ 及最大高度 $H=\dfrac{(v_0\sin\theta_0)^2}{2g}$ ，其中 v_0 表拋體初速，而 θ_0 表拋體初速與水平地面的夾角。

習　題

●延伸思考習題：(※不列入考試，僅列入加分題)

1. 請申述真實拋體(Real Projectiles)可能會受到哪些因子的影響。(※提示：可參閱Special Topics！)

2. 請申述本章相對運動理論(涉及伽利略轉換)與愛因斯坦狹義相對論(涉及勞侖茲轉換)的相關性。(※提示：狹義相對論可參閱教科書CH39！)