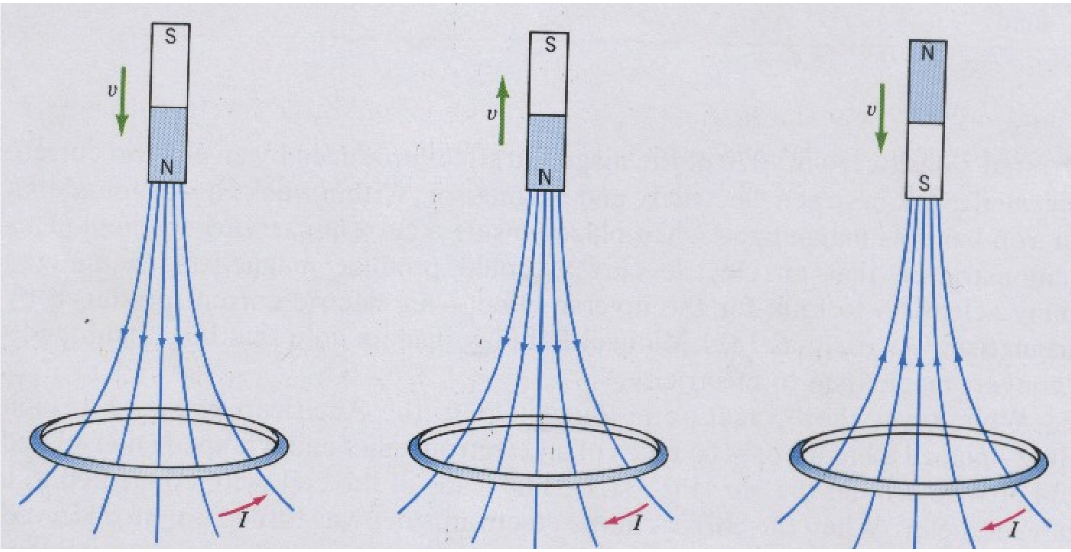
電磁感應(Electromagnetic Induction)－磁生電

✦電磁感應現象
- 導體相對於磁場運動，導致感應電流的產生。
- 磁場隨時間變化，導致感應電場，進而形成感應電流。



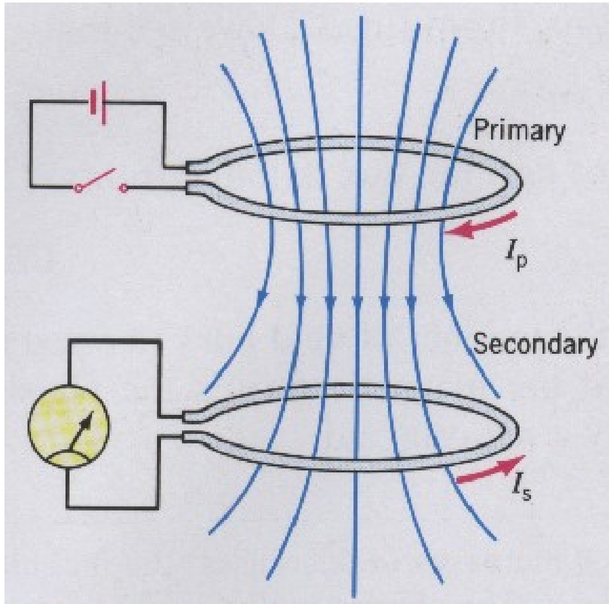


Fig.31.3

Fig.31.4

●產生電磁感應現象的主要實驗

- 磁場強度變化 (change in field strength)
- 線圈面積變化 (change in area)
- 線圈方向的改變 (change in orientation)

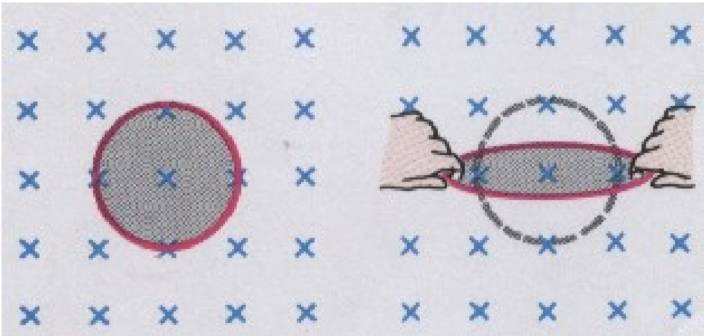


Fig.31.5

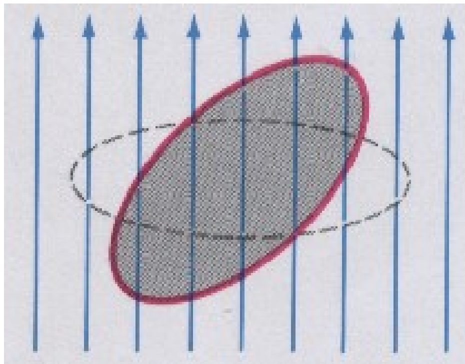


Fig.31.6

• Magnetic flux (磁通量) $\begin{cases} \Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta & \text{(Uniform B)} \\ \Phi_B = \int \vec{B} \cdot d\vec{A} & \text{(nonuniform B or the surface isn't flat)} \end{cases}$

The SI unit $\Rightarrow$ weber (Wb) $\Rightarrow 1\ \mathrm{T} = 1\ \mathrm{Wb/m^2}$

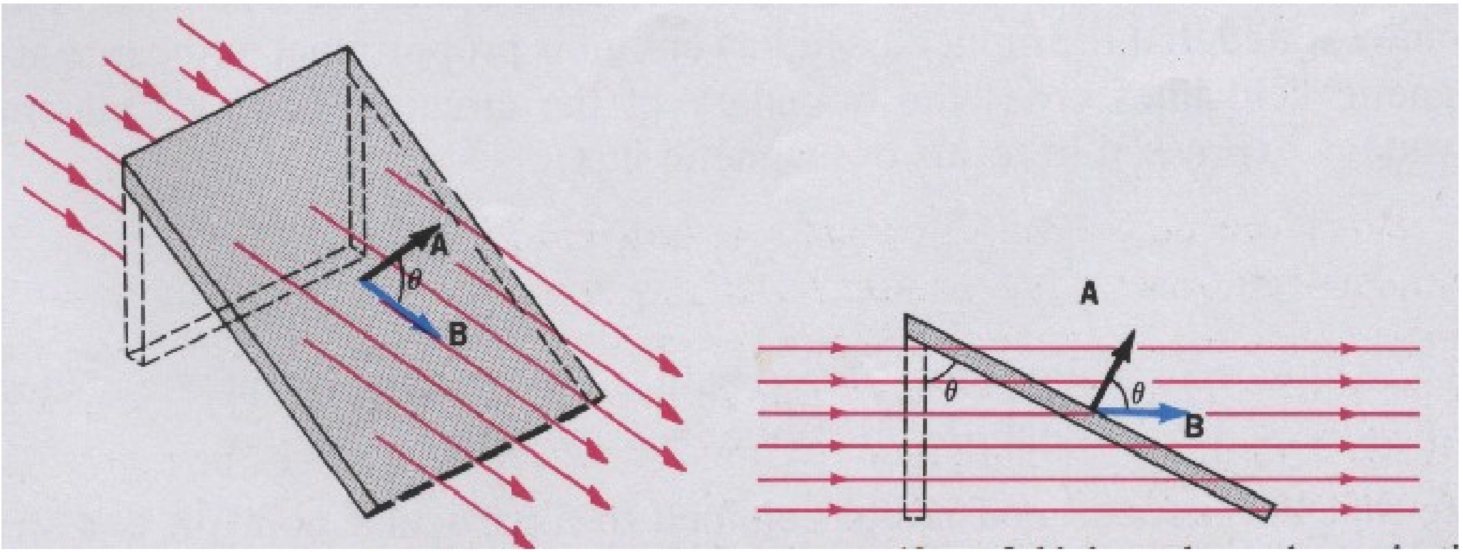


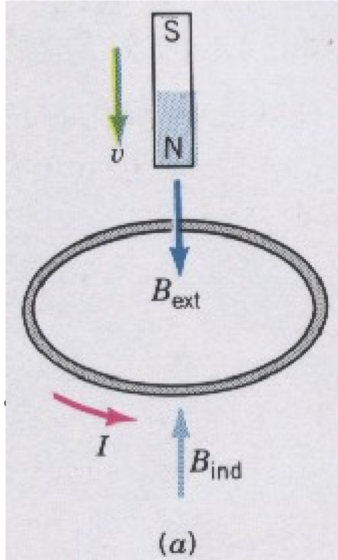
Fig.31.7

•Faraday's Law (法拉第定律)—描述感應電動勢(induced emf)產生原因。

沿任意封閉路徑的感應電動勢(The induced emf)大小正比於穿越此路徑所圍面積之磁通量 變化率。

$$\Rightarrow \xi \propto \frac{d\Phi}{dt} = \frac{dB}{dt} A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt}$$

(包括: B,A,θ的變化率，符合前述實驗)



•Lenz's Law(楞次定律)—判斷感應電流或感應電動勢方向

感應電動勢的產生係反對磁通量變化，分析 $\begin{cases} B_{ext}: \text{外部磁場} \\ B_{int}: \text{感應磁場} \end{cases}$

$$\Rightarrow \begin{cases} \text{當磁鐵靠近線圈}(B_{ext}\text{增加}) \Rightarrow B_{int}\text{成反向，抵消}B_{ext}\text{的增加。} \\ \text{當磁鐵離開線圈}(B_{ext}\text{減小}) \Rightarrow B_{int}\text{成同向，彌補}B_{ext}\text{的減小。} \end{cases}$$

$$\Rightarrow \begin{cases} \mathrm{B_{int}}\text{反向可推知感應電流I為反時針方向。} \\ \mathrm{B_{int}}\text{同向可推知感應電流I為順時針方向。} \end{cases}$$

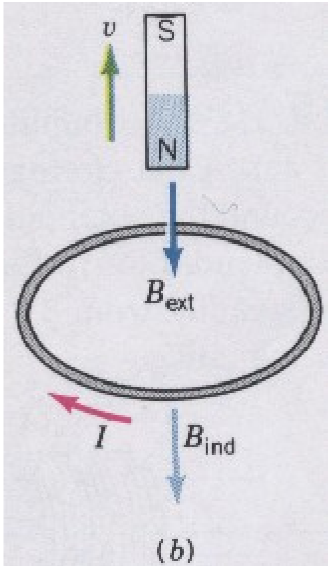


Fig.31.9

➢楞次定律是能量守恆的結果(Helmholtz)

－若B_{int}係用來增強B_{ext}，則此增強的磁場會增大感應電流I，而增大的感應電流I將導致更大的磁場，又引發更大的感應電流I，如此一直下去是不可能的。

➢感應電動勢方向恆與磁通量變化相反。

感應電動勢方向的定義－由右手定則(right-hand rule)判定產生如Fig.31.10的磁場之迴路電流方向為正。

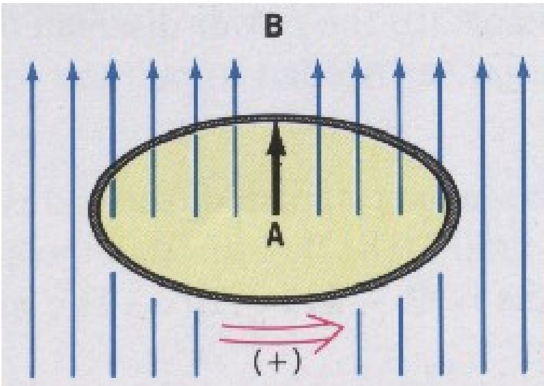


Fig.31.10

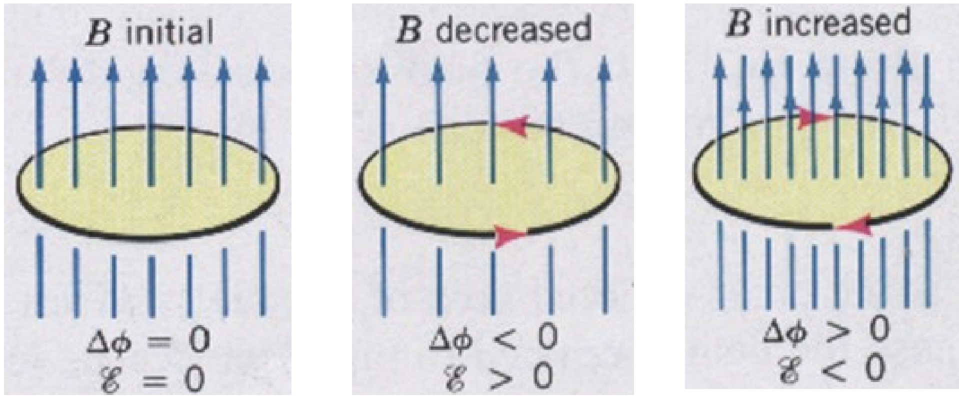


Fig.31.11

➢ Faraday's Law (+Lenz's Law) $\Rightarrow \xi = -\dfrac{d\Phi}{dt}$ (modern statement)

若考慮N匝 $\Rightarrow \xi = -N\dfrac{d\Phi}{dt}$ (such as solenoid or toroid)

Example 31.3 Find (a)the current in the resistor; (b) the power dissipated in the resistor; (c) the mechanical power needed to pull the rod.

$$\because \Phi = BA = B\ell x$$

$$\therefore |\xi| = \frac{d\Phi}{dt} = \frac{d}{dt}(B\ell x) = B\ell \frac{dx}{dt} = B\ell v$$

$$I = \frac{|\xi|}{R} = \frac{B\ell v}{R} \quad \text{—Ans } (a)$$

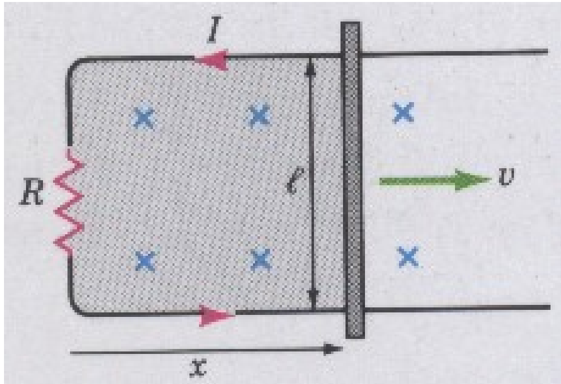


Fig.31.13

$$P_{elec} = I^2 R = \frac{(B\ell v)^2}{R^2} \cdot R = \frac{(B\ell v)^2}{R} \text{—Ans } (b)$$

$$Rod \Rightarrow \begin{cases} \text{磁力向左} \Rightarrow \vec{F}_{mag} = I\vec{\ell} \times \vec{B} \Rightarrow F_{mag} = I\ell B \\ \text{外力向右} \xrightarrow{\text{因}rod\text{等速移動}} |\vec{F}_{ext}| = |\vec{F}_{mag}| = I\ell B \end{cases}$$

$$P_{mech} = \vec{F}_{ext} \cdot \vec{v} = I\ell Bv = (\frac{B\ell v}{R})\ell Bv = \frac{(B\ell v)^2}{R} \text{—Ans (c)}$$

•Generators(發電機)

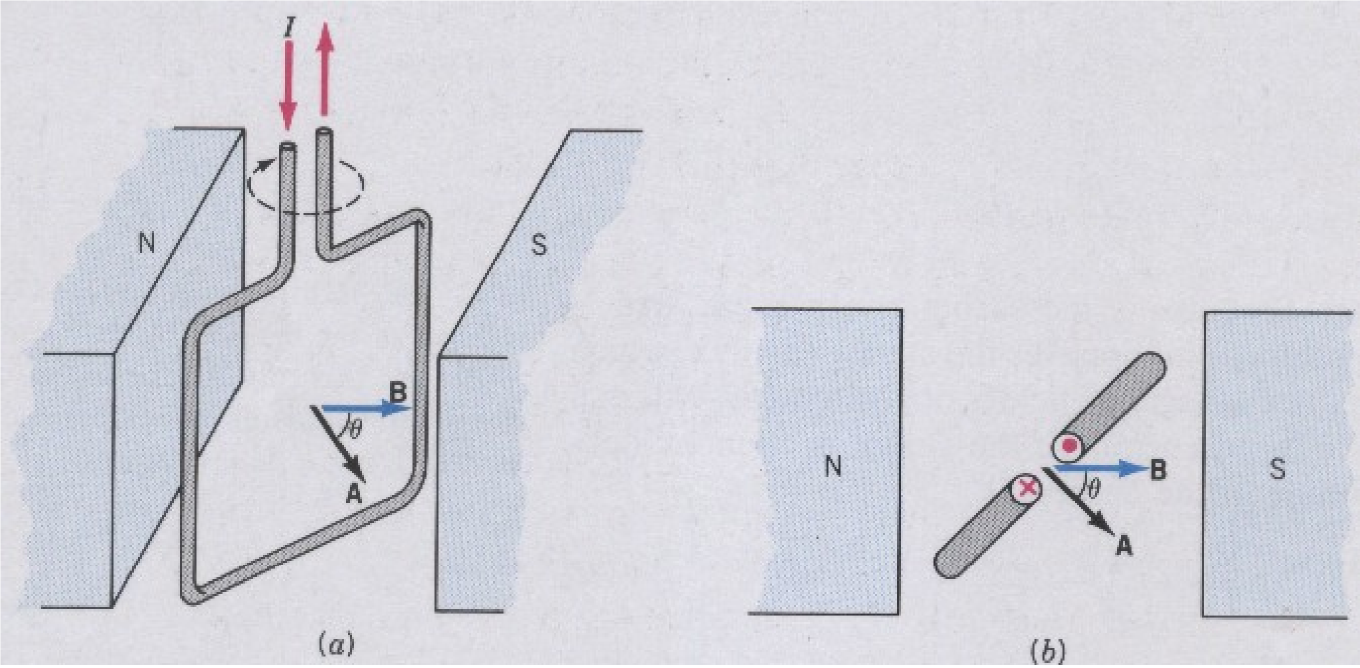
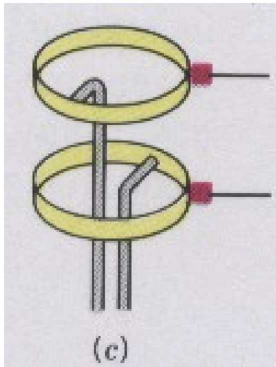


Fig.31.16



磁通量$\Rightarrow \Phi = BA\cos(\theta) = BA\cos(\omega t)$

感應電動勢$\Rightarrow \xi = -N\dfrac{d\Phi}{dt} = NAB\omega\sin(\omega t)$

$\Rightarrow \xi = \xi_0 \sin(\omega t) \qquad (\text{where } \xi_0 = NAB\omega)$

一產生極性相反的交流電(alternating current)

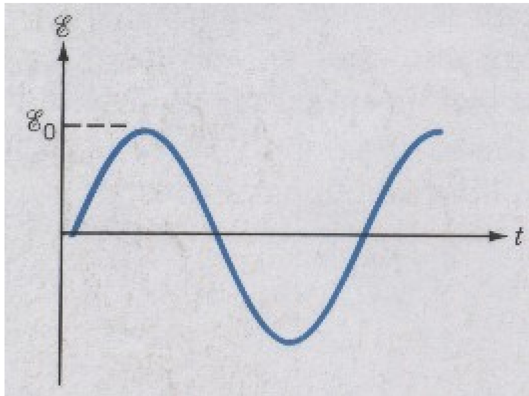


Fig.31.17

ac generators（交流發電機） ⇨ Commutator（換向器） ⇨ multiple coils纏繞成圓柱體 / multiple commutator (多重換向器) ⇨ dc generator (直流發電機)

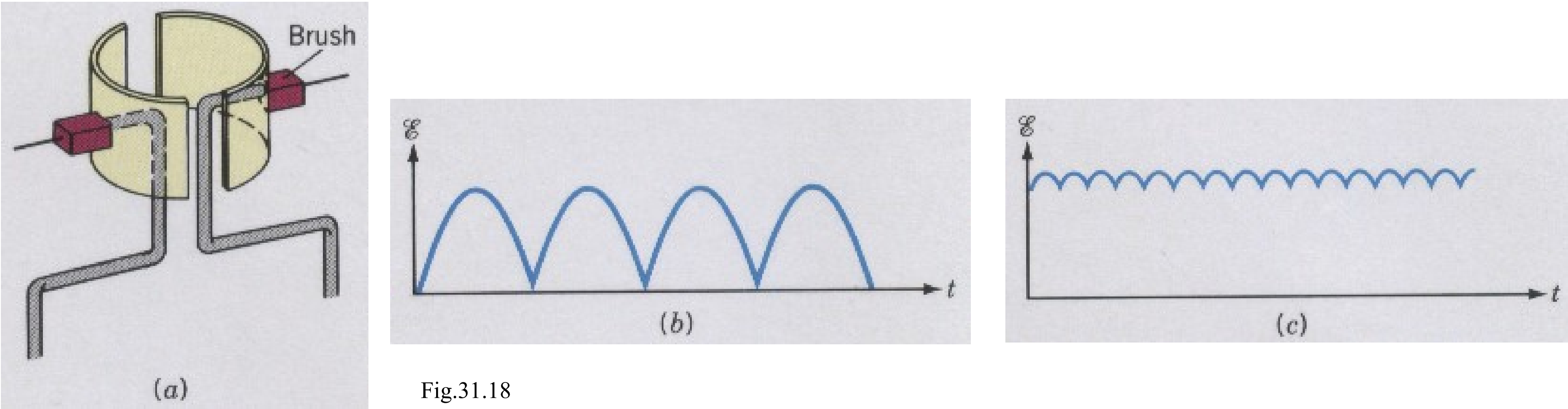


Fig.31.18

•The back emf of motors (電動機的反電動勢)

電動機原理－利用載流線圈在磁場中產生力矩來運轉。

反電動勢－線圈在磁場中運轉也會產生感應電動勢，但其方向與外部電動勢相反，即所謂的反電動勢(the back emf)，其大小與電動機運轉角速度(ω)成正比。

➢反電動勢(back emf)對電動機運作的影響

啟動瞬間電流相當大－電動機啟動瞬間，線圈靜止，無反電動勢產生，而線圈內阻又小，故啟動電流(startup current)相當大。

線圈轉動，電流減小－當線圈轉動將引發感應電動勢(即反電動勢)，可抵消部份之外部原有電動勢，導致淨電動勢(net emf)減小，故線圈通過的電流將減小。

No load (無負載)－電動機轉速會隨輸入能量(由外部電動勢產生)而增大，但反電動勢亦將隨轉速增大，造成淨電動勢(net emf)仍相當小，故線圈通過的電流不會大幅增加，因而線圈不致燒毀。

Load (接負載)－電動機轉速會減小，若負載太大促使轉速太小，感應的反電動勢就會相當小，則外部電動勢將形成足夠大的淨電動勢， 造成線圈通過的電流大幅增加，最後導致線圈燒毀。

The origins of the induced emf (感應電動勢的起因)

$$\xi=\frac{W_{ne}}{q}=\frac{1}{q}\oint\vec{F}\cdot d\vec{\ell}\xrightarrow{\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})}\xi=\oint(\vec{E}+\vec{v}\times\vec{B})\cdot d\vec{\ell}$$

$$\Rightarrow\xi=\oint\vec{E}\cdot d\vec{\ell}+\oint(\vec{v}\times\vec{B})\cdot d\vec{\ell}$$

$$\Rightarrow\begin{cases}\xi=\oint\vec{E}\cdot d\vec{\ell}\ \text{(}E\text{ 為感應電場)一若線圈沒有相對於磁場運動，則磁場需隨時間變化}\\ \xi=\oint(\vec{v}\times\vec{B})\cdot d\vec{\ell}\ \text{(動生電動勢)一若磁場不隨時間改變，則線圈需相對於磁場運動}\end{cases}$$

- Induced electric field (感應電場)一線圈未運動、B≠const (v=0)

$$\xi=\oint\vec{E}\cdot d\vec{\ell}=-\frac{d\Phi}{dt}=-A\frac{dB}{dt}$$

$$\left(\frac{d\Phi}{dt}=\frac{d}{dt}(BA\cos\theta)=A\frac{dB}{dt}+B\cancelto{0}{\frac{dA}{dt}}-BA\cancelto{0}{\sin\theta\frac{d\theta}{dt}}\right)$$

因線圈截面未變　　因線圈未旋轉運動

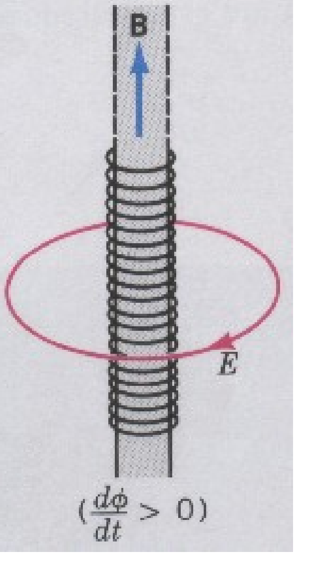


Fig.31.19

➢感應電場與靜電場的差異

- 感應電場的電力線(場線)是封閉的，而靜電場則有可能開放，如: 靜電場起始或終止於點電荷。
- 感應電場屬於非保守場(nonconservative field)，沿封閉路徑的積分不等於零，而靜電場則為零，因靜電場為保守場。

Example 31.6 Find the induced electric field (a) inside (b) ouside the solenoid.

When $r < R$ $\Rightarrow \oint \vec{E} \cdot d\vec{\ell} = E \oint d\ell = E(2\pi r)$

$$= -A\frac{dB}{dt} = -(\pi r^2)\frac{dB}{dt}$$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt}$$

When $r > R$ $\Rightarrow \oint \vec{E} \cdot d\vec{\ell} = E(2\pi r) = -(\pi R^2)\frac{dB}{dt}$

$$\Rightarrow E = -\frac{R^2}{2r}\frac{dB}{dt}$$

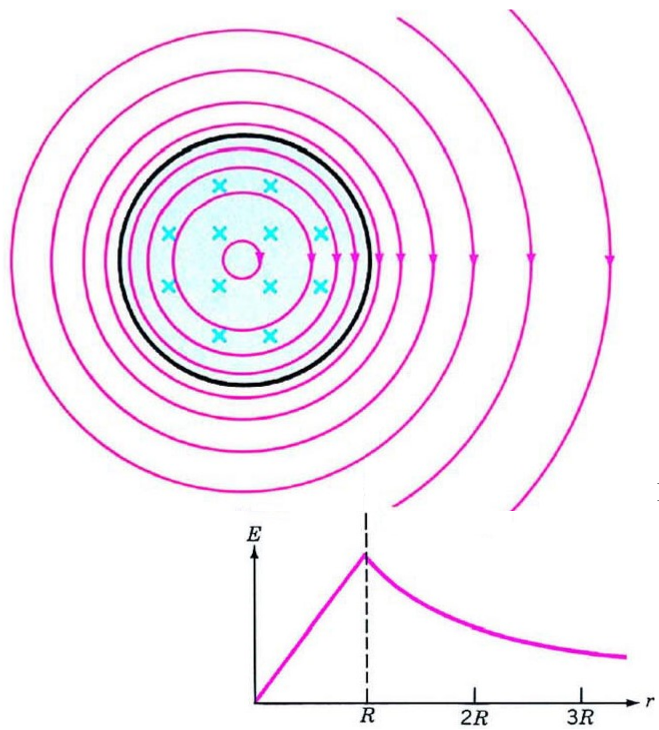


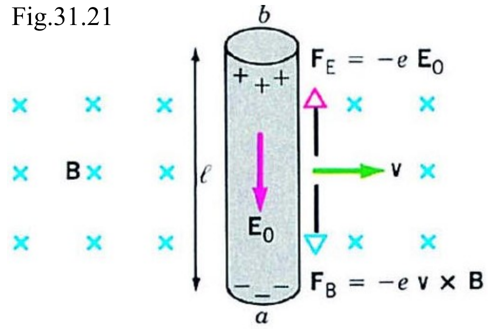
Fig.31.20

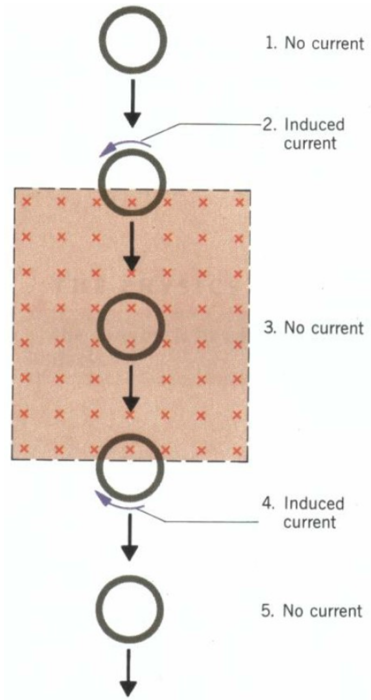
Note: 螺線管外的感應電場係由電荷產生，因時變(time-depend)的磁場與感應電場會導致電荷加速。

• Motion emf (動生電動勢)—線圈相對於磁場運動、B=const.

$$\xi = \oint (\vec{v}\times\vec{B})\cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$

Fig.31.21





➢導體棒移動產生電動勢(emf)

$$\vec{F}_B = -e\vec{v}\times\vec{B} \xrightarrow{\text{驅使電子向下}} \vec{F}_E = -eE_0$$

$$\xrightarrow{\vec{F}_B\text{與}\vec{F}_E\text{平衡(電子不再移動)}\Rightarrow F_E = F_B} E_0 = vB$$

$$\xrightarrow{\text{由兩端聚集正負電荷所形成的靜電場}E_0\text{產生端電位差}} V_B - V_A = E_0\ell = B\ell v$$

$$\xrightarrow{\text{端電位差=動生電動勢}} \xi = B\ell v$$

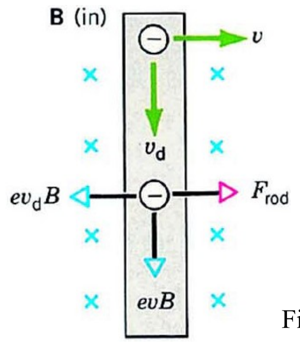


Fig.31.24

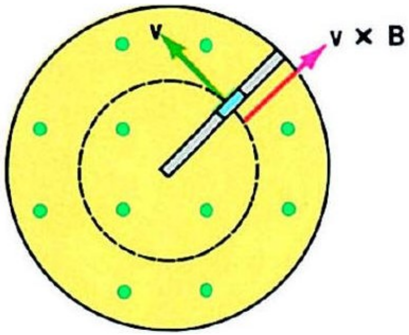
➢磁力不會作功：

假設電子 $\begin{cases}\text{隨導體棒水平運動速率為v。}\\ \text{沿導體棒垂直漂移速率為v}_d\text{。}\end{cases}$

$\boxed{\text{磁力產生的功率 } P = \vec{F}\cdot\vec{v}} \Rightarrow \begin{cases}\text{垂直方向：} P_{\text{垂直}} = evBv_d \ (\because F_B = evB,\ \vec{F}_B\text{與}\vec{v}_d\text{方向相同})\\ \text{水平方向：} P_{\text{水平}} = -ev_dBv \ (\because F_B = ev_dB,\ \vec{F}_B\text{與}\vec{v}_d\text{方向相反})\end{cases}$

$\boxed{\text{磁力的淨功率}} \Rightarrow P_{net} = P_{\text{垂直}} + P_{\text{水平}} = 0$ （如此可證實磁力不作功）

Example 31.7 Find the emf of a homopolar generator(單極發電機).



$$\xi = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell} = \int vBdr$$

$$= \int r\omega Bdr \quad = \omega B\int_0^R rdr = \frac{1}{2}\omega BR^2$$

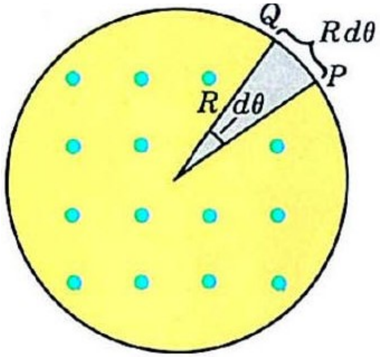


Fig.31.23

$$|\xi| = \frac{d\Phi}{dt} = \frac{BdA}{dt} \quad = \frac{B\left[\frac{1}{2}R\left(Rd\theta\right)\right]}{dt}$$

$$= \frac{1}{2}BR^2\frac{d\theta}{dt} \quad = \frac{1}{2}BR^2\omega = \frac{1}{2}\omega BR^2$$

✦渦電流 (Eddy current)

Case(a) 磁棒垂直導體平板移動

－磁棒靠近導體平板，因導體平板上任意迴路的磁通量發生改變，故感應出逆時針的渦電流；反之，若遠離則感應順時針渦電流。

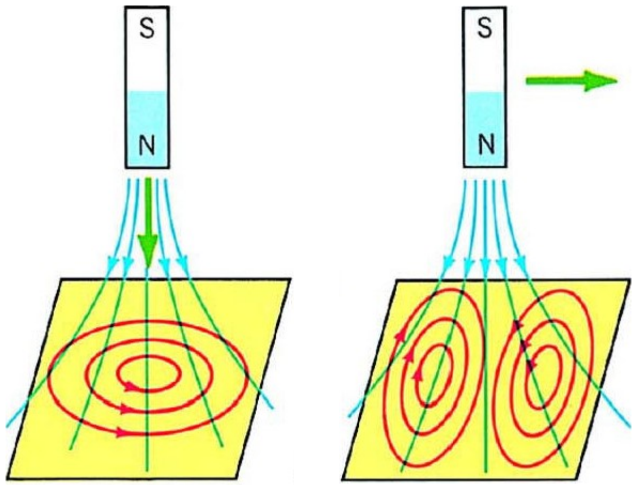


Fig.31.25

Case(b) 磁棒平行導體平板移動

遠離區域的感應渦電流為順時針
（磁通量減小，產生吸力）

靠近區域的感應渦電流為逆時針
（磁通量增加，產生斥力）

Case(c) 導體平板在均勻磁場中移動

－導體平板遠離磁場運動會產生順時針的感應電流，導致平板受到向左的吸力(可由 $\vec{F}=I\vec{\ell}\times\vec{B}$ 判定)，此力與其運動方向相反。

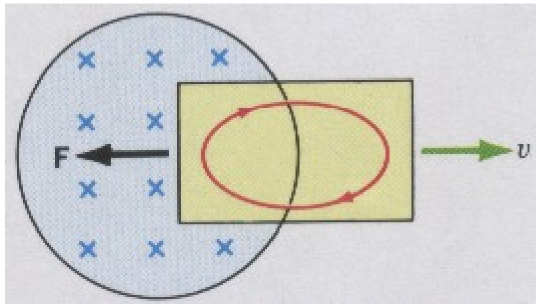


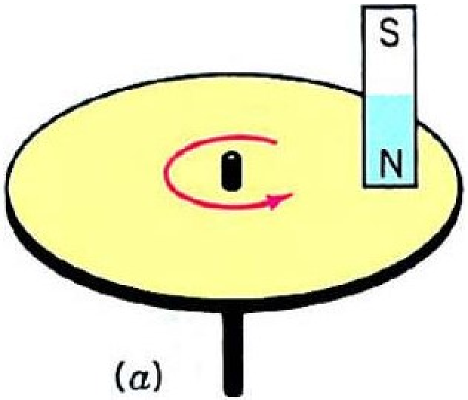
Fig.31.26

- 感應渦電流磁力的應用－電子天平或檢流計之線圈振盪減緩、火車煞車系統、汽車速率錶。
  (吸力)
- 渦電流在導體內運動時，會產生熱能，如：半導體的熔冶及精化、電磁爐。

➢磁鐵懸浮實驗－繞鉛直軸快速旋轉的導體圓盤邊上的磁鐵，渦電流的磁力會導致磁鐵沿盤緣運動方向移動。

➢跳彈實驗－將螺線管纏繞在鐵棒，再將銅環套在鐵棒上，當AC電流通上螺線管，銅環會因渦電流的斥力而被推開垂直向上飛去。

➢感應渦電流磁力的應用－火車的磁浮與推進上。
(斥力)



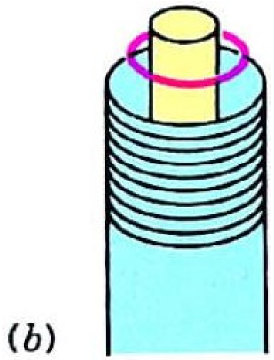
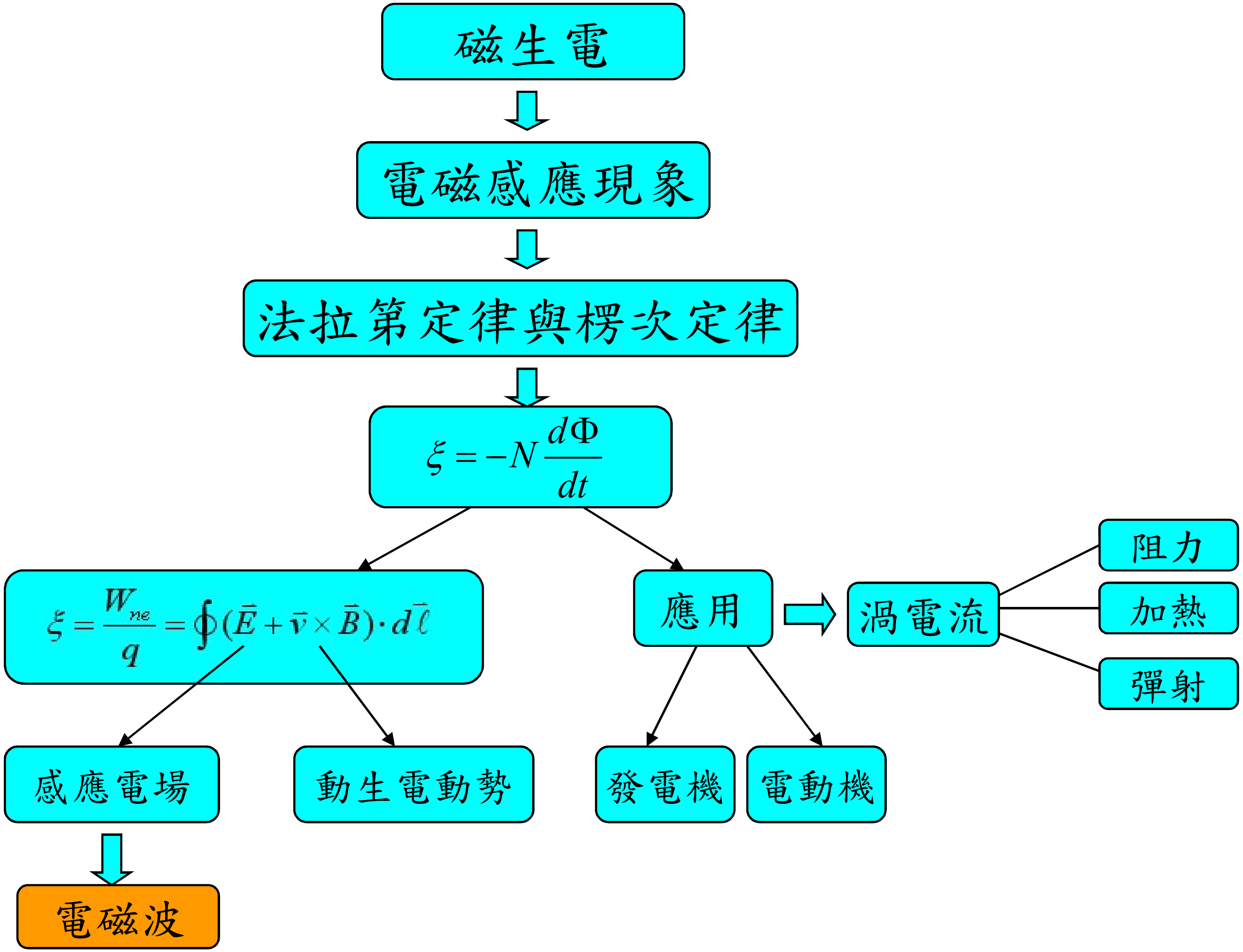


Fig.31.27

本章單元重要觀念發展脈絡彙整



習 題

- 教科書習題 (p.647~p.651)

  Exercise: 5, 9, 11, 21, 25, 29, 35

- 基本觀念問題：

  1.請說明法拉第定律(Faraday’s Law)與楞次定律(Lenz’s Law)。

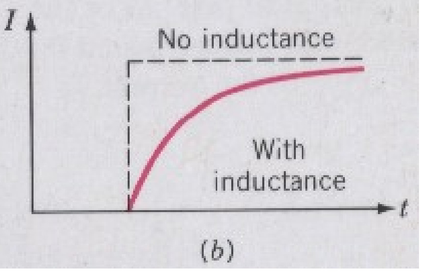
  2.請解釋電動機(motors)在高負載時，線圈易燒毀的原因。

※期末考範圍至此為止，以下列為專題講授！

電感(Inductance)－線圈因電流變化而產生感應電動勢(儲存磁能)

$$\begin{cases}\text{自感(Self-Induction)}\Rightarrow\text{線圈因本身電流變化造成磁通量改變而產生感應電動勢。}\\ \text{互感(Mutual-Induction)}\Rightarrow\text{線圈的磁通量改變係由其他線圈造成。}\end{cases}$$

$$\xi = -\frac{d(N\Phi)}{dt}$$



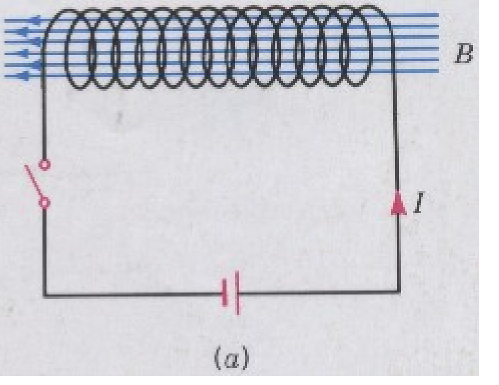


Fig.32.2

其中NΦ為磁通量或磁交鏈數(The flux likage)

Example：如圖32.3

$$N_1\Phi_1 = N_1(\Phi_{11} + \Phi_{12})$$

$$\Rightarrow \xi_1 = -N_1\frac{d}{dt}(\Phi_{11} + \Phi_{12})$$

$$= -N_1\frac{d}{dt}\Phi_{11} - N_1\frac{d}{dt}\Phi_{12}$$

（自感）　（互感）

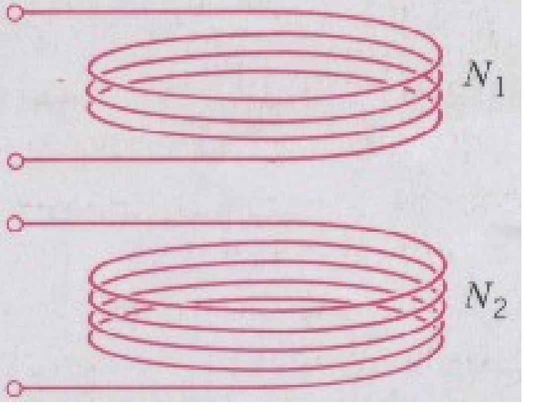


Fig.32.3

• 自感 $\Rightarrow N_1\Phi_{11} = L_1 I_1$ (L_1為比例常數，稱為線圈1的自感值)

$$\Rightarrow \xi_{11} = -N_1\frac{d\Phi_{11}}{dt} = -L_1\frac{dI_1}{dt}$$ (ξ_{11}極性僅與 $\frac{dI_1}{dt}$ 有關)

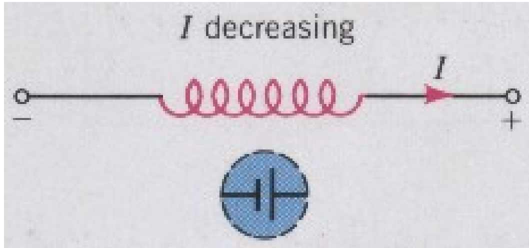


Fig.32.4

• 互感 $\Rightarrow N_1\Phi_{12} = M_{12}I_2$ or $N_2\Phi_{21} = M_{21}I_1$ ($M_{12}=M_{21}=M$， M稱為互感值)

$$\Rightarrow \xi_{12} = -N_1\frac{d\Phi_{12}}{dt} = -M\frac{dI_2}{dt}$$

In sum,

$$\Rightarrow \xi_1 = -N_1\frac{d}{dt}(\Phi_{11}+\Phi_{12}) = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = \xi_{11}+\xi_{12}$$

Example 32.1 A long solenoid of length ℓ and cross-sectional area A has N turns. Find its self-inductance.

$$\Phi = BA = \mu_0 nIA \ \Rightarrow L = \frac{N\Phi}{I} = N\mu_0 nA = n\ell\mu_0 nA = \mu_0 n^2 A\ell$$

$$(\because n = N/\ell)$$

Example 32.3 A small coil placed within a solenoid. What is their mutual inductance?

已知： A small coil $\Rightarrow A_1$, N_1
A long solenoid $\Rightarrow A_2$, n_2

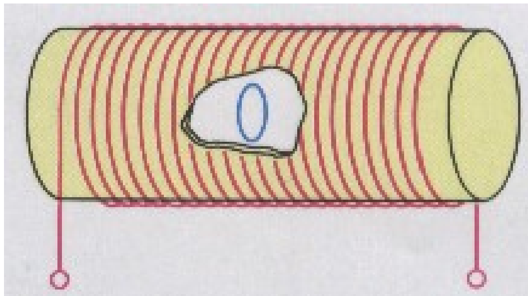


Fig.32.6

$$\Phi_{12} = B_2 A_1 = \left(\mu_0 n_2 I_2\right) A_1 \ \Rightarrow M = \frac{N_1\Phi_{12}}{I_2} = \mu_0 n_2 N_1 A_1$$

LR 電路 (LR circuits)

- 充電(Rise) $\Rightarrow \left[\frac{dI}{dt} > 0 \Rightarrow \xi_L = -L\frac{dI}{dt} = V_a - V_b > 0 \right]$

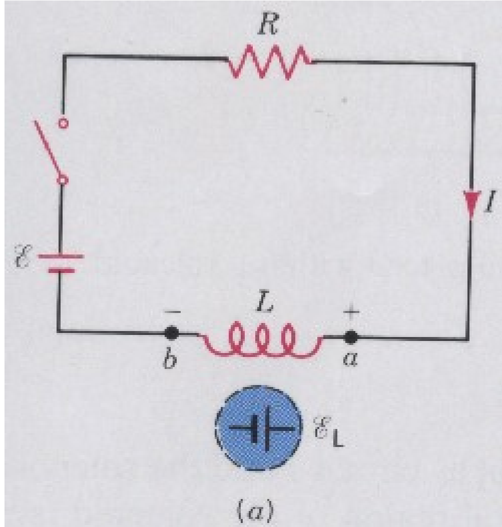


Fig.32.7

From Kirchhoff's loop rule： $\xi - IR - L\frac{dI}{dt} = 0$

$$\Rightarrow \frac{\xi}{R} - I - \frac{L}{R}\frac{dI}{dt} = 0 \xrightarrow{Let\ y=\frac{\xi}{R}-I,\frac{dy}{dt}=-\frac{dI}{dt}} y + \frac{L}{R}\frac{dy}{dt} = 0$$

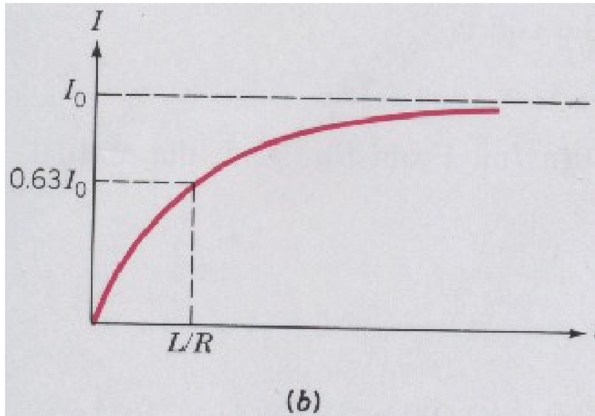
$$\Rightarrow \frac{dy}{y} = -\frac{R}{L}dt \Rightarrow \ln y = -\frac{R}{L}t + \ln y_0 \Rightarrow y = y_0 e^{-Rt/L}$$

$$\Rightarrow \frac{\xi}{R} - I = y_0 e^{-Rt/L} \xrightarrow[initial\ cond.]{I=0\ at\ t=0} y_0 = \frac{\xi}{R}$$

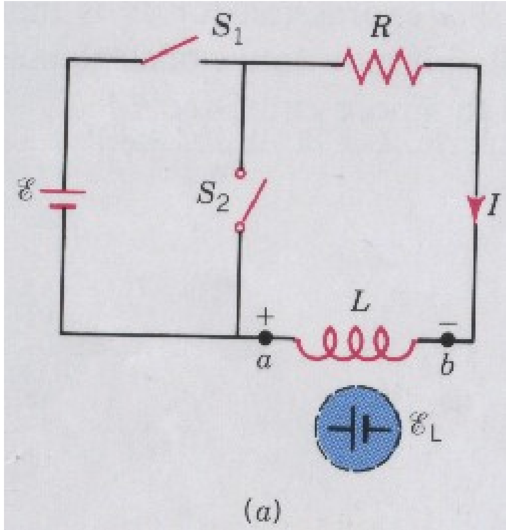
$$\Rightarrow \frac{\xi}{R} - I = \frac{\xi}{R}e^{-Rt/L} \Rightarrow I = \frac{\xi}{R}(1 - e^{-Rt/L})$$

$$\xrightarrow{I_0=\frac{\xi}{R}} I = I_0(1 - e^{-Rt/L}) \xrightarrow{\tau=L/R} I = I_0(1 - e^{-t/\tau})$$

$$When\ t = \tau \Rightarrow I = I_0(1 - e^{-1}) \approx 0.63 I_0$$



$$\bullet\text{放電(Decay)} \Rightarrow \left[\frac{dI}{dt}<0 \Rightarrow \xi_L = -L\frac{dI}{dt} = V_a - V_b > 0\right]$$



From Kirchhoff's loop rule： $-IR - L\dfrac{dI}{dt} = 0$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L}dt \Rightarrow \ln I = -\frac{R}{L}t + \ln I_0$$

$$\xrightarrow[\text{initial cond.}]{I=I_0=\xi/R\ at\ t=0} I = \frac{\xi}{R}e^{-Rt/L} \Rightarrow I = I_0 e^{-t/\tau}$$

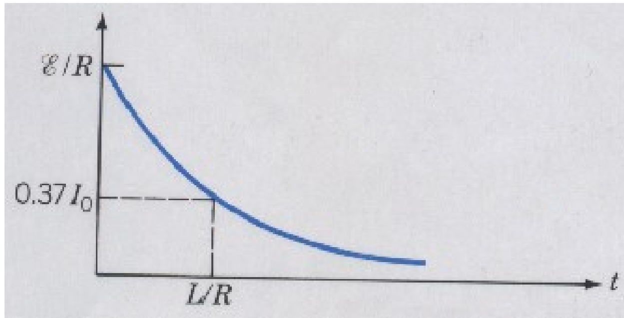


Fig.32.8

$$Initial\ condition \Rightarrow \begin{cases} Rise \Rightarrow I = 0 \text{（斷路）} \\ \text{Decay} \Rightarrow I = I_0 \text{（短路）} \end{cases}$$

$$Steady\ state \Rightarrow \begin{cases} Rise \Rightarrow I = I_0 \text{（短路）} \\ \text{Decay} \Rightarrow I = 0 \text{（斷路）} \end{cases}$$

儲存於電感的能量(Energy stored in an inductor)

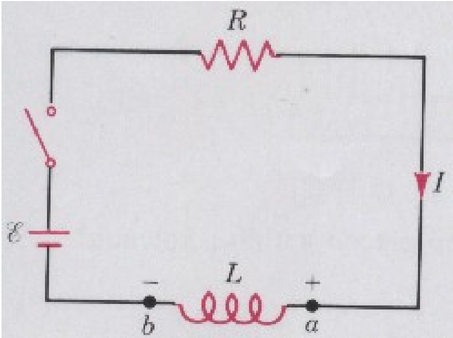


Fig.32.7

$$\xi - iR - L\frac{di}{dt} = 0 \quad \Rightarrow \xi = iR + L\frac{di}{dt}$$

$$\Rightarrow i\xi = i^2R + Li\frac{di}{dt} \quad \begin{cases} i\xi \Rightarrow \textit{the power supplied by battery} \\ i^2R \Rightarrow \textit{the power dissipated by resistor} \\ Li\dfrac{di}{dt} \Rightarrow \textit{the power supplied by inductor} \end{cases}$$

$$\frac{dU_L}{dt} = Li\frac{di}{dt} \quad \Rightarrow \textit{The total energy} \ : \ U_L = L\int_0^I i\,di = \frac{1}{2}LI^2$$

$$\xrightarrow[Solenoid]{L=\mu_0 n^2 A\ell} U_L = \frac{1}{2}(\mu_0 n^2 A\ell)I^2 \xrightarrow[Solenoid]{B=\mu_0 nI} U_L = \frac{B^2}{2\mu_0}A\ell$$

$$\textit{Energy density} \ : \ u_B = \frac{U_L}{A\ell} = \frac{B^2}{2\mu_0}$$

Example 32.6: The self-inductance of a toroid.

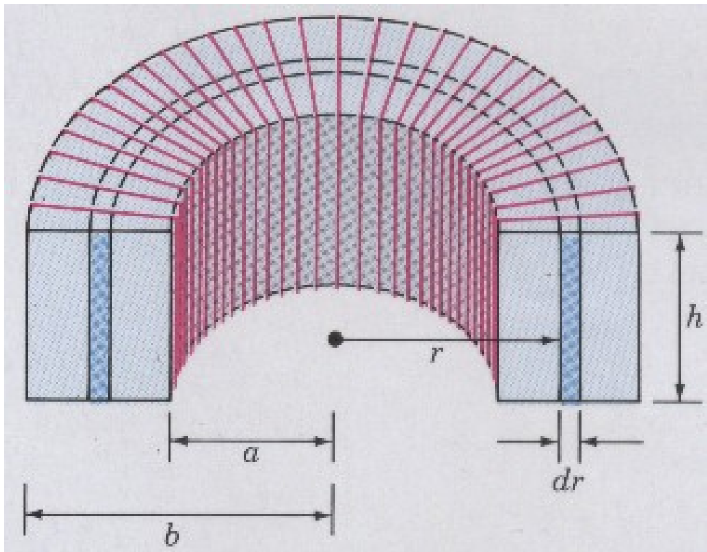
$$B=\frac{\mu_0 NI}{2\pi r},\ u_B=\frac{B^2}{2\mu_0},\ dV=h(2\pi r dr)$$

$$dU=u_B dV=(\frac{B^2}{2\mu_0})h(2\pi r dr)=\frac{\mu_0 N^2 I^2 h}{4\pi}\frac{dr}{r}$$

$$U=\int dU=\frac{\mu_0 N^2 I^2 h}{4\pi}\int_a^b\frac{dr}{r}=\frac{\mu_0 N^2 I^2 h}{4\pi}\ln\frac{b}{a}$$

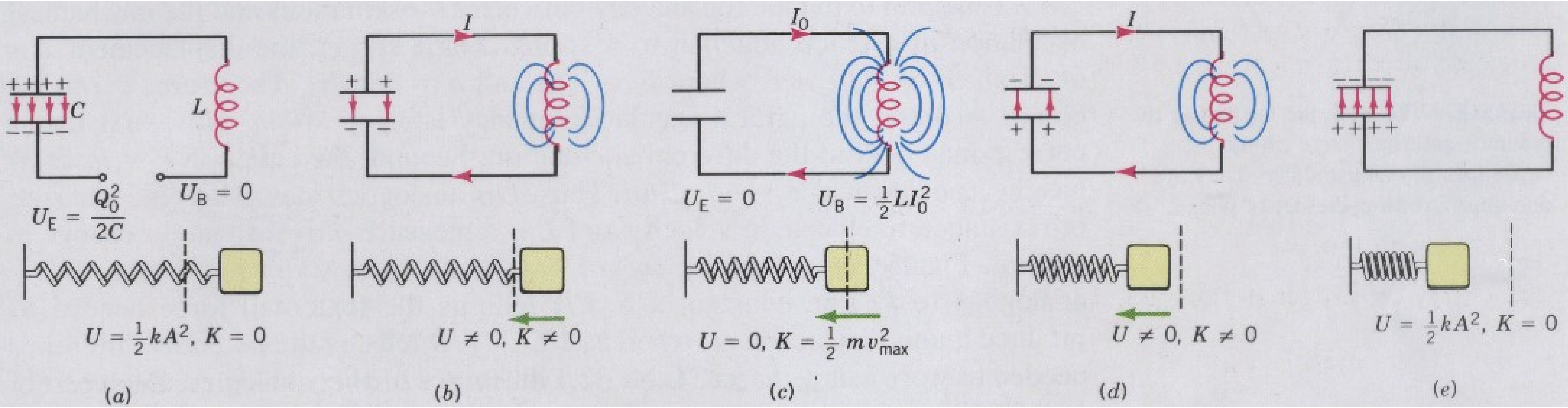
$$From U=\frac{1}{2}LI^2\Rightarrow L=\frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a}$$

Fig.32.9



LC振盪(LC oscillations)

Fig.32.10



From Kirchhoff's loop rule： $\dfrac{Q}{C} - L\dfrac{dI}{dt} = 0 \xrightarrow{\because I = -\frac{dQ}{dt}} \dfrac{Q}{C} + L\dfrac{d^2Q}{dt^2} = 0$

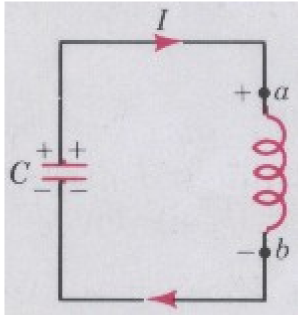


Fig.32.11

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0 \xrightarrow[\text{簡諧振盪}]{\frac{d^2x}{dt^2}+\omega^2x=0} Q = Q_0\sin(\omega_0 t + \phi)$$

where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ (natural angular frequency)

$$\xrightarrow[initial\ cond.]{Q=Q_0\ at\ t=0} \phi = \pi/2 \Rightarrow Q = Q_0\cos\left(\omega_0 t\right)$$

$$\xrightarrow{I=-\frac{dQ}{dt}} I = I_0\sin\left(\omega_0 t\right) \quad \text{, where } I_0 = \omega Q_0$$

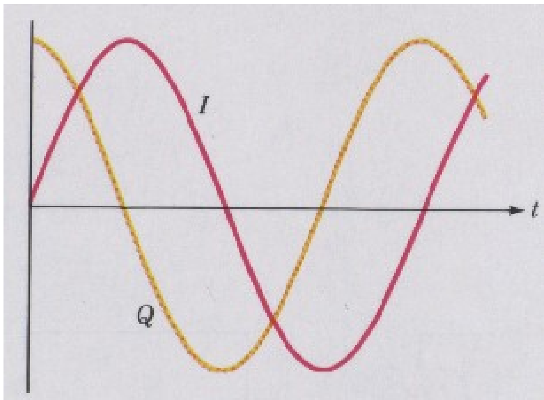


Fig.32.12

$$\text{the total energy} \Rightarrow U = U_E + U_B = \frac{Q_0^2}{2C}\cos^2(\omega_0 t) + \frac{LI_0^2}{2}\sin^2(\omega_0 t)$$

$$\because I_0 = \omega_0 Q_0 \;\; and \;\; \omega_0 = \frac{1}{\sqrt{LC}} \qquad \therefore U = \frac{Q_0^2}{2C} = \frac{LI_0^2}{2} = constant$$

TABLE 32.1 ANALOGIES BETWEEN MECHANICAL AND ELECTRICAL QUANTITIES

Mechanical:	x	v	m	$\frac{1}{2}mv^2$	k	$\frac{1}{2}kx^2$	F	$P = Fv$
Electrical:	Q	I	L	$\frac{1}{2}LI^2$	$\frac{1}{C}$	$\frac{1}{2}\frac{Q^2}{C}$	V	$P = VI$

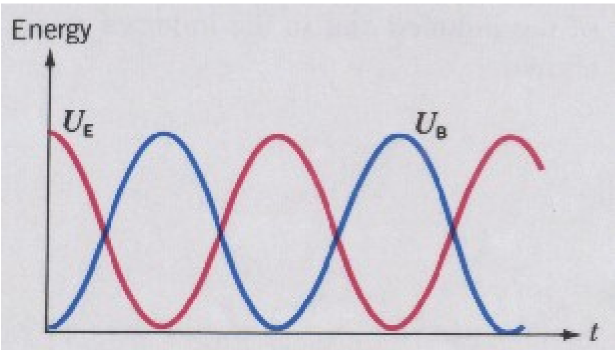


Fig.32.13

純LC振盪是不合理的，理由如下：

1.任何電感器皆有電阻。

2.能量會以電磁波方式輻散，不能維持定值。

• RLC振盪(Damped LC oscillations)

From Kirchhoff's loop rule：$\dfrac{Q}{C} - IR - L\dfrac{dI}{dt} = 0 \xrightarrow{I=-\frac{dQ}{dt}}$

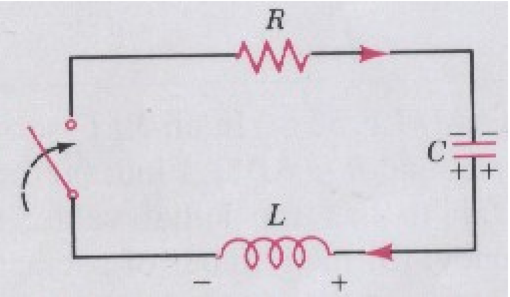


Fig.32.14

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \xrightarrow[\text{damped harmonic motion}]{m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=0} Q = Q_0 e^{-Rt/2L}\cos(\omega' t + \delta)$$

where $\omega' = \sqrt{\omega_0^2 - \left(\dfrac{R}{2L}\right)^2}$

$$\begin{cases} \omega_0 > R/2L \Rightarrow R < 2\omega_0 L & \text{(under-damped)-次阻尼} \\ \omega_0 = R/2L \Rightarrow R = 2\omega_0 L & \text{(critically damped)-臨界阻尼} \\ \omega_0 < R/2L \Rightarrow R > 2\omega_0 L & \text{(over-damped)-過阻尼} \end{cases}$$

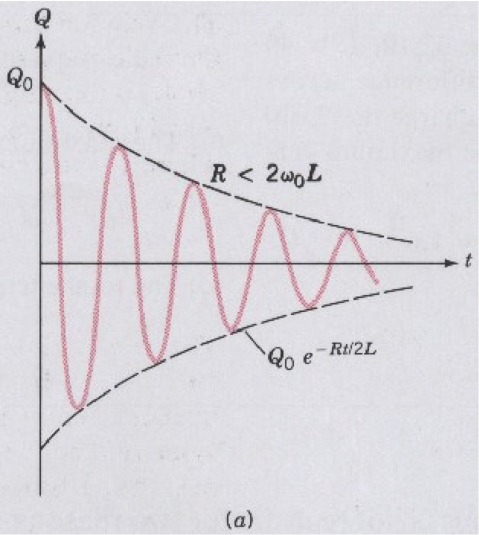
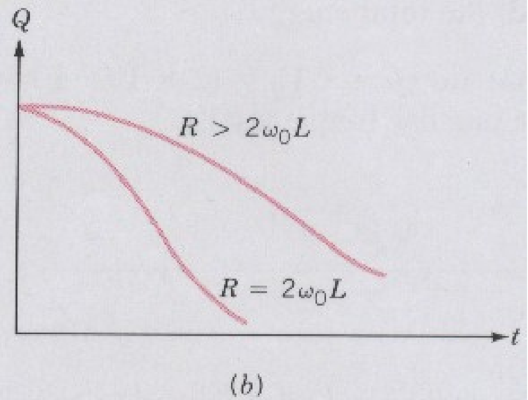


Fig.32.15



本章單元重要觀念發展脈絡彙整

電感

⇓

自感與互感現象

⇓

電感值的定義與分析

⇓

$$\xi_1 = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = \xi_{11} + \xi_{12}$$

⇓

電路應用

LR電路

充電→短路
放電→斷路

儲存磁能

$$u_B = \frac{B^2}{2\mu_0}$$

RLC電路

LC振盪(簡諧振盪)
RLC振盪(阻尼振盪)