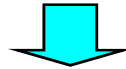


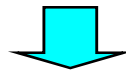
# 磁學 (Magnetism)

## 29~32章探究的主要問題架構

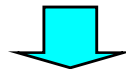
磁場如何形成？其影響為何？



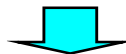
磁力如何產生？其應用為何？



磁場如何估算？



磁如何生電？



磁能如何儲存？

# 磁學 (Magnetism)

## ✦ 磁場(The magnetic field)

磁化物質  
電流 }  $\Rightarrow$  Magnetic effect  $\Rightarrow$  Compass needle  
(磁效應) (羅盤針)

### ● 磁場特性

磁場線由北極出，南極入，形成封閉環路。

地球上無磁單極(monopole)。

地理南北極與地磁南北極相反。

磁場表示法

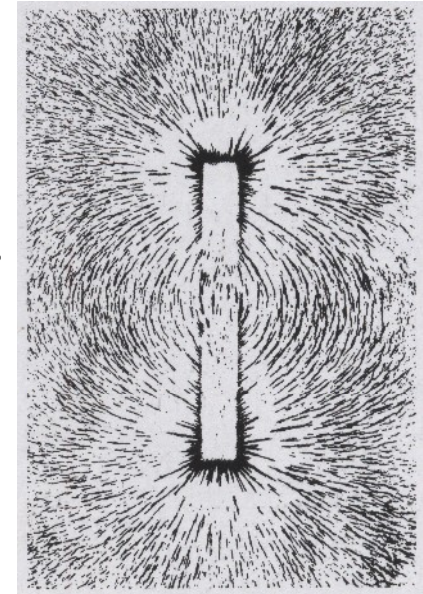


Fig.29.2

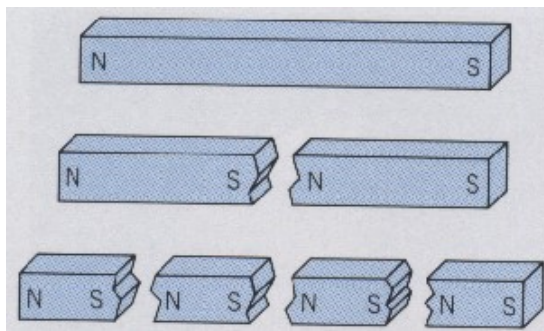


Fig.29.3

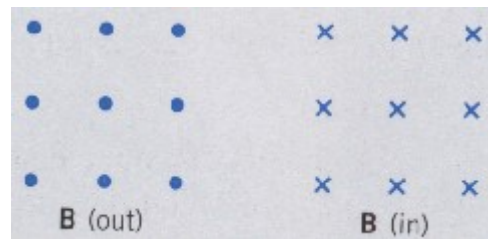
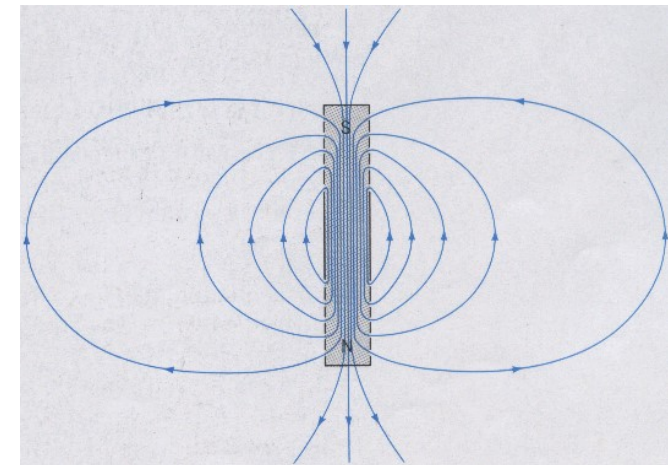


Fig.29.4



- Definition of the Magnetic Field

— 以運動電荷測試磁場 (仿電荷測試電場)

$$\begin{cases} F \propto \sin \theta \\ F \propto qv \end{cases} \Rightarrow F \propto qv \sin \theta$$

$\therefore F \propto B \Rightarrow$  定義  $B$  為比例常數

$$\Rightarrow F = qvB \sin \theta$$

$$\Rightarrow \vec{F} = q\vec{v} \times \vec{B} \quad (\text{向量式})$$

$$(\because \vec{F} \perp \vec{v} \text{ and } \vec{F} \perp \vec{B})$$

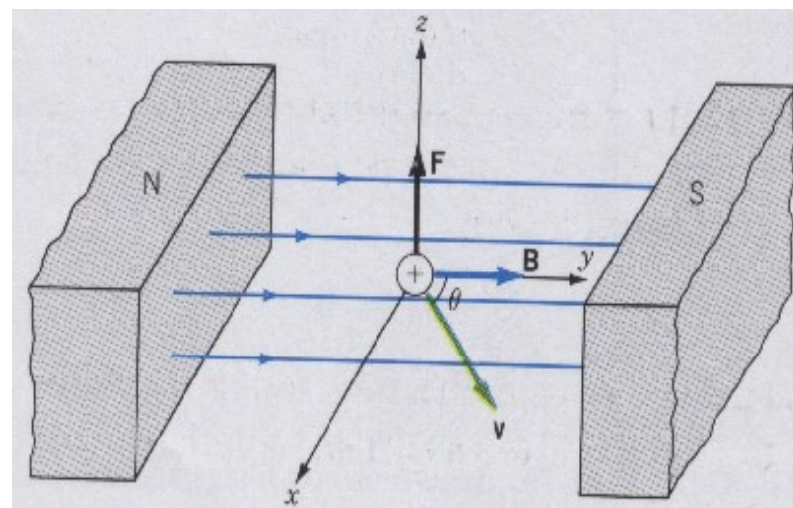


Fig.29.5

(※注意：磁場無法驅動靜止的帶電粒子)

磁力不會作功，不能改變粒子的動能。 $(\because \vec{F} \perp \vec{v})$

SI unit: M.K.S  $\Rightarrow$  tesla (T) ; C.G.S  $\Rightarrow$  Gauss (G) — 常用

$$1 \text{ T} = 10^4 \text{ G}$$

- 載流導體上所受的磁力 (Force on a current-carrying conductor)

If current-carrying wire  $\perp \vec{B}$

$$\Rightarrow F = qvB = (nA\ell)ev_d B \quad (\because \begin{cases} q = (nA\ell)e \\ v = v_d \end{cases})$$

$$= I\ell B \quad (\because I = nAev_d)$$

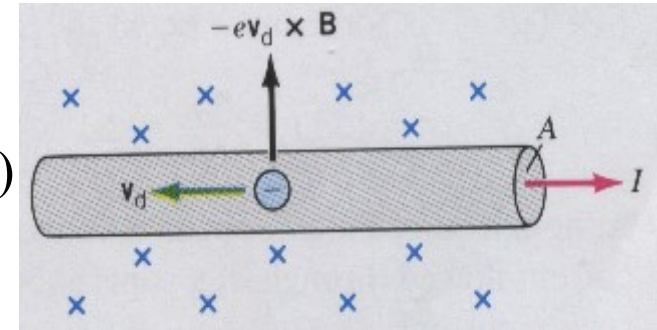


Fig.29.7

If current-carrying wire  $\nmid \vec{B} \Rightarrow F = I\ell B \sin \theta \Rightarrow \vec{F} = I\vec{\ell} \times \vec{B}$

If the wire is not straight or the field is not uniform  $\Rightarrow d\vec{F} = Id\vec{\ell} \times \vec{B}$

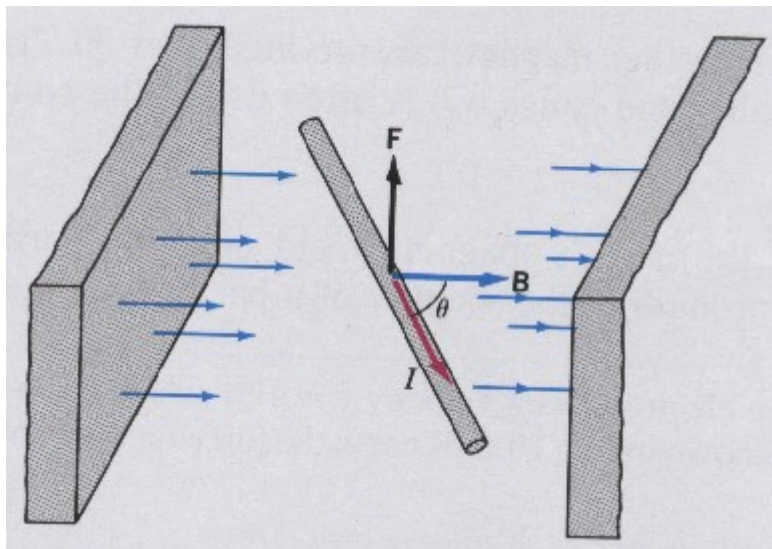


Fig.29.8

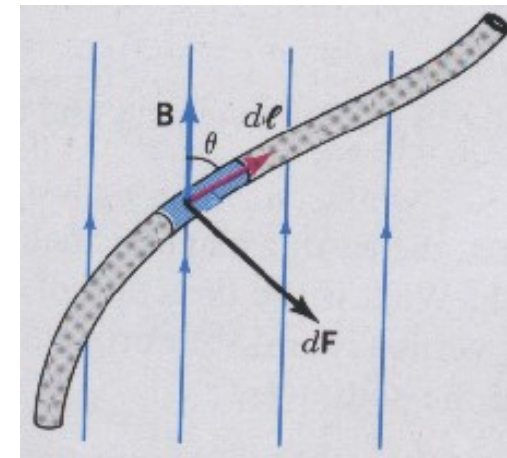


Fig.29.9

Example 29.4: Find the force on the loop for Fig.29.12.

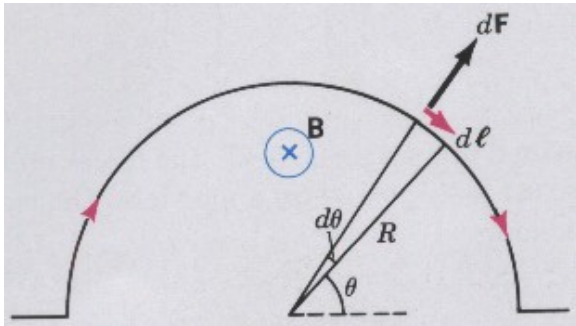


Fig.29.12

電流素(current element)所受磁力

{	x 分量成對抵消
	y 分量會疊加

➤沿半圓環的總力：

$$dF_y = dF \sin \theta = I(Rd\theta)B \sin \theta$$

$$\Rightarrow F_y = IRB \int_0^\pi \sin \theta d\theta = IRB [-\cos \theta]_0^\pi = 2IRB$$

➤直線連接半圓環(或Fig.29.13之AB)兩端的總力：

$$F_y = \int_0^{2R} IB d\ell = IB \int_0^{2R} d\ell = 2IRB$$

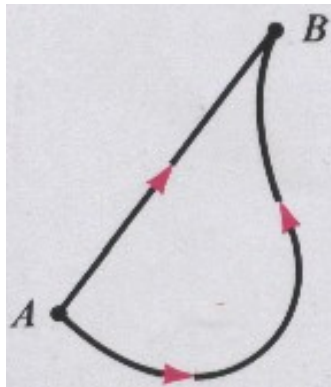


Fig.29.13

➤延伸應用：

1. A→B任一路徑所受磁力皆為 $2IRB$ ，B→A則為 $-2IRB$ 。
2. 在一均勻磁場內，任一封閉載流線圈的淨力為零。



# ✦ 單匝載流線圈所受的力矩(Torque on a current loop)

- 均勻磁場  $\left\{ \begin{array}{l} \text{淨力(net force)或合力為零。} \\ \text{淨轉矩(net torque)或合力矩不一定為零。} \end{array} \right.$

考慮Fig.29.14的矩形迴路 (邊長 $a$ 與 $c$ )  $\xrightarrow[\text{僅考慮}c\text{邊作用力}]{z\text{軸為轉軸}}$   $\left\{ \begin{array}{l} \vec{F}_1 = I(-c\hat{k}) \times (B\hat{i}) = -IcB\hat{j} \\ \vec{F}_2 = I(c\hat{k}) \times (B\hat{i}) = IcB\hat{j} \end{array} \right.$

$$\tau_{net} = r_{\perp} F_1 + r_{\perp} F_2 = 2(IcB) \left( \frac{a}{2} \sin \theta \right) = IAB \sin \theta$$

( $\because r_{\perp} = \frac{a}{2} \sin \theta$ ) (where  $A=ac$ )

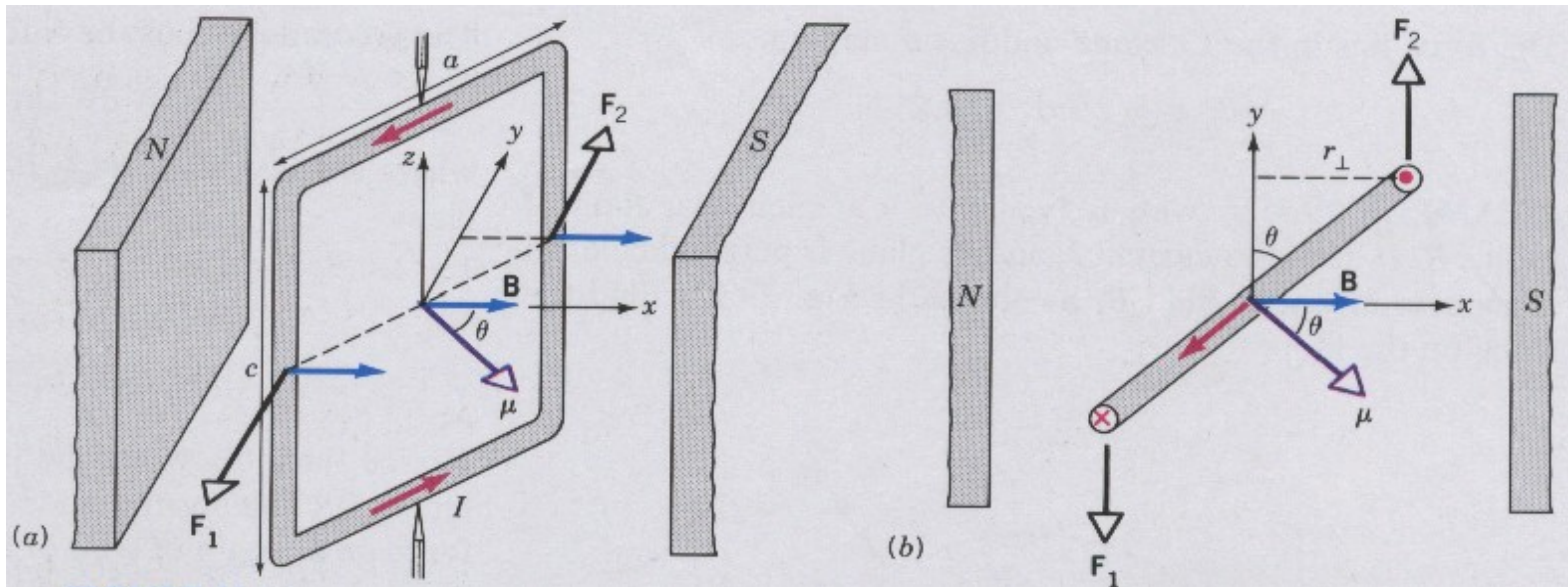


Fig.29.14

Magnetic dipole moment (磁偶極矩)  $\xrightarrow{\text{is defined as}} \vec{\mu} = NIA\hat{n}$

<其中  $\hat{n}$  為線圈平面法線方向(可依右手定則判定)，N表匝數，SI unit:  $\text{A}\cdot\text{m}^2$ >

$$N\text{匝線圈} \Rightarrow \tau = NIAB \sin \theta \Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$$

(此力矩使磁偶極矩沿磁場方向排列)

$$\Delta U = \int \tau d\theta = \int_{\theta_1}^{\theta_2} \mu B \sin \theta d\theta$$

$$\Rightarrow U_2 - U_1 = \mu B \cos \theta_1 - \mu B \cos \theta_2$$

$$\text{令 } U_1 = 0, \quad \theta_1 = \pi/2 \Rightarrow U = -\vec{\mu} \cdot \vec{B}$$

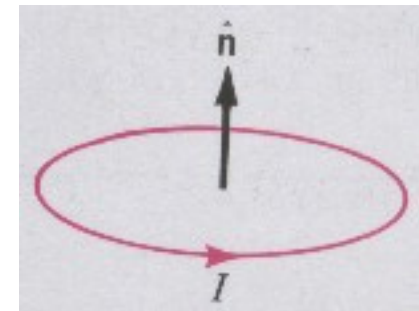


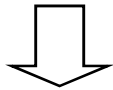
Fig.29.15

$$\text{近似電偶極矩在均勻電場的形式} \Rightarrow \begin{cases} \vec{\tau} = \vec{p} \times \vec{E} \\ U = -\vec{p} \cdot \vec{E} \end{cases}$$

●應用：

➤檢流計(Galvanometer)

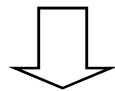
Moving-coil galvanometer  
(圈動檢流計)



$$\tau_B = \mu B \sin \theta \quad (\text{from magnetic field})$$

$$\tau_{sp} = \kappa \phi \quad (\text{from a coiled spring})$$

$$NIAB \sin \theta = \kappa \phi$$



為了刻度線性化(消去 $\sin \theta$ )，將磁極改為圓柱形，則 $\theta = \pi/2$ 。

$$NIAB = \kappa \phi \Rightarrow \phi = \frac{NAB}{\kappa} I$$

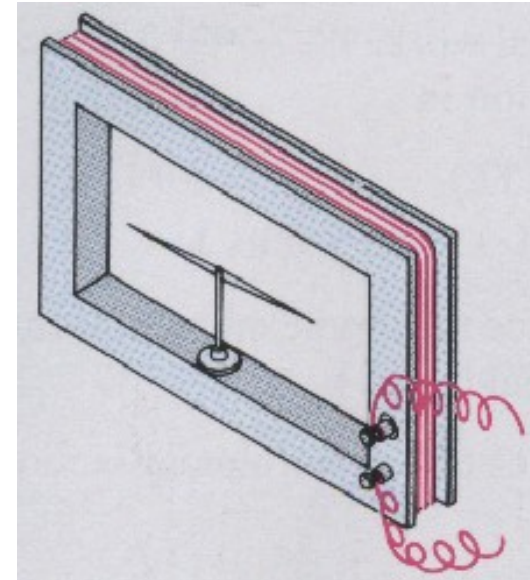


Fig.29.17

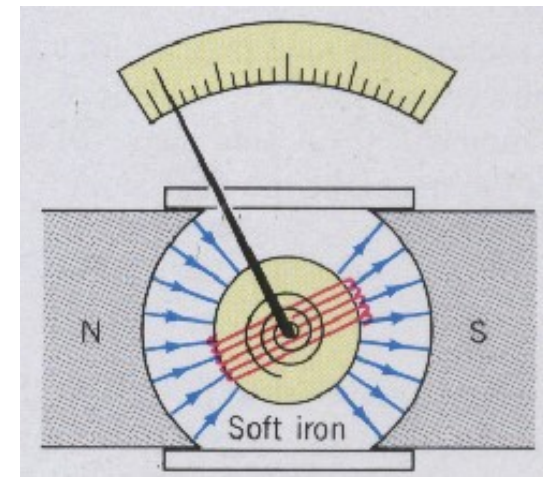


Fig.29.18



➤DC motor (直流電動機) — 利用磁場中載流線圈的轉矩作功，並藉由換向器 (commutator)讓線圈朝某一方向持續旋轉，其中還需利用慣性克服換向器的空隙無電流區。

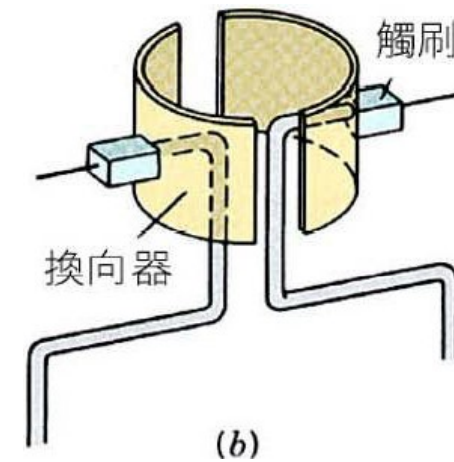
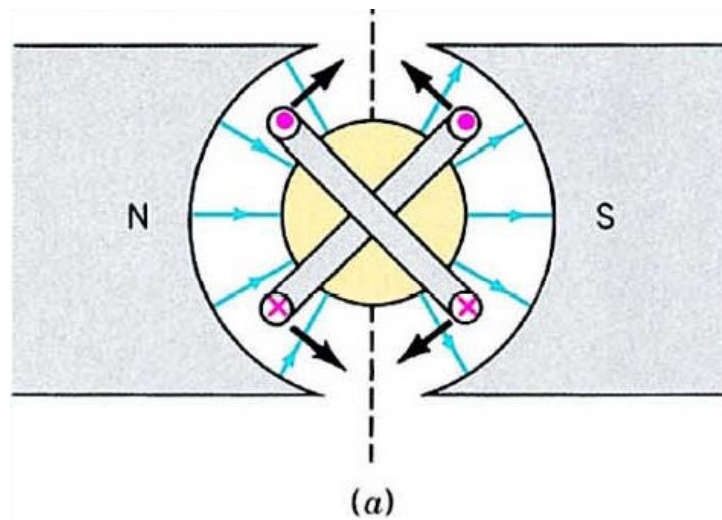


Fig.29.20



# 帶電粒子在磁場中的運動

(the motion of charged particles in magnetic fields)

一般的應用

TV tube(電視印象管的聚焦)  
fusion control(核融合控制)  
地磁對高能粒子的束縛

## ●均勻磁場：

➤Case(A): 若  $\vec{v} \perp \vec{B}$  (即垂直入射)，則粒子為圓周運動。

$$\Rightarrow F_B(\text{磁力}) = qvB = \frac{mv^2}{r} (\text{向心力}) \Rightarrow r = \frac{mv}{qB}$$

$$\Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \Rightarrow f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

$\left\{ \begin{array}{l} r \text{ 稱為迴旋半徑(cyclotron radius or gyroradius)} \\ f_c \text{ 稱為迴旋頻率(cyclotron frequency)} \end{array} \right.$

Conclusion：

(i)週期與頻率皆與粒子速率無關。

(ii)荷值比(q/m)相同的粒子亦具有相同的迴旋週期與頻率。

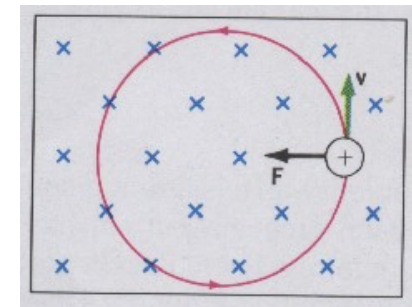
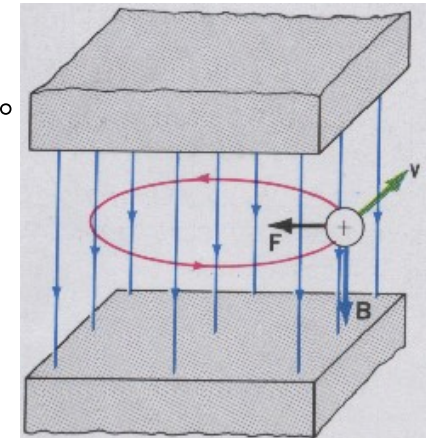


Fig.29.22

➤Case( B ): 若  $\vec{v} \not\perp \vec{B}$  (即傾斜入射,  $\vec{v}$ 與 $\vec{B}$ 夾有角度 $\theta$ ) , 則粒子為螺旋運動。  
(Helical motion)

$$\Rightarrow \begin{cases} \vec{v}_{\parallel} \text{表平行}\vec{B}\text{的分量} \xrightarrow{\text{不受磁場}\vec{B}\text{影響}} \text{直線運動} \\ \vec{v}_{\perp} \text{表垂直}\vec{B}\text{的分量} \xrightarrow{\text{磁力為向心力}} \text{圓周運動} \end{cases}$$

螺旋線的一螺距(pitch)即一週期的粒子位移大小。

$$\Rightarrow d = v_{\parallel} T = v_{\parallel} \frac{2\pi m}{qB}$$

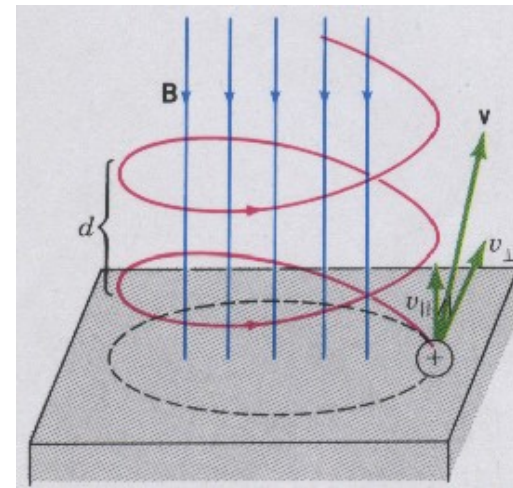


Fig.29.23

## ●非均勻磁場

⇒ { 迴旋半徑與磁場強度成反比，即  $r \propto 1/B$ 。  
粒子所受磁力朝著磁場較弱區域。

Magnetic bottles(磁瓶) effect

— 當粒子朝向磁場強度較強的區域運動，則會在某一點停止前進並開始倒逆其行進方向。

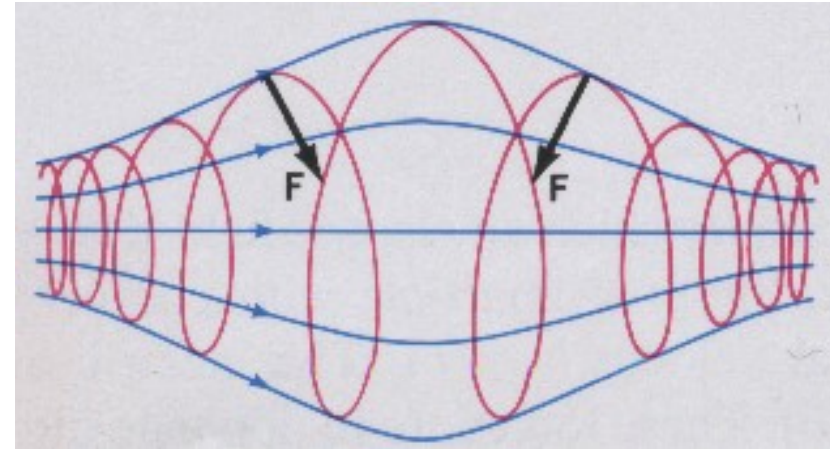


Fig.29.24

應用 { 控制高溫電漿(plasma)，  
如：核融合反應研究。  
將高能粒子控制在地磁區域而形成凡阿倫輻射帶。  
(Van Allen radiation belts)

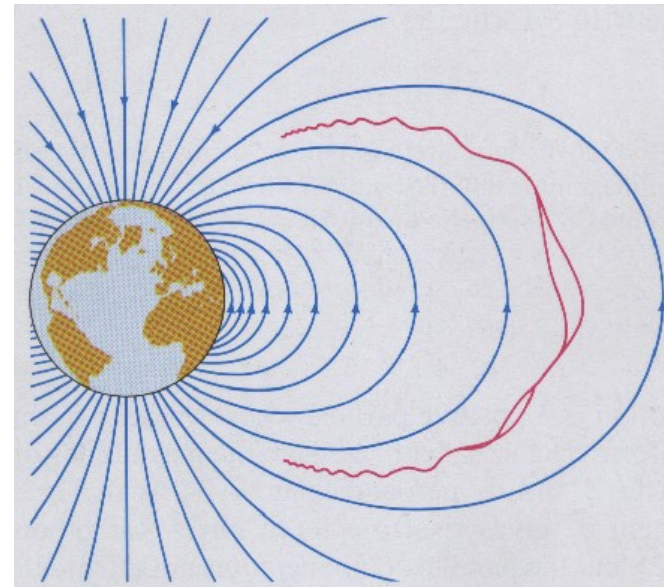


Fig.29.25



# ✦ 帶電粒子受電場與磁場的交叉場(crossed fields)聯合作用

$$\Rightarrow \vec{F} = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{— Lorentz force (勞倫茲力)}$$

Case:  $\vec{E} = -E\hat{j}$ ,  $\vec{B} = -B\hat{k}$  (shown as Fig.29.26)

$$\Rightarrow \begin{cases} \vec{F}_E = -qE\hat{j} \\ \vec{F}_B = qvB\hat{j} \end{cases} \Rightarrow \text{If } \vec{F}_E + \vec{F}_B = -qE\hat{j} + qvB\hat{j} = 0$$

(帶電粒子進入crossed fields未偏向)

$$\Rightarrow \vec{E} + \vec{v} \times \vec{B} = 0 \Rightarrow \vec{E} = -\vec{v} \times \vec{B} \xrightarrow{\text{if } \vec{v} \perp \vec{B}} v = \frac{E}{B}$$

## ● 應用 $\Rightarrow$ Mass spectrometer(質譜儀)

— 可分離同位素，偵測污染物或雜質

$$r = \frac{mv}{qB} \Rightarrow \frac{m}{q} = \frac{Br}{v}$$

$$\xrightarrow{v=E/B_1} \frac{m}{q} = \frac{B_1 B_2}{E} r$$

Fig.29.28

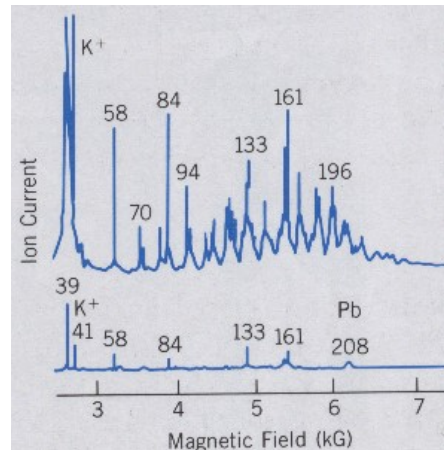


Fig.29.26

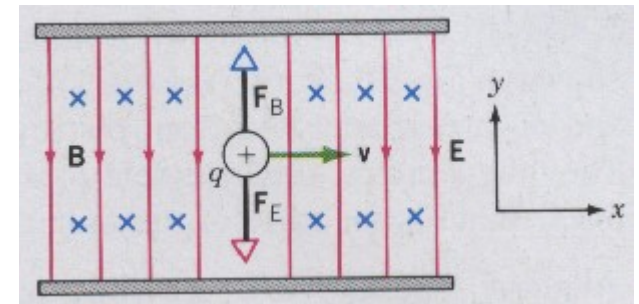
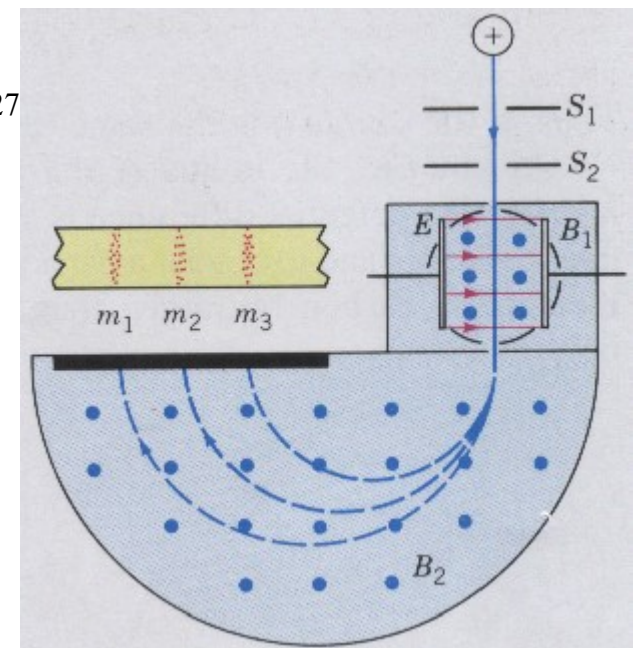


Fig.29.27





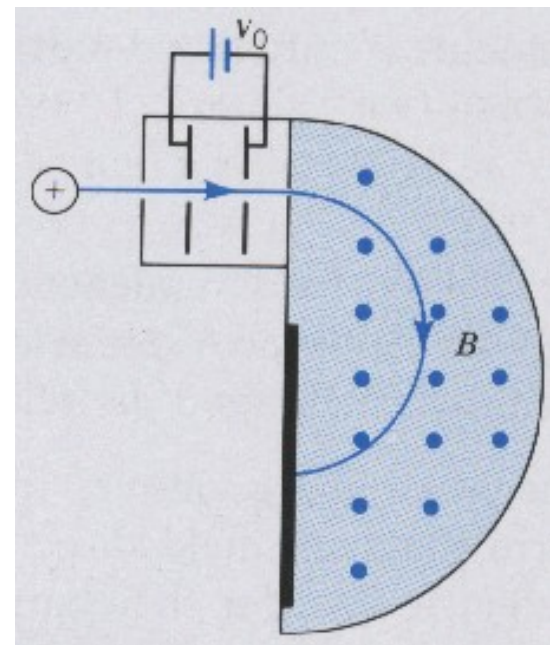
### Example 29.9: (A.J. Dempster's mass spectrometer)

Two isotopes of an element with  $m_1$  and  $m_2$  are accelerated from rest by a potential difference  $V$ . They then enter a uniform field  $B$  normal to the magnetic field lines. What is the ratio of the radii of their paths?

$$\text{Kinetic energy} \Rightarrow \frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$\text{the radius of the path} \Rightarrow r = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}$$



# ★ The Hall effect (霍[哈]耳效應)

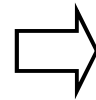
磁力作用於載流導線  
(因不論磁場是否存在，導體  
內電流分佈皆相同)—Maxwell



磁力與電流成正比，跟導線  
size無關。(因電流為零，磁力便  
消失)—Hall



在磁場中，fluid被磁力拉向  
導線一邊，促使有效截面減  
小，電阻將增大。



在磁場中，電荷 $+q$ 以 $v_d$ 沿金屬片移  
動時，會受到向上的磁力，造成金  
屬片頂部帶正電，底部帶負電，形  
成向下的Hall電場，導致一個向下的  
靜電力。當更多電荷移往金屬頂  
部與底部，靜電力會增大，最後與  
磁力平衡後，電荷便不再偏向。

$$\begin{cases} \vec{F}_B = qv_d B \hat{k} \\ \vec{F}_E = -qE \hat{k} \end{cases} \Rightarrow F_B = F_E \Rightarrow E = v_d B$$

$$\text{霍耳電壓} \Rightarrow V_H = EW = v_d BW$$

(Hall potential difference)

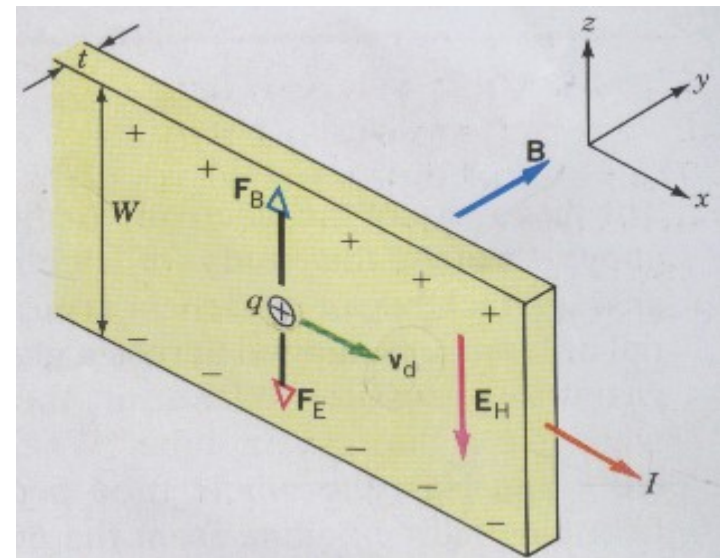


Fig.29.34

$$\therefore \begin{cases} I = nqv_d A \\ A = Wt \end{cases} \Rightarrow v_d = \frac{I}{nqWt} \quad , \quad \therefore V_H = v_d BW = \frac{IB}{nqt}$$

應用

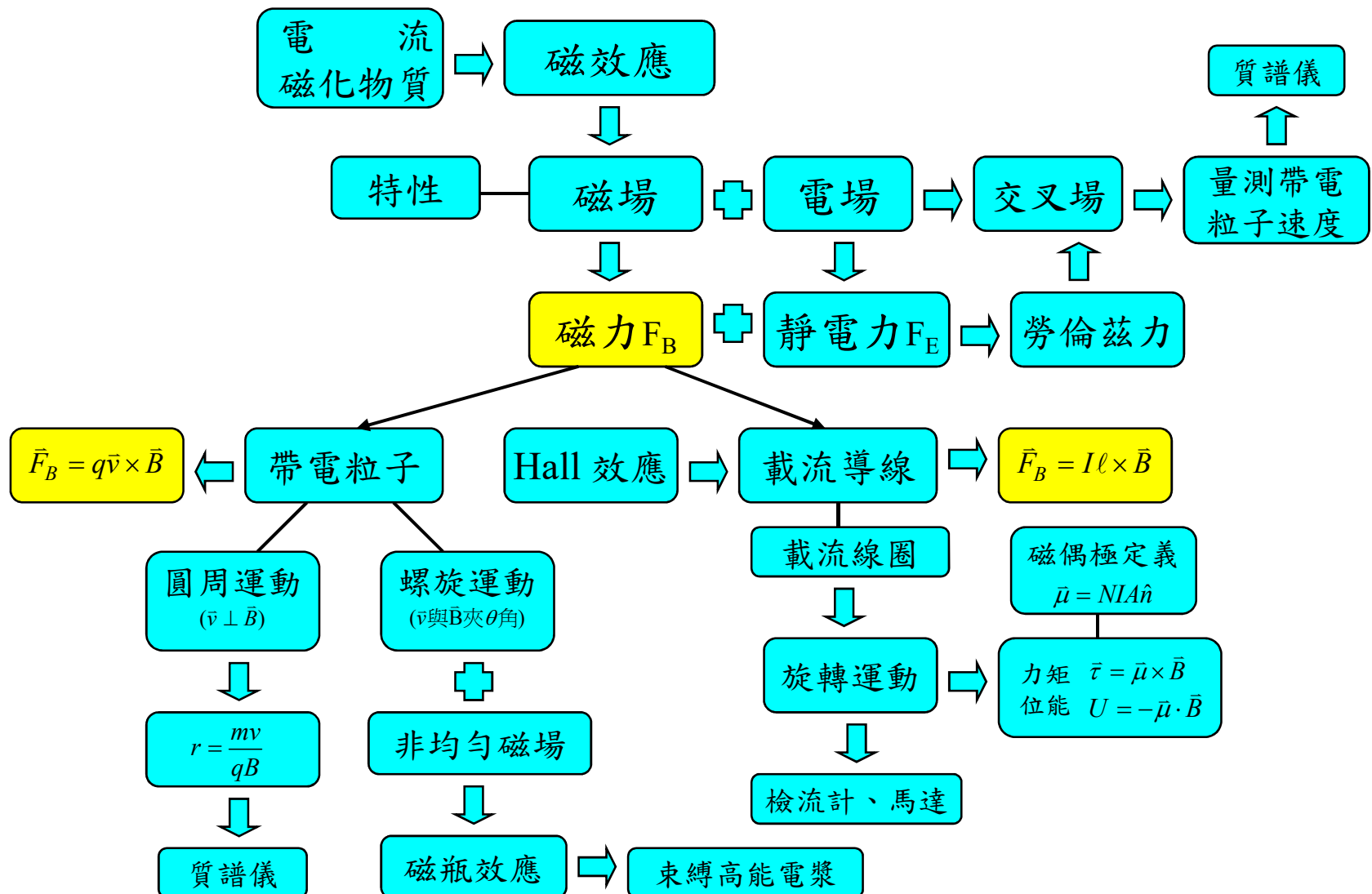
霍耳電壓( $V_H$ )極性可判知導體內移動電荷為負。  
 磁場已知，可求漂移速度 $v_d$ ；或 $v_d$ 已知，可求磁場 $B$ 。  
 霍耳探針(Hall probe)－利用哈耳電壓量測磁場。  
 霍耳電壓( $V_H$ )反比於載流電荷密度( $n$ )。

補充

Au, Cu, Ag, Pt, Al 的 $V_H$ 極性偏向負電荷載體。  
 Co, Zn, Pb, Fe 及半導體Si, Ge的 $V_H$ 極性偏向正電荷載體。

**作用於載流導線上的磁力性質** (The nature of the magnetic force on a current-carrying wire)－磁力並非直接作用於導線上，因導線晶格上的正離子幾乎靜止不動(除熱運動之外)，不會產生磁力，但導線內漂移的負電荷卻可由 Hall effect 在導線兩側形成電場，促使導線正離子產生靜電力( $F=qE$ ，靜止於電場中的電荷仍具靜電力)，此力相當於磁力，作用於導線晶格的正離子上。

# 本章單元重要觀念發展脈絡彙整



## 習題

- 教科書習題 (p.599~p.604)

Exercise: 7,11,21,25,31,35,39,59,75,77

Problem: 5

- 基本觀念問題：

- 1.請說明磁偶極矩(magnetic dipole moment) $\mu$ 的定義(包括：大小與方向)，並寫出均勻磁場 $B$ 對磁偶極矩 $\mu$ 所造成的力矩及位能向量式。
- 2.請說明磁瓶效應。
- 3.請利用霍爾效應(Hall effect)解釋載流導線受磁力的作用。



## ✦ 磁場的來源(Source of magnetic field) — 由運動的電荷(電流)產生

- 無窮長直載流導線形成的磁場  $B \Rightarrow \begin{cases} B \propto 1/R \\ B \propto I \end{cases} \xrightarrow{\text{SI unit}} B = \frac{\mu_0 I}{2\pi R}$   
(Infinite wire)

其中  $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A} \rightarrow$  permeability constant (導磁常數)

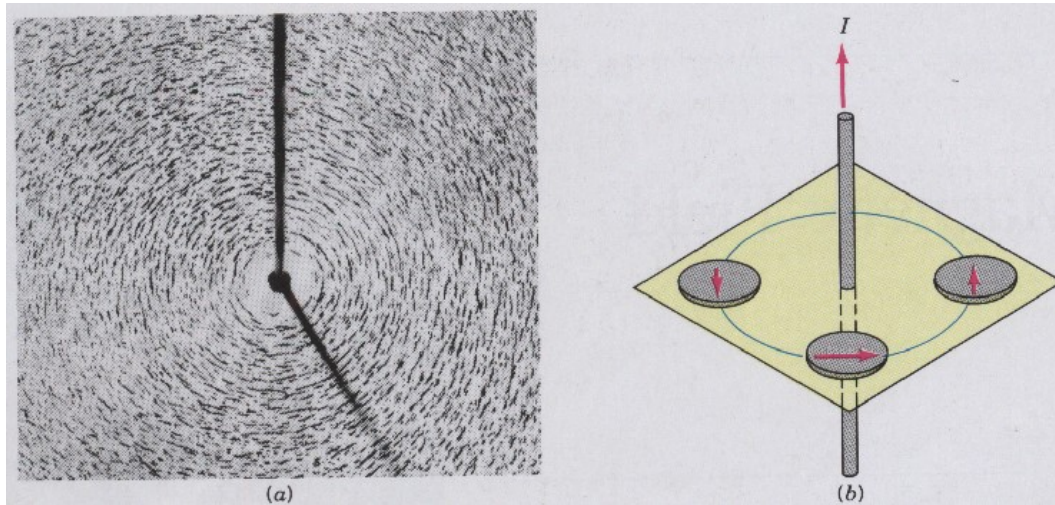


Fig.30.1

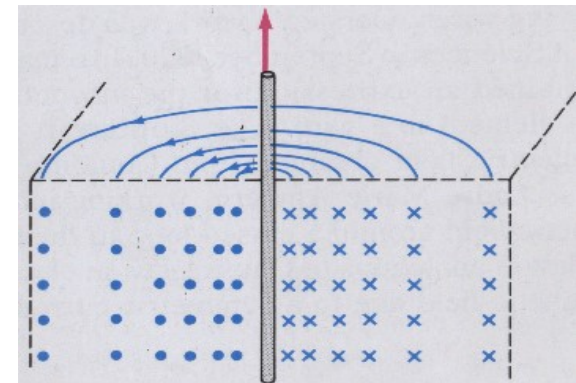
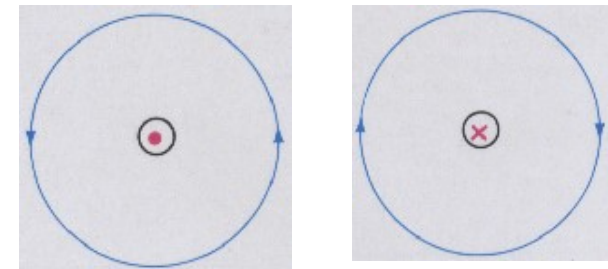


Fig.30.2

## ● 平行導線間的磁力 (Magnetic force between parallel wires)

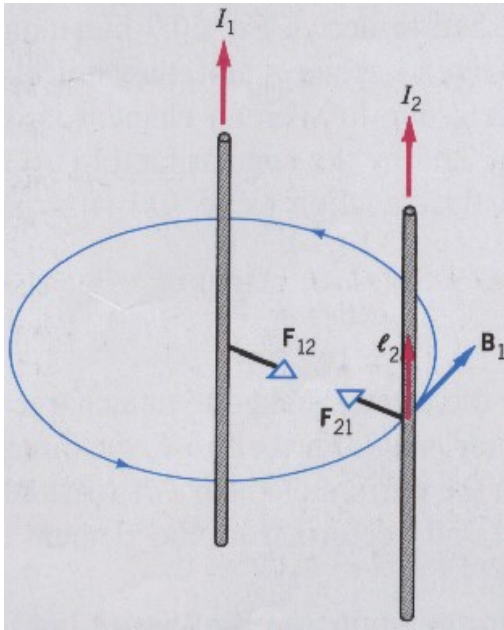


Fig.30.3

若考慮 wire 1 形成的磁場  $B_1$ ，則 wire 2 部份長度  $\ell_2$  所受磁力為  $\vec{F}_{21}$

或考慮 wire 2 形成的磁場  $B_2$ ，則 wire 1 部份長度  $\ell_1$  所受磁力為  $\vec{F}_{12}$

$$\Rightarrow \vec{F}_{21} = I_2 \vec{\ell}_2 \times \vec{B}_1 \quad \text{或} \quad \vec{F}_{12} = I_1 \vec{\ell}_1 \times \vec{B}_2$$

➤ 考慮  $\ell_1 = \ell_2$

$$\Rightarrow F_{21} = I_2 \ell_2 B_1 = I_2 \ell_2 \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I_1 I_2 \ell_2}{2\pi d} = F_{12}$$

➤ 考慮  $\ell_1 = \ell_2 = \ell$

$$\text{單位長度所受磁力} \Rightarrow \frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

結論

安培定義的基礎  $\Rightarrow \ell = 1\text{m}, d = 1\text{m}, I_1 = I_2 = 1\text{A} \Rightarrow F = 2 \times 10^{-7} \text{N}$   
 若  $I_1$  與  $I_2$  同向，則兩導線相吸；若  $I_1$  與  $I_2$  反向，則兩導線相斥。

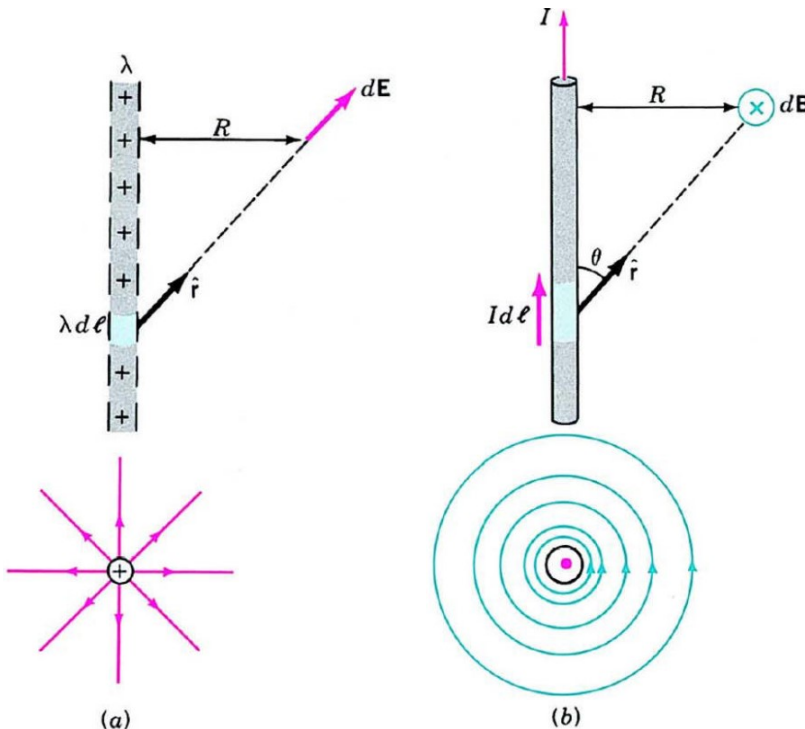
✦ 磁場計算  $\Rightarrow \begin{cases} \text{Biot-Savart Law (Current element) to Coulomb Law (point charge)} \\ \text{Ampere's Law (closed loop) to Gauss's Law (closed surface)} \end{cases}$

• **Biot-Savart Law (必歐沙伐定律)** — 類似庫倫定律

$$\text{無窮長直導線} \Rightarrow E = \frac{2k\lambda}{R} \xrightarrow{\text{對照}} B = \frac{2k'I}{R} \quad (k' = \mu_0 / 4\pi)$$

$$\text{推導應用方法} \Rightarrow d\vec{E} = k \frac{\lambda d\ell}{r^2} \hat{r} \xrightarrow{\text{對照}} d\vec{B} = k' \frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

$$(\because dq = \lambda d\ell)$$



$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

(向量表示式)

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{Id\ell \sin \theta}{r^2}$$

(純量式—求磁場大小)

Example 30.2 An infinite straight wire carries a current  $I$

$$|d\vec{\ell} \times \hat{r}| = d\ell \sin \theta = d\ell \sin(\pi - \theta) = d\ell \cos \alpha$$

$$\because \ell = R \tan \alpha \Rightarrow d\ell = R \sec^2 \alpha d\alpha ; r = R \sec \alpha$$

$$\begin{aligned} \therefore B &= \int dB = \frac{\mu_0 I}{4\pi} \int_{-\alpha_1}^{\alpha_2} \frac{|d\vec{\ell} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \int_{-\alpha_1}^{\alpha_2} \frac{d\ell \cos \alpha}{R^2 \sec^2 \alpha} \\ &= \frac{\mu_0 I}{4\pi} \int_{-\alpha_1}^{\alpha_2} \frac{(R \sec^2 \alpha d\alpha) \cos \alpha}{R^2 \sec^2 \alpha} \\ &= \frac{\mu_0 I}{4\pi R} \int_{-\alpha_1}^{\alpha_2} \cos \alpha d\alpha = \frac{\mu_0 I}{4\pi R} (\sin \alpha_1 + \sin \alpha_2) \end{aligned}$$

$$\text{For an infinite wire} \Rightarrow B = \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha$$

$$= \frac{\mu_0 I}{4\pi R} [\sin(\pi/2) + \sin(\pi/2)] = \frac{\mu_0 I}{2\pi R}$$

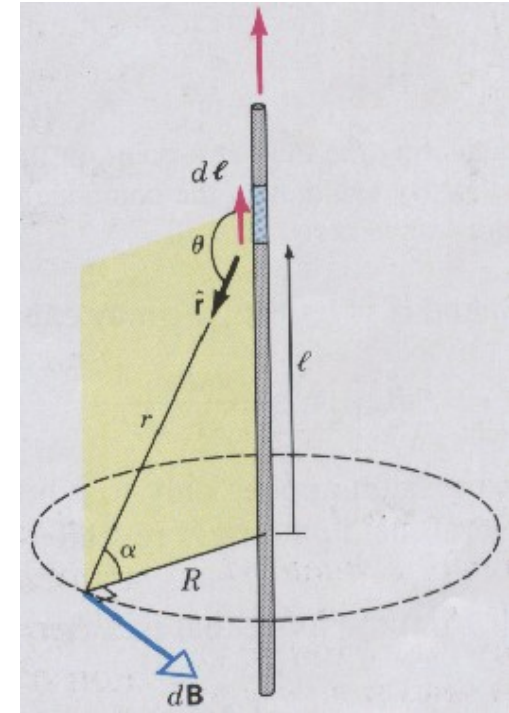


Fig.30.6

Example 30.3 A circular loop of radius  $a$  carries a current  $I$

$$\begin{cases} dB_{\perp} \text{ 分量會成對抵消} \\ dB_{\text{axis}} \text{ 分量可重疊累加} \end{cases}$$

$$\Rightarrow dB_{\text{axis}} = dB \sin \alpha = \left( \frac{\mu_0 I d\ell}{4\pi r^2} \right) \left( \frac{a}{r} \right)$$

$$\left( \because d\vec{\ell} \perp \hat{r}, \therefore dB = \frac{\mu_0 I}{4\pi} \frac{|d\vec{\ell} \times \hat{r}|}{r^2} = \frac{\mu_0 I d\ell}{4\pi r^2} \right)$$

$$\begin{aligned} B_{\text{axis}} &= \int dB_{\text{axis}} = \frac{\mu_0 I a}{4\pi r^3} \int_0^{2\pi a} d\ell = \frac{\mu_0 I a}{4\pi r^3} (2\pi a) \\ &= \frac{\mu_0 I a^2}{2r^3} = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \end{aligned}$$

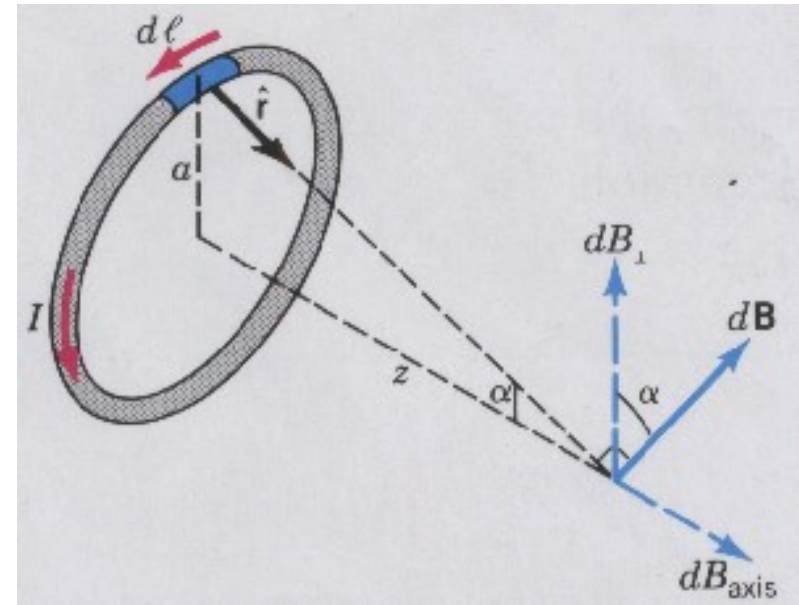


Fig.30.7

$$\text{For far field, that is, } z \gg a \Rightarrow B_{\text{axis}} = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \approx \frac{\mu_0 I a^2}{2z^3} = \frac{\mu_0 I \pi a^2}{2\pi z^3} = \frac{2k' \mu}{z^3}$$

(相當於dipole moment的磁場)

$$(\text{where } k' = \frac{\mu_0}{4\pi}, \mu = IA = I\pi a^2)$$



$$E = \frac{2kp}{z^3} \quad (\text{沿電偶極極軸的遠處電場}) \text{— 如圖(a)}$$

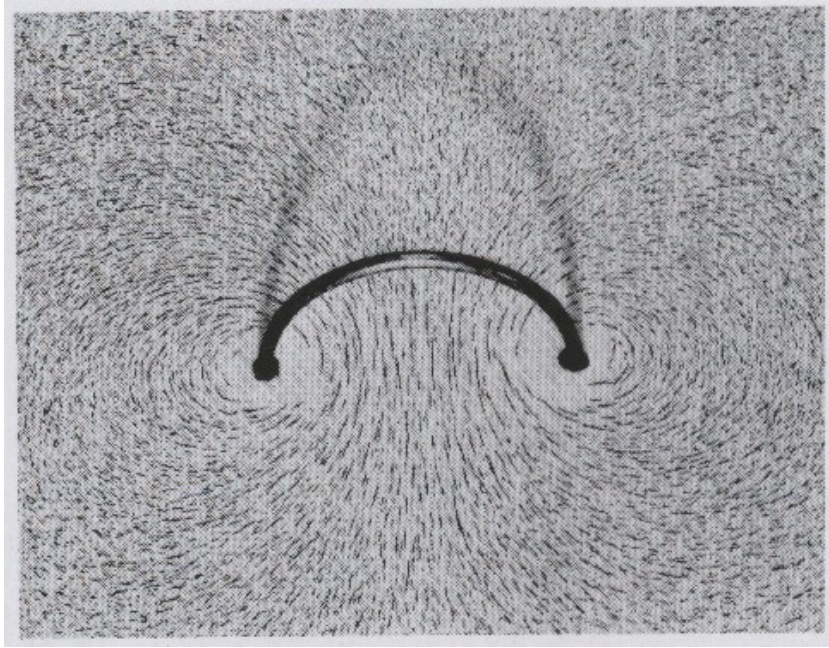


Fig.30.8

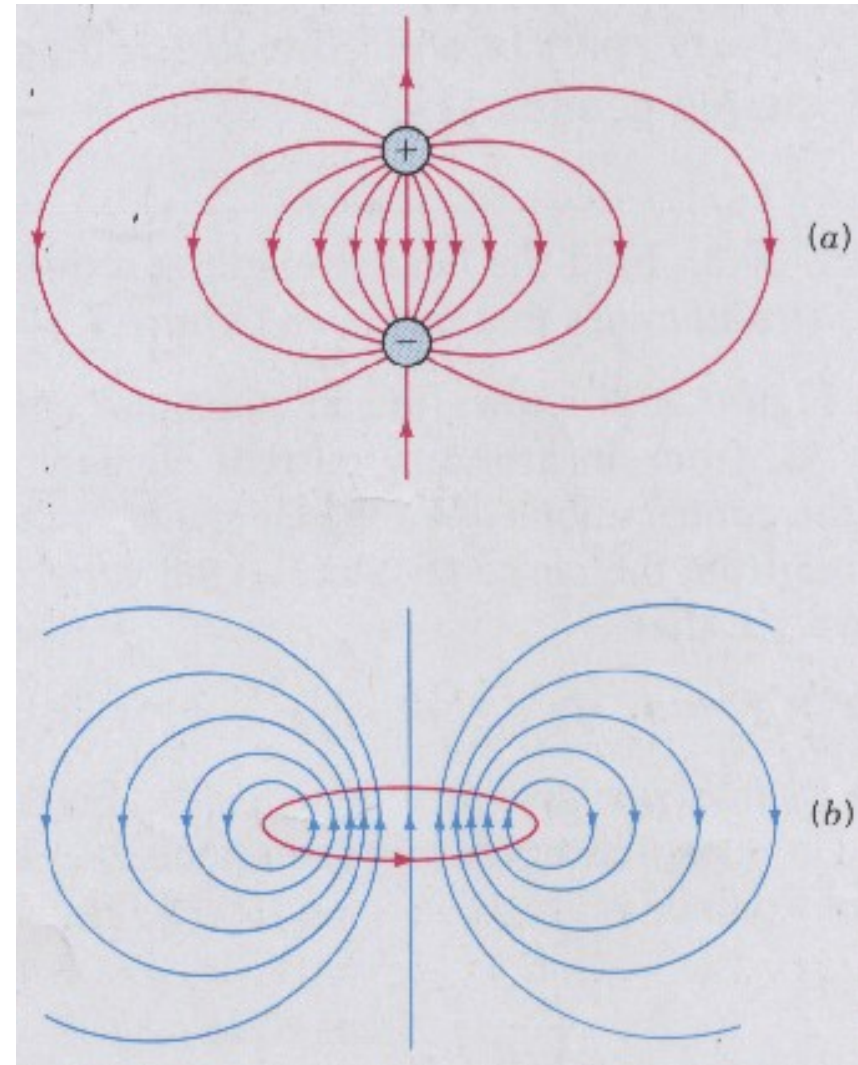


Fig.30.9

$$B = \frac{2k'\mu}{z^3} \quad (\text{沿環形線圈中央軸的遠處磁場}) \text{— 如圖(b)}$$

單一環形線圈  
(遠場近似dipole)



多匝環形線圈  
(線圈內部磁場最強  
且近似均勻，線圈  
外部磁場最弱)



螺旋線管(solenoid)  
(線圈匝數更多更密)  
—管內磁場均勻且最  
大，管外近似為零

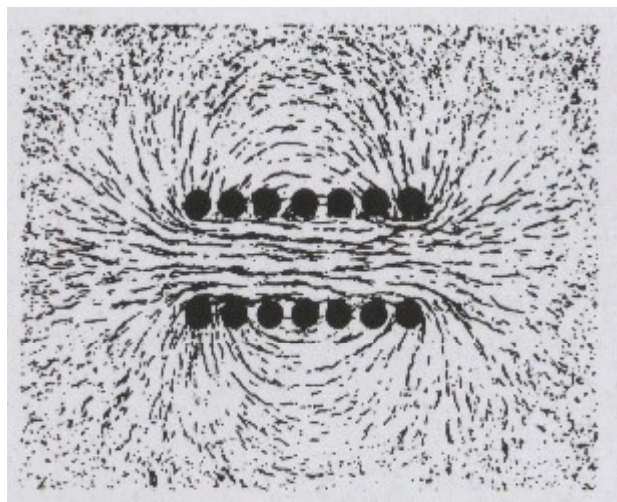


Fig.30.10

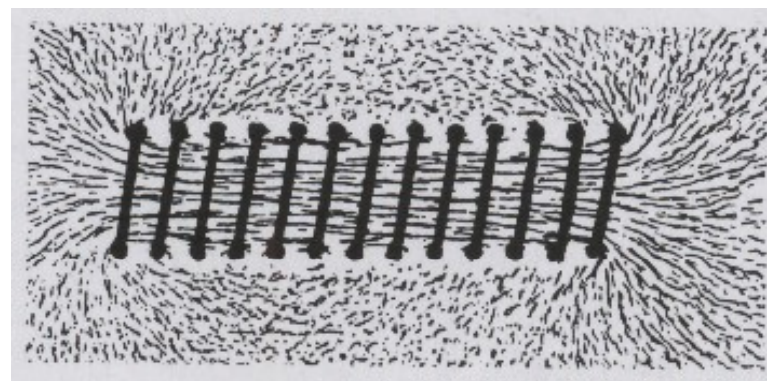
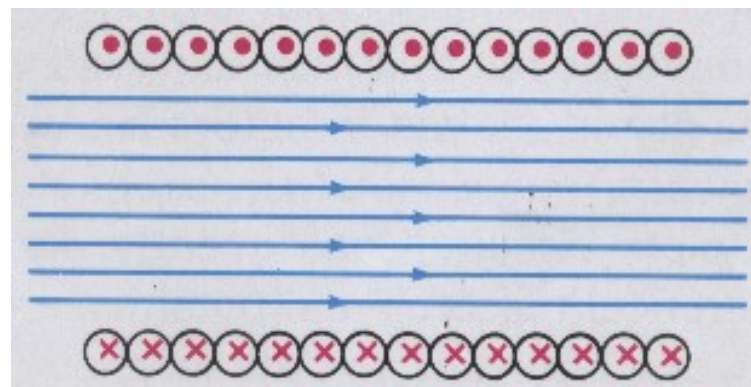
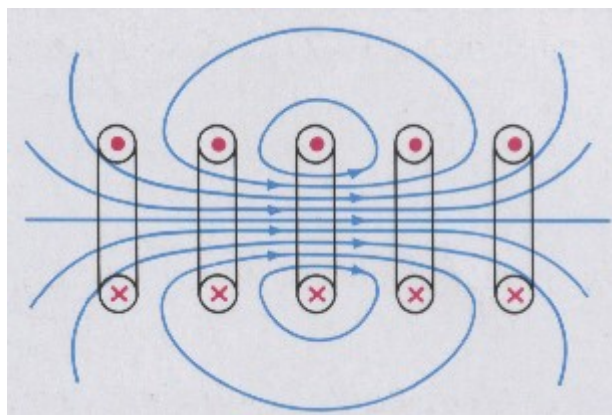


Fig.30.12





**Example 30.4** A solenoid of length  $l$  and radius  $a$  has  $N$  turns of wire and carries a current  $I$

$$z = a \tan \theta \Rightarrow dz = a \sec^2 \theta d\theta$$

The current of a loop  $\xrightarrow{n=N/\ell} nI dz = nI a \sec^2 \theta d\theta$

$$\text{From circular loop: } B_{\text{axis}} = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$$

$$\Rightarrow dB = \frac{\mu_0 (nI dz) a^2}{2(a^2 + z^2)^{3/2}} = \frac{\mu_0 nI a^3 \sec^2 \theta d\theta}{2(a^2 + a^2 \tan^2 \theta)^{3/2}}$$

$$= \frac{\mu_0 nI a^3 \sec^2 \theta d\theta}{2a^3 \sec^3 \theta} = \frac{1}{2} \mu_0 nI \cos \theta d\theta$$

$$B = \frac{1}{2} \mu_0 nI \int_{\theta_1}^{\theta_2} \cos \theta d\theta = \frac{1}{2} \mu_0 nI (\sin \theta_2 - \sin \theta_1)$$

For a infinite solenoid  $\xrightarrow{\theta_1 = -\pi/2, \theta_2 = \pi/2} B = \mu_0 nI$

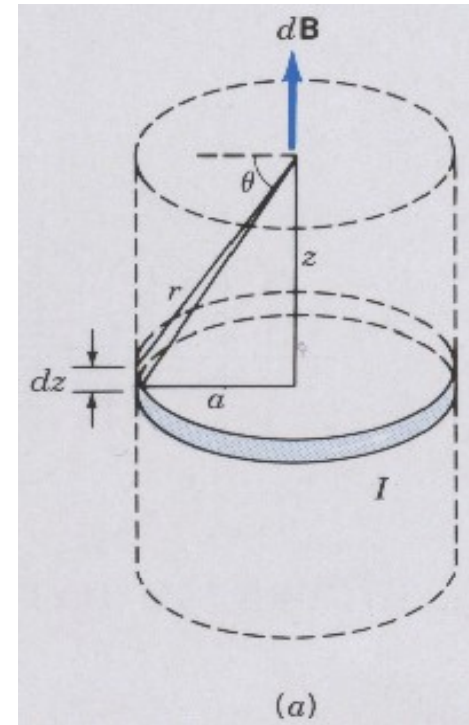
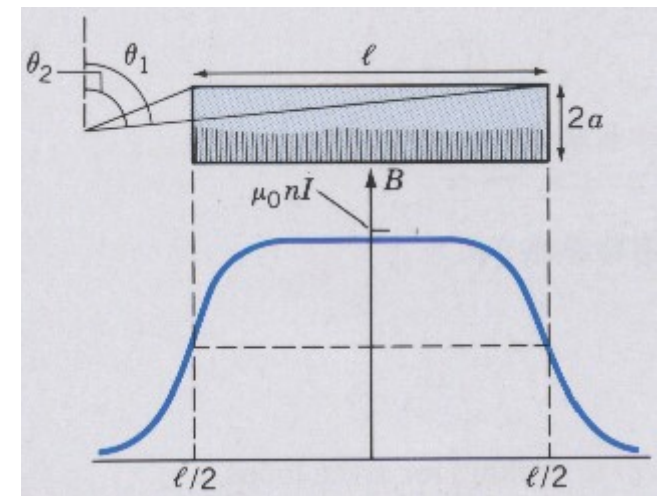


Fig.30.13



## ●Ampere's Law (安培定律) — 類似高斯定律

主要應用於無窮長直導線  $\Rightarrow B = \frac{\mu_0 I}{2\pi R} \Rightarrow B(2\pi r) = \mu_0 I$  (考慮圓形路徑)  
(磁力線為同心圓)

$$\boxed{\text{安培定律}} \Rightarrow \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

(考慮任意封閉路徑)

注意

I 為流過封閉路徑包圍的截面淨電流。

I 需穩流且為非磁化物質才成立。

I 可為帶電粒子束(a beam of charged particles)。

B並非封閉路徑內的電流貢獻，而是所有鄰近電流的貢獻。

封閉路徑(或積分路徑)的選取需考慮磁場的分佈及電流流動的幾何形狀。

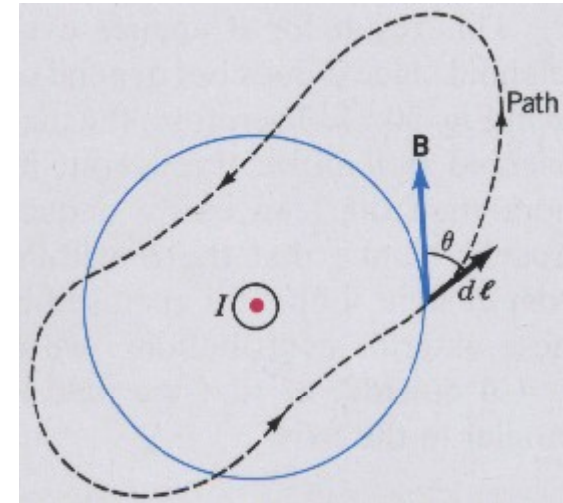


Fig.30.15

Example 30.5 An infinite straight wire of radius  $R$  carries a current  $I$

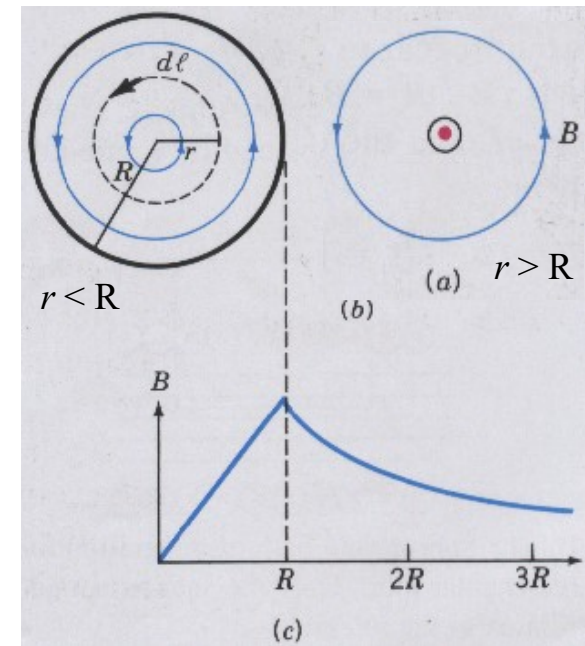
$$\text{For } r > R \Rightarrow \oint \vec{B} \cdot d\vec{\ell} = B \oint d\ell = \mu_0 I$$

$$\Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\text{For } r < R \Rightarrow B(2\pi r) = \mu_0 \frac{\pi r^2}{\pi R^2} I$$

$$\Rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

Fig.30.16



Example 30.6 An ideal infinite solenoid carries a current  $I$

$$\oint \vec{B} \cdot d\vec{\ell} = \int_a^b \vec{B} \cdot d\vec{\ell} + \int_b^c \vec{B} \cdot d\vec{\ell} + \int_c^d \vec{B} \cdot d\vec{\ell} + \int_d^a \vec{B} \cdot d\vec{\ell}$$

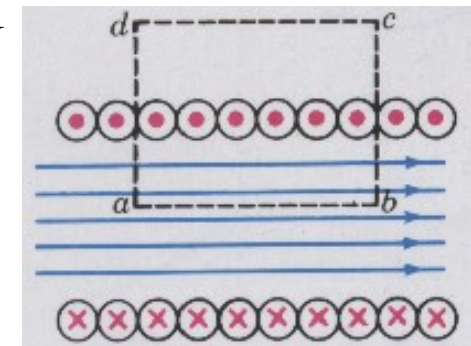
( $\because$  管外  $\vec{B} = 0$ )

( $\because \vec{B} \perp d\vec{\ell} \Rightarrow \int \vec{B} \cdot d\vec{\ell} = 0$ )

(note:  $n = N/L$ )

Fig.30.17

$$\oint \vec{B} \cdot d\vec{\ell} = \int_a^b \vec{B} \cdot d\vec{\ell} = B \int_0^L d\ell = \mu_0 n L I \Rightarrow B L = \mu_0 n L I \Rightarrow B = \mu_0 n I$$





Example 30.7 An toroidal coil (with  $N$  turns) carries a current  $I$

$$\oint \vec{B} \cdot d\vec{\ell} = B \oint d\ell = \mu_0 NI$$

$$\Rightarrow B(2\pi r) = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

注意  $\Rightarrow \begin{cases} \text{管內磁場為非均勻。} \\ \text{管外磁場為零。} \end{cases}$

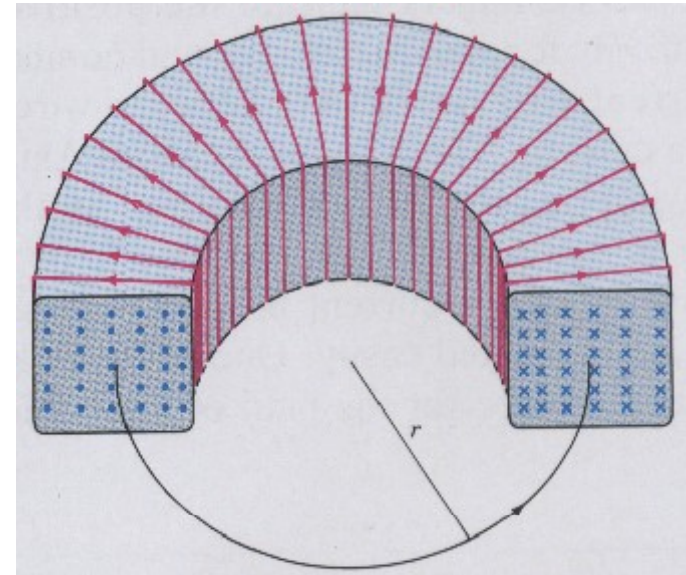
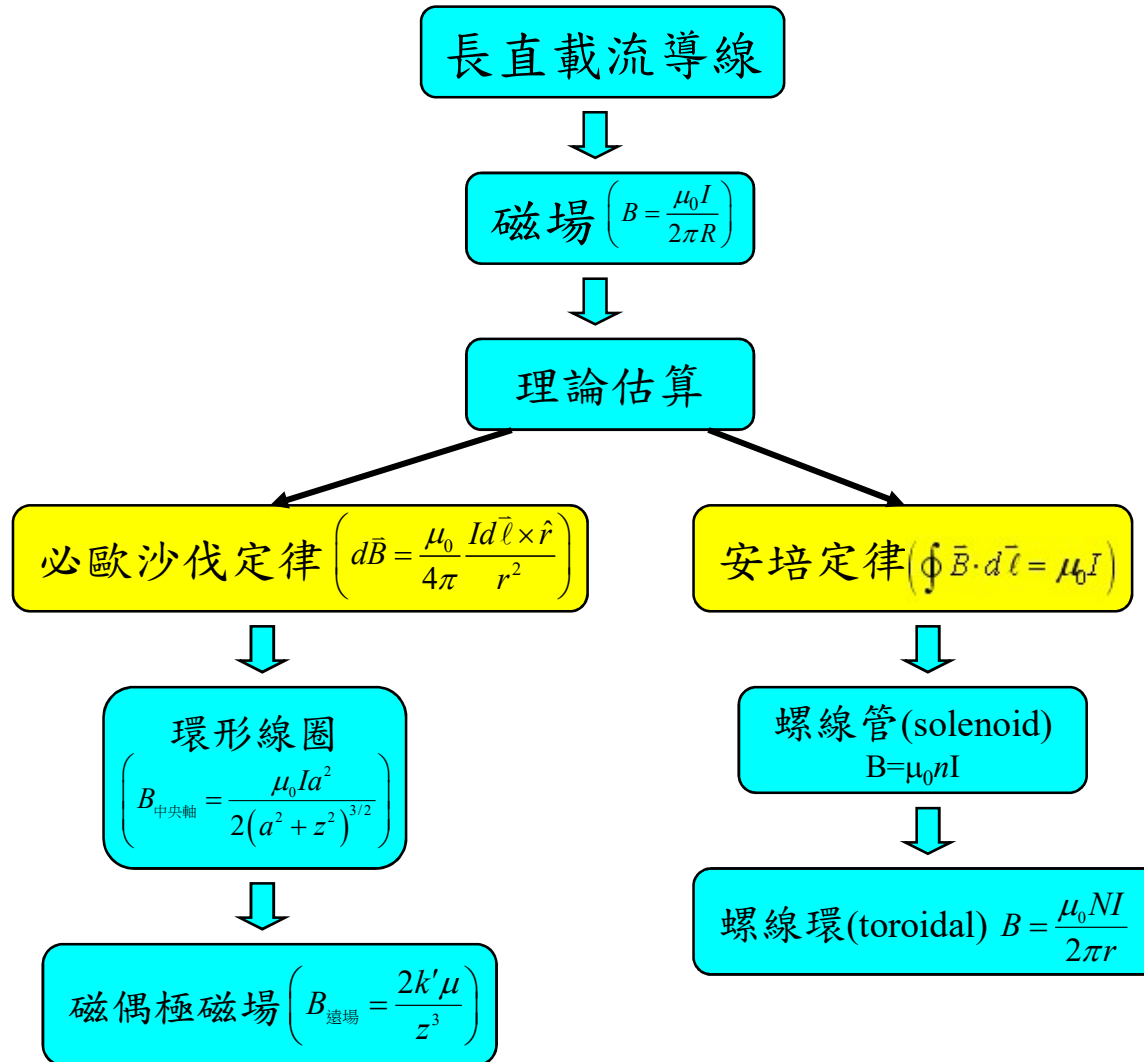


Fig.30.18



An unusual magnet at Lawrence Livermore Labs which is designed as part of a scheme to confine a hot plasma (ionized gas) in experiments for harnessing the energy released by the fusion of nuclei.

## 本章單元重要觀念發展脈絡彙整



## 習題

- 教科書習題 (p.618~p.623)

Exercise: 1,11,13,17,23,25,29,35,37,39,49

- 基本觀念問題：

- 1.請說明必歐沙伐定律與安培定律。