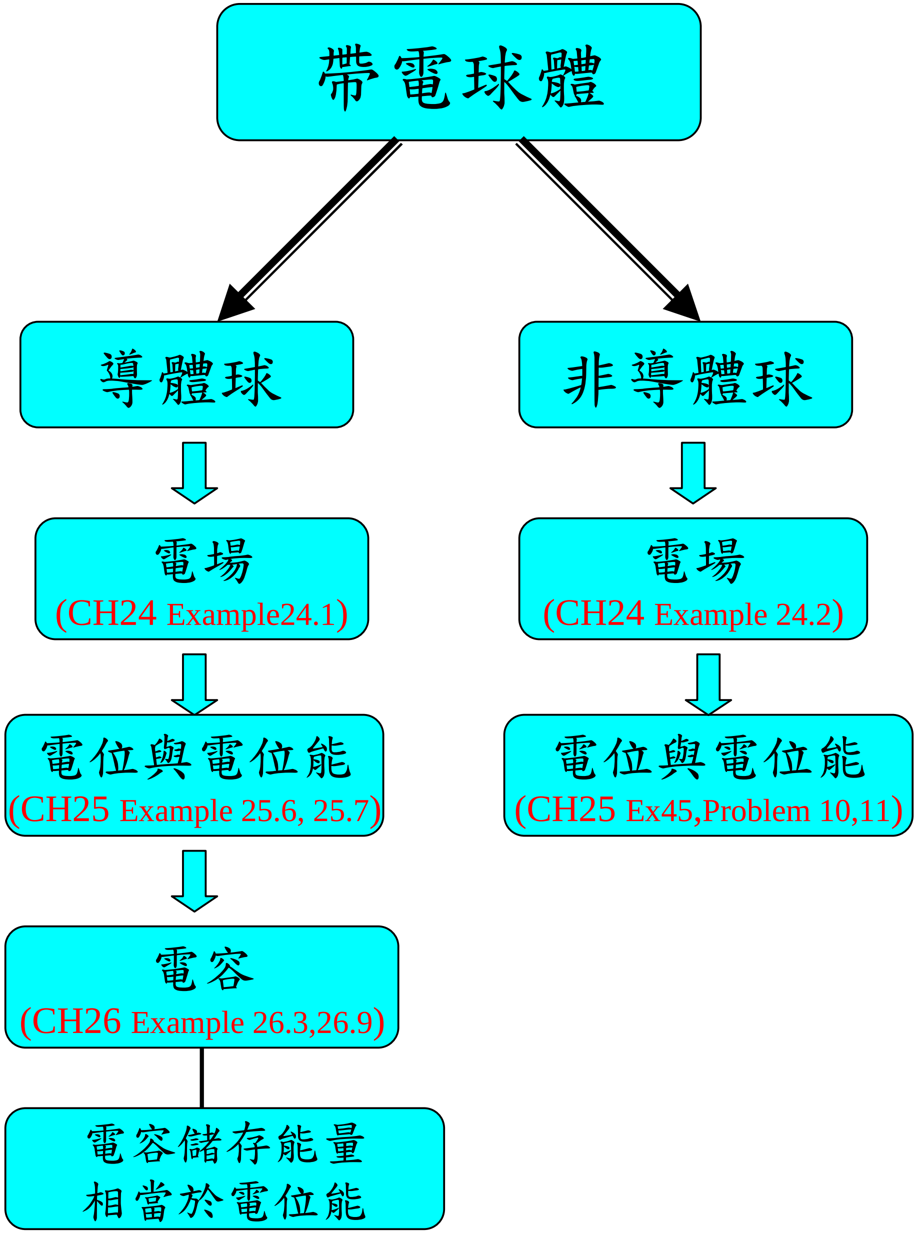
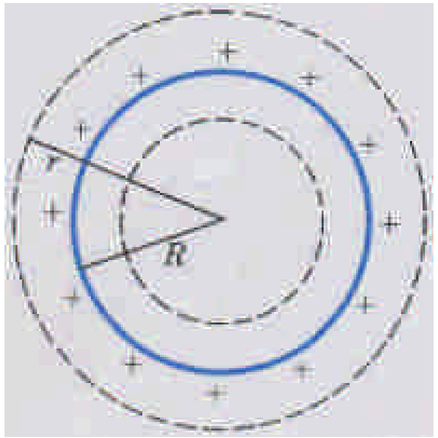

帶電球體
導體球
非導體球
電場
(CH24 Example24.1)
電場
(CH24 Example 24.2)
電位與電位能
(CH25 Example 25.6, 25.7)
電位與電位能
(CH25 Ex45,Problem 10,11)
電容
(CH26 Example 26.3,26.9)
電容儲存能量
相當於電位能

電 場

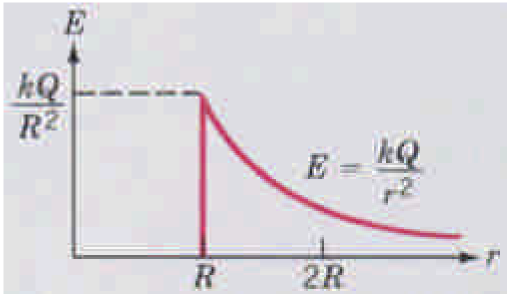
Example 24.1 電荷均勻分佈球殼表面(導體球)



球殼外部 ($r>R$)：

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E dA = E\oint dA = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

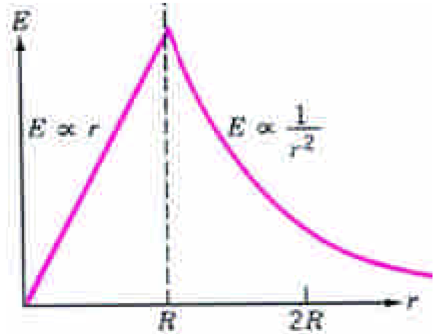
$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$



球殼內部 ($r<R$)：

$$\Phi_E = E(4\pi r^2) = 0 \qquad \Rightarrow E = 0$$

Example 24.2 電荷均勻分佈於整個球體(非導體球)



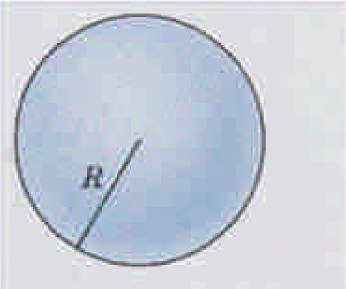
球體外部 ($r>R$) $\quad \Phi_E = E(4\pi r^2) = \dfrac{Q}{\varepsilon_0} \Rightarrow E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{kQ}{r^2}$

球體內部 ($r<R$) $\quad \Phi_E = E(4\pi r^2) = \dfrac{(r^3/R^3)Q}{\varepsilon_0} \Rightarrow E = \dfrac{kQr}{R^3}$

$$\left(\because \frac{4}{3}\pi r^3 \cdot \frac{Q}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3}Q\right)$$

電位與電位能

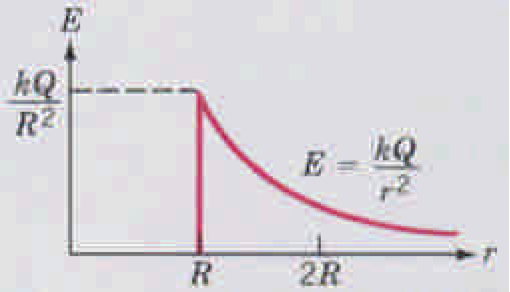
Example 25.6：（導體球電位）



When $r > R \Rightarrow E = \dfrac{kQ}{r^2}$ ； $V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$

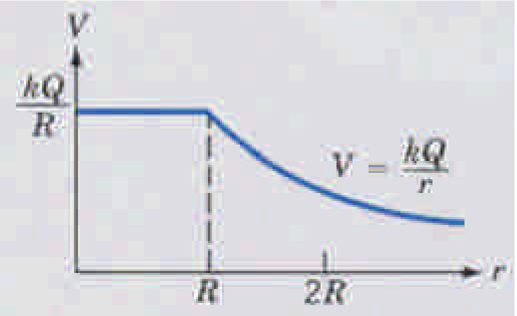
$$= -\int_{\infty}^{r} \left(\frac{kQ}{r^2}\hat{r}\right) \cdot d\vec{r} = -\int_{\infty}^{r} \frac{kQ}{r^2} dr = -kQ\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{kQ}{r}$$

$$\because V(\infty) = 0 \Rightarrow V(r) = \frac{kQ}{r}$$



When $r < R \Rightarrow E = 0$ ； $V(r) - V(R) = 0$

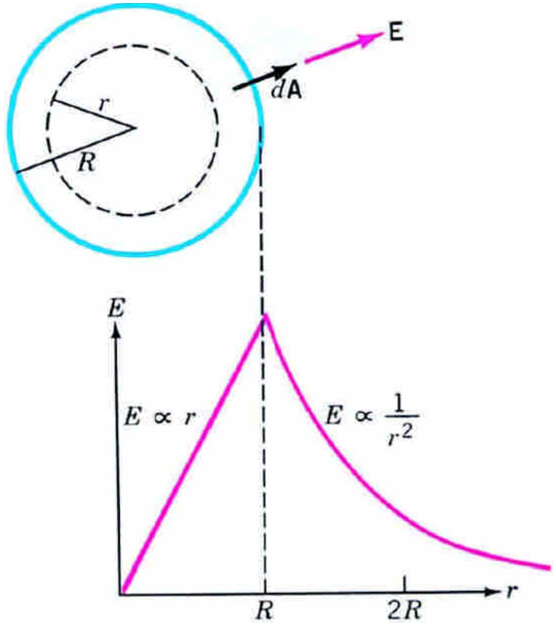
$$\because V(R) = \frac{kQ}{R} \Rightarrow V(r) = \frac{kQ}{R}$$



Example 25.7：（導體球電位能）

導體球的電位能$\Rightarrow$ $dW = Vdq = \left(\dfrac{kq}{R}\right)dq \Rightarrow W = \int_0^Q \dfrac{kq}{R} dq = \dfrac{kQ^2}{2R} = \dfrac{1}{2}QV$

●非導體球電位 (習題 Ex.45 或 Problem 10)



When $r > R \Rightarrow E = \dfrac{kQ}{r^2}$; $V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$

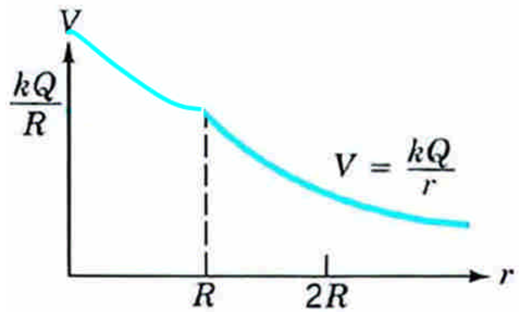
$$= -\int_{\infty}^{r} \left(\frac{kQ}{r^2}\hat{r}\right) \cdot d\vec{r} = -\int_{\infty}^{r} \frac{kQ}{r^2} dr = -kQ\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{kQ}{r}$$

$$\because V(\infty) = 0 \Rightarrow V(r) = \frac{kQ}{r}$$

When $r < R \Rightarrow E = \dfrac{kQr}{R^3}$; $V(r) - V(R) = -\int_{R}^{r} \vec{E} \cdot d\vec{r}$

$$= -\int_{R}^{r} \left(\frac{kQr}{R^3}\hat{r}\right) \cdot d\vec{r} = -\int_{R}^{r} \frac{kQr}{R^3} dr = -\frac{kQ}{R^3}\int_{R}^{r} r dr$$

$$= -\frac{kQ}{R^3}\left[\frac{r^2}{2}\right]_{R}^{r} = -\frac{kQ}{R^3}\left[\frac{r^2 - R^2}{2}\right] = \frac{kQ(R^2 - r^2)}{2R^3}$$



$$\Rightarrow V(r) - V(R) = V(r) - \frac{kQ}{R} = \frac{kQ(R^2 - r^2)}{2R^3} \Rightarrow V(r) = \frac{kQ(R^2 - r^2)}{2R^3} + \frac{kQ}{R}$$

$$\Rightarrow V(r) = \frac{kQ(3R^2 - r^2)}{2R^3}$$

●非導體球的電位能 (Problem 11)

$$dU = Vdq$$

$$q(r) = \frac{4\pi r^3}{3}\rho \quad = \frac{4\pi r^3}{3}\left(\frac{Q}{4\pi R^3/3}\right) = \frac{Qr^3}{R^3} \qquad (\text{其中 } \rho = \frac{Q}{4\pi R^3/3})$$

$$V(r) = \frac{kq(r)}{r} = \frac{k\left(\dfrac{Qr^3}{R^3}\right)}{r} = \frac{kQr^2}{R^3} \quad ; \quad dq = \frac{3Qr^2}{R^3}dr$$

$$dU = Vdq = \frac{kQr^2}{R^3}\cdot\frac{3Qr^2}{R^3}dr = \frac{3kQ^2r^4}{R^6}dr$$

$$U = \int Vdq = \frac{3kQ^2}{R^6}\int_0^R r^4 dr = \frac{3kQ^2}{R^6}\left(\frac{r^5}{5}\right)_0^R = \frac{3kQ^2}{5R^5}$$