電位 (Electric Potential)

- 電位與位能有密切關係，電位即每單位電荷的位能。
- 電位為純量，運算不需考慮向量問題。

$$W_{EXT} = +\Delta U = U_f - U_i = -W_c \quad (\text{考慮} v = \text{constant} \Rightarrow W_{net} = \Delta K = 0 = W_{EXT} + W_c)$$

(位能變化)

- 電位(electric potential)之定義：（電位與電荷大小無關，僅與位置有關）

$$\Delta V = \frac{\Delta U}{q} \Rightarrow \Delta V = \frac{W_{EXT}}{q} = -\frac{W_c}{q} = -\frac{\vec{F}_c \cdot \Delta\vec{s}}{q} \quad (\text{SI單位} \Rightarrow 1\ \text{V} = 1\ \text{J/C})$$

(電位差或電壓)

$$\text{考慮微量變化} \Rightarrow dV = \frac{dU}{q} = -\frac{W_c}{q} = -\frac{\vec{F}_c \cdot d\vec{s}}{q}$$

$$W_c\text{以靜電力作功表示} \Rightarrow dV = -\frac{q\vec{E} \cdot d\vec{s}}{q} = -\vec{E} \cdot d\vec{s}$$

$$\Rightarrow \Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

若要進一步定義電位，則必須設定零位面！

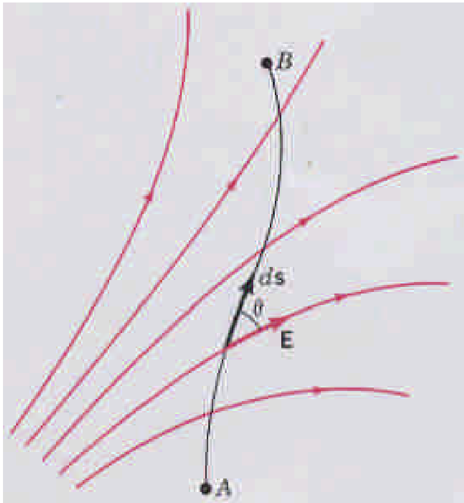


Fig.25.2

重力場與電場之比較 (考慮均勻場情況)

$$\Delta U = U_f - U_i = W_{EXT} = -W_c = -(-m\vec{g}\cdot\vec{s})\text{ 或 }-(-q\vec{E}\cdot\vec{s})$$

重力位能： $U_f = U_g = mgy$ ⇒重力位(U_g/m)： $V_g = gy$
(將地面設為零位面，即$U_i = 0$； s = y)

電位能⇒ $U_f = U_E = qEy$ ⇒電位(U_E/q) ：$V_E = Ey$
(將–板設為零位面，即$U_i = 0$； s=y)

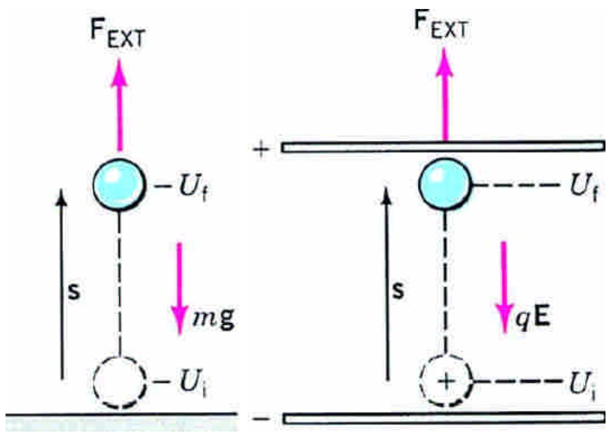


Fig.25.2

•均勻電場(uniform field)

$$\Delta V = -\int \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\int d\vec{s} = -\vec{E}\cdot\Delta\vec{s}$$

⇨ 電荷沿等位線移動不會造成電位差。
電位沿電場方向呈線性遞減。

如 Fig. 25.3 ⇒ $\Delta V = \pm Ed$ (其中正電荷為–，負電荷為+)

電場E的等效單位 ⇒ 1 V/m = 1 N/C

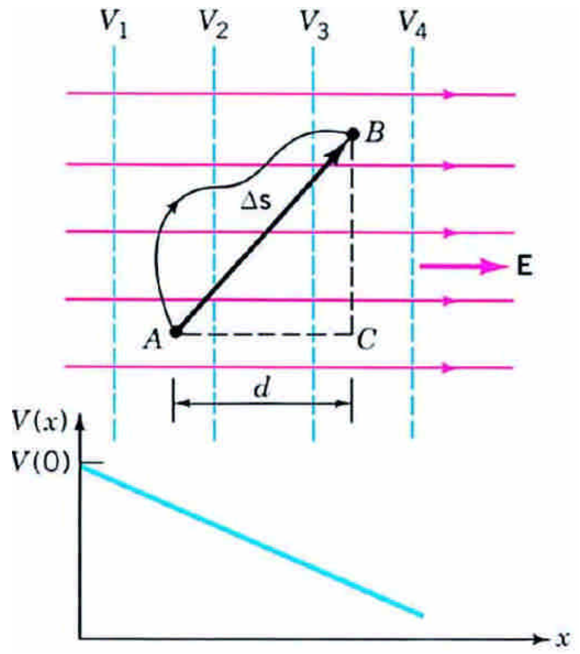


Fig.25.3

●等位線或等位面(equipotential)

$\Rightarrow \begin{cases} \text{1.電場線必垂直於等位線。} \\ \text{2.沿等位線移動的電荷不作功。} \end{cases}$

●帶電粒子運動的能量

$$\Delta K + \Delta U = 0 \;\Rightarrow \Delta K = -\Delta U = -q\Delta V$$

$$\Rightarrow \Delta K = e\Delta V \quad (1\ eV = 1.602\times 10^{-19}\ J)$$

●點電荷的電位(potential of point charge)

$$\vec{E} = E_r \hat{r} = \frac{kQ}{r^2}\hat{r} \quad \text{（非均勻電場）}$$

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_A^B E_r dr = -\int_A^B \frac{kQ}{r^2} dr$$

$$= -\left[-\frac{kQ}{r}\right]_A^B = kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

考慮$V_B = 0$ at $r \to \infty$， 則：$V = \dfrac{kQ}{r}$

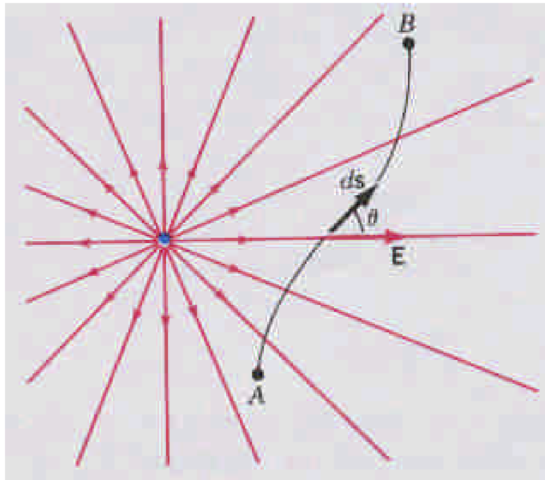


Fig.25.7

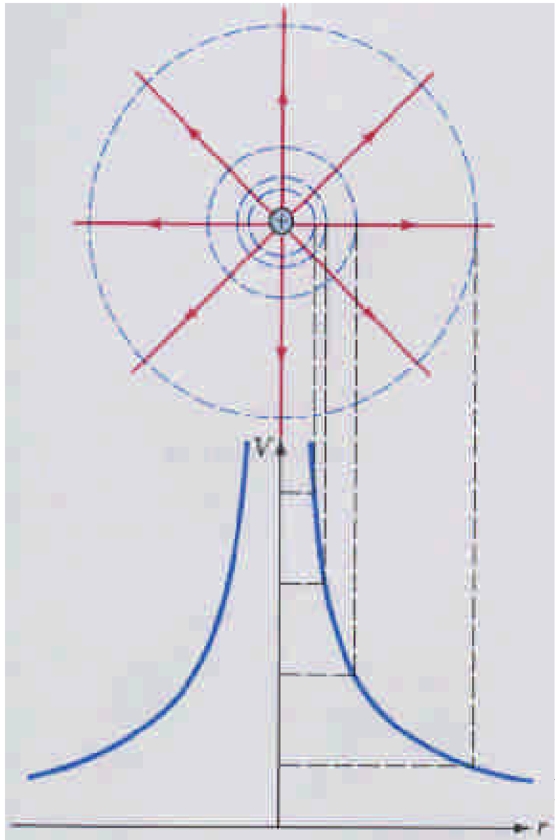


Fig.25.8

- 多個點電荷的電位
  - 一 利用疊加原理(電位為純量，可直接進行加減) $\Rightarrow V = \sum \frac{kQ_i}{r_i}$
    - ➢Case (A) $\Rightarrow$V= 0 (如Fig.25.9)，但E$\neq$ 0 (如Fig.25.10)

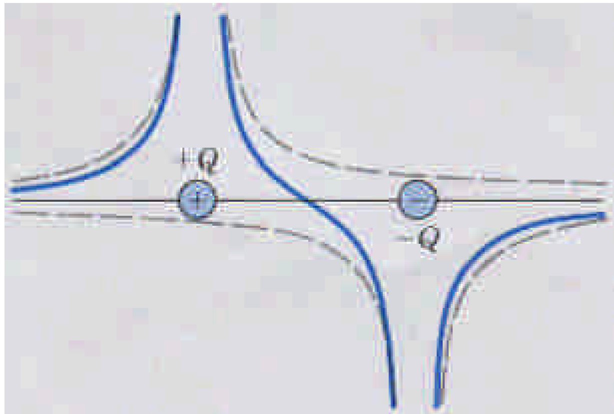


Fig.25.9

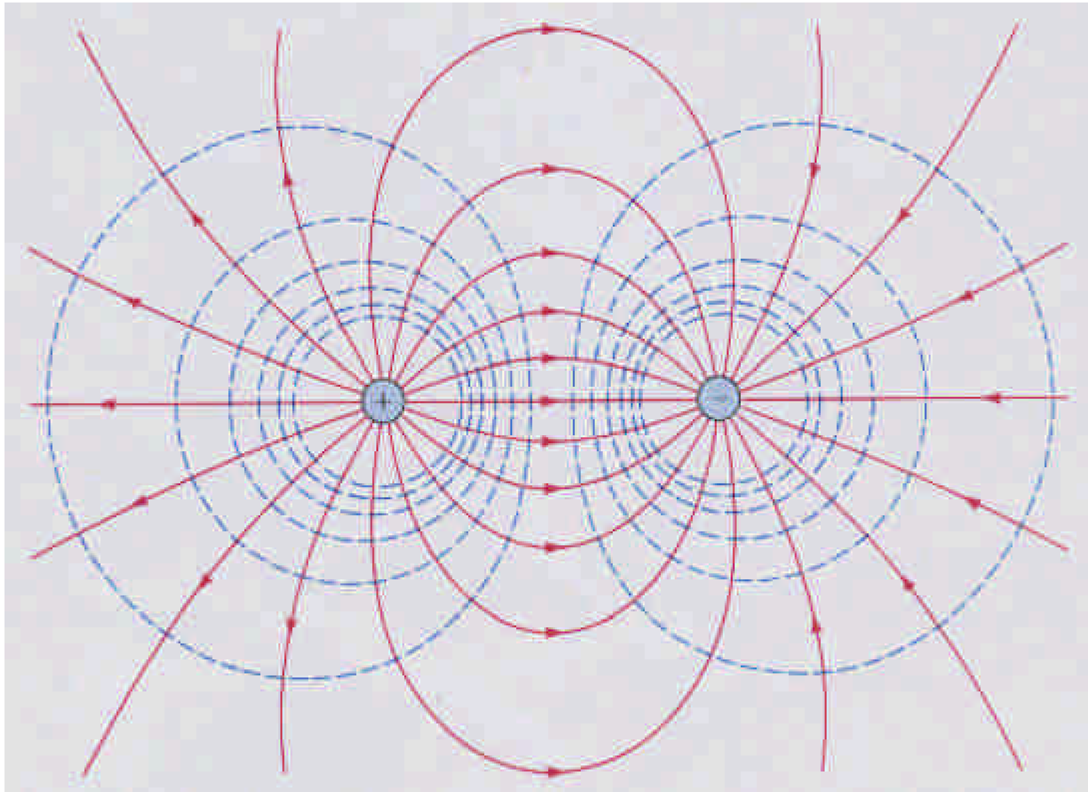


Fig.25.10

➢Case (B) $\Rightarrow$E= 0 (如Fig.25.11)，但V$\neq$ 0 (如Fig.25.12)

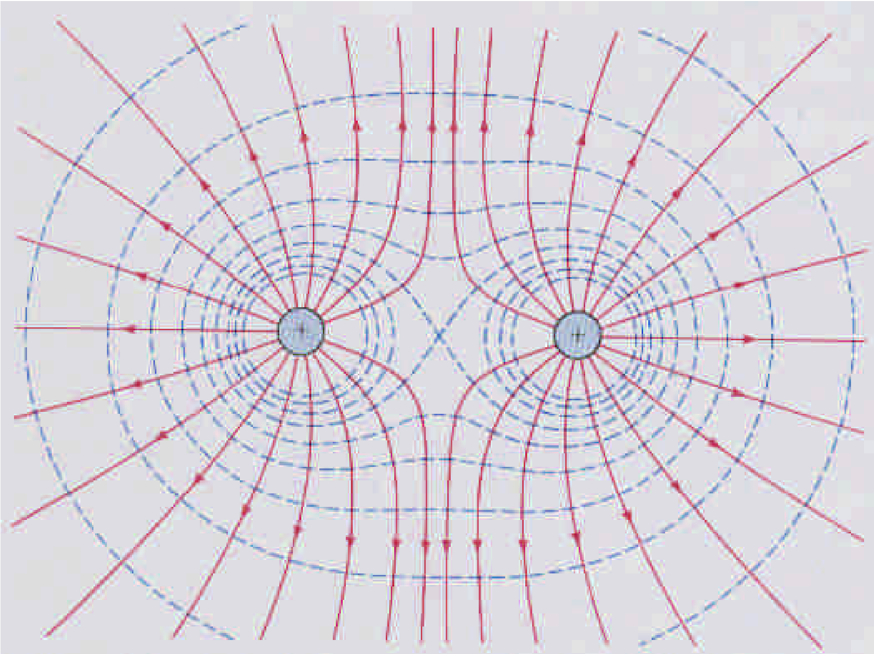


Fig.25.11

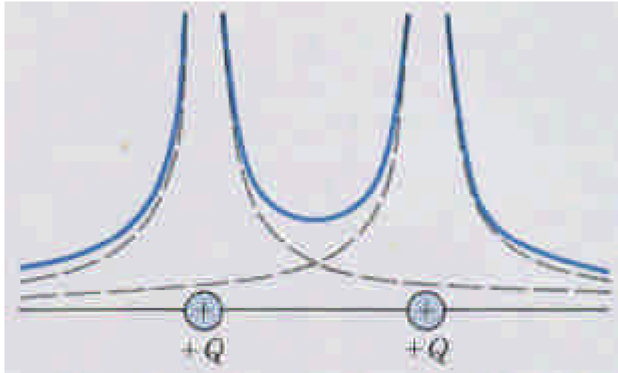


Fig.25.12

●兩點電荷的電位能

$$\Rightarrow U = qV = q\left(\frac{kQ}{r}\right) = \frac{kqQ}{r}\begin{cases}\text{若極性相同，則}U>0\text{，表外力作正功(即保守力作負功)}\\ \text{若極性相異，則}U<0\text{，表外力作負功(即保守力作正功)}\end{cases}$$

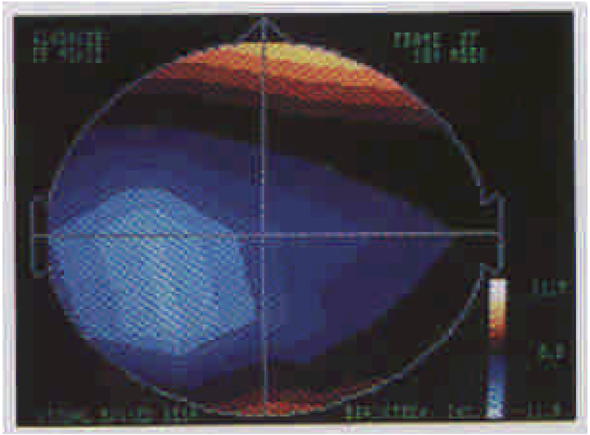
(相當於將兩電荷自無窮遠處等速移至相距 r 處外力所作的功)

$$\Delta U = U(r) - U(\infty) = q\Delta V = q\left(-\int_{\infty}^{r}\vec{E}\cdot d\vec{r}\right) = -q\int_{\infty}^{r}\frac{kQ}{r^2}dr = \frac{kqQ}{r} \quad \Rightarrow U(r) = \frac{kqQ}{r} \quad (\because \mathrm{U}(\infty)=0)$$

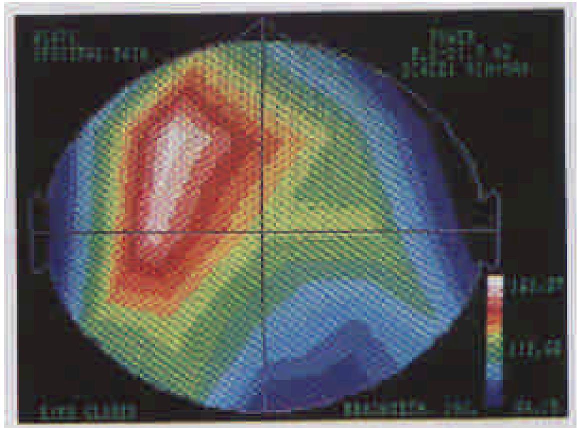
● 多個點電荷的電位能

$$U_{ij} = \frac{kq_i q_j}{r_{ij}}, \quad \text{but } i \neq j \qquad (※計算總電位能須注意 i\neq j)$$

➢腦部經閃光刺激後，可激發其電位的分佈

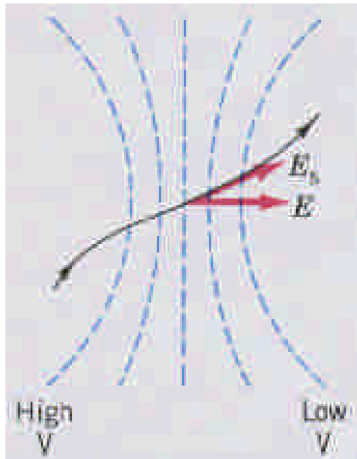


(tumor)



(Epilepsy)

Fig.25.14



● 電位推導電場

$$dV = -\vec{E}\cdot d\vec{s} = -Eds\cos\theta = -E\cos\theta ds = -E_s ds$$

$$\Rightarrow E_s = -\frac{dV}{ds}$$

考慮三維直角座標系$\Rightarrow$ $dV=-\vec{E}\cdot d\vec{s}=-(E_x\hat{i}+E_y\hat{j}+\vec{E}_z\hat{k})\cdot(dx\hat{i}+dy\hat{j}+dz\hat{k})$

$$=-(E_x dx+E_y dy+E_z dz)$$

其中 $E_x=-\left(\frac{dV}{dx}\right)_{y,z=const.}=-\left(\frac{\partial V}{\partial x}\right)\xrightarrow{\text{同理可知}}\vec{E}=-\left(\frac{\partial V}{\partial x}\right)\hat{i}-\left(\frac{\partial V}{\partial y}\right)\hat{j}-\left(\frac{\partial V}{\partial z}\right)\hat{k}$

●連續電荷分佈(continuous charge distribution)

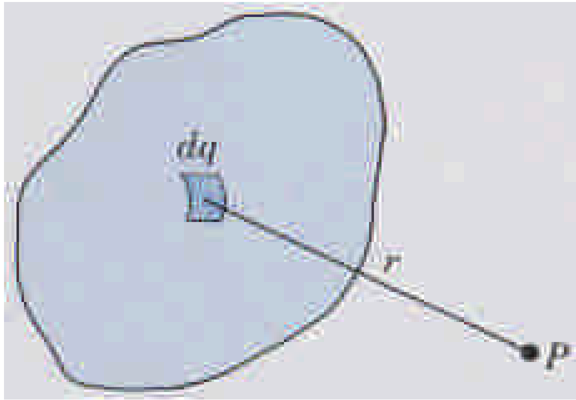


Fig.25.15

電位計算方式 $\Rightarrow$

(1) $dV=\frac{kdq}{r}\Rightarrow V=k\int\frac{dq}{r}$

（零電位假設在無窮遠）

(2) $V_B-V_A=-\int_A^B\vec{E}\cdot d\vec{s}$

(零電位可任意假設，但電場須藉由Gauss’s Law求出)

Example 25.5：

單位環形面積所帶電荷$\Longrightarrow$ $dq = \sigma(2\pi x dx)$

$$dV = \frac{kdq}{r} = \frac{k\sigma(2\pi x dx)}{(x^2+y^2)^{1/2}}$$

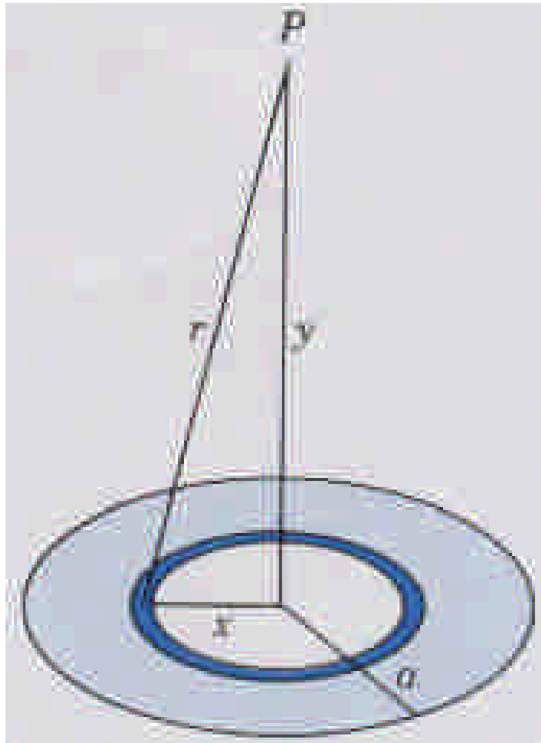


Fig.25.16

$$V = 2\pi k\sigma \int_0^a \frac{xdx}{(x^2+y^2)^{1/2}} = 2\pi k\sigma \int_0^a \frac{(1/2)dx^2}{(x^2+y^2)^{1/2}}$$

$$= 2\pi k\sigma \left[\left(x^2+y^2 \right)^{1/2} \right]_0^a$$

$$= 2\pi k\sigma \left[\left(a^2+y^2 \right)^{1/2} - y \right]$$

$(\because$ 令$H = x^2+y^2$ & $dH = dx^2 \Rightarrow \int (\frac{1}{2}) H^{-1/2} dH = H^{1/2} + c)$

討論:

1.有限圓盤在無窮遠處可近似點電荷，即 $y >> a \to V \approx \dfrac{kQ}{y}$

$$V = 2\pi k\sigma\left[\left(a^2+y^2\right)^{1/2}-y\right] = 2\pi k\sigma\left[y+\frac{a^2}{2y}-y\right] = \pi k\sigma\cdot\frac{a^2}{y} = \frac{k\sigma\pi a^2}{y} = \frac{kQ}{y}$$

$(\because \sigma\pi a^2 = \mathrm{Q})$

其中 $(a^2+y^2)^{1/2} = y\left(1+\dfrac{a^2}{y^2}\right)^{1/2} \approx y\left(1+\dfrac{a^2}{2y^2}+\cdots\right)$ (二項式展開)

$(\because y >> a \Rightarrow a/y << 1)$

2.推導電場(習題 Ex.47)

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left\{2\pi k\sigma\left[\left(a^2+y^2\right)^{1/2}-y\right]\right\} = -2\pi k\sigma\frac{d}{dy}\left[\left(a^2+y^2\right)^{1/2}-y\right]$$

$$= -2\pi k\sigma\left[y\left(a^2+y^2\right)^{-1/2}-1\right] = 2\pi k\sigma\left[1-y\left(a^2+y^2\right)^{-1/2}\right]$$

(同Example 23.8結果)

Example 25.6：

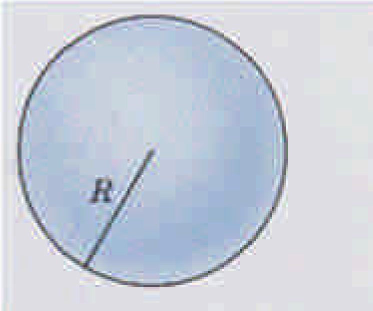
When $r > R \Rightarrow E = \dfrac{kQ}{r^2}$ ； $V(r) - V(\infty) = -\int_\infty^r \vec{E} \cdot d\vec{r}$

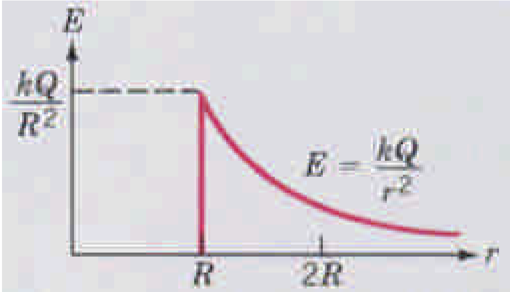
$$= -\int_\infty^r (\frac{kQ}{r^2}\hat{r}) \cdot d\vec{r} = -\int_\infty^r \frac{kQ}{r^2} dr = -kQ\left[-\frac{1}{r}\right]_\infty^r = \frac{kQ}{r}$$

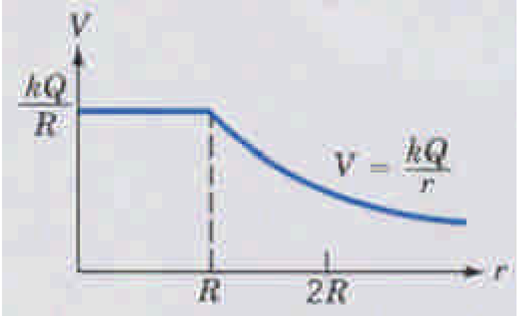
$$\because V(\infty) = 0 \Rightarrow V(r) = \frac{kQ}{r}$$

When $r < R \Rightarrow E = 0$ ； $V(r) - V(R) = 0$

$$\because V(R) = \frac{kQ}{R} \Rightarrow V(r) = \frac{kQ}{R}$$







Example 25.7：

導體球的電位能$\Rightarrow dW = Vdq = (\dfrac{kq}{R})dq \Rightarrow W = \int_0^Q \dfrac{kq}{R} dq = \dfrac{kQ^2}{2R} = \dfrac{1}{2}QV$

(考慮外力將dq自無窮遠移至導體球上所作的功)

Disscussion： *U=QV* 表單一電荷 ； *U=1/2 QV* 表系統電荷

導體(conductors)

- 在靜電平衡狀態下，導體內部與表面的電位皆相同。
  - 導體內部空腔之電場為0，即所謂的屏蔽效應。

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} \quad ; \qquad If \quad V_B = V_A \Rightarrow \vec{E} = 0$$

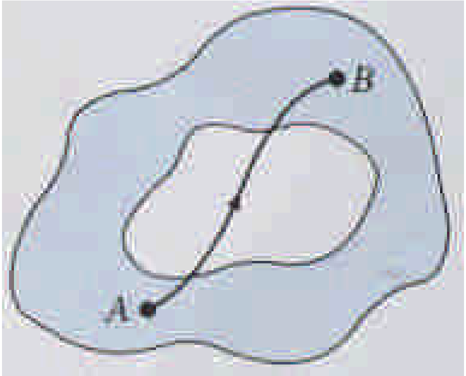


Fig.25.18

  - 在均勻電場中的電中性導體，接近球面等位面為圓形，電力線呈輻射狀。

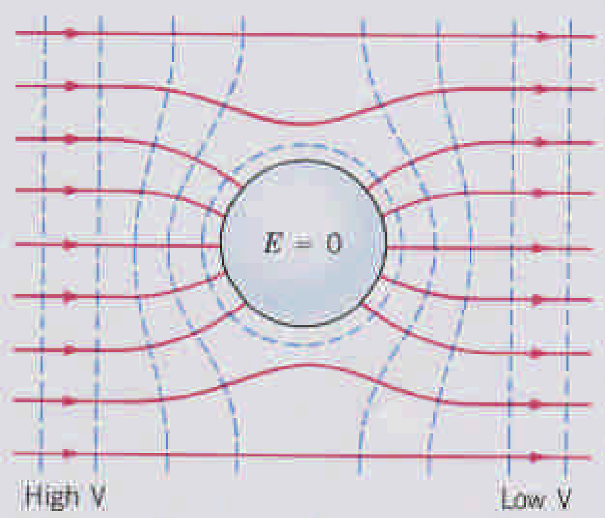


Fig.25.19

- 導體曲率半徑愈小，則電荷密度愈大。
  - 導線相連的導體球

$$V_1 = V_2 \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2} \Rightarrow \frac{4\pi R_1^2\sigma_1}{R_1} = \frac{4\pi R_2^2\sigma_2}{R_2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} \Rightarrow \sigma_1 R_1 = \sigma_2 R_2$$

Fig.25.20

➢尖端放電(discharge)

因靠近導體表面 $E=\sigma/\varepsilon_0$，由此推知，愈尖銳處會有較大的電場E，若電場足夠大 ($\sim 3\times 10^6$ V/m)，則在空氣中造成放電現象。

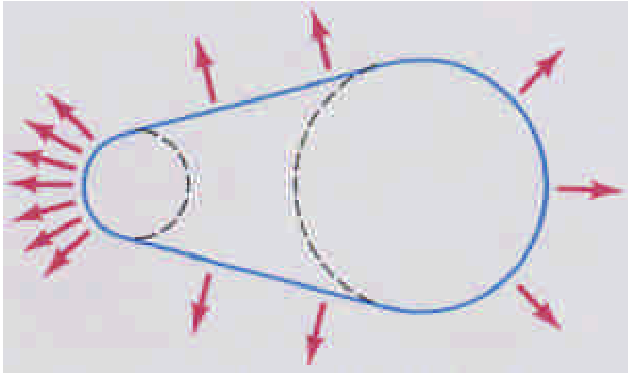


Fig.25.21

崩潰(breakdown)電場的發生

－因宇宙射線及大地輻射導致空氣分子部份解離，而電場 作用會導致已解離的電子加速撞擊其他分子形成更多離子，致使空氣失去絕緣特性變成導體，造成電暈放電 (corona discharge)。

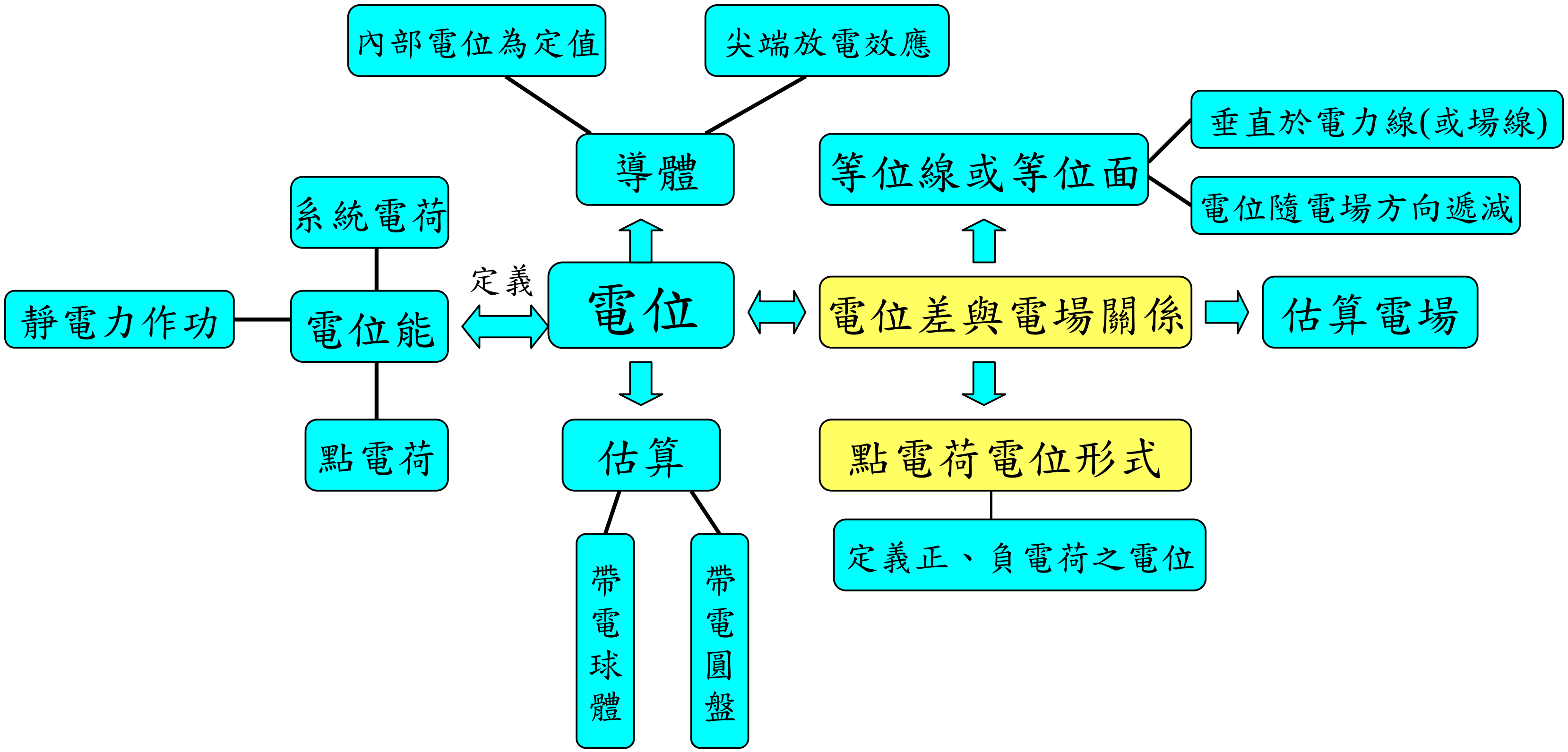
應用

避雷針(lightning rod)、飛機機翼的拖曳短線、場離子顯微鏡(field-ion microscopy)、高壓設備的平緩表面(防止放電)。

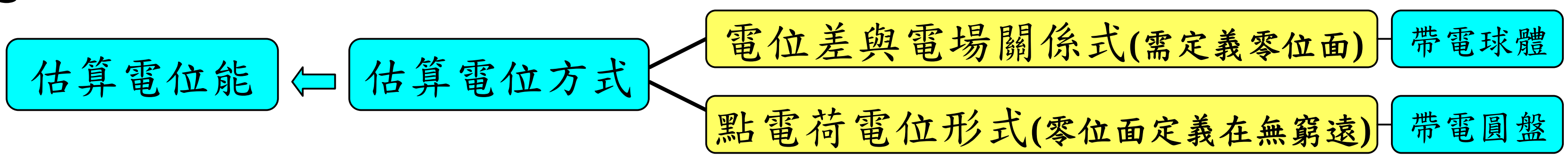
塵爆(dust explosion)－半徑0.05 mm的灰塵粒子在150V即能放電。

$$V=kQ/R,\ E=kQ/R^2 \Rightarrow V=ER \Rightarrow V\propto R$$

本章重要觀念發展脈絡彙整



注意:



習 題

- 教科書習題 (p.503~p.508)

  Exercise: 5,11,15,17,21,23,25,37,43,47,51,57,61,65,73

  Problem: 5,7,10,11,12,13

  Problem 12 Ans. (b) $E_r = 2kp\cos\theta/r^3$; $E_\theta = kp\sin\theta/r^3$

- 基本觀念習題：

1. 請寫出電位差與電場關係式。
2. 若定義無窮遠的電位為零，則請推導點電荷+Q的電位 $V = \dfrac{kQ}{r}$，其中r為相距點電荷的距離。
3. 請說明等位線(或等位面)的特性。
4. 請推導導體球與非導體球內外的電位。
5. 請說明導體尖端放電原理。

●延伸思考習題：（※不列入考試，僅列入加分題）

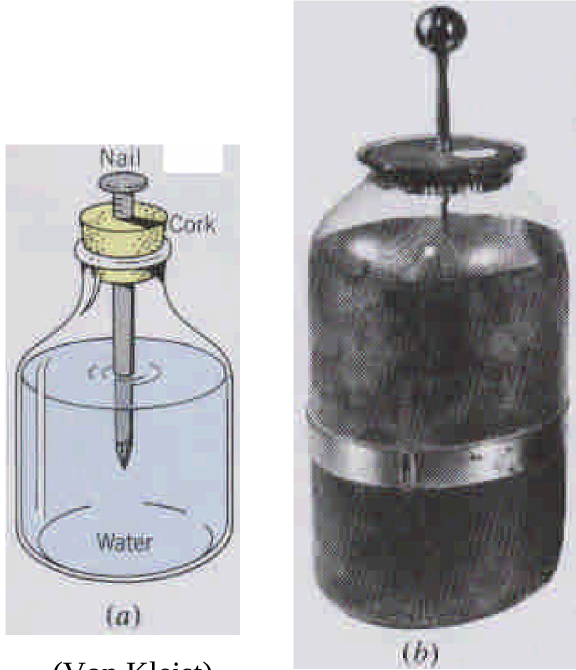
1.請探討Example 25-5帶電圓盤表面的電位。

2.請列舉電位在生活與科技上的應用 。

✦電容器(Capacitor)

儲存電荷 ⇨ 來登瓶 (Leyden jar) ⇨ 金屬平板電容

⇨ 應用
1. Radios & Tvs (濾波器應用)
2. 電子計時
3. 高能粒子加速器
4. 防止傳輸線過載



(Von Kleist) Fig.26.1

- 電容 (Capacitance)－相當於一比例常數

$Q = C\Delta V$ －儲存電荷(Q)正比於兩板間的電位差ΔV

$= CV$ (將ΔV視為一參數V)

$\Rightarrow C = \dfrac{Q}{V}$ $\Rightarrow$ SI unit: 1 farad=1 coulomb/volt

一般電容器的電容值約$1pF(10^{-12}F)$或 $1\mu F(10^{-6}F)$。

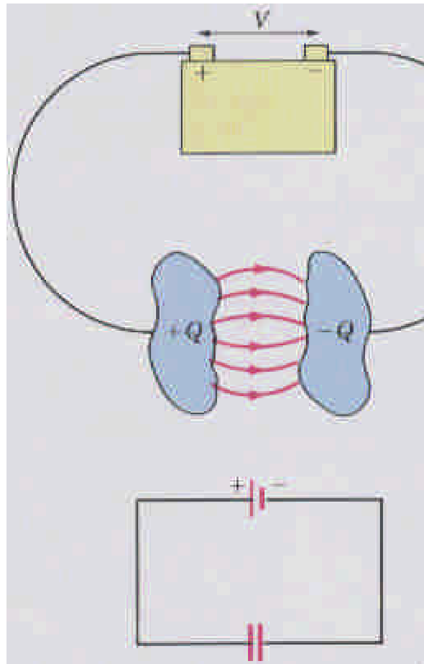


Fig.26.

- 平行電板電容(Parallel-Plate Capacitor)

$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 A} \qquad (\because \sigma=\frac{Q}{A})$$

$$C=\frac{Q}{V}=\frac{Q}{Ed}=\frac{Q}{(\frac{Q}{\varepsilon_0 A})d}=\frac{\varepsilon_0 A}{d}$$

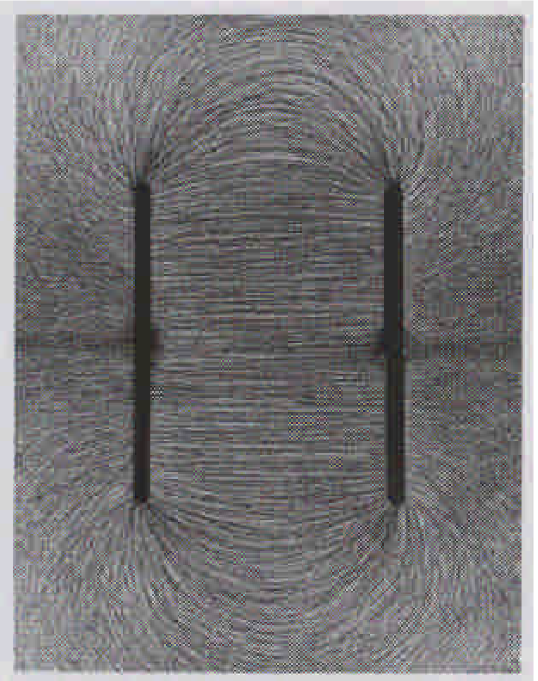


Fig.26.3

電容C與電容的幾何性質有關：

$$\begin{cases} If \quad V=const. \Rightarrow C\propto Q \Rightarrow \begin{cases} Q=\sigma A \Rightarrow Q\propto A \\ Q=\varepsilon_0 EA=\varepsilon_0 VA/d \Rightarrow Q\propto 1/d \end{cases} \\ If \quad Q=const. \Rightarrow C\propto 1/V \Rightarrow V=Ed \quad (\because E=\frac{Q}{\varepsilon_0 A}=const.) \Rightarrow V\propto d \Rightarrow C\propto 1/d \end{cases}$$

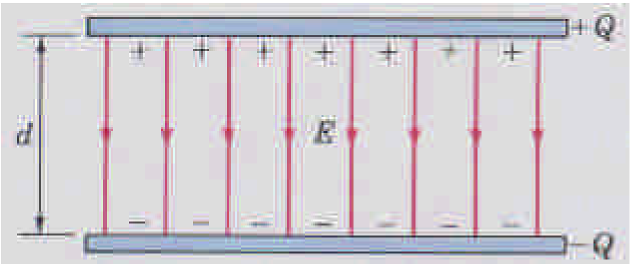


Fig.26.4

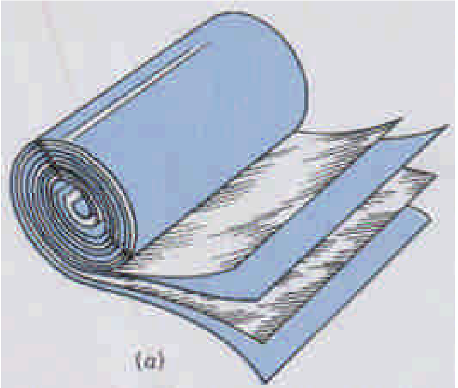


Fig.26.5

Example 26.3：孤立導體球的電容？

假設導體球表面帶電+Q，而地面為電容的另一面

$$C=\frac{Q}{V}=\frac{Q}{|V(R)-V(\infty)|}=\frac{Q}{kQ/R}=\frac{R}{k}=4\pi\varepsilon_0 R \qquad (\because k=\frac{1}{4\pi\varepsilon_0})$$

Example 26.4：球形電容器(spherical capacitor)

$$V_2-V_1=-\int_{R_1}^{R_2}E_r dr=-\int_{R_1}^{R_2}\frac{kQ}{r^2}dr=-\left[-\frac{kQ}{r}\right]_{R_1}^{R_2}=kQ\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$$

$$C=\frac{Q}{V}=\frac{Q}{|V_2-V_1|}=\frac{Q}{kQ\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}=\frac{R_1R_2}{k(R_2-R_1)}$$

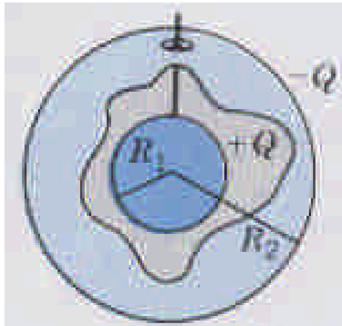


Fig.26.6

Discussion:

(a) $If\ \ R_2>>R_1 \Rightarrow C=\frac{R_1R_2}{k(R_2-R_1)}\approx\frac{R_1R_2}{kR_2}=\frac{R_1}{k}=4\pi\varepsilon_0 R_1$ (近似孤立導體球電容)

(b) $If\ \ R_2\approx R_1=R$, $R_2-R_1=d \Rightarrow C=\frac{R_1R_2}{k(R_2-R_1)}\approx\frac{R^2}{kd}=\frac{4\pi\varepsilon_0R^2}{d}=\frac{\varepsilon_0 A}{d}$ $(\because A=4\pi R^2)$

(近似平行電板電容)

Example 26.5： 圓柱形電容器(cylindrical capacitor)

$$C = \frac{Q}{|V_b - V_a|} = \frac{\lambda L}{2k\lambda ln(b/a)} = \frac{L}{2kln(b/a)} = \frac{2\pi\varepsilon_0 L}{ln(b/a)}$$

$$V_b - V_a = -\int_a^b E_r dr = -\int_a^b (\frac{2k\lambda}{r})dr = -2k\lambda\int_a^b \frac{dr}{r} = -2k\lambda ln\frac{b}{a}$$

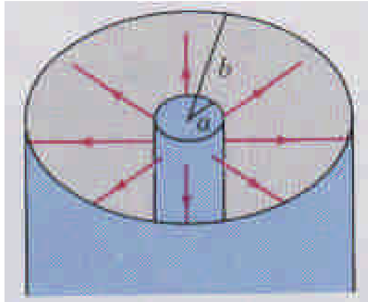


Fig.26.7

- 電容串聯(series) $\begin{cases} Q = Q_1 = Q_2 \\ V = V_1 + V_2 \end{cases}$

$$\Rightarrow V = V_1 + V_2 \Rightarrow \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$(series) \qquad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdot\cdot\cdot + \frac{1}{C_N}$$

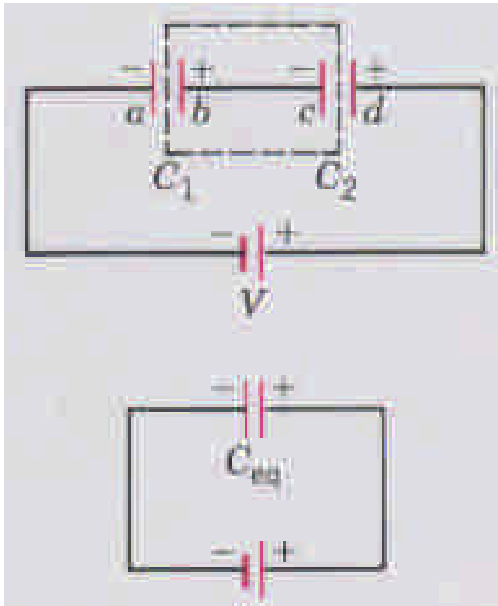


Fig.26.8

• 電容並聯(parallel) $\begin{cases} V = V_1 = V_2 \\ Q = Q_1 + Q_2 \end{cases}$

$$\Rightarrow Q = Q_1 + Q_2 \Rightarrow C_{eq}V = C_1V_1 + C_2V_2 \Rightarrow C_{eq} = C_1 + C_2$$

$$(parallel) \quad C_{eq} = C_1 + C_2 + \cdot\cdot\cdot + C_N$$

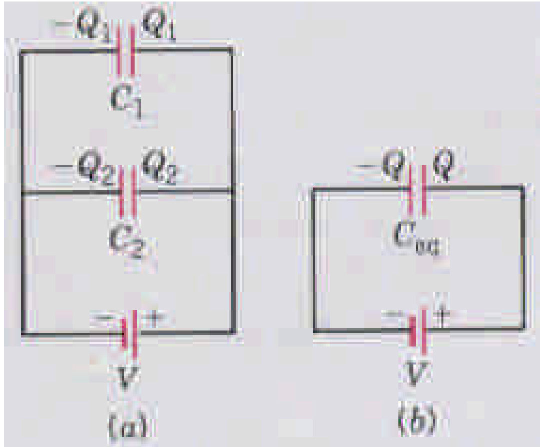


Fig.26.9

• 電容儲存的能量(Energy stored in a capacitor)

$dW = Vdq = (q/C)dq$ (假設dq 經導線自負極板移至正極板)

$$\Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = U_E$$

考慮平行電板電容 $\Rightarrow \quad C = \dfrac{\varepsilon_0 A}{d}, V = Ed$

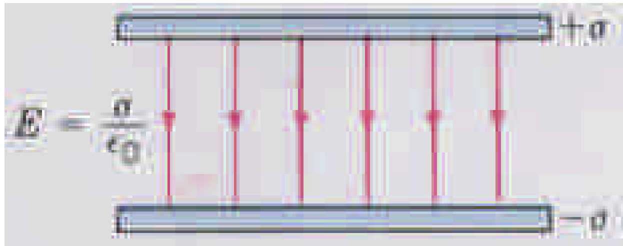


Fig.26.12

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}\cdot(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2(Ad)$$

$$u_E(energy\ density) = U_E / Ad(\text{體積}) = \frac{1}{2}\varepsilon_0 E^2$$

(能量以電場形式儲存，亦即電容係儲存電能。)

Example 26.7： 如Fig.26.11，求初始狀態(a)及末狀態(b)的兩電容器上的電荷、電位差及其儲存能量大小。

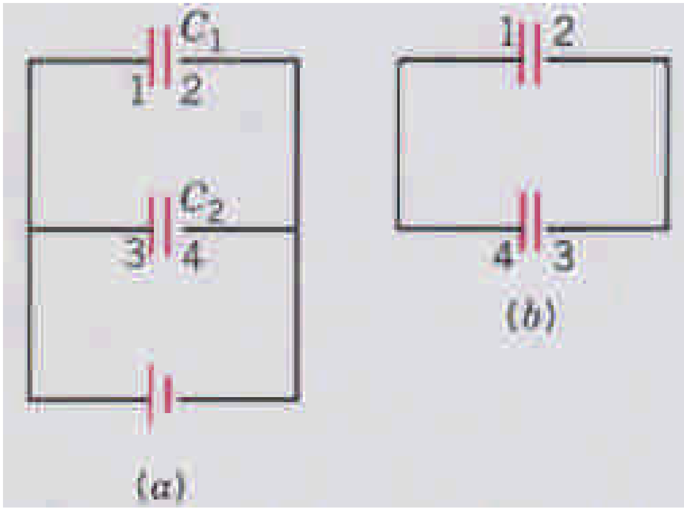


Fig.26.11

初狀態(a)

$$\begin{cases} Q_1 = C_1V_1 = 5\mu \times 12 = 60\mu C \\ Q_2 = C_2V_2 = 3\mu \times 12 = 36\mu C \end{cases}$$

$$V_1 = V_2 = 12V$$

$$\begin{cases} U_1 = \frac{1}{2}Q_1V_1 = \frac{1}{2}(60\mu C)(12V) = 360\mu J \\ U_2 = \frac{1}{2}Q_2V_2 = \frac{1}{2}(36\mu C)(12V) = 216\mu J \end{cases}$$

重新組合並聯後，假設兩電容的電荷分別為 Q_1', Q_2'

末狀態(b)

$$\left.\begin{aligned} &Q_1' + Q_2' = 60\mu C - 36\mu C = 24\mu C && (1) \\ &V_1' = V_2' \Rightarrow \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{5\mu F} = \frac{Q_2'}{3\mu F} \Rightarrow 3Q_1' - 5Q_2' = 0 && (2) \end{aligned}\right\} \Rightarrow Q_1' = 15\mu C,\ Q_2' = 9\mu C$$

$$V_1' = V_2' = \frac{Q_2'}{C_2} = \frac{9\mu C}{3\mu F} = 3V \quad ; \quad \begin{cases} U_1 = \frac{1}{2}Q_1'V_1' = \frac{1}{2}(15\mu C)(3V) = 22.5\mu J \\ U_2 = \frac{1}{2}Q_2'V_2' = \frac{1}{2}(9\mu C)(3V) = 13.5\mu J \end{cases}$$

Example 26.9： 孤立金屬球的電位能

$$dU_E = u_E(dV_{volume}) = \frac{1}{2}\varepsilon_0 E^2(4\pi r^2 dr)$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{kQ}{r^2}\right)^2\left(4\pi r^2 dr\right) = \frac{kQ^2}{2r^2}dr$$

$$\left[\because E = \frac{kQ}{r^2} \quad , \quad as\ \ r > R\right]$$

$$U_E = \frac{kQ^2}{2}\int_R^\infty r^{-2}dr = \frac{kQ^2}{2}\left[-\frac{1}{r}\right]_R^\infty = \frac{kQ^2}{2R}$$

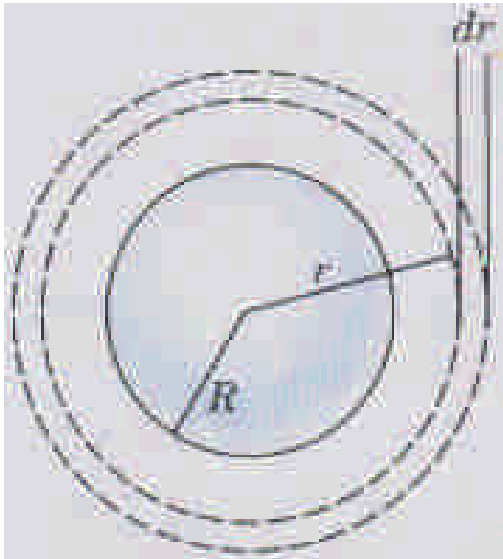


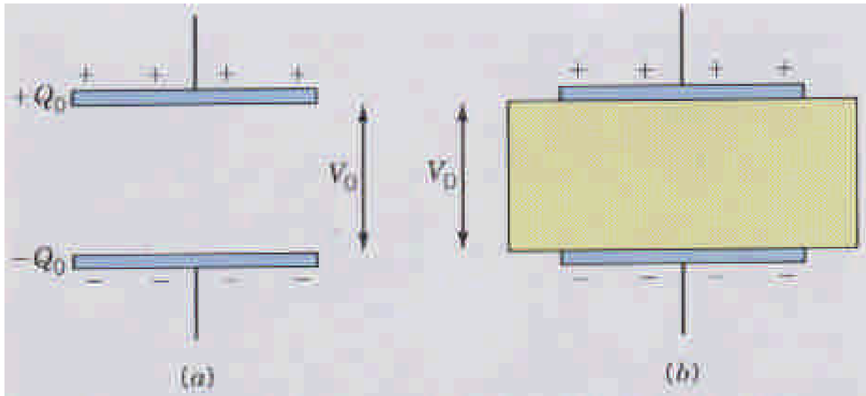
Fig.26.13

介電質(Dielectrics)

– 將電容兩平板間插入非導電物質會導致電容增加，而此非導電物質即介電質，但也有例外，如：水。

(i)未接電池⇒電位差V會改變，電荷Q不變。

Fig.26.14



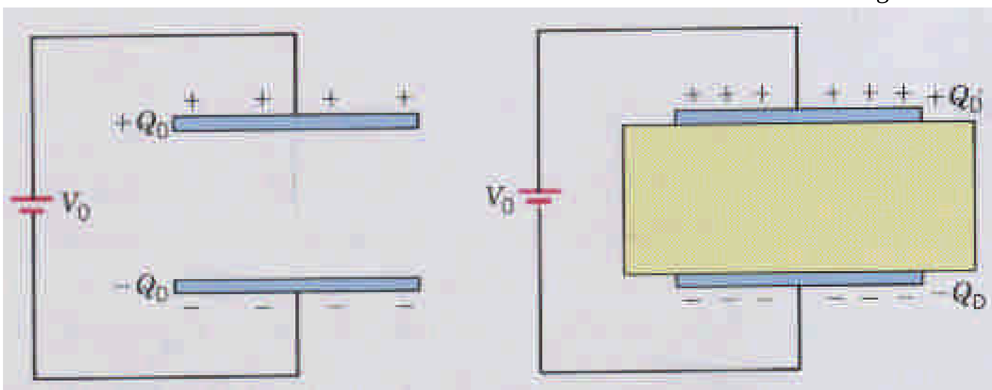
$$\Rightarrow V_D = \frac{V_0}{\kappa} \quad (\kappa\text{為介電質常數，}\kappa \geq 1)$$

$$\Rightarrow E_D = \frac{E_0}{\kappa} \quad (\because V = Ed)$$

$$\Rightarrow C_D = \frac{Q_0}{V_D} = \frac{Q_0}{V_0/\kappa} = \kappa C_0$$

(ii)連接電池⇒電位差V不變，電荷Q會改變。

Fig.26.15



$$\Rightarrow Q_D = \kappa Q_0$$

$$\Rightarrow C_D = \frac{Q_D}{V_0} = \frac{\kappa Q_0}{V_0} = \kappa C_0$$

$$(\because E = \sigma/\varepsilon_0 \Rightarrow E = \kappa E_0\,,$$

$$\therefore E_D = E/\kappa = \kappa E_0/\kappa = E_0)$$

- 介電質的優點：

  1.可增加電容。

  2.可減小電容的體積。

  3.可增加臨界電位差(或介電強度)，使電容不致放電崩潰。

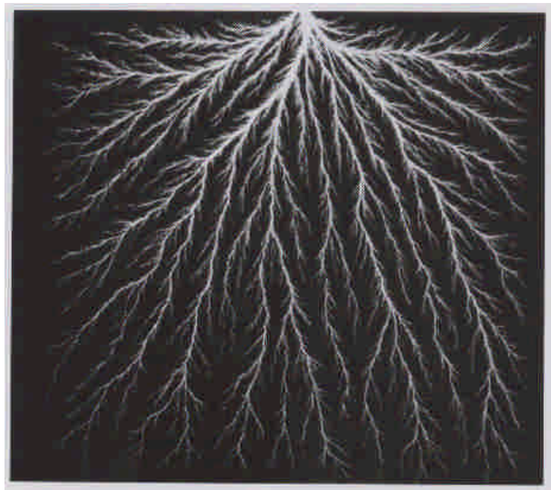


Fig.26.16

- 介電質強度(dielectric strength)

  －使介電質喪失絕緣特性而崩潰放電的最大電場強度。

TABLE 26.1 DIELECTRIC CONSTANTS AND STRENGTHS

Material	Dielectric Constant	Dielectric Strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Glass	4–6	9
Paraffin	2.3	11
Rubber	2–3.5	30
Mica	6	150
Water	80	—

- **介電質的原子觀點**(Atomic view of dielectrics)

  ➢介電質常數(Dielectric Constant) κ 相當於該電介質內部電荷對外部電場的反應程度。

  ➢一般分子密度較低，其介電質常數 κ 亦較小，如氣體。

  ➢介電質若為非極性分子，則由於外加電場的作用會形成感應電偶極矩(induced dipole moment)。

➢感應電偶極矩及永久電偶極矩皆會沿外加電場方向排列，最後在介電質兩端造成電荷區隔現象，即所謂的電極化(polarization)現象，如Fig.26.17。其中區隔的感應電荷極性與平板極性相反，在介電質內部形成與外部電場E_0指向相反的感應電場E_i，導致介電質的淨電場減小，即：

$$E_D = E_0 - E_i = \frac{E_0}{\kappa}$$

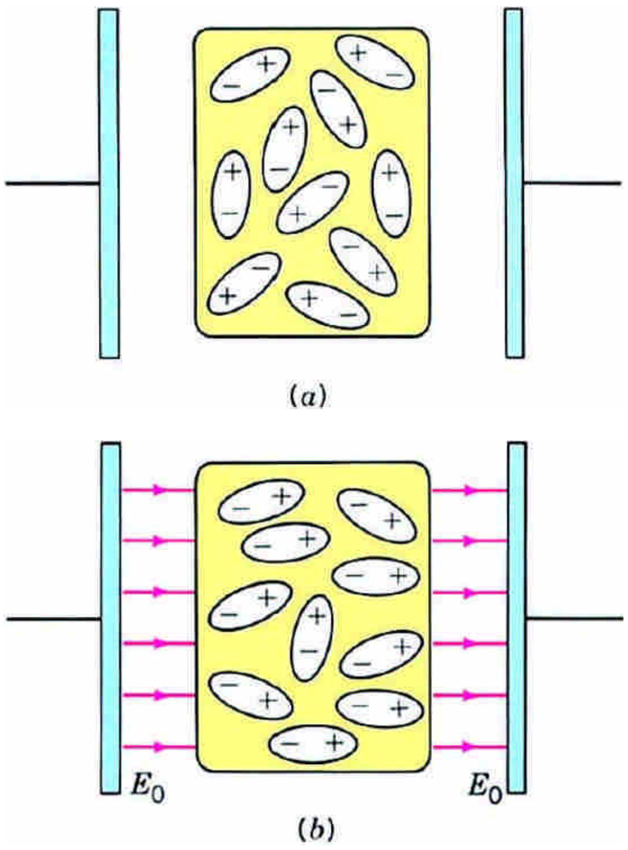


Fig.26.17

說明

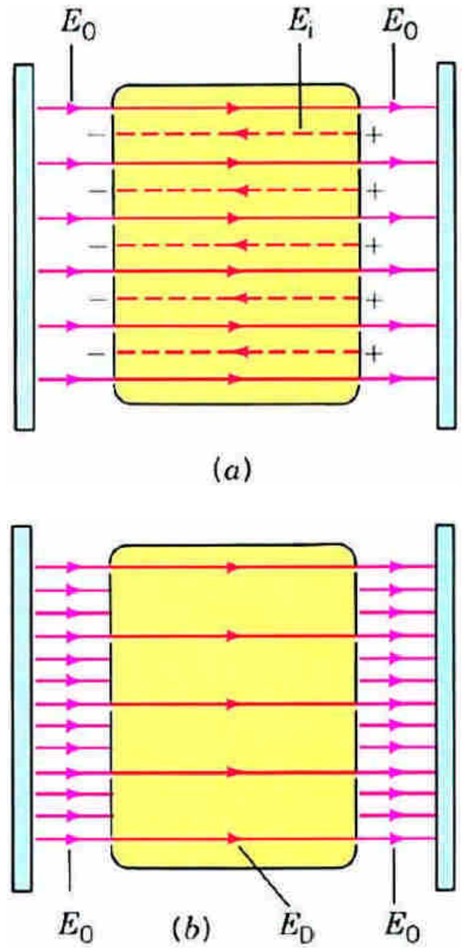


Fig.26.18

Example 26.10 已知介電質的厚度t、介電質常數κ、平行電板的面積A及分隔距離d，求Fig.26.19的電容？

$$E_0 = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad E_D = \frac{E_0}{\kappa}$$

$$\Rightarrow V = E_0(d-t) + E_D t$$

$$= \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\kappa}\right]$$

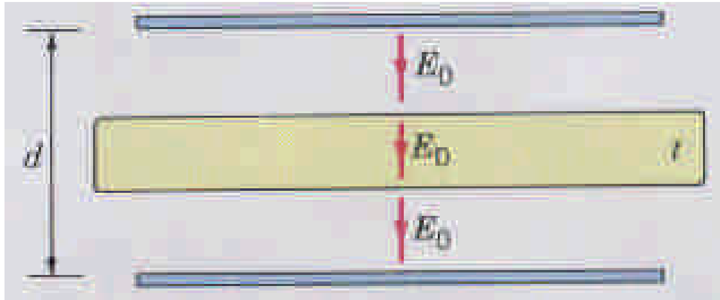


Fig.26.19

$$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\kappa}\right]} = \frac{\varepsilon_0 A}{d + t(\frac{1}{\kappa} - 1)}$$

$$(note: If\ \ \kappa = 1,\ then\ C = \frac{\varepsilon_0 A}{d})$$

- 考慮介電質的高斯定律 $\Rightarrow \oint \vec{E}\cdot d\vec{A} = \dfrac{Q_f}{\varepsilon}$ （※以下推導證明僅供參考，不列入考試範圍）

$$E_D = E_0 - E_i = \frac{E_0}{\kappa} \Rightarrow \frac{\sigma_f}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0} = \frac{\sigma_f}{\kappa\varepsilon_0} \Rightarrow \sigma_f A - \sigma_b A = \frac{\sigma_f A}{\kappa} \Rightarrow Q_f - Q_b = \frac{Q_f}{\kappa}$$

考慮如右圖虛線的封閉曲面積分：

$$\oint \vec{E}\cdot d\vec{A} = \int \vec{E}_D \cdot d\vec{A} = (E_0 - E_i)A = \left(\frac{\sigma_f}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}\right)A$$

$$= \frac{1}{\varepsilon_0}(\sigma_f A - \sigma_b A) = \frac{1}{\varepsilon_0}(Q_f - Q_b) = \frac{Q_f}{\kappa\varepsilon_0} = \frac{Q_f}{\varepsilon}$$

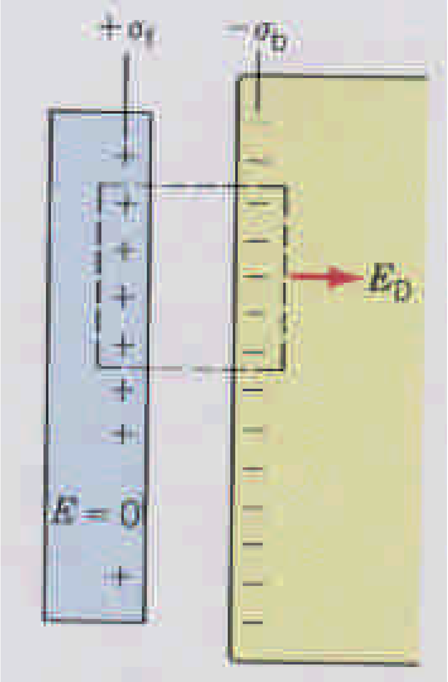
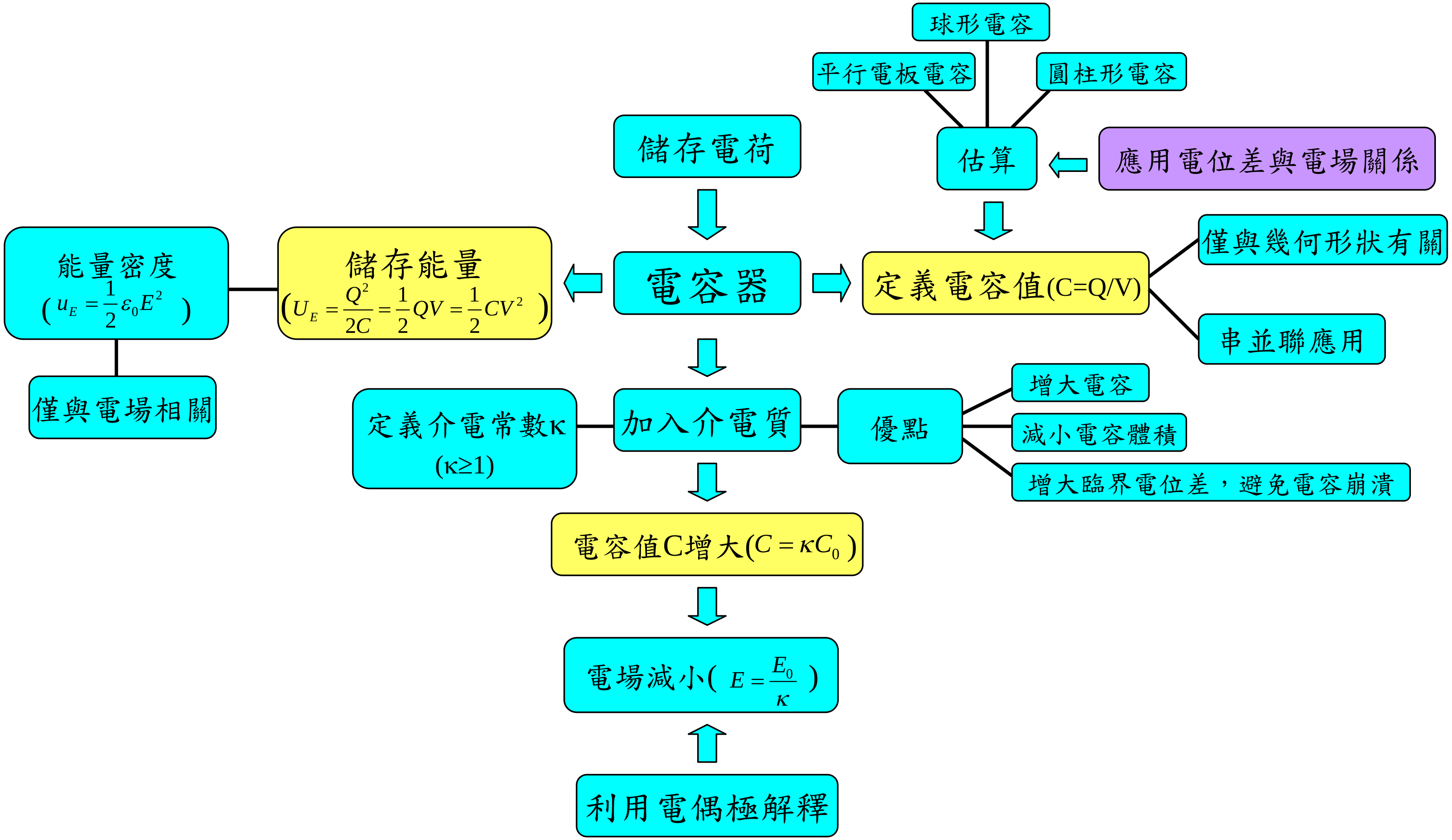


Fig.26.20

本章重要觀念發展脈絡彙整



習 題

- 教科書習題 (p.525~p.529)

Exercise:

7,11,13,15,19,21,25,29,31,33,34,35,41,42,43,47,57,59,63

Problem: 1,11

Ex34 Ans. (a)None; (b)Halved; (c)Halved

Ex42 Ans. $\dfrac{2\kappa_1\kappa_2 C_0}{\kappa_1+\kappa_2}$

- 基本觀念習題：

1.請推導平行電板電容、圓柱形電容及球形電容之電容值C。

2.請推導電容器儲存的能量為$U_E=\dfrac{Q^2}{2C}$及能量密度為$u_E=\dfrac{1}{2}\varepsilon_0 E^2$，其中$E$表電場。

3.請問電容器加入介電質的優點為何？

習 題

4.何謂介電質強度與介電質常數？

5.請以原子觀點說明介電質內部淨電場減小現象。