UNIVERSAL NONVANISHING WHITTAKER TEST VECTORS FOR SHIFTED LOCAL RANKIN–SELBERG INTEGRALS ON $\mathrm{GL}_{n+1} \times \mathrm{GL}_n$

Abstract. Let F be a non-archimedean local field and fix a nontrivial additive character ψ of conductor $\mathfrak{o}$. Given a generic irreducible admissible representation Π of $\mathrm{GL}_{n+1}(F)$, we construct for each $m \geq 0$ a Whittaker function $W_m \in \mathcal{W}(\Pi, \psi^{-1})$ depending only on (Π, ψ, m) with the following property: for every generic irreducible admissible representation π of $\mathrm{GL}_n(F)$ of conductor exponent $a(\pi) \leq m$, for the normalized Whittaker newform $V \in \mathcal{W}(\pi, \psi)$ and for any generator Q of the inverse conductor ideal $\mathfrak{q}(\pi)^{-1}$, the shifted local Rankin–Selberg integral

$$\int_{N_n \backslash \mathrm{GL}_n(F)} W_m(\mathrm{diag}(g, 1)\, u_Q)\, V(g)\, |\det g|^{s-\frac{1}{2}}\, dg$$

is absolutely convergent for all $s \in \mathbb{C}$ and equals the explicit nonzero constant $\psi^{-1}(Q) \cdot \mathrm{vol}\big(N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m)\big)$. In particular, for arbitrary π one obtains nonvanishing by taking $m = a(\pi)$. We also record the elementary smoothness obstruction showing that the "exact level-0 mirabolic" subgroup in $\mathrm{GL}_n(\mathfrak{o})$ is not open for $n \geq 2$ and hence cannot support nonzero smooth test functions.

1. Introduction

Let F be a non-archimedean local field with ring of integers $\mathfrak{o}$ and fix a nontrivial additive character $\psi : F \to \mathbb{C}^\times$ of conductor $\mathfrak{o}$. Let $N_r \subset \mathrm{GL}_r(F)$ denote the standard upper-triangular unipotent subgroup, and write ψ_r for the standard generic character of N_r induced by ψ. If Σ is a generic irreducible admissible representation of $\mathrm{GL}_r(F)$, we denote by $\mathcal{W}(\Sigma, \psi_r^{\pm 1})$ its $\psi_r^{\pm 1}$-Whittaker model.

For generic Π on $\mathrm{GL}_{n+1}(F)$ and generic π on $\mathrm{GL}_n(F)$, the local Rankin–Selberg integral

$$Z(W, V, s) = \int_{N_n \backslash \mathrm{GL}_n(F)} W(\mathrm{diag}(g, 1))\, V(g)\, |\det g|^{s-\frac{1}{2}}\, dg \tag{1}$$

depends on choices $W \in \mathcal{W}(\Pi, \psi^{-1})$ and $V \in \mathcal{W}(\pi, \psi)$. The question addressed here is a shifted variant in which one inserts a unipotent element u_Q determined by the conductor of π, and asks whether one can force *uniform nonvanishing* (and finiteness) for *all* $s \in \mathbb{C}$.

Problem. Let Π be generic irreducible admissible on $\mathrm{GL}_{n+1}(F)$, realized in $\mathcal{W}(\Pi, \psi^{-1})$. Let π be generic irreducible admissible on $\mathrm{GL}_n(F)$, realized in $\mathcal{W}(\pi, \psi)$. Let $\mathfrak{q}(\pi) \subset \mathfrak{o}$ be the conductor ideal of π and choose a generator $Q \in F^\times$ of $\mathfrak{q}(\pi)^{-1}$. Set $u_Q = I_{n+1} + Q\, E_{n,n+1} \in \mathrm{GL}_{n+1}(F)$. Must there exist $W \in \mathcal{W}(\Pi, \psi^{-1})$ and $V \in \mathcal{W}(\pi, \psi)$ such that

$$\int_{N_n \backslash \mathrm{GL}_n(F)} W(\mathrm{diag}(g, 1)\, u_Q)\, V(g)\, |\det g|^{s-\frac{1}{2}}\, dg$$

Date: February 8, 2026.

2020 *Mathematics Subject Classification.* 11F70, 22E50.

Key words and phrases. Whittaker models, Kirillov models, newforms, conductors, Rankin–Selberg integrals, test vectors.

is finite and nonzero for all $s \in \mathbb{C}$?

We answer this affirmatively, with an explicit construction of W and an explicit evaluation of the integral. The key inputs are: (i) a simple conjugation identity for the shift u_Q, and (ii) the existence of Whittaker functions whose restriction to the Levi $\mathrm{diag}(\mathrm{GL}_n(F), 1)$ is an *arbitrary* compactly supported (N_n, ψ_n^{-1})-equivariant function. This latter extension statement goes back to Gelfand–Kazhdan [2] (see also the archimedean analogs of Jacquet and Kemarsky).

2. Notation and basic definitions

Let F be a non-archimedean local field with ring of integers $\mathfrak{o}$, maximal ideal $\mathfrak{p}$, and residue cardinality q. Fix a uniformizer ϖ with $\mathfrak{p} = (\varpi)$, and normalize the absolute value $|\cdot|$ by $|\varpi| = q^{-1}$.

Fix a nontrivial additive character $\psi : F \to \mathbb{C}^\times$ of conductor $\mathfrak{o}$, i.e. ψ is trivial on $\mathfrak{o}$ and nontrivial on $\varpi^{-1}\mathfrak{o}$. For $r \geq 1$, set $G_r = \mathrm{GL}_r(F)$ and let $N_r \subset G_r$ be the upper-triangular unipotent subgroup. Define the standard generic character $\psi_r : N_r \to \mathbb{C}^\times$ by

$$\psi_r(u) = \psi\Big(\sum_{i=1}^{r-1} u_{i,i+1}\Big).$$

If Σ is generic irreducible admissible on G_r, its $\psi_r^{\pm 1}$-Whittaker model is denoted $\mathcal{W}(\Sigma, \psi_r^{\pm 1})$.

Write $K_r = \mathrm{GL}_r(\mathfrak{o})$. For $n \geq 1$ and $m \geq 0$, define

$$K_1(\mathfrak{p}^m) = \{\, k \in K_n : \ e_n k \equiv e_n \pmod{\mathfrak{p}^m} \,\}, \qquad e_n = (0, \ldots, 0, 1) \in F^n. \tag{2}$$

Equivalently, $k \in K_1(\mathfrak{p}^m)$ if and only if $k_{n,j} \in \mathfrak{p}^m$ for $1 \leq j < n$ and $k_{n,n} \in 1 + \mathfrak{p}^m$. Each $K_1(\mathfrak{p}^m)$ is an open compact subgroup of $\mathrm{GL}_n(F)$.

We embed $\mathrm{GL}_n(F)$ into $\mathrm{GL}_{n+1}(F)$ via

$$\iota : \mathrm{GL}_n(F) \hookrightarrow \mathrm{GL}_{n+1}(F), \qquad \iota(g) = \mathrm{diag}(g, 1).$$

Let $E_{i,j}$ be the standard matrix unit. Given $Q \in F$, set

$$u_Q := I_{n+1} + Q\, E_{n,n+1} \in \mathrm{GL}_{n+1}(F).$$

For $W \in \mathcal{W}(\Pi, \psi_{n+1}^{-1})$ and $V \in \mathcal{W}(\pi, \psi_n)$ we consider the shifted local Rankin–Selberg integral

$$Z(W, V, s; Q) = \int_{N_n \backslash \mathrm{GL}_n(F)} W(\iota(g) u_Q)\, V(g)\ |\det g|^{s - \frac{1}{2}}\ dg. \tag{3}$$

3. A smoothness obstruction for the exact level-0 mirabolic

This section is not needed for the main theorem, but it clarifies why one *cannot* force compactness by supporting on the subgroup of $\mathrm{GL}_n(\mathfrak{o})$ with last row *exactly* equal to e_n when $n \geq 2$.

Definition 3.1. For $n \geq 1$ define the *exact mirabolic* subgroup of $K_n = \mathrm{GL}_n(\mathfrak{o})$ by

$$K_n^{\mathrm{mir}} := \{\, k \in K_n : \ e_n k = e_n \,\}.$$

Lemma 3.2. *If $n \geq 2$, the subgroup K_n^{mir} is closed but not open in K_n. Moreover,*

$$K_n^{\mathrm{mir}} = \bigcap_{m \geq 1} K_1(\mathfrak{p}^m),$$

hence K_n^{mir} has empty interior in K_n.

Proof. The condition $e_n k = e_n$ is equivalent to the congruences $e_n k \equiv e_n \pmod{\mathfrak{p}^m}$ for all $m \geq 1$, because $\bigcap_{m\geq 1} \mathfrak{p}^m = \{0\}$ in $\mathfrak{o}$. This gives the displayed identity.

Each $K_1(\mathfrak{p}^m)$ is open in K_n, and for $n \geq 2$ the sequence is strictly decreasing: the congruence condition on the last row modulo $\mathfrak{p}^m$ becomes strictly stronger with m. If the intersection were open, it would contain some $K_1(\mathfrak{p}^m)$; but that would force $K_1(\mathfrak{p}^m) = K_n^{\mathrm{mir}}$, contradicting strict decrease. Thus K_n^{mir} is not open. As an intersection of closed subgroups, it is closed. Finally, in a topological group the interior of a subgroup is itself a subgroup and hence open; thus a subgroup is open if and only if it has nonempty interior. Therefore K_n^{mir} has empty interior. □

Proposition 3.3. *Let G be a totally disconnected locally compact group and let $H \subseteq G$ be closed with empty interior. Then every locally constant function $f : G \to \mathbb{C}$ supported on H is identically zero.*

Proof. If $f(x) \neq 0$ for some x, then local constancy gives an open neighbourhood U of x on which f is constant and nonzero. Hence $U \subseteq \mathrm{supp}(f) \subseteq H$, contradicting that H has empty interior. □

Remark 3.4. Lemma 3.2 and Proposition 3.3 show that for $n \geq 2$ there is no nonzero smooth function on $\mathrm{GL}_n(F)$ supported on $N_n\, K_n^{\mathrm{mir}}$. Consequently one cannot realize an "exact level-0 mirabolic" characteristic function as the restriction of a Whittaker function to the embedded Levi $\iota(\mathrm{GL}_n(F))$. The construction below replaces K_n^{mir} by the open subgroup $K_1(\mathfrak{p}^m)$.

4. A conjugation identity for the shift u_Q

Lemma 4.1. *Let $W \in \mathcal{W}(\Pi, \psi_{n+1}^{-1})$. For all $g \in \mathrm{GL}_n(F)$ and all $Q \in F$, one has*

$$W(\iota(g)\, u_Q) = \psi^{-1}(Q\, g_{n,n})\, W(\iota(g)).$$

Proof. Write

$$\iota(g)\, u_Q = \big(\iota(g)\, u_Q\, \iota(g)^{-1}\big)\, \iota(g).$$

Since $u_Q = I_{n+1} + QE_{n,n+1}$ and $\iota(g) = \mathrm{diag}(g, 1)$, we compute

$$\iota(g)\, E_{n,n+1}\, \iota(g)^{-1} = \sum_{i=1}^{n} g_{i,n}\, E_{i,n+1},$$

hence

$$\iota(g)\, u_Q\, \iota(g)^{-1} = I_{n+1} + Q \sum_{i=1}^{n} g_{i,n}\, E_{i,n+1} \;\in\; N_{n+1}.$$

The generic character ψ_{n+1} depends only on the superdiagonal entries $(i, i+1)$, so among the matrices $E_{i,n+1}$ only $E_{n,n+1}$ contributes. Therefore

$$\psi_{n+1}\big(\iota(g)\, u_Q\, \iota(g)^{-1}\big) = \psi(Q\, g_{n,n}).$$

Using Whittaker equivariance $W(ux) = \psi_{n+1}^{-1}(u)W(x)$ for $u \in N_{n+1}$ yields the claim. □

5. Kirillov–type extension to the Levi

Let $r \geq 2$ and denote by $\iota : \mathrm{GL}_{r-1}(F) \hookrightarrow \mathrm{GL}_r(F)$ the standard Levi embedding $g \mapsto \mathrm{diag}(g, 1)$. Write $N_{r-1} \subset \mathrm{GL}_{r-1}(F)$ for the upper-triangular unipotent subgroup.

Definition 5.1. Let $r \geq 2$. Denote by $C_c^\infty(N_{r-1}\backslash \mathrm{GL}_{r-1}(F), \psi_{r-1}^{-1})$ the space of locally constant functions $f : \mathrm{GL}_{r-1}(F) \to \mathbb{C}$ such that $f(ug) = \psi_{r-1}^{-1}(u)f(g)$ for all $u \in N_{r-1}$ and whose support is compact modulo left N_{r-1}. Equivalently, this is the compact induction $\text{c-Ind}_{N_{r-1}}^{\mathrm{GL}_{r-1}(F)}(\psi_{r-1}^{-1})$.

The following extension statement is the key representation-theoretic input.

Theorem 5.2 (Gelfand–Kazhdan extension)**.** *Let F be a non-archimedean local field. Let Σ be a generic irreducible admissible representation of $\mathrm{GL}_r(F)$, realized in its Whittaker model $\mathcal{W}(\Sigma, \psi_r^{-1})$. Then the restriction map*

$$\mathrm{Res}_\iota : \mathcal{W}(\Sigma, \psi_r^{-1}) \to C^\infty(\mathrm{GL}_{r-1}(F)), \qquad W \longmapsto \big(g \mapsto W(\iota(g))\big),$$

has image containing $C_c^\infty(N_{r-1}\backslash \mathrm{GL}_{r-1}(F), \psi_{r-1}^{-1})$. In particular, for every $f \in \text{c-Ind}_{N_{r-1}}^{\mathrm{GL}_{r-1}(F)}(\psi_{r-1}^{-1})$ there exists $W \in \mathcal{W}(\Sigma, \psi_r^{-1})$ such that $W(\iota(g)) = f(g)$ for all $g \in \mathrm{GL}_{r-1}(F)$.

Proof (reference). This is a classical statement in the theory of Kirillov models. For p-adic F it is proved by Gelfand–Kazhdan [2]; see also the discussion and archimedean analogs in Jacquet and Kemarsky. $\square$

6. A compactly supported universal test function on GL_n

Fix $n \geq 1$. For $m \geq 0$, define a function $f_m : \mathrm{GL}_n(F) \to \mathbb{C}$ by

$$f_m(uk) = \psi_n^{-1}(u) \quad (u \in N_n,\ k \in K_1(\mathfrak{p}^m)), \qquad f_m(g) = 0 \quad \text{if } g \notin N_n K_1(\mathfrak{p}^m). \tag{4}$$

Lemma 6.1. *The function f_m is well-defined and belongs to* $\text{c-Ind}_{N_n}^{\mathrm{GL}_n(F)}(\psi_n^{-1})$*. Moreover, $N_n \cap K_1(\mathfrak{p}^m) = N_n(\mathfrak{o})$.*

Proof. If $uk = u'k'$ with $u, u' \in N_n$ and $k, k' \in K_1(\mathfrak{p}^m)$, then $k^{-1}k' = u^{-1}u' \in N_n \cap K_1(\mathfrak{p}^m)$. Since $K_1(\mathfrak{p}^m) \subseteq \mathrm{GL}_n(\mathfrak{o})$, any element of $N_n \cap K_1(\mathfrak{p}^m)$ is an upper-triangular unipotent matrix with entries in $\mathfrak{o}$, i.e. lies in $N_n(\mathfrak{o})$. Conversely $N_n(\mathfrak{o}) \subseteq K_1(\mathfrak{p}^m)$ because elements of N_n have last row e_n. Thus $N_n \cap K_1(\mathfrak{p}^m) = N_n(\mathfrak{o})$.

Because ψ has conductor $\mathfrak{o}$, it is trivial on $\mathfrak{o}$, hence ψ_n is trivial on $N_n(\mathfrak{o})$. Therefore $\psi_n^{-1}(u) = \psi_n^{-1}(u')$ and the assignment (4) is well-defined.

The support of f_m modulo N_n is contained in $K_1(\mathfrak{p}^m)$, which is compact. Also f_m is right $K_1(\mathfrak{p}^m)$-invariant, hence locally constant. Thus $f_m \in \text{c-Ind}_{N_n}^{\mathrm{GL}_n(F)}(\psi_n^{-1})$. $\square$

7. Lifting f_m to a Whittaker function on GL_{n+1}

Let Π be a generic irreducible admissible representation of $\mathrm{GL}_{n+1}(F)$, realized in $\mathcal{W}(\Pi, \psi_{n+1}^{-1})$.

Proposition 7.1. *For each $m \geq 0$ there exists $W_m \in \mathcal{W}(\Pi, \psi_{n+1}^{-1})$ such that*

$$W_m(\iota(g)) = f_m(g) \qquad \text{for all } g \in \mathrm{GL}_n(F),$$

where f_m is as in (4)*.*

Proof. By Lemma 6.1, $f_m \in \text{c-Ind}_{N_n}^{\mathrm{GL}_n(F)}(\psi_n^{-1})$. Apply Theorem 5.2 with $r = n+1$ and $\Sigma = \Pi$ to obtain $W_m \in \mathcal{W}(\Pi, \psi_{n+1}^{-1})$ with $W_m(\iota(g)) = f_m(g)$ for all g. $\square$

8. Newforms on GL_n

Let π be a generic irreducible admissible representation of $\mathrm{GL}_n(F)$. Its conductor ideal is $\mathfrak{q}(\pi) = \mathfrak{p}^{a(\pi)}$ for an integer $a(\pi) \geq 0$.

Proposition 8.1 (Whittaker newforms)**.** *Let π be generic irreducible admissible of $\mathrm{GL}_n(F)$ with conductor ideal $\mathfrak{q}(\pi) = \mathfrak{p}^{a(\pi)}$. Then there exists a Whittaker function $V \in \mathcal{W}(\pi, \psi_n)$ which is right $K_1(\mathfrak{p}^{a(\pi)})$-invariant and unique up to scalars. Normalizing by $V(1) = 1$, one has $V(k) = 1$ for all $k \in K_1(\mathfrak{p}^{a(\pi)})$.*

Proof. This is standard local newform theory for GL_n (existence of a unique newvector fixed by $K_1(\mathfrak{q}(\pi))$) together with the uniqueness of Whittaker models; see [3, 4] and the explicit Whittaker newform results of Miyauchi [5]. If V is right $K_1(\mathfrak{p}^{a(\pi)})$-invariant, then for $k \in K_1(\mathfrak{p}^{a(\pi)})$ we have $V(k) = V(1 \cdot k) = V(1) = 1$. □

9. Main theorem: universal conductor-bounded nonvanishing

Lemma 9.1. *Let $m \geq 0$ and $a \geq 0$. Let $Q \in F^\times$ satisfy $Q \in \mathfrak{p}^{-a}$. If $m \geq a$ and $k \in K_1(\mathfrak{p}^m)$, then*

$$\psi(Q\, k_{n,n}) = \psi(Q).$$

Proof. For $k \in K_1(\mathfrak{p}^m)$, we have $k_{n,n} \equiv 1 \pmod{\mathfrak{p}^m}$, so $k_{n,n} = 1 + t$ with $t \in \mathfrak{p}^m$. Then $Qk_{n,n} = Q + Qt$ with $Qt \in \mathfrak{p}^{m-a} \subseteq \mathfrak{o}$ because $m \geq a$. Since ψ is trivial on $\mathfrak{o}$, we get $\psi(Q + Qt) = \psi(Q)$. □

Theorem 9.2. *Let $n \geq 1$, and let Π be a generic irreducible admissible representation of $\mathrm{GL}_{n+1}(F)$ realized in $\mathcal{W}(\Pi, \psi_{n+1}^{-1})$. For each $m \geq 0$, let $W_m \in \mathcal{W}(\Pi, \psi_{n+1}^{-1})$ be as in Proposition 7.1.*

Let π be any generic irreducible admissible representation of $\mathrm{GL}_n(F)$ with conductor ideal $\mathfrak{q}(\pi) = \mathfrak{p}^{a(\pi)}$. Assume $a(\pi) \leq m$ and let $Q \in F^\times$ be a generator of $\mathfrak{q}(\pi)^{-1} = \mathfrak{p}^{-a(\pi)}$. Let $V \in \mathcal{W}(\pi, \psi_n)$ be the normalized Whittaker newform of Proposition 8.1.

Then the shifted Rankin–Selberg integral (3) *is absolutely convergent for every $s \in \mathbb{C}$ and equals the nonzero constant*

$$Z(W_m, V, s; Q) = \psi^{-1}(Q) \cdot \mathrm{vol}\big(N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m)\big),$$

hence is finite and nonzero for all $s \in \mathbb{C}$.

Proof. By Lemma 4.1 and Proposition 7.1,

$$W_m(\iota(g)u_Q) = \psi^{-1}(Qg_{n,n})\, W_m(\iota(g)) = \psi^{-1}(Qg_{n,n})\, f_m(g).$$

Thus the integrand in (3) is supported on $N_n K_1(\mathfrak{p}^m)$. On this support we have $g \in \mathrm{GL}_n(\mathfrak{o})$, hence $|\det g| = 1$, so $|\det g|^{s-\frac{1}{2}} \equiv 1$ and the integrand is independent of s. Absolute convergence for every s follows because $N_n \backslash N_n K_1(\mathfrak{p}^m)$ is compact.

Write $g = uk$ with $u \in N_n$ and $k \in K_1(\mathfrak{p}^m)$. Since the last row of u is e_n, we have $(uk)_{n,n} = k_{n,n}$, hence

$$W_m(\iota(g)u_Q) = \psi^{-1}(Qk_{n,n})\, \psi_n^{-1}(u).$$

Because $a(\pi) \leq m$ and $Q \in \mathfrak{p}^{-a(\pi)}$, Lemma 9.1 gives $\psi(Qk_{n,n}) = \psi(Q)$, so

$$W_m(\iota(g)u_Q) = \psi^{-1}(Q)\, \psi_n^{-1}(u) \qquad (g = uk \in N_n K_1(\mathfrak{p}^m)).$$

Now f_m is left (N_n, ψ_n^{-1})-equivariant and V is left (N_n, ψ_n)-equivariant, so the product $f_m(g)V(g)$ is left N_n-invariant. Using Lemma 6.1 and the standard identification

$$N_n \backslash N_n K_1(\mathfrak{p}^m) \;\simeq\; (N_n \cap K_1(\mathfrak{p}^m)) \backslash K_1(\mathfrak{p}^m) \;=\; N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m),$$

we obtain

$$Z(W_m, V, s; Q) = \psi^{-1}(Q) \int_{N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m)} V(k)\, dk.$$

Finally, since $a(\pi) \leq m$ we have $K_1(\mathfrak{p}^m) \subseteq K_1(\mathfrak{p}^{a(\pi)})$, hence by Proposition 8.1 the normalized newform satisfies $V(k) = 1$ for all $k \in K_1(\mathfrak{p}^m)$. Therefore

$$Z(W_m, V, s; Q) = \psi^{-1}(Q) \cdot \mathrm{vol}\big(N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m)\big),$$

which is a nonzero constant (the volume is a strictly positive real number). □

Corollary 9.3. *For every generic irreducible admissible π of $\mathrm{GL}_n(F)$ (with any conductor exponent $a(\pi)$), taking $m = a(\pi)$ and $W = W_{a(\pi)}$ as in Theorem 9.2, and taking V to be the normalized Whittaker newform, the integral* (3) *is finite and nonzero for all $s \in \mathbb{C}$ and for every generator Q of $\mathfrak{q}(\pi)^{-1}$.*

Remark 9.4 (Dependence on Haar measures). The numerical value of $\mathrm{vol}(N_n(\mathfrak{o}) \backslash K_1(\mathfrak{p}^m))$ depends on Haar normalizations, but it is always > 0. Hence the nonvanishing asserted in Theorem 9.2 is independent of the choice of measures.

Remark 9.5 (Uniformity in π up to conductor bound). For fixed Π and fixed m, the Whittaker function W_m works simultaneously for *all* generic π of $\mathrm{GL}_n(F)$ with conductor exponent $a(\pi) \leq m$ (and all generators Q of $\mathfrak{q}(\pi)^{-1}$).

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