# Optimization of energy production of a CHP plant with heat storage

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Abstract— Combined heat and power (CHP) production is a very efficient technique to produce power and heat in an integrated process. In CHP plants, generation of heat and power follows a joint characteristic, which means that production planning of both commodities must be done in coordination. The hourly produced power can be sold to the grid at market price, but heat must be produced to meet the local demand of district heating or heat for specific industrial processes. Typically, the most profitable operation of a CHP system can be planned by using an optimization model.

The high efficiency and profitability of CHP plants can be further improved by utilization of energy storage units. Heat storages make it possible to relax the constraint to produce heat each hour to exactly match the local demand. This allows satisfying the variable heat demand more cheaply by storing heat during low demand and discharging heat when demand is high. By relaxing the connection between heat and power production, heat storages also allow producing more electricity to the power market when the spot price is high and reducing the power generation when spot price is low. The aim of this study is to develop a model for optimizing the operation of a CHP plant together with a heat storage. The model is a linear programming (LP) model consisting of hourly models connected together with dynamic storage constraints. The objective is to minimize the production (fuel) costs subtracted by revenue from selling power to the market. The model is demonstrated using modified reallife data of a Finnish city. The results are useful for planning efficient operation of the plant. The model can also be adapted for determining the optimal size of the storage.

Keywords: Combined heat and power (CHP); linear programming (LP); economic optimization; heat storage; energy efficiency.

# I. INTRODUCTION

A combined heat and power (CHP) plant can convert about 90% of the fuel into energy, whereas conventional thermal power plants operate with 40% efficiency [1]. A heat storage can further enhance the cost and energy efficiency of a CHP plant [2]. Consequently, heat storages are widely used in district heating systems [3].

Development of CHP is typically based on heat demand; however, its economic advantage relies also on the potential to sell the produced power on the power market [4]. In Finland, Sweden, Norway and Denmark the power market is operated by NordPool [5]. Optimal production planning of CHP in the presence of a power market requires considering both forecasts

of spot price for power and the demand of heat. Both heat demand and power price fluctuate hourly and seasonally. Thus, an operation analysis is required for CHP systems [6].

Many studies have been done to address the problem, and different methods have been applied to optimize the operation of CHP systems. Because heat cannot be distributed over long distances, the primary constraint is that the local heat demand must at all times be met. Economic optimization of the system then tries to minimize the production (fuel) costs. Several studies have proposed solution methods for economic dispatch problems of cogeneration systems [6-12]. Based on some studies, utilization of heat storages can improve the CHP performance and save costs [2, 10, 13, 14]. Economic analysis of energy storages always poses a complicated problem due to its temporal nature. The way a storage unit is operated in one time step affects how it can be used in future. Because considerable uncertainty about the future conditions is associated with the planning problem, optimal operation of a storage is a challenging decision problem [15]. Different techniques may be used to solve the CHP production problem with energy storages. For example, the Lagrangian relaxation technique has been used to solve trigeneration planning problems with storages [16].

In this study, we model the CHP planning problem with heat storage as a linear programming (LP) model. The objective is to minimize the production costs while gaining maximal income from power sales to the market. Linear formulation of the feasible operating region of the CHP plant is described in detail. The methodology leads to most cost-efficient operation of the system. The model is demonstrated using slightly modified production and demand data of a Finnish city and power price information from NordPool. The model is solved using the efficient linear programming software LP2 [17, 18]. The results are validated by comparing with existing energy planning software EnergyPRO [19] and EnergyPLAN [20].

#### II. THE CHP MODEL

To satisfy the variable heat demand is the primary target of the CHP system. Lack of heat is not permitted and excess production of heat leads usually to a small cost due to disposing the heat e.g. in an auxiliary cooler. The objective is to minimize the overall net acquisition costs for heat and power. The net acquisition costs include production costs e.g. fuel costs subtracted by revenue from selling power to the market [8]. In principle also revenue from selling heat should be included in the objective function. However, because heat sales must exactly meet the demand, revenue from selling heat will be a constant and does not affect the optimization.

The planning horizon may be anything from a few days to several years. In practice, fairly reliable forecasts for power price and heat demand are available for about one week, since there is a possible pattern in heat and power consumption with the weekly duration and they differ between working days and weekend [21]. Because the spot market for power operates on an hourly basis, the CHP planning model is decomposed into a sequence of hourly models. Furthermore, the switching on and off of CHP takes more than one hour, and it is between 2 and 12 hours for full power of steam turbine plants [22]. Depending on what kind of dynamic constraints (linking periods together) exist, the planning model can be solved efficiently using various decomposition techniques [23-28]. However, in this study we formulate and solve the model as a single LP problem. The advantage with an LP formulation is that it is easy to extend for different kinds of analyses.

# A. Individual CHP plant modeling

The characteristic of a CHP plant can be considered as a surface in 3-dimensional space, corresponding to different combinations of heat and power production (p,q) and corresponding production cost(c). In this study we assume that the CHP characteristic is convex. The characteristic is convex exactly when the feasible operating region in the (p, q) plane is convex, and the production cost is a convex function of p and q. [The operating region is convex, if given any two points within the region, also the connecting line segment is within the region. The cost function is convex if it curves upward or is flat everywhere in the operating region.] The convexity assumption is reasonable for many types of CHP plants. Even if the characteristic is not convex, it may be possible to run the plant fractions of an hour in different modes, which in effect yields a convex characteristic. When convexity cannot be assumed, mixed integer encoding of the plant characteristic can be applied [11,23], but that is out of the scope of this paper.

A convex CHP characteristic in terms of heat, power, and cost (q, p, c) is illustrated in Fig. 1. The operating region area in (p, q) plane shows the adjustable cogeneration of heat and power. For this convex CHP plant, extreme points of a convex combination [28, 29] can represent the characteristic operating zone (all corner points  $(q_i, p_i, c_i)$  of the polygon in Fig. 1) [23].

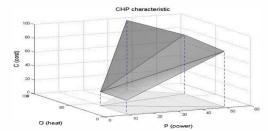


Fig. 1. Feasible operating region of a convex CHP plant [23]

Based on the convexity assumption, the relation between the production level of heat and power and hourly operating costs (Q, P, C) of a CHP plant can be represented as the convex combination of corner points  $(\mathbf{q}_i, p_i, c_i)$ :

$$C = \sum_{j \in J} c_j x_j$$

$$P = \sum_{j \in J} p_j x_j$$

$$Q = \sum_{j \in J} q_j x_j,$$

$$\sum_{j \in J} x_j = 1,$$

$$x_j \ge 0, j \in J$$
(1)

Here J is the index set related to the extreme points of the plant characteristic and the variables  $x_j$  are used to form the convex combination. [A convex combination is a weighted average with non-negative weights.] The extreme point formulation allows running the plant anywhere within the 3-dimensional polyhedron spanned by the extreme points. However, minimization of costs will always yield solutions on the lower envelope, which is the characteristic illustrated in Fig. 1. The lower envelope corresponds to operating the CHP plant with minimum cost with each combination of heat and power production.

The variables in the hourly CHP model should have a time index (t), but for simplicity they are omitted. In addition, the characteristic corner points  $(\P_i, p_i, c_i)$  can be time-dependent parameters. Using this formulation, the characteristic shape can change hourly. The symbols used in the model formulation are presented in Table I.

TABLE I. MODEL NOMENCLATURE

Symbol	Description			
$c_j, p_j, q_j$	Production cost, power production, and heat production at			
	characteristic point $j \in J$			
P,Q,C	Production of power and heat, production cost			
Х	Variable used to encode convex combination of operating			
	region			
j	Subscript of extreme point in characteristic			
t	Index for time step (hour)			
S	Storage content			
dis	Discharge			
η	Efficiency			
p, q, stor	Subscript for power and heat products, and storage			
J	Set of extreme points of the operating regions of all plants			
T	Set of time periods			

The extreme point modeling technique applies also to separate heat and power production components. Such components include condensing power plants, heat pumps, or heat only boilers. The separate production components can be modelled with either  $p_i = 0$  in heat components and  $q_i = 0$  in power components.

#### B. Linear programming model

The hourly model of the CHP for minimization of overall costs of the net acquisition is as follows:

$$\min \sum_{t=1}^{T} ((\sum_{j \in J} c_j^t x_j^t) - c_p^t P^t)$$
 (2)

$$\sum_{i \in J} x_j^t = 1 \tag{3}$$

$$\sum_{j \in J} x_j^t = 1$$

$$\sum_{j \in J} p_j^t x_j^t = P^t$$
(3)

$$\sum_{j \in J} \boldsymbol{q}_{j}^{t} \boldsymbol{x}_{j}^{t} - \boldsymbol{q}_{stor}^{t} + \eta_{dis} \boldsymbol{q}_{dis}^{t} = Q^{t}$$

$$\tag{5}$$

$$S_{\mathbf{q}}^{t} = \eta_{stor} S_{\mathbf{q}}^{t-1} + \mathbf{q}_{stor}^{t} - \mathbf{q}_{dis}^{t}$$
 (6)

$$0 \le S_{\bullet}^{t} \le S_{\bullet}^{\max} \tag{7}$$

$$x_i^t \ge 0, j \in J \tag{8}$$

$$t = 1, \dots, T \tag{9}$$

The objective function (2) minimizes the overall net acquisition costs consisting of fuel costs  $c^t$  subtracted by revenue from selling power  $c_{\scriptscriptstyle n}P$  to the spot market. Index t corresponds to the time step, S is the storage level variable,  $q_{\it dis}$  is the discharge from the storage and  $q_{\it stor}$  is the amount of heat stored during a period. Because some heat may be lost during storage and also discharging, this is considered with efficiency ratios  $\eta_{stor}$  and  $\eta_{dis}$  . The power balance (4) determines the power production  $P^t$  sold to the market at price  $c_n$  in every hour. The heat balance (5) indicates that combined production, storing and discharge from the storage must equal the demand  $O^t$  each hour.

### III. COMPUTATIONAL RESULTS

Solving the model can serve different purposes. Primarily, the solution can help in short or medium term production planning based on power price and heat demand forecasts. The model can also be used in evaluating the benefit of a heat storage, or to finding an ideal size for the storage.

The model can be defined for an arbitrary time horizon. In the following we consider a one week model (168 hours). Table II lists the key input parameters. The heat demand is slightly modified real-life data for a city in Finland. In this example the characteristic points remain constant over time.

TABLE II. OPERATING POINTS AND INPUT DATA

	Values
Maximun capacity of	Power= 35 MW
CHP plant	Heat=70 MW
	Fuel= 127.27 MW
Heat demand	Weekly heat demand of a finnish city (MWh)
Power price	NordPool spot price in Finland 2013 (€/MWh)
Fuel price	15 (€/MWh)
Storage capacity 1	406 MWh
Storage capacity 2	90 MWh
Storage capacity 3	80 MWh

The power price is actual hourly data provided by NordPool spot [5] for Finland in 2013 in one week. The fuel price is considered constant. We have solved the model with different storage capacities to see the benefit of a heat storage.

To validate the results, we have also modeled and solved the system with two existing software packages, EnergyPRO [19] and EnergyPlan [20]. Figure 2 shows the schematic of the model in EnergyPRO. It is a modeling software package for technical and economic optimization and analysis for cogeneration or trigeneration systems. Figure 3 also shows the schematic of the model in EnergyPLAN. In these software, the modelling of CHP is a little more restricted than a general LP model. We have chosen for this study a model that can be handled by each of the three systems.

Solving the model with a large heat storage of 406 MWh gives the following results. For hourly output variables, Figs. 4 and 5 display the fluctuations of two variables in one week. The weekly heat storage content variation is shown in Fig. 4. Based on the hourly heat demand fluctuations, the CHP plant produces heat to supply the heat demand and after that as much electricity as can be sold profitably to the market.

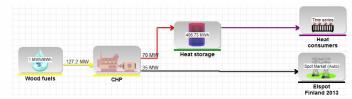


Fig. 2. Schematic of the CHP plant by EnergyPRO

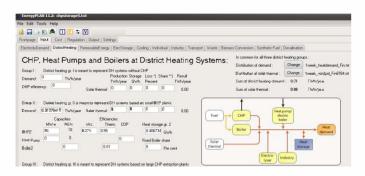


Fig. 3. Schematic of the CHP plant by EnergyPLAN

The excess produced heat is stored in the heat storage so that it can be discharged later, when it is profitable to do so. As can be seen from Fig. 4, there is difference between the initial heat storage levels for EnergyPLAN with other two models in which initial heat storage is considered zero. In EnergyPLAN the initial storage is equal to the total capacity. Thus, the fluctuations of LP and EnergyPRO are similar, but different from EnergyPLAN in the early time periods, since it starts from the maximum amount of heat storage content. LP also has the potential to have diverse heat storage tanks with different initial contents which is crucial when there are more than one heat reservoirs in the CHP plant with different capacities. Fig. 5 shows the variations in fuel consumption in one week by LP. It is a crucial parameter because of the pollutant emission and also production cost.

Table III shows the optimized values for power production, fuel consumption, and objective function using LP and the two software packages. The results are very similar, and the comparison of results for three models is highly satisfactory.

Solving the model with a smaller heat storage of 90 MWh gives a total cost of 101 994 €, which is 278 € more than with the large storage. Thus the larger storage is more profitable in operation compared to the smaller one. In investment planning this cost difference should be compared with the investment costs of the corresponding heat storages, and also the simulation should be extended over a full year. This is outside the scope of this paper.

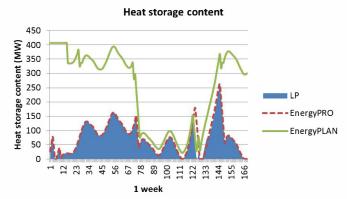


Fig. 4. Heat storage content in one week

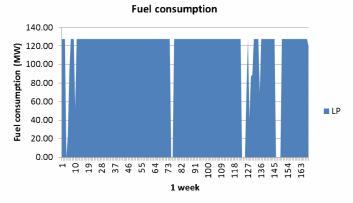


Fig. 5. Fuel consumption in a week by LP

Finally, solving the model using the smallest storage of 80 MWh using LP reveals that the system is infeasible, i.e. the production capacity is not able to satisfy the demand at some peak hours. With a larger storages it was possible to accumulate enough heat during hours of low demand to manage the peak hours, but a storage of 80 MWh is too small for this. EnergyPlan did not detect the infeasibility. EnergyPRO gave a warning message "Heat demand not met", but did give an (infeasible) solution.

TABLE III. THE VALIDATION OF DECISION VARIABLES

Decision variables	LP	EnergyPRO	EnergyPLAN
Power production (MW)	5 392	5 392	5 338
Fuel consumption (MW)	19 607	19 607	20 000
Total cost (€)	101 716	101 765	110 000

#### IV. CONCLUSION

Production planning of CHP with a heat storage is a difficult task, but it can be done using an optimization model leading to significant savings in operating costs. A short term production plan is needed for trading the CHP power on the spot market. This study concentrated on the economic optimization of CHP production in the weekly time horizon. The heat storage improves the flexibility of the CHP system by relaxing the connection between heat and power production, allowing heat to be produced in advance during low demand to be used when the demand is high. This can also improve the revenue from power sales, as more power can be produced during price peaks. The problem was formulated as an LP model and solved using an general purpose sparse simplex code. The model was tested using actual heat demand and power price data and with three different sizes for the heat storage. The proposed model can optimize the CHP efficiently and reliably. The advantage with an LP formulation in comparison to dedicated energy optimization systems is that the LP model can be flexibly extended with new kinds of production technologies, and it is easy to perform various analyses.

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#### REFERENCES

- S. Kelly and M. Pollitt, "An assessment of the present and future opportunities for combined heat and power with district heating (CHP-DH) in the United Kingdom," *Energy Policy*, vol. 38 no.11, pp.38-48, 2010.
- [2] L. Majic, I. Krzelj, and M. Delimar, "Optimal scheduling of a CHP system with energy storage," 36th International Convention on Information & Communication Technology Electronics & Microelectronics (MIPRO), IEEE, 2013, pp. 1253-1257.
- 3] H. Wang, J. Wenling, R. Lahdelma, Z. Pinghua, and Z. Shuhui, "Atmospheric environmental impact assessment of a combined district heating system," *Building and Environment*, vol. 64, pp. 200-212, 2013.

- [4] L. Stankeviciute and A. K. Riekkola, "Assessing the development of combined heat and power generation in the EU," *International Journal* of Energy Sector Management, vol. 8, pp.76-99, 2014.
- [5] Nord pool spot:Website: http://www.nordpoolspot.com/Market-data1/Elspot/Area-Prices/ALL1/Hourly/
- [6] A. Costa and A. Fichera, "A mixed-integer linear programming (MILP) model for the evaluation of CHP system in the context of hospital structures," *Applied thermal engineering*, In Press, Corrected Proof, DOI: 10.1016, 02.051, ISSN: 13594311, 2014.
- [7] M. A. Gonzalez Chapa and J. R. Vega Galaz, "An Economic Dispatch Algorithm For Cogeneration Systems," *Power Engineering Society General Meeting*, *IEEE*, 2014, vol.1, pp. 989 -994.
- [8] A. Rong, R. Lahdelma, M. Grunow, "An improved unit decommitment algorithm for combined heat and power systems," *European Journal of Operational Research*, vol. 195, pp. 552-562, 2009.
- [9] A.K. Basu, Electr. Eng. Dept., C.I.E.M., Kolkata, India; Bhattacharya, A.; Chowdhury, S.; Chowdhury, S.P., "Planned Scheduling for Economic Power Sharing in a CHP-Based Micro-Grid," *IEEE Transactions on Power Systems*, vol. 27, pp. 30-38, 2012.
- [10] C. Marnay, G. Venkataramanan, M.Stadler, A.S. Siddiqui, R. Firestone, and B. Chandran, "Optimal Technology Selection and Operation of Commercial-Building Microgrids," *IEEE TRANSACTIONS ON POWER SYSTEMS*, vol. 23, no. 3, aug. 2008.
- [11] S. Makkonen and R. Lahdelma, "Non-convex power plant modeling in energy optimization," *European Journal of Operational Research*, vol. 171, pp. 1113-1126, 2006.
- [12] H.Wang, W. Jiao, R. Lahdelma, and P. Zou, "Techno-economic analysis of a coal-fired CHP based combined district heating system with gasfired boilers for peak load compensation," *Energy Policy*, vol. 39, pp. 7950-7962, 2011.
- [13] J. Vandewalle, N. Keyaerts, and W. D'haeseleer, "The role of thermal Storage and Natural Gas in a Smart Energy System," 9th International Conference on the European Energy Market (EEM), IEEE, 2012, pp. 1-9.
- [14] D. HENNING, "Cost minimization for a local utility through chp, heat storage and load management," *International journal of energy* research, vol. 22, pp. 691-713, 1998.
- [15] C. Marnay, H. Asano, S. Papathanassiou, and G. Strbac, "Policymaking for microgrids," *Power and Energy Magazine*, *IEEE*, vol. 6, pp. 66-77, 2008.
- [16] A. Rong, R. Lahdelma, and P.B. Luh, "Lagrangian relaxation based algorithm for trigeneration planning with storages," European Journal of Operational Research, vol. 188, pp. 240–257, 2008.
- [17] R. Lahdelma, J. Nurminen, and S. Ruuth, "Implementations of LP- and MIP systems," EURO VIII, Helsinki University of Technology, Systems Analysis Laboratory Research Reports A18, Lisbon 1986.
- [18] R. Lahdelma and H. Hakonen, "An efficient linear programming algorithm for combined heat and power production," *European Journal* of Operational Research, vol 148, pp. 141-151, 2003.
- [19] EnergyPRO, EMD International A/S Niels Jernes Vej 10 9220 Aalborg Ø Denmark.
  - Website: http://www.emd.dk/
- [20] EnergyPLAN 11.2, Aalborg University, Denmark. Website: <a href="http://www.energyplan.eu/">http://www.energyplan.eu/</a>
- [21] E. Dotzauer, "Simple model for prediction of loads in district-heating systems," Applied Energy, Vol. 73, pp. 277-284, 2002.
- [22] Asko Vuorinen, "Planning of optimal power systems," publisher: Ekoenergo Oy, Finland, ISBN: 978-952-67057-1-2, 2008.
- [23] A. Rong, R. Lahdelma, "Efficient algorithms for combined heat and power production planning under the deregulated electricity market," *European Journal of Operational Research*, vol. 176, pp. 1219–1245, 2007
- [24] N. Alguacil, and A.J. Conejo, "Multiperiod optimal power flow using Benders Decomposition," *Transactions on Power Systems, IEEE*, vol. 15, pp. 196–201, 2000.

- [25] R. Baldick, "The generalized unit commitment problem," Transactions on Power Systems, IEEE vol. 10, pp. 465–475, 1995.
- [26] X. Guan, P. Luh, and L. Zhang, "Nonlinear approximation method in Lagrangian relaxation-based algorithms for hydrothermal scheduling," *Transaction on Power Systems, IEEE*, vol. 10, pp. 772–778, 1995.
- [27] S.J. Wang, S.M. Shahidehpour, D.S. Kirschen, S. Mokhtari, and G.D. Irisarri, "Short-term generation scheduling with transmission and environmental constraints using an augmented Lagrangian relaxation," *Transaction on Power Systems, IEEE*, vol. 10, pp. 1294–1301, 1995.
- [28] M.S. Bazaraa and C.M. Shetty, "Nonlinear Programming Theory and Algorithms." Wiley, New York, NY, 1993.
- [29] G. Dantzig, "Linear Programming and Extensions," Princeton University Press, Princeton, NJ, 1963.