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Adaptive numerical methods for computing the Greeks of barrier options

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Received: date / Accepted: date

Abstract We consider space adaptive finite difference methods for the Black–Scholes partial differential equation for computing the prices and Greeks of barrier options. Space adaptivity is based on a monitoring function from the discrete first and second derivatives of the numerical solution. Computational results are given for barrier option problems. Using these space adaptivity techniques, we can significantly increase the accuracy and decrease the computational time of the numerical solutions.

Keywords Black–Scholes equation · barrier option · finite difference method · adaptive space

1 Introduction

In this paper, we develop an accurate and efficient numerical method to solve the Black–Scholes (BS) equation for the barrier option problems:

$$\frac{\partial u}{\partial \tau} = \frac{1}{2}(\sigma x)^2 \frac{\partial^2 u}{\partial x^2} + rx\frac{\partial u}{\partial x} - ru, \text{ for } (x,\tau) \in \Omega \times (0,T], \tag{1}$$

where $u(x,\tau)$ is the value of the derivative security, x is the value of the underlying security, $\tau = T - t$ is the time to maturity, t is the current time, T is the expiry date, r is the risk-free interest rate, and σ is the volatility of the underlying asset [2,8]. $\Omega = (0, L_x)$ is the spatial computational domain and the initial condition is given by $u(x,0) = p(x)$.

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In this paper, we propose a space-time adaptive technique for the prices and Greeks of barrier options: space adaptivity which is based on a monitoring function from the discrete first and second derivatives of the numerical solution.

This paper is organized as follows. In Section 2, we describe the space-time adaptivity algorithm in detail. Section 3 provides numerical results. Conclusions are given in Section 4.

2 Numerical solution

In this paper, we focus on the up and out call option. The payoff function is $p(x) = \max(x - K, 0)$ and the boundary condition is $u(0, t) = u(H, t) = 0$, where K is the strike price and H is the knock out barrier (see Fig. 1).

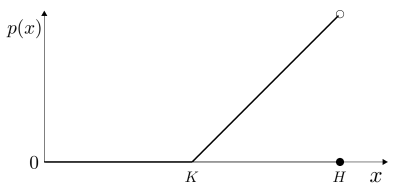


Fig. 1 Payoff function $p(x)$ of an up and out call option.

2.1 Discretization with finite differences

The finite difference method has been applied to pricing financial contracts. The BS equation is discretized on a grid defined by $x_0 = 0$ and $x_{i+1} = x_i + h_i$

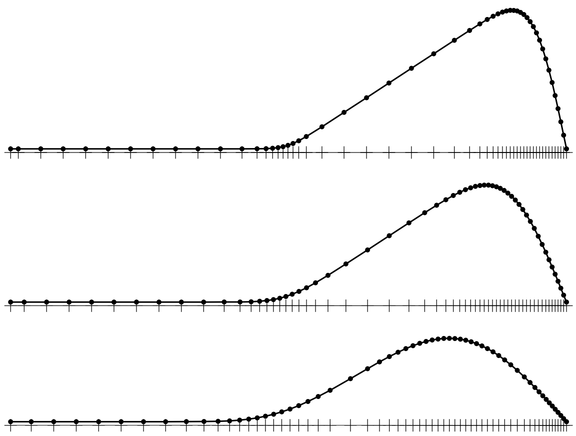
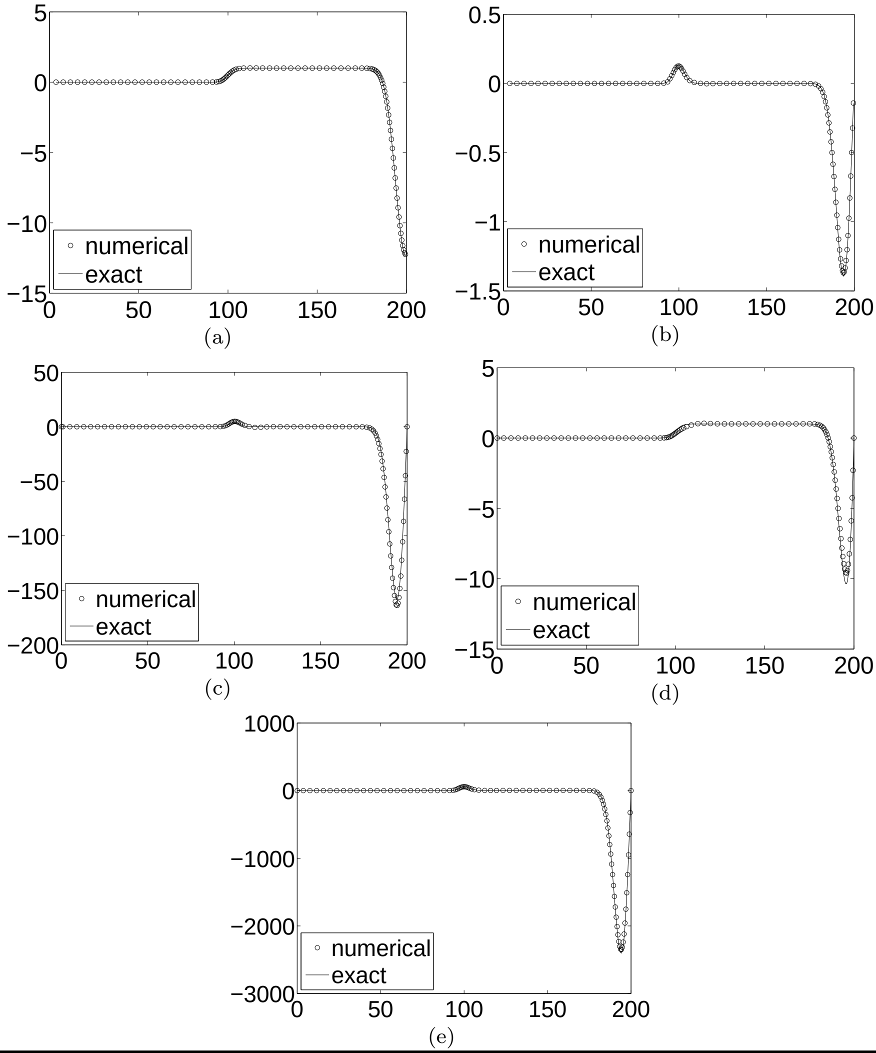


Fig. 6 Adaptive grids which are generated by $h(x)$ and the solution on the grids as time increases (from up to bottom).



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