VICI: Option Task Overview

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Task 1: ATM IV & Slope

1. 篩選 TXO 與 TX data (expiry: 202308)
2. 將 tick 資料 resample 為 1min k
3. 依時間逐步計算 Implied Volatility
4. 使用 SVI 模型進行 IV Calibration
5. 使用 SVI 計算 ATM IV 與 ATM Slope
6. 繪製圖表：Future 與 ATM IV、Future 與 ATM Slope

Task 2: Realized Volatility

1. 將 TX tick 資料 resample 為 10s k (為了降噪不使用 tick)
2. 計算 5min rolling 的 Realized Vol & Rogers-Satchell Vol
3. 將結果 resample 為 1m 級別並繪製圖表

Task 1: ATM IV & Slope

Compute ATM IV & Slope

- Use SVI calibration:
  - Smooth
  - Well-behaved at large strike, has a linear tail
  - Can extrapolation, unlike linear/cubic spline
  - Butterfly arbitrage-free (theor)

Basic Setting

Set:

- $r = 0$
- $T = \text{ts} - \text{“2023-08-16 13:30:00”}$ and annualized (252 days)
- $F =$ TX futures price (as the forward for convenience)

BS model:

$$\text{Call} = e^{-rT}\left[F\Phi(d_1) - K\Phi(d_2)\right]$$
$$\text{Put} = e^{-rT}\left[K\Phi(-d_2) - F\Phi(-d_1)\right]$$

with $d_1 = \frac{\log(F/K)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$

SVI Model

On a total implied variance curve (obtained from an IV smile), the curve is modeled as:

$$w_{\text{SVI}}(k; a, b, \rho, m, \sigma) = a + b\left(\rho\,(k-m) + \sqrt{(k-m)^2 + \sigma^2}\right)$$

- Model parameters: (a, b, ρ, m, σ)
- $k = \ln \frac{K}{F}$, log-moneyness
- Find the *best* params (a, b, ρ, m, σ) to **calibrate the total implied variance curve** $w_{\text{imp}}(k)$

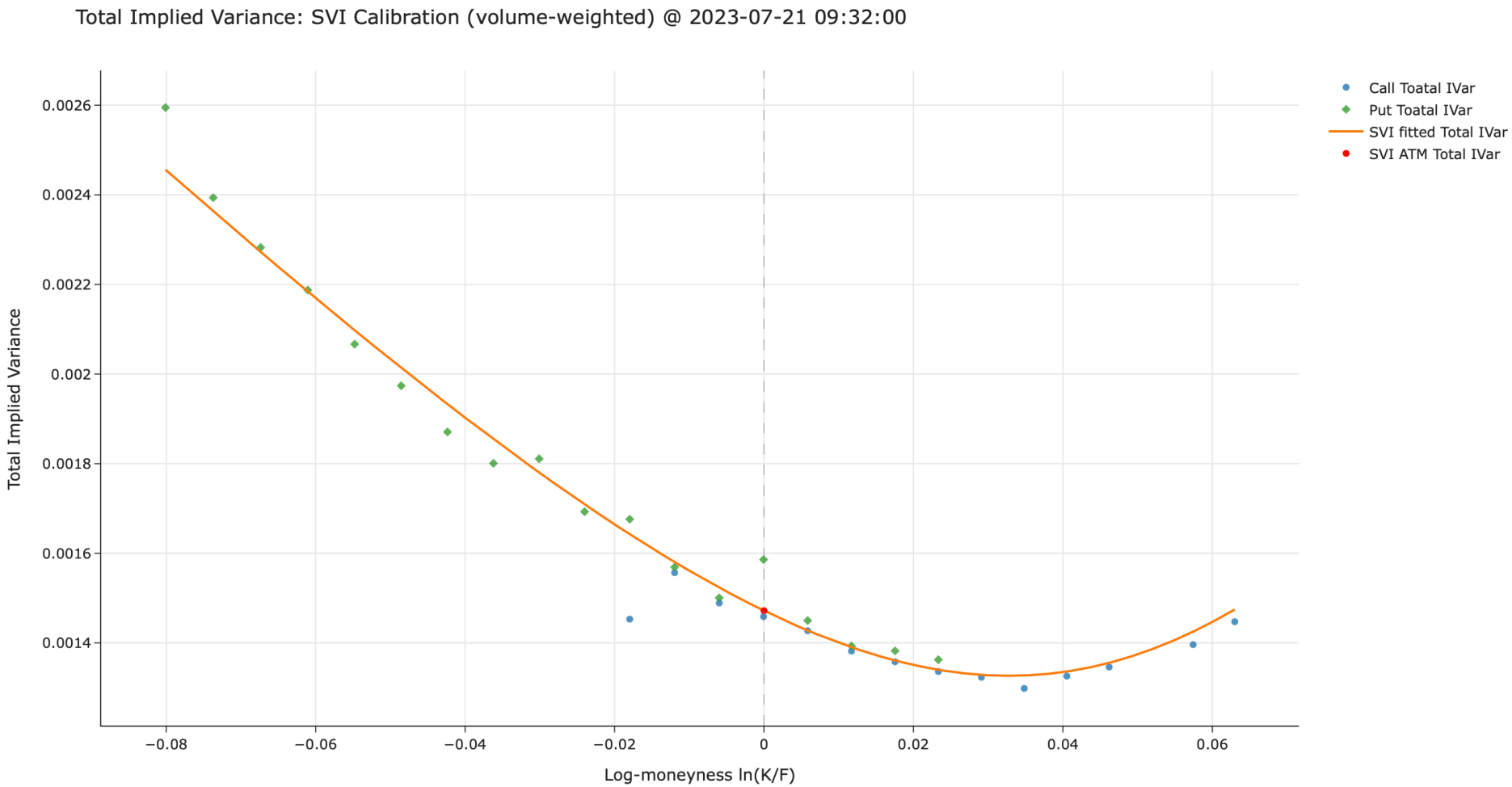
SVI Model: Calibration

1. Select OTM and near-ATM options:
   - Calls: $k > -0.1$
   - Puts: $k < 0.2$
2. Calculate their IVs
3. Get total implied variances: $(k_i, w_{\text{imp}}(k_i))$
4. Define a volume-weighted loss function:

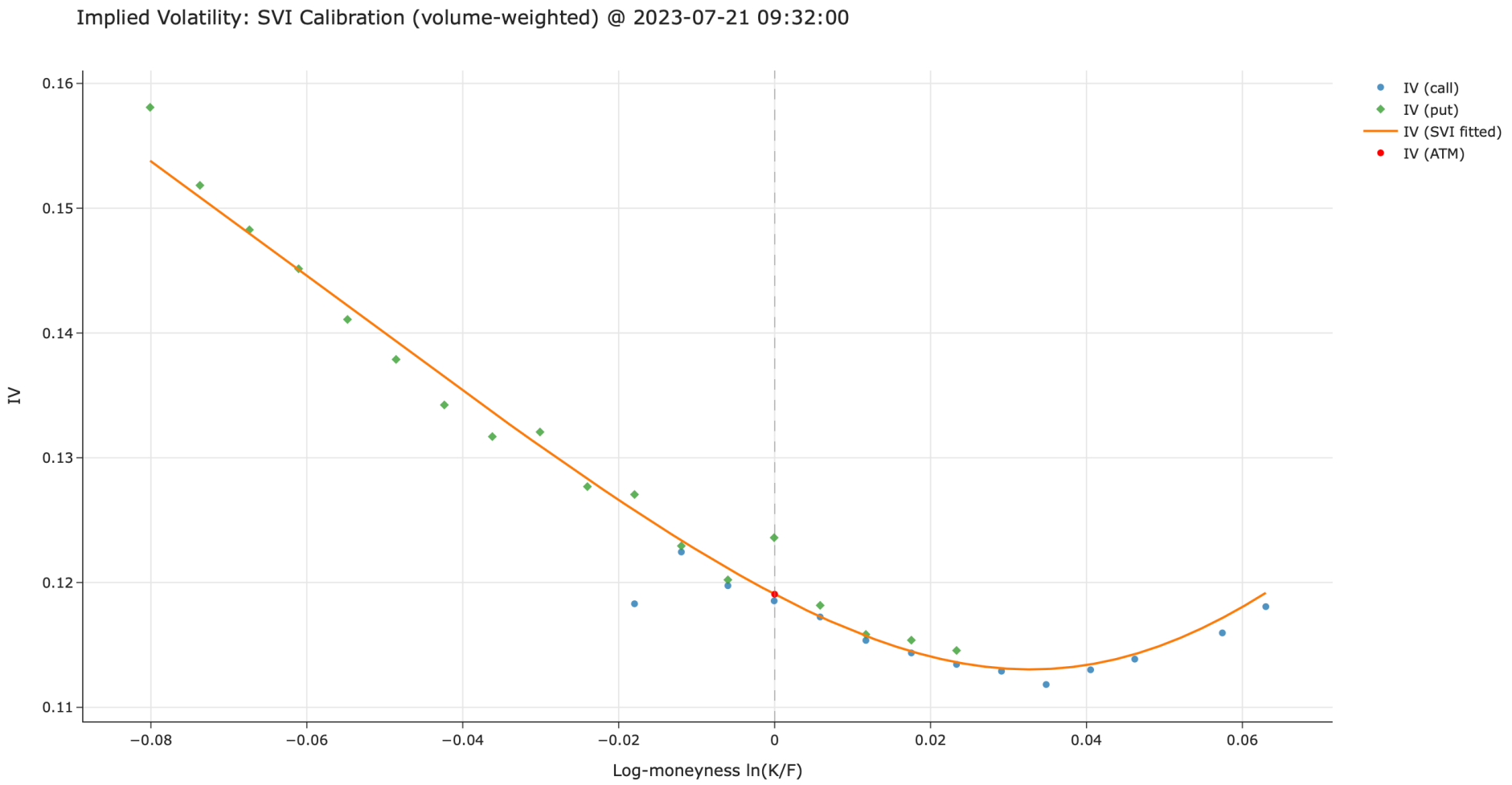
$$L(a, b, \rho, m, \sigma) = \sum_i \text{volume}_i \cdot \left(w_{\text{imp}}(k_i) - w_{\text{SVI}}(k_i; a, b, \rho, m, \sigma)\right)^2$$

and find the best params (a, b, ρ, m, σ)

SVI Model: Total Implied Variance



SVI Model: Implied Volatility Smile



SVI Model: ATM metrics

- **ATM IV**:

$$\sigma_{\text{SVI}}(k)\Big|_{k=0} = \sqrt{\frac{w_{\text{SVI}}(0)}{T}} = \sqrt{\frac{a + b\left(-\rho m + \sqrt{m^2+\sigma^2}\right)}{T}}$$

- **ATM Slope**:

$$\frac{d\sigma_{\text{SVI}}}{dk}(k)\Big|_{k=0} = \frac{d}{dk}\sqrt{\frac{w_{\text{SVI}}(k)}{T}}\Big|_{k=0} = \frac{b\left(\rho - \frac{m}{\sqrt{m^2+\sigma^2}}\right)}{2T\sigma_{\text{SVI}}(0)}$$

Task 2: Realized Volatility