Driver assistance system design A

Lane keeping - part 1

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Outline

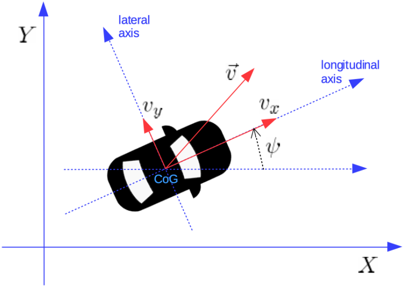
1. Introduction
2. Dynamic single-track model
3. Lane keeping
4. Simulation results

Introduction

- The goal of lane keeping systems is to maintain the vehicle within the lane through a control action on the steer.
- Indeed, lateral dynamics is unstable and control is necessary to keep the vehicle in the lane (in manual driving, controller = driver).
- The control system is not intended to replace the driver.
- The control system is aimed to improve safety:
  - help the driver in emergency situations, e.g. in the case of tiredness, lack of attention, critical road conditions, etc.
  - reduce the driver's tiredness/drowsiness.

Dynamic single-track model

- To investigate the lane keeping problem and design suitable controllers, we recall the dynamic single-track (DST) model.



- Vehicle variables:

  X, Y: coordinates of the vehicle CoG in an inertial reference frame

  ψ: yaw angle

  $\omega_\psi \doteq \dot{\psi}$: yaw rate

  $\vec{v} \equiv V$: velocity vector in the inertial frame

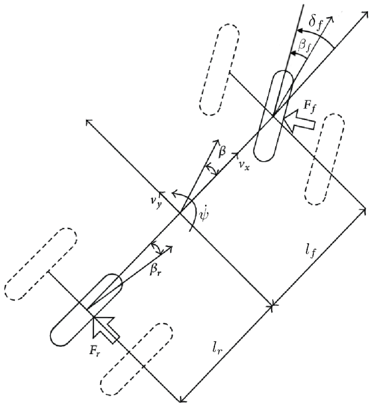
  v_x: longitudinal speed $= \vec{v}$ component along the longitudinal axis

  v_y: lateral speed $= \vec{v}$ component along the lateral (transverse) axis

  a_x: longitudinal acceleration in the inertial frame.

Dynamic single-track model

- The DST model (or bicycle model) is characterized by a single-track (two wheels: one rear wheel, one front wheel), which is equivalent to a vehicle with four wheels, where the left part is equal to the right part.



- Vehicle variables:
  δ_f: steering angle
  β: vehicle slip angle = angle between the vehicle longitudinal axis and velocity
  β_f, β_r: tire slip angles = angles between the tire longitudinal axis and velocity.
- Vehicle parameters:
  CoG: center of gravity
  m, J: mass and moment of inertia
  l_f: distance CoG - front wheel center
  l_r: distance CoG - rear wheel center
  c_f, c_r: front/rear cornering stiffnesses.

Dynamic single-track model

- The state equations of the DST model are

$$
\begin{aligned}
\dot{X} &= v_x \cos\psi - v_y \sin\psi \\
\dot{Y} &= v_x \sin\psi + v_y \cos\psi \\
\dot{\psi} &= \omega_\psi \\
\dot{v}_x &= v_y \omega_\psi + a_x \\
\dot{v}_y &= -v_x \omega_\psi + \frac{2}{m}\left(F_{yf} + F_{yr}\right) \\
\dot{\omega}_\psi &= \frac{2}{J}\left(l_f F_{yf} - l_r F_{yr}\right)
\end{aligned}
$$

  where F_{yf} and F_{yr} are the lateral forces exchanged between tire and road. Different tire models can be considered (next slide).

- State: $\zeta = (X, Y, \psi, v_x, v_y, \omega_\psi)$, input: $u = (a_x, \delta_f)$.

Dynamic single-track model

- Linear (for $v_x = const$) tire model:

$$\begin{aligned} F_{yf} &= -c_f \beta_f, \quad F_{yr} = -c_r \beta_r \\ \beta_f &= \frac{v_y + l_f \omega_\psi}{v_x} - \delta_f, \quad \beta_r = \frac{v_y - l_r \omega_\psi}{v_x}. \end{aligned} \tag{1}$$

- Nonlinear simplified tire model:

$$\begin{aligned} F_{yf} &= -c_f \beta_f \cos \delta_f, \quad F_{yr} = -c_r \beta_r \\ \beta_f &= \operatorname{atan}\left(\frac{v_y + l_f \omega_\psi}{v_x}\right) - \delta_f, \quad \beta_r = \operatorname{atan}\left(\frac{v_y - l_r \omega_\psi}{v_x}\right). \end{aligned} \tag{2}$$

- Nonlinear Pacejka's tire model:

$$F_{yf} = -f_P(\beta_f) \cos \delta_f, \qquad F_{yr} = -f_P(\beta_r) \tag{3}$$

where β_f and β_r are those in (2) and $f_P(\beta)$ is given by the Pacejka's magic formula (next slide).

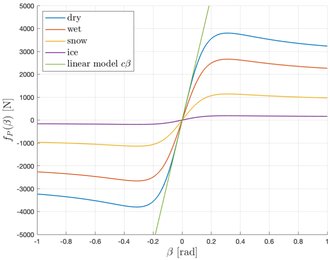
Dynamic single-track model

- Pacejka's magic formula:

$$f_P(\beta) \doteq p_1 \sin\left(p_2 \operatorname{atan}\left(p_3\beta - p_4\left(p_3\beta - \operatorname{atan}(p_3\beta)\right)\right)\right)$$

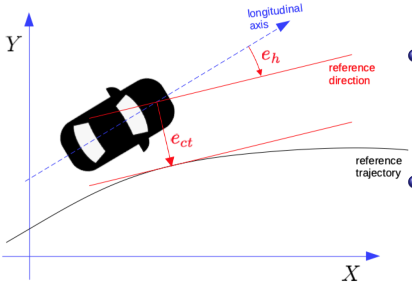
p_1: peak value, p_2: shape factor, p_3: stiffness factor, p_4: curvature factor. Linearizing this formula, we find $p_1 p_2 p_3 = c_f$ (or c_r).

  - In real applications, these parameters are difficult to measure/estimate.
  - They change in function of the road conditions.



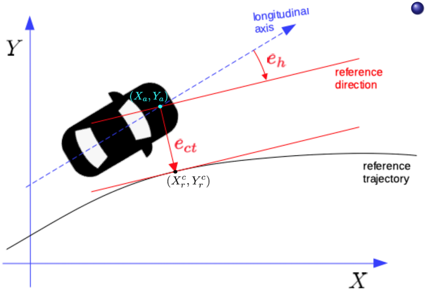
Lane keeping

- The goal of lane keeping is to maintain the vehicle within the lane or, more precisely, to have the vehicle track a given reference trajectory.
- The lateral dynamics is unstable. For constant v_x, the linearized DST model have unstable eigenvalues.
- The following tracking errors can be suitably used for lane keeping control.



- Heading error e_h: angle between the vehicle longitudinal axis and the reference direction.
- Cross-track error e_{ct}: displacement from the center of the vehicle front axle to the closest point on the trajectory.

Lane keeping



- To compute the two errors, we define the following quantities:

  $p_a \doteq (X_a, Y_a, \psi)$: vehicle front axle pose

  $P_r \doteq \{p_r^1, \dots, p_r^N\}$: reference trajectory

  $p_r^i \doteq (X_r^i, Y_r^i, \psi_r^i) \in P_r$: reference pose

  (X_r^c, Y_r^c): trajectory point closest to the vehicle: $c = \arg\min_i \left\|(X_r^i, Y_r^i) - (X_a, Y_a)\right\|$

  ψ_r^c: corresponding reference yaw angle.

- Heading error: $e_h \doteq \psi_r^c - \psi$.
- Cross-track error: $e_{ct} \doteq (Y_r^c - Y_a)\cos\psi_r^c - (X_r^c - X_a)\sin\psi_r^c$.

  e_{ct} is the signed distance between the vehicle and the reference trajectory (see next slide).

Lane keeping

- Define the 3D vectors

$$\rho \doteq \begin{bmatrix} X_r^c - X_a \\ Y_r^c - Y_a \\ 0 \end{bmatrix} \qquad \text{vector from } (X_a, Y_a) \text{ to } (X_r^c, Y_r^c)$$

$$\xi \doteq \begin{bmatrix} \cos\psi_r^c \\ \sin\psi_r^c \\ 0 \end{bmatrix} \qquad \text{reference direction of motion.}$$

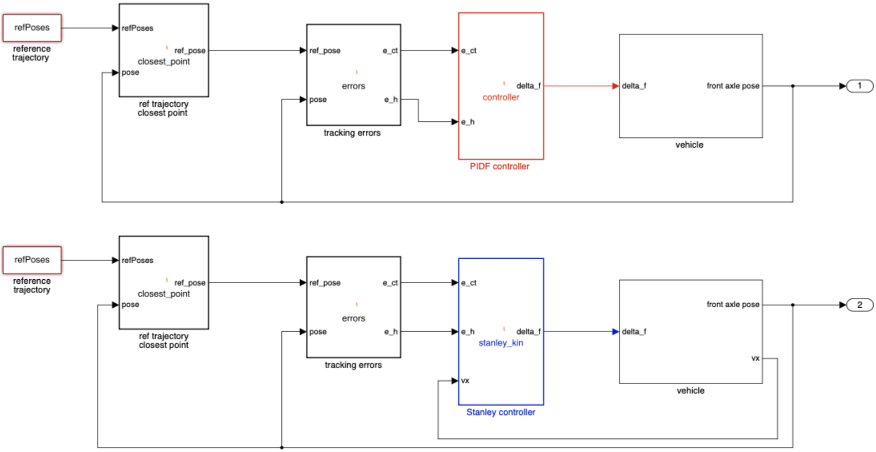
- Their cross-product is $\xi \times \rho = \begin{bmatrix} 0 \\ 0 \\ e_{ct} \end{bmatrix}$.
- Since ξ and ρ are orthogonal, $\|\xi \times \rho\| = \|\xi\| \, \|\rho\| = \|\rho\|$. Hence,

$$|e_{ct}| = \|\rho\| = \|(X_r^c, Y_r^c) - (X_a, Y_a)\| \,.$$

- The sign of e_{ct} is as follows:
  - vehicle on the left of reference trajectory $\iff e_{ct} < 0$
  - vehicle on the right of reference trajectory $\iff e_{ct} > 0$.

Lane keeping

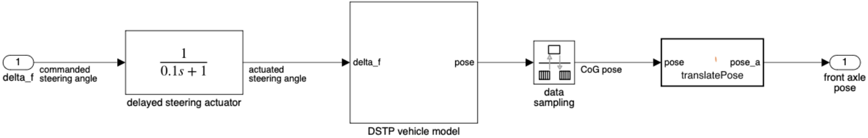
Closed-loop schemes



- Two controller types considered: PIDF, Stanley.

Lane keeping
Vehicle model

- DSTP: dynamic single-track vehicle model with Pacejka's tire formula.
  - Vehicle parameters: $l_f = 1.2\,\mathrm{m}$, $l_r = 1.6\,\mathrm{m}$, $m = 1575\,\mathrm{kg}$, $J = 4000\,\mathrm{kg\,m^2}$, $c_f = c_r = 27e3\,\mathrm{N/rad}$.
  - Tire parameters: $p_1 = 3863\,\mathrm{N}$, $p_2 = 1.5$, $p_4 = -0.5$, and $p_3 = c_f/p_1/p_2$ (front tire) or $p_3 = c_r/p_1/p_2$ (rear tire).
  - A constant speed v_x is directly imposed in "DSTP vehicle model".
- Other blocks:
  - steering actuator dynamics
  - pose translation from CoG to front axel
  - data sampling blocks (sampling time $= 0.05\,\mathrm{s}$).



Lane keeping

Designed controllers

- **PIDF1** (discrete-time):
  - Uses the cross-track error as the feedback variable.
  - Designed using the Simulink PID tuner (sampling time $T_s = 0.05\,\mathrm{s}$).
- **PIDF2** (discrete-time):
  - Uses the cross-track error and heading error as feedback variables.
  - Designed using the Simulink PID tuner (sampling time $T_s = 0.05\,\mathrm{s}$).
- **Stanley kinematic controller**:
  - "Classical" geometric path tracking controller, consisting of a simple nonlinear feedback law (winner of a DARPA challenge).
  - The variables used for feedback are the cross-track error and the heading error.

Lane keeping

PID controllers

- Discrete-time PIDF controller general form:

$$C(z) = K_P + K_I \tfrac{T_s}{z-1} + K_D \tfrac{z-1}{T_f(z-1)+T_s}.$$

- Consider the following controllers:
  - $C_{ct}(z)$: PIDF controller using e_{ct} as the feedback variable.
  - $C_h(z)$: PIDF controller using e_h as the feedback variable.
- PIDF1 control law: $\delta_f = C_{ct}(z)\, e_{ct}$.
- PIDF2 control law: $\delta_f = C_{ct}(z)\, e_{ct} + C_h(z)\, e_h$.
- Controller design: PIDF controllers' parameters designed using the Simulink PID tuner, assuming a constant speed $v_x = 40\,\mathrm{km/h}$.

Lane keeping

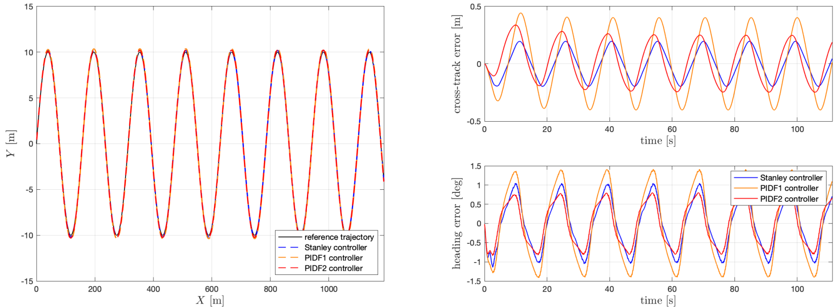
Stanley's kinematic controller

- Define
$$\delta_o = e_h + \operatorname{atan}\left(\frac{K_S\, e_{ct}}{v_b + v_x}\right)$$
where v_x is the vehicle long. speed, v_b is a "small" term introduced to avoid a null denominator, and K_S is a gain to be tuned.
- Stanley's kinematic control law:
$$\delta_f = \operatorname{sat}(\delta_o, \delta_m) \doteq \begin{cases} \delta_m, & \delta_o \geq \delta_m \\ \delta_o, & -\delta_m < \delta_o < -\delta_m \\ -\delta_m, & \delta_o \leq -\delta_m \end{cases}$$
where δ_m is the maximum steering angle.
- When applied to the kinematic single-track model, this control law guarantees convergence to zero of e_{ct} and e_h.
- Parameters: $K_S = 2$, $v_b = 1$, $\delta_m = 35^o$ (from a Matlab tutorial).

Lane keeping

Simulation results for $v_x = 40\,\mathrm{km/h}$

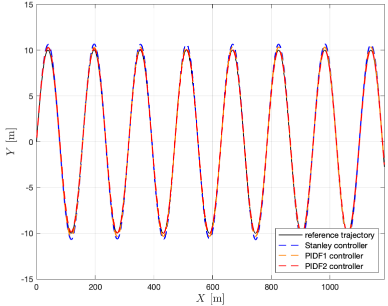
- Relatively challenging reference trajectory: $Y_r = 10\sin(0.04X_r)$.

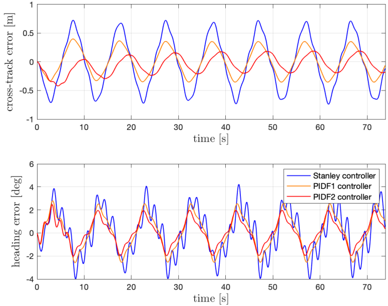


- For all the controllers $|e_{ct}| < 0.5\,\mathrm{m}$.
- Stanley provides the best performance: $|e_{ct}| < 0.2\,\mathrm{m}$.
- PDF2 shows improvements wrt PDF1.

Lane keeping

Simulation results for $v_x = 60\,\mathrm{km/h}$

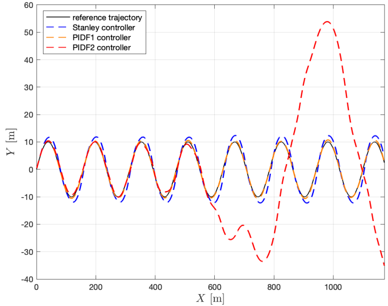


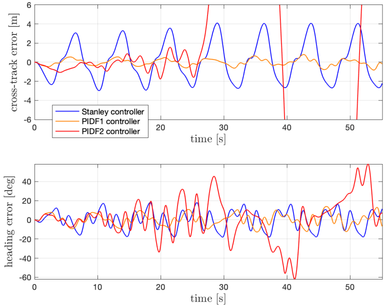


- The performance of all controllers worsen.
- Stanley shows the worst degradation.
- PDF2 shows improvements wrt PDF1.
- In (more realistic) scenarios with smaller curvatures, smaller errors are obtained.

Lane keeping

Simulation results for $v_x = 80\,\mathrm{km/h}$





- PDF2 does not work anymore (failure depends on the tuning).
- PDF1 provides the best performance $|e_{ct}| < 1\,\mathrm{m}$. However, values $|e_{ct}| \cong 1\,\mathrm{m}$ are not acceptable in real scenarios.
- **Remark:** LTI controllers may only work in certain speed ranges and fail in other ranges $\rightarrow$ gain-scheduling/nonlinear control.