Driver assistance system design A

Observers

Carlo Novara
Politecnico di Torino
Dip. Elettronica e Telecomunicazioni

Observer design – Introduction

- In most practical situations, the state of the system to control (plant P) cannot be measured, and thus full state feedback cannot be performed.

- In these situations, the state can be estimated by an observer.

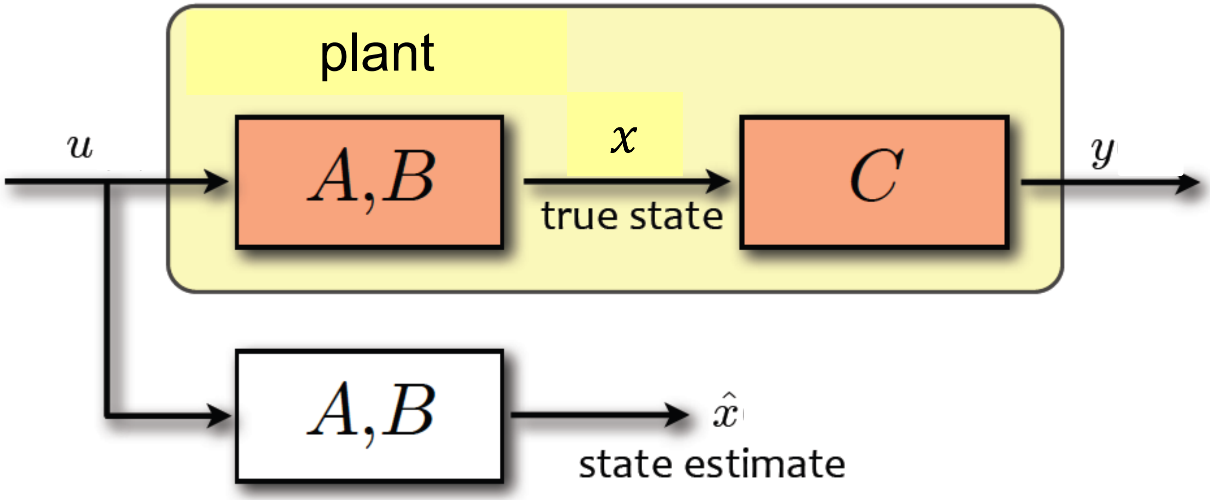
- Two possible options:

  - open-loop observer.

  - closed-loop observer.

Observer design – Open-loop observer

- A copy (or a model) of the plant is used as the observer.



Sistem A, B: $\dot{x} = Ax + Bu$

Observer A, B: $\dot{\hat{x}} = A\hat{x} + Bu$

C: gain matrix

$D = 0$ (only in the figure)

Observer design – Open-loop observer: Estimation error dynamics

- Plant: $\dot{x} = Ax + Bu$

  $y = Cx + Du$

  Observer: $\dot{\hat{x}} = A\hat{x} + Bu$

  Estimation error: $e = x - \hat{x}$

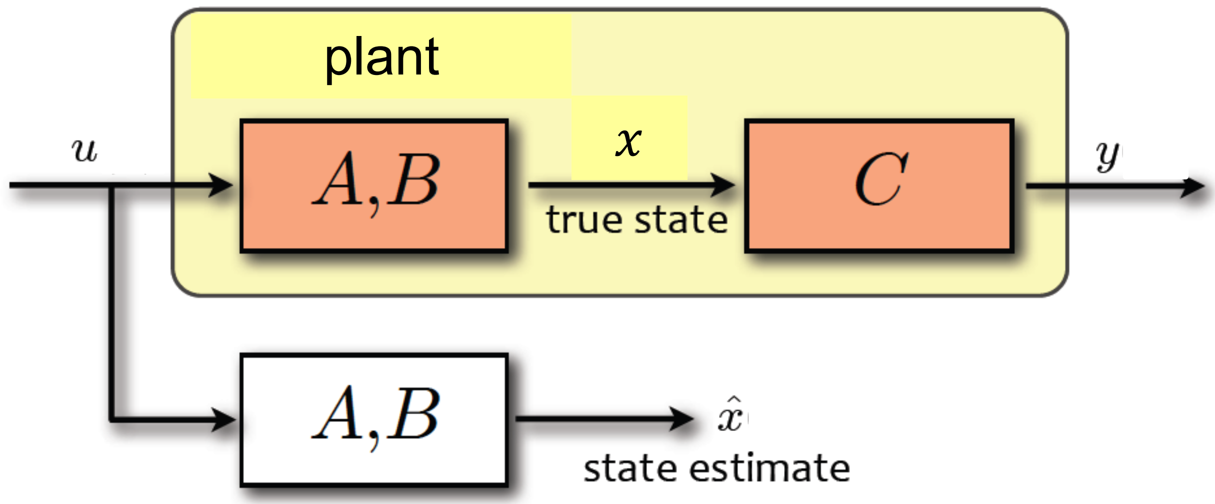
- Dynamics of the estimation error:

$$\dot{e} = Ae$$

← does not depend on u

← can be unstable

Observer design – Open-loop observer



- Drawbacks:
  - divergent estimation error if the plant is unstable;
  - less accurate estimate;
  - no noise attenuation.

Observer design – Closes-loop observer: basic idea

- Plant:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du.$$

- Observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$
$$\hat{y} = C\hat{x} + Du$$

The idea is to improve the estimate $\hat{x}$ by using the measured y. The matrix L has to be suitably designed (as discussed later).

- Combining the above equations:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x} - Du)$$
$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + (B - LD)u.$$

The latter equation is the so-called Luenberger observer.

Observer design – Closes-loop observer: Estimation error dynamics

- Plant: $\dot{x} = Ax + Bu$

  $y = Cx + Du$

  Observer: $\dot{\hat{x}} = (A - LC)\hat{x} + Ly + (B - LD)u$

  Luenberger observer

  Estimation error: $e = x - \hat{x}$

- Dynamics of the estimation error:

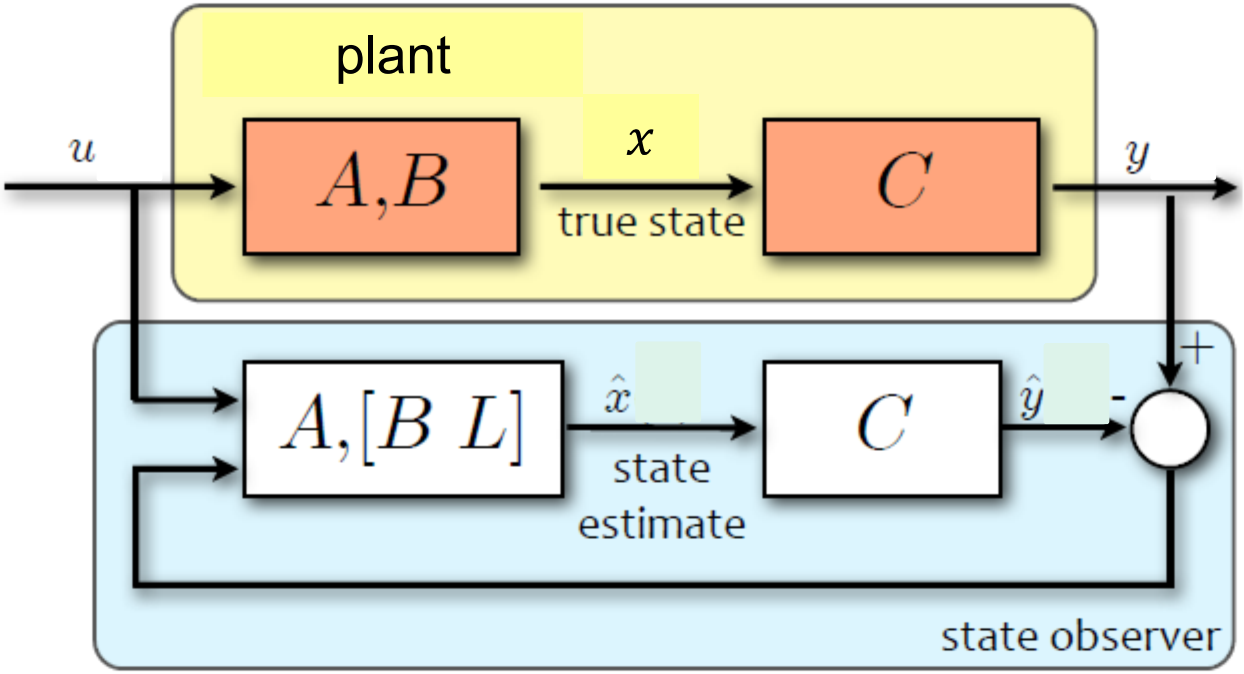
$$\dot{e} = (A - LC)e$$

← does not depend on u

← L is chosen to stabilize the error dynamics

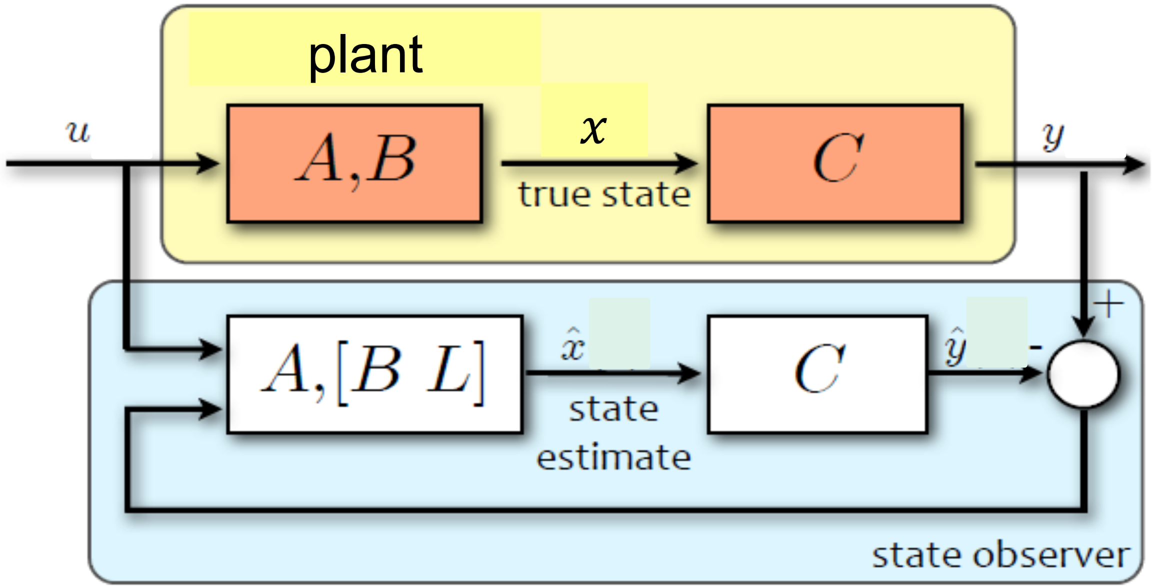
$$\lim_{t \to \infty} \hat{x}(t) = x(t)$$

Observer design – Closed-loop observer



- The estimation is corrected at each time t by means of a feedback from the estimation error.

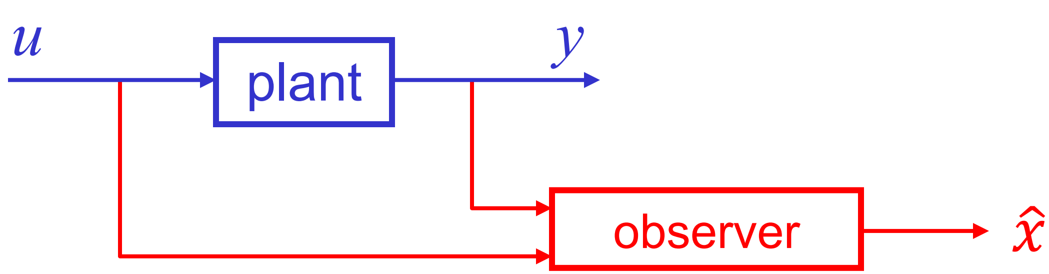
Observer design – Closed-loop observer



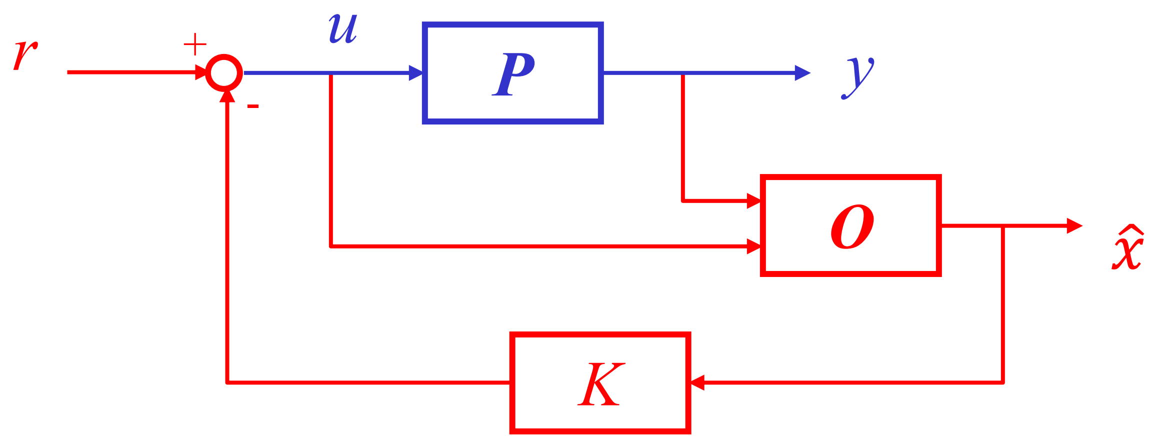
- A closed-loop observer is a dynamic system with

  - input: (u,y)

  - output: an estimate $\hat{x}$ of the system state



Observer design – Feedback control form the estimated state



- The eigenvalues of the observer ***O*** (of $A\text{-}LC$) should be stable and placed in such a way that $\mathrm{Re}[\mathrm{eig}(A\text{-}LC)] \ll \mathrm{Re}[\mathrm{eig}(A\text{-}BK)]$,
  - ➢ with not too large (in absolute value) real part, in order to avoid amplification of high-frequency noises and numerical problems.

- **Matlab**: `L = acker(A',C',Po_des)';` (`place`)

  `Po_des` = vector with the desired observer eigenvalues.

- The eigenvalues of $A\text{-}LC$ can be arbitrarily placed if and only if (A,C) is completely observable.

Observer design – Separation principle

Theorem (separation principle). The eigenvalues of the closed-loop system are $\operatorname{eig}(A-BK)\cup\operatorname{eig}(A-LC)$.

Proof. Consider the closed-loop system with $r=0$.

The state of the closed-loop system is given by $\begin{bmatrix} x \\ \hat{x} \end{bmatrix}$ or, equivalentely, by $\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} x \\ x-\hat{x} \end{bmatrix}$.

First, we have that $\dot{x} = Ax + Bu, \quad u = -K\hat{x}.$

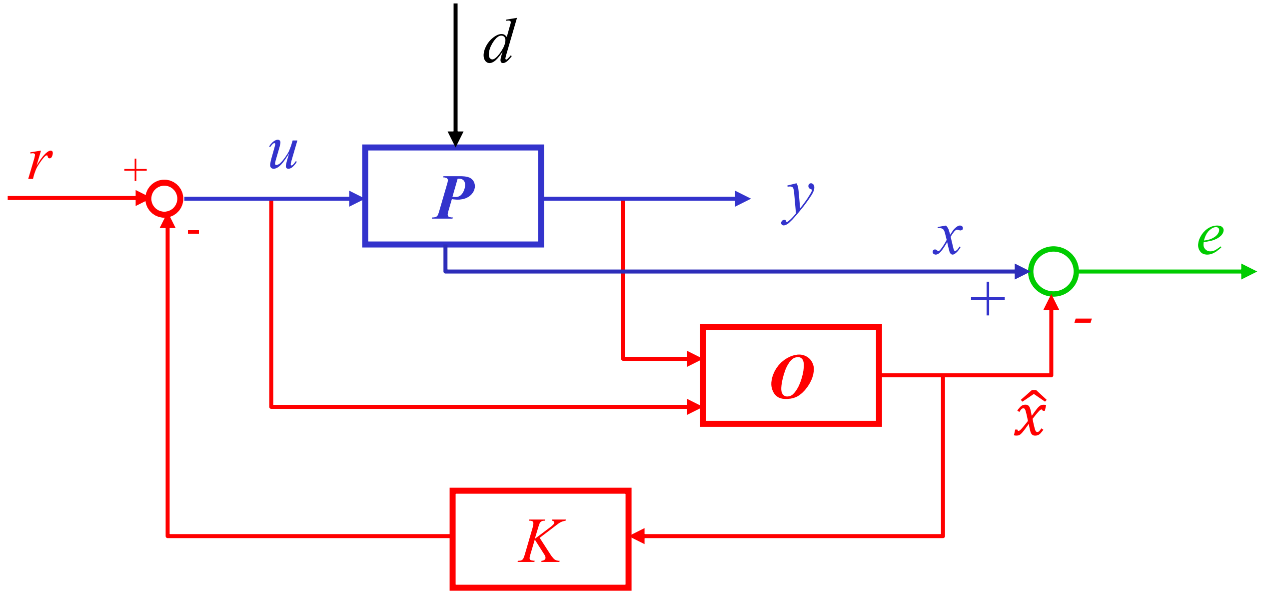
Then, $\dot{x} = Ax - BK\hat{x} = Ax - BK(x-e) = (A-BK)x + BKe.$

Second, we have that $\dot{\hat{x}} = (A-LC)\hat{x} + Ly + Bu, \quad y = Cx.$

Then, $\dot{e} = (A-LC)e.$

It follows that $\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$. ■

Observer design – Estimation error dynamics



- In the presence of noise:

$$\dot{e} = (A - LC)e + w$$

where w accounts for the disturbance d.

If A-LC is asymptotically stable and the disturbance is bounded, then $x - \hat{x}$ is bounded.

Observer design– Kalman-Bucy filter

- Plant: $\dot{x} = Ax + Bu + B_d d_a$ $\quad\quad d = \begin{bmatrix} d_a \\ d_y \end{bmatrix}$

  $y = Cx + d_y$

- Steady-state Kalman-Bucy filter:

$$\dot{\hat{x}}(t) = \left(A - LC\right)\hat{x}(t) + Ly(t) + Bu(t)$$

- The KB filter is a particular case of Luenberger observer, where L is designed to obtain an optimal filter:

  - the KB filter is optimal in the sense that it minimizes the estimation error variance for white Gaussian disturbances.

- Problem: the KB filter may not be fast enough when the estimated state is used for feedback control → robustness problems.

Observer design– Kalman-Bucy filter

- Plant: $\dot{x} = Ax + Bu + B_d d_a$

  $y = Cx + d_y$

  $$d = \begin{bmatrix} d_a \\ d_y \end{bmatrix}$$

- Steady-state Kalman-Bucy filter:

$$\dot{\hat{x}}(t) = \left(A - LC\right)\hat{x}(t) + Ly(t) + Bu(t) \quad \leftarrow \text{KB observer}$$

$$L = AP_d C^T R_d^{-1}$$

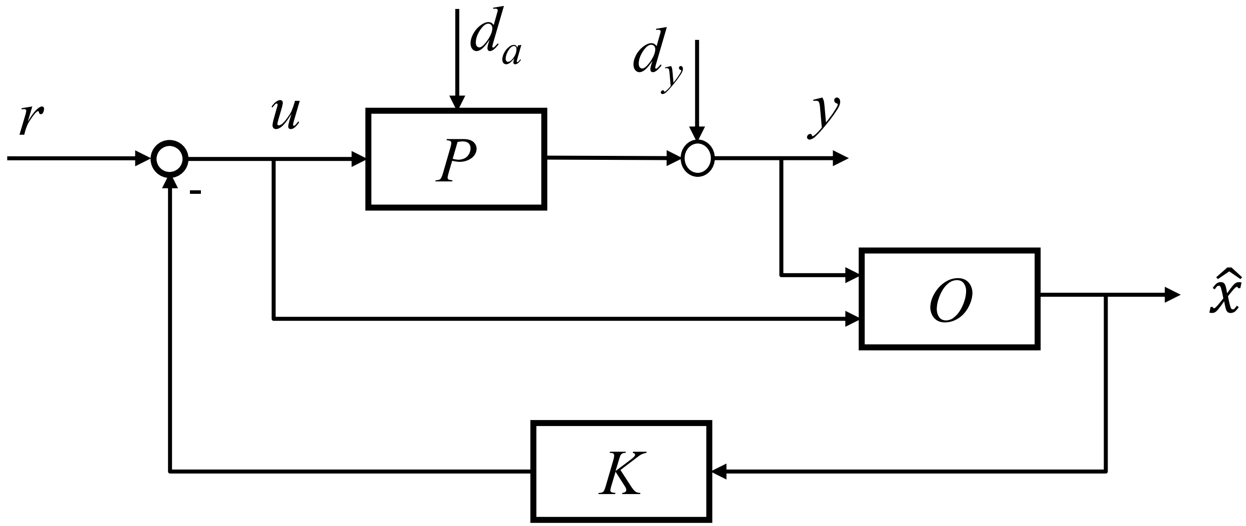
$$AP_d + P_d A^T + Q_d - P_d C^T R_d^{-1} C P_d = 0 \quad \leftarrow \text{Algebraic Riccati Equation}$$

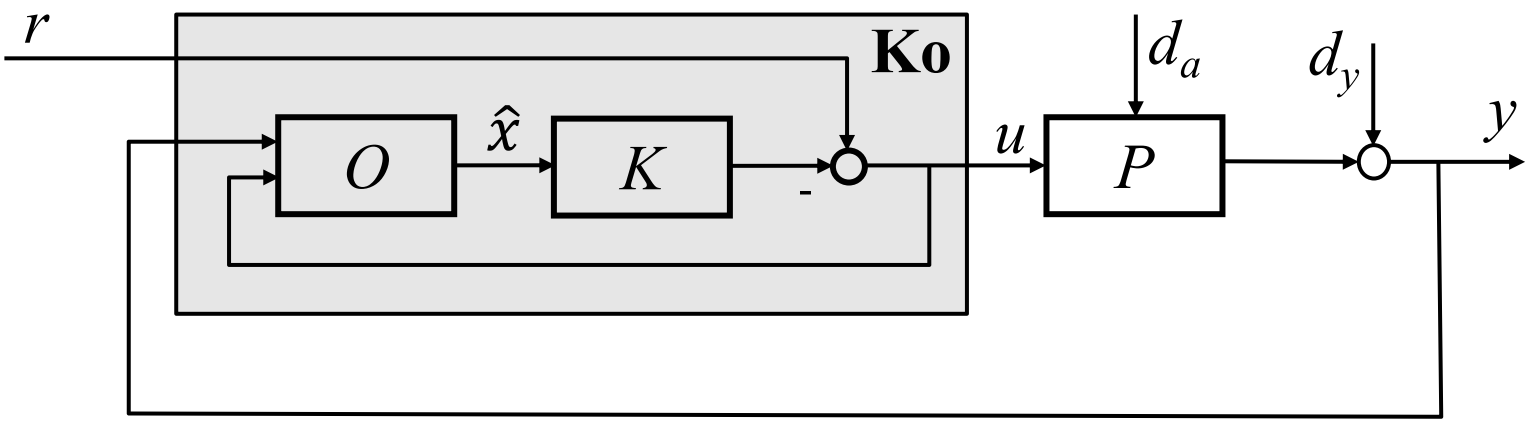
Q_d and R_d are the covariance matrices of d_a and d_y.

- The Kalman-Bucy filter can be designed using Matlab:

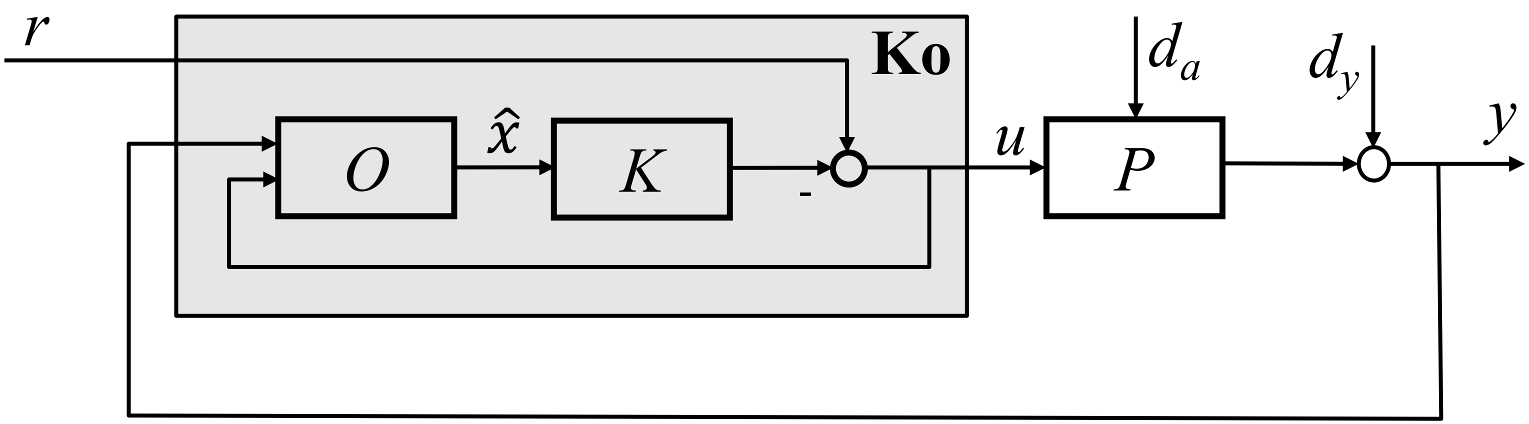
```
P = ss(A,[B Bd],C,[D zeros(ny,1)]);
[O,L] = kalman(P,Qd,Rd);
```

Observer design – Overall controller





Observer design – Overall controller



$$\mathbf{Ko}: \begin{cases} \dot{\hat{x}}(t) = \left(A - BK - LC\right)\hat{x}(t) + Br(t) + Ly(t) \\ u(t) = -K\hat{x}(t) + r(t) \end{cases}$$

- The overall controller is defined in Matlab as

```
Ak = A-B*K-L*C;
Bk = [B L];
Ck = -K;
Dk = [1 0];
Ko = ss(Ak,Bk,Ck,Dk);
```