

# Driver assistance system design A

## Lane keeping part 2

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# 1 Introduction

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# Introduction

- The goal of lane keeping systems is to **maintain the vehicle within the lane** through a control action on the steer.
- Indeed, lateral dynamics is unstable and control is necessary to keep the vehicle in the lane (in manual driving, controller = driver).
- The control system is not intended to replace the driver.
- The control system is aimed to improve safety:
  - ▶ help the driver in emergency situations, e.g. in the case of tiredness, lack of attention, critical road conditions, etc.
  - ▶ reduce the driver's tiredness.



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# Dynamic single-track error model

- In order to design an effective state feedback controller, it is convenient to introduce a new model of the vehicle lateral dynamics, written in terms of **orientation error** and **lateral error**.
- To define these errors, we recall the variables and parameters used for the dynamic single-track models (DST and DSTP):

$X, Y$ : coordinates of the vehicle CoG in an inertial reference frame

$\psi$ : yaw angle

$\omega_\psi \doteq \dot{\psi}$ : yaw rate

$\vec{v}$ : velocity vector in the inertial frame

$v_x$ : longitudinal speed =  $\vec{v}$  component along the longitudinal axis

$v_y$ : lateral speed =  $\vec{v}$  component along the transverse axis

$a_x$ : longitudinal acceleration

$a_y$ : lateral acceleration

$m, J$ : mass and moment of inertia

$l_f, l_r$ : distance CoG - front/rear wheel center

$c_f, c_r$ : front/rear cornering stiffnesses.

# Dynamic single-track error model

- The **orientation error** is defined as

$$e_\psi \doteq \psi - \psi_r.$$

Note that  $e_\psi = -e_h$ , where  $e_h$  is the heading error. We introduced both the quantities just for the sake of coherence with the literature.

- Suppose that the vehicle has to track a given reference trajectory and let  $\rho$  be the radius of curvature of this path at a given point.
- If the vehicle is exactly tracking the reference with longitudinal speed  $v_x$ , its yaw rate is given by  $\omega_{\psi_r} = \dot{\psi}_r = v_x / \rho$ .
  - ▶ Hence,  $\omega_{\psi_r}$  is the reference yaw rate.
- If the vehicle is not exactly tracking the reference, we have a yaw rate error  $\omega_\psi - \omega_{\psi_r}$ .

# Dynamic single-track error model

- From classical mechanics, we recall that  $a_y = \dot{v}_y + v_x \omega_\psi$ .
- If we replace  $\omega_\psi$  with the yaw rate error  $\omega_\psi - \omega_{\psi r}$ , we obtain the following lateral acceleration error:

$$\ddot{e}_y = \dot{v}_y + v_x(\omega_\psi - \omega_{\psi r}).$$

- With  $v_x \cong \text{const}$ , by integration we obtain

$$\dot{e}_y = v_y + v_x(\psi - \psi_r) = v_y + v_x e_\psi.$$

The quantity  $e_y$  is called the **lateral error** and quantifies the lateral deviation of the vehicle CoG from the reference trajectory.

It is similar to the cross-track error  $e_{ct}$  but not exactly the same.

- We can now write a lateral dynamics model in terms of these two errors.



## Dynamic single-track error model

- The model, called the dynamic single-track error (DSTE) model, is described by the following state equation<sup>1</sup>:

$$\dot{\mathbf{e}} = A_e \mathbf{e} + B_\delta \delta_f + B_\psi \dot{\psi}_r$$
$$A_e \doteq \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2\frac{c_f+c_r}{m} & -2\frac{c_f+c_r}{mv_x} & -2\frac{c_f l_f - c_r l_r}{mv_x} \\ 0 & 2\frac{c_f l_f - c_r l_r}{J} & -2\frac{c_f l_f - c_r l_r}{Jv_x} & -2\frac{c_f l_f^2 + c_r l_r^2}{Jv_x} \end{bmatrix}$$
$$B_\delta \doteq \begin{bmatrix} 0 \\ 0 \\ \frac{2c_f}{m} \\ \frac{2c_f l_f}{J} \end{bmatrix}, \quad B_\psi \doteq \begin{bmatrix} 0 \\ 0 \\ -v_x - 2\frac{c_f l_f - c_r l_r}{mv_x} \\ -2\frac{c_f l_f^2 + c_r l_r^2}{Jv_x} \end{bmatrix}$$

- ▶ state:  $\mathbf{e} \doteq (e_y, e_\psi, \dot{e}_y, \dot{e}_\psi)$
- ▶ command input:  $\delta_f$  (steering angle)
- ▶ disturbance:  $\dot{\psi}_r$  (reference yaw rate).

<sup>1</sup>Rajamani, Vehicle Dynamics and Control, Springer, 2012.

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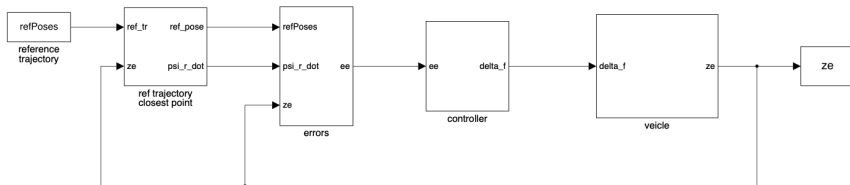
# Lane keeping

## Designed controllers

- **Kraja**: State feedback controller designed from the DSTE model; closed-loop eigenvalues  $\{-5 + 3j, -5 - 3j, -7, -10\}$  taken from Rajamani's book.
- **Klqr**: LQR controller designed from the DSTE model, with  $R = 5$ ,  $Q = \text{diag}(1, 1, 0, 0)$ .
- **Klqi**: LQI controller designed from the DSTE model, with  $R = 1$ ,  $Q = 10I$ . Output:  $(e_y, e_\psi)$ .
- **PIDF** (discrete-time):
  - ▶ Using  $e_y + e_\psi$  as the feedback variable.
  - ▶ Designed by means of the Simulink PID tuner.
- All controllers designed assuming a constant speed  $v_x = 40 \text{ km/h}$ .
- DSTE model used for control design. The controllers are tested using the DSTP model.

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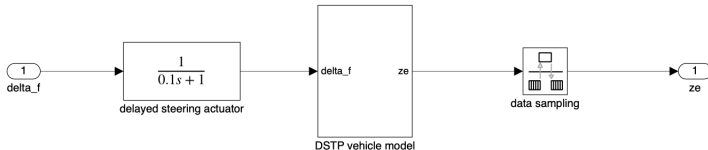
## Closed-loop scheme



- vehicle: DSTP model and steering actuator dynamics.
- controller: one of the controllers described above.
- errors: computation of lateral and orientation errors.
- reference trajectory: set of points giving the reference trajectory.
- ref trajectory closest point: on-line computation of the closest point to the vehicle.

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## Vehicle model



- DSTP: dynamic single-track vehicle model with Pacejka's tire formula.
  - ▶ Vehicle parameters:  $l_f = 1.2$  m,  $l_r = 1.6$  m,  $m = 1575$  kg,  $J = 4000$  kg m<sup>2</sup>,  $c_f = c_r = 27e3$  N/rad.
  - ▶ Tire parameters:  $p_1 = 3863$  N,  $p_2 = 1.5$ ,  $p_4 = -0.5$ , and  $p_3 = c_f/p_1/p_2$  (front tire) or  $p_3 = c_r/p_1/p_2$  (rear tire).
  - ▶ A constant speed  $v_x$  is directly imposed in "DSTP vehicle model".
- Other blocks:
  - ▶ steering actuator dynamics
  - ▶ data sampling block.

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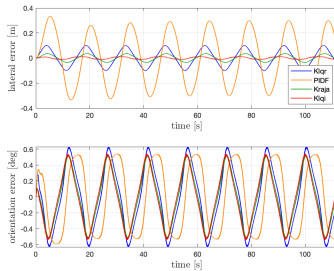
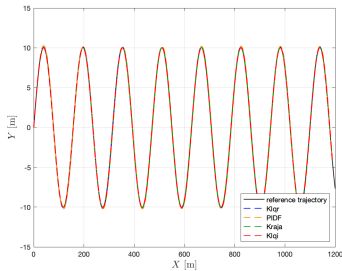
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# Lane keeping

Simulation results for  $v_x = 40$  km/h

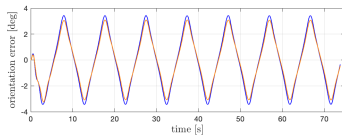
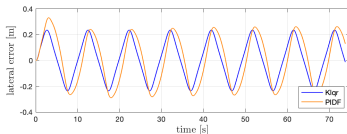
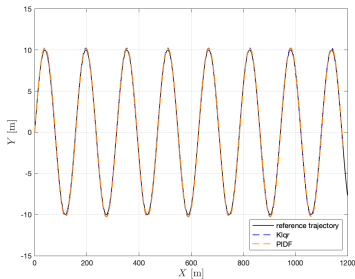
- Relatively challenging reference trajectory:  $Y_r = 10 \sin(0.04X_r)$ .



- Data sampling time 0.01 s: All controllers work. Klqi provides an excellent performance, PIDF the worst performance.
- Data sampling time 0.05 s: Kraja leads to an unstable behavior; Klqi works but gives rise to chattering.
- In the following, only the results of the Klqr and PIDF controllers are shown, assuming a data sampling time of 0.05 s.

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Simulation results for  $v_x = 60$  km/h

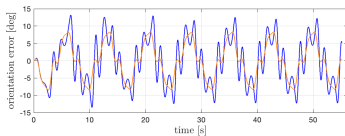
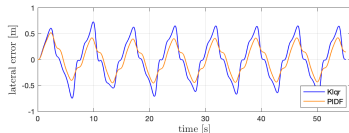
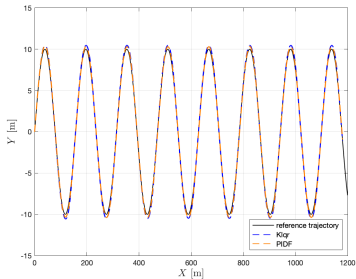


- The performance of the controllers gets worse but still rather good.
- In (more realistic) scenarios with smaller curvatures, smaller errors are obtained.



# Lane keeping

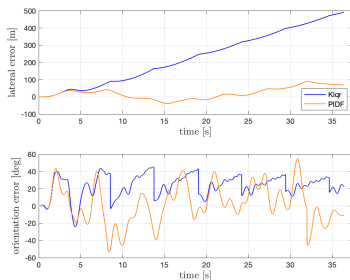
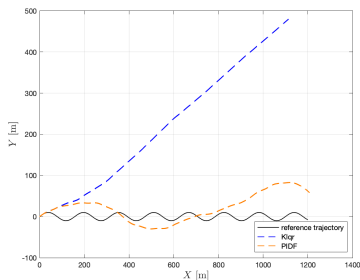
Simulation results for  $v_x = 80$  km/h



- The performance of the controllers gets worse but still not too bad.
- These controllers seem to work better than the Stanley controller and the PIDF controllers based on the cross-track error and heading error.

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Simulation results for  $v_x = 120 \text{ km/h}$



- The controllers do not work anymore.
- **Remark:** Relatively simple controllers may work for different speed ranges but not all  $\rightarrow$  gain-scheduling/nonlinear control may provide better results on the whole speed range.