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Exercise Ind. 2

Argument for the correctness of exp<u>Recursive</u> using Strong Induction.

<u>Basis:</u> We have already proven that for any given $n \in \mathbb{Z}^+$, expIterative gives the correct result. So we can be certain that for $n \leq 4$ expRecursive gives a correct result because it implements expIterative.

<u>Inductive Step:</u> Lets assume that for $0 \leq n \leq 4$ expRecursive holds true. Lets prove for some arbitriarly chosen even and odd numbers greater than 4.

<u>Inductive Proof:</u>

<u>Even numbers as exponents:</u> When an even number is used as exponent the recursive call will return $\frac{n}{2} \cdot \left(\frac{n+1}{2} - 0{,}5\right)$. So the code expRecursive $(x, \frac{n+1}{2})$ is basically doing the same job as expRecursive $(x, \frac{n}{2})$. The second Recursive call will return $\frac{n}{2^2} \cdot \left(\frac{n+1}{2^2} - 0{,}5\right)$ and so on. In general, after the j^{th} recursive call in expRecursive, the exponents will be evaluated as $\frac{n}{2^j} \leq 4$. Thus we conclude expRecursive is correct for even exponents.

<u>Odd numbers as exponents:</u> When an odd number is used as exponent, the Recursive call will return $\left(\frac{n}{2} - 0{,}5\right) \cdot \left(\frac{n+1}{2}\right)$. So the code expRecursive $(x, \frac{n}{2})$ and expRecursive $(x, \frac{n+1}{2})$ return numbers with a difference of 1. The second Recursive call will return $\left(\frac{n}{2^2} - 0.5\right) \cdot \left(\frac{n+1}{2^2}\right)$ and so on. In general, after the j^{th} recursive call in expRecursive, the exponents will be evaluated as $\left(\frac{n}{2^j}\right) \cdot \left(\frac{n+1}{2^j}\right)$, with both of them ≤ 4. Thus we conclude expRecursive is correct for odd exponents.

$T(n)$ for expRecursive is:

$$T(n) = 2T(n/2) + 1. \quad \text{, where } a=2,\ b=2 \text{ and } c=0.$$

<u>Via Master's Thorem:</u> $2^0 < 2$, so $a > b^c$

$$T(n) = O(n^{\log_2^2})$$

$$\underline{\underline{T(n) = O(n)}}$$