Exercise Ind.2

· Argument for the correctness of expRecursive using Strong Induction

Basis: We have already proven that for any given $n \in \mathbb{Z}_+$ that expIteratives gives the correct result. So we can be certain that for $n \leq 4$ expRecursive gives a correct result because it implements expIterative.

Inductive Step: Lets assume that for $0 \leq i \leq k$ expRecursive holds true. Lets prove for k+1 and k+2.

Inductive Proof: Lets assume k+1 in the former step is equal to 5. Because $5 \geq 4$ the line expRecursive$(x, \frac{n}{2})$ • expRecursive$(x, \frac{n+1}{2})$ is executed, which gives the result 2 and 3 respectively. Then expRecursive is called again but this time expIterative is called because $2,3 \leq 4$. And we know expIterative is correct, so we conclude expRecursive for k+1 is also correct.
We follow the same steps for k+2 which we assume to be equal to 6. We get the result 3 and 3 from the recursive line in our algorithm. Again expIterative is called because $3 \leq 4$ which is going to be correct. So we conclude expRecursive for k+2 is also correct.

∴ We conclude that expRecursive for any given n (whether even or odd is true.

T(n) for expRecursive is:

$$T(n) = 2T(n/2) + n$$

$a = 2$, $b = 2$, $c = 1$

Via Master's Thorem: $2^1 = 2$, so $a = b^c$

$$T(n) = O(n^1 \log n)$$
$$= O(n \log n)$$