Induction Exercises

2). $\sum_{j=1}^{n} (2j-1) = n^2$

Base Case: $2j-1 = 2(1)-1 = \underline{\underline{1}}$

$\qquad\qquad\quad 1^2 = \underline{\underline{1}}$

Inductive Step!. Lets assume that $p(k) = k^2$ holds true.
Is it true for $p(k+1)$?

We know that,

$2.1-1 + 2.2-1 \cdots + 2k-1 = k^2$

$2.1-1 + 2.2-1 \cdots + 2k-1 + 2(k+1)-1 = k^2 + 2(k+1)-1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = k^2 + 2k + 2 - 1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \underline{\underline{k^2 + 2k + 1}}$

$(k+1)^2 = \underline{\underline{k^2 + 2k + 1}}$

∴So we know that $p(n)$ holds true for $n \geq 1$.