Induction Exercises

1) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(n+2)}{6}$

Base Case: $i^2 = 1^2 = 1$

$$\frac{1(1+1)(2.1+1)}{6} = \frac{2.3}{6} = 1$$

Inductive Step: Lets assume that $p(k) = \dfrac{k(k+1)(2k+1)}{6}$ holds true.
Is it true for $p(k+1)$?

$$p(k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\to p(k) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{2k^3 + k^2 + 2k^2 + k}{6} + k^2 + 2k + 1$$

$$= \frac{2k^3 + k^2 + 2k^2 + k + 6k^2 + 12k + 6}{6}$$

$$= \frac{2k^3 + 9k^2 + 13k + 6}{6}$$

$$=$$

$$\frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k^2+3k+2)(2k+3)}{6}$$

$$= \frac{2k^3 + 3k^2 + 6k^2 + 9k + 4k + 6}{6}$$

$$= \frac{2k^3 + 9k^2 + 13k + 6}{6}$$

$$=$$

$\therefore$ So we know $P(n)$ holds true for $n \geq 1$.