Introduction Contents

Introduction Contents

* What / loor / shry





So, what is it that I will be to you talking about? Let's break it down:

- 1. First of all we are interested in controlling multiple systems, that operate in the same space, at the same time. . .
- $2.\ldots$ these systems do not obey the commands of a central, shared control authority, but decide their actions on their own \ldots
- 3. ... under the presence of a disturbance of some sort
- 4. Lastly, these systems are somehow binded in a shared activity

—Cooperation realizations (1/2)



For instance, DARPA is interested in controlling swarms of drones whose members may need to act as relays of information, due to the fact that communication ranges are not limitless.



In any case, we find that there are two fundamental prerequisites of cooperation in decentralized settings: an arbitrary agent needs to maintain connectivity with a set of agents, and avoid colliding with them, and any obstacle that it may come across.

In this context, there are two interesting areas of control:

Control in what sense? (1/2)



The first is trajectory tracking, where either an agent all the whole multi-agent system needs to follow a given trajectory as close as possible, and stabilization. . .

—Control in what sense? (2/2)



where for instance each agent is assigned a desired configuration to which he must arrive at some point and stay, regardless of disturbances affecting his dynamic behaviour. I found this video of stabilization on the internet to be quite impressive.

And this is what the goal of this thesis is: to stabilize a multi-agent system subject to disturbances to predefined configurations for all agents. More or less we are interested in achieving this:

In particular: stabilization

What we see here are three agents that have to somehow bypass two obstacles 1. without the blue agent losing connectivity with the other two, and 2. without all of them crashing into agents or obstacles prior to stabilizing themselves in some chosen configuration.





Now, how did we achieve this? We used the control strategy of Receding Horizon (or Model Predictive) Control, where the connectivity and collision avoidance mandates are incorporated as constraints



Why MPC?

- direct incorporation of connectivity / collision constraints

- direct incorporation of input / state constraints

- theret incorporation of input / state constraints

- theretic incorporation of stability

- theretic incorporation of stability

- theretic incorporation extended are not

Why choose MPC over any other strategy?

MPC has the unique advantage that it can directly incorporate

- 1. the type of constraints we are interested in imposing 2. input \slash state constraints
- a. we can procure theoretic guarantees of stability b. it can be effective in aspects other methods are not

Prior approaches

Navigation Functions
 Cost-coupled Decentralized MPC

Prior approaches

For instance the same problem was approached using navigation functions, but this way proved to be not as robust as MPC.

The superiority of MPC has been exploited previously, and some approaches consider imposing the kinds of constraints we are interested in by coupling systems through the cost functions that MPC uses, instead of through their constraints.

The specifics: Contents

The specifies: Contents

* Terminology - Natains

* Paskers

* Sindation

* Simulation

 $\ensuremath{\mathsf{OK}}$ so next I'll talk about the problem and start formalizing it

└─Model in Space



We model agents in space as spherical rigid bodies, of radius r, which have a limited connectivity range of d, moving in space with p denoting their position through time.



In terms of dynamics, all agents are described by continuous-time, non-linear differential equations that include additive and bounded disturbance terms that are unknown.

 \square Solution requirements (0/III)

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Solution requirements (0/10)

Drops u_i \in \mathcal{U}_i such that

\bullet Collisions are satisfied for all i,j \in V_i, \ell \in \mathcal{L}, \ell \geq 0

\|u_i(t) - \mu_j(t)\| \ge \delta_{i,j},
\|u_i(t) - \mu_j(t)\| \le \delta_{i,j},
\|u_i(t) - \mu_j(t)\| \le
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 $\|\mathbf{x}_i(\infty) - \mathbf{x}_{i,des}\| = 0$

Formally then, the problem lies in designing a control regime such that all input signals stay bounded, no collisions occur, all neighbours stay well-connected at all times, and that all agents are stabilized in steady-state

Solution requirements (I/III): constraint design

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Solution requirements (I/III): constraint dusign Collision on a solution for all <math>i,j \in V, \ell \in \mathcal{L}, t \geq 0 \|[\mathbf{x}_i(t) - \mathbf{p}_j(t)]\| \geq d_{\mathbf{x}_{i,j}} \|[\mathbf{y}_i(t) - \mathbf{p}_j(t)]\| \geq d_{\mathbf{x}_{i,j}} \|[\mathbf{y}_i(t) - \mathbf{p}_j(t)]\| \geq d_{\mathbf{x}_{i,j}} p, component of all as k. In Singhtham i \in \mathcal{N}_{i,j} \in \mathcal{K}_i matrix correctivity at all time \ell \geq 0 \|[\mathbf{p}_i(t) - \mathbf{p}_j(t)]\| \leq m_i \mathbf{n}_i^{-1} \mathbf{n}_j^{-1}. (Considering) System in stable at dense at all \|[\mathbf{x}_i(t) - \mathbf{x}_{i,j}]\| \leq m_i \mathbf{n}_j^{-1} \mathbf{n}_j^{-1}.
```

I will first focus on the second requirement and then go through the other two.

State constraint set

State constraint will $Enote all state respicements on agent <math>\varepsilon$ through time varying and Z_{ε} $Z_{\varepsilon} = \{x_{\varepsilon}(t) \in X_{\varepsilon}(\mathbf{p}_{\varepsilon}(t) - \mathbf{p}_{\varepsilon}(t))\} \times \int_{\Omega_{\varepsilon}} v_{\varepsilon} \in Z_{\varepsilon}(t)$ $\|\mathbf{p}_{\varepsilon}(t) - \mathbf{p}_{\varepsilon}(t)\| < Z_{\varepsilon}, \quad \forall \varepsilon \in X_{\varepsilon}, \quad \forall \varepsilon \in X_{\varepsilon}$ $\|\mathbf{p}_{\varepsilon}(t) - \mathbf{p}_{\varepsilon}\|^{2} \times \mathcal{L}_{\varepsilon} \quad \forall \varepsilon \in X_{\varepsilon}$ $\|\mathbf{p}_{\varepsilon}(t) - \mathbf{p}_{\varepsilon}\|^{2} \times \mathcal{L}_{\varepsilon} \quad \forall \varepsilon \in X_{\varepsilon}$ $\|\mathbf{p}_{\varepsilon}(t) - \mathbf{p}_{\varepsilon}\|^{2} \times \mathcal{L}_{\varepsilon} \quad \text{where } x \text{ where } x \text{ as time } t$ X_{ε} assigned resignation agent z. The problem requires $x_{\varepsilon}(t) \in Z_{\varepsilon}$ for $t \ge 0$

The good thing with MPC as I mentioned is that we can encode all constraints in a set, say Z, which in our case is time-varying. The problem merely requires that the trajectories of each agent are kept within Z.

error-wise



If we look at the system in terms of error, we can get its dynamics and express the corresponding constraint set by using the minkowski set difference. We will be speaking in terms of error for the following slides. Solution requirements (II/III): control input

Solution requirements ((I/III)), control input $\begin{aligned} & \text{Design } \mathbf{x}_i \in \mathcal{U}_i \text{ such that} \\ & \text{- Collisions we needed for all } i, j \in \mathcal{V}_i \in \mathcal{L}_i \succeq 0 \\ & \|\mathbf{x}_i(t) - \mathbf{p}_i(t)\| \geq \underline{\omega}_i, \\ & \|\mathbf{x}_i(t) - \mathbf{p}_i(t)\| \geq \underline{\omega}_i, \end{aligned}$ p, component of state \mathbf{x}_i .

• Neighbours $i \in \mathcal{N}_j$, $j \in \mathcal{N}_i$ maintain connectivity at all times $t \geq 0$ $\|\mathbf{p}_t(t) - \mathbf{p}_j(t)\| < \min(\overline{d}_i, \overline{d}_j)$ • (Closed-loop) System is stable at desired state

 $\|\mathbf{x}_i(\infty) - \mathbf{x}_{i,\mathsf{dex}}\| = 0$

If we now focus on the control input requirements. . .

Solution requirements (II/III): control imput

Designed control input us, must

result in states that satisfy the state constraints

*One trajectory to / statisfies at desired state (low close?)

**cit and for facility 20 (s. G. V.J.)

...Such an input can be obtained by solving an optimization problem at regular sampling intervals.

The optimization problem



it is the input that minimizes a standard MPC criterion over a period of T_p time units. This is the typical formulation of the MPC solution. However, there is a big difference here. The disturbance is unaccounted for in the dynamics of the solution because it is unknown.

LIssue #1



And so the first question we are asking is: how can we guarantee that the error does not escape the set in which we want it to be?

The optimization problem



What we do in this case is manipulate the set in which the error should lie in within the solution of the optimization problem. And that is what this (shows red) constraint is. At each step we progressively tighten the original constraint set during the solution of the optimization problem, so that, irrespective of the actual magnitude of the disturbance affecting the real system, its real trajectory always stays within the prescribed constraints. And we can do this because we know the supremum of the disturbance

Restricted constraint set (1/5)



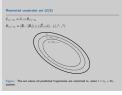
What is the logic behind this manipulation? Let's depict the real constraint set of the error with this ellipse. When solving the optimization problem, we consider the evolution of the trajectory of the error for a number of time units. If disturbances are present, then, as time goes by, the true state becomes more and more unpredictable as more and more disturbance has affected it in the course of time, since the input given to the system does not know anything about the disturbances. The more time goes by and the more disturbance is present, the more unpredictable the system becomes, and the more likely it is that its states will escape the constraint set. If instead we restrict the predicted trajectory in a reverse manner at each step to be futher and further away from the boundary of the constraints, then we can guard the system from violating its constraints, provided that the horizon has been selected so that the last constrained set is not empty.

Restricted constraint set (2/5)



In the first step, when $t=t_{k+1}$, the real constraint set is reduced to the states that will not violate the constraints if at $t=t_k$ the disturbance hits its maximum.

Restricted constraint set (3/5)



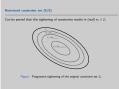
In the second step the same happens for t_{k+2}

Restricted constraint set (4/5)

Restricted constraint set $\{\xi/S\}$: $\mathcal{E}_{n-n} = \{\mathcal{E}_i: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} = \{\mathcal{E}_i: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \} = \{\mathcal{E}_i: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \} = \{\mathcal{E}_i: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \} = \{\mathcal{E}_i: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n} \} = \{\mathcal{E}_n: \mathcal{E}_{n-n} \\ \in \mathcal{E}_{n-n$

until we reach the horizon

Restricted constraint set (5/5)



We can prove that if we restrict the predicted trajectory in this way within the optimization problem, then the real state will never violate the constraints.

Issue #2



This would be sufficient if we were only considering one agent. But we are considering multiple agents that need to cooperate with each other.

—How can agents plan...



suppose for instance that agent i solves its optimization problem and he must sustain connectivity with agent j. within the optimization problem, agent j moves, and its position needs to be known to agent i at every step

igspace They access the open-loop trajectories of agents in \mathcal{R}_i

They access the open loop trajectains of agents in Σ .

When or time t_1 agent t where a finite horizon optimization problem, he has a second of the states $x_1(t_1)$ and $x_2(t_2)$ and $x_3(t_3)$ of all agents $x_3(t_4)$ and $x_3(t_4)$ are a second or $x_3(t_4)$ and $x_3(t_4)$ and x

therefore, at time t_k when an agent solves its optimization problem, he needs to know where all agents within its sensing range are, and where they predict they will be for the next T_p seconds.

Inter-agent intra-horizon constraint design



Figure : The inter-agent constraint regime for two agent, i. j. fully custined circles denote measured configurations, while party outline circles denote predicted configurations. During the solution to the individual optimization problems, the predicted configuration of each regime at each timestep is constrained by the predicted configuration of each regime at each timestep is constrained by the predicted configuration of the other agent at the same timestep (hence the homologously identical colours at each discrete timestep).

Now that agent i knows where agent j thinks he will move, we can constrain the states of agent i at each time step by the predicted state of agent j at the same time step. then, the constraint tightening takes care of the rest.

Steering to the desired state

Steading to the designed control impact α , must α with a colors that such a control impact α , must α with a colors that such the control control α with a colors that such the control control α with replicitly α / stables as the desired state (box closel) α was and the families $M \ge 0$ (α , $c(A_1)$).

That takes care of the state feasibility. The question now is: how does the input steer the state to the equilibrium?

Road-trip to equilibrium (1/7)



The constraint set tightening concludes at some set

Road-trip to equilibrium (2/7)



and suppose that this is it

Road-trip to equilibrium (3/7)



if this set is not empty, and if the linearized model of the system is stabilizable, then we can find a feasible input whose form is linear feedback with respect to the state, which Road-trip to equilibrium (4/7)



if we apply it to a state that lives in a set $\boldsymbol{\Psi}$

Road-trip to equilibrium (5/7)



it will move the state to the terminal set

Road-trip to equilibrium (6/7)



and since Ω is a subset of $\Psi,$ it will always stay inside $\Omega.$

 \square Road-trip to equilibrium (7/7)



the size of the terminal set depends on the how strong the disturbance is and on the hierarchy of all of these sets. Provided that the supremum is bounded by a certain threshold that depends on the size of the terminal set and Ψ , and the length of the horizon.

Recursive feasibility

Recorder feasibility

Personshor: the designed control input as, must

* must be stated that saidly for once constraints

* design the supprisery / calciding as all the design of these closes?)

* what and be feasible VIZ & (bq. CLR)

Can show the supprisery / calciding as the design of the closes?)

Can show the design is designed as a constraint of the control o

as for the feasibility of the input, we can patch together past control sequences with the feedback controller and come up with a feasible control signal

Solution requirements (III/III): stabilization

Solution requirements (10), (02), with distance as solution for all $z_1\in V,\ \ell\in \mathcal{L},\ \ell\geq 0$, $\|p_1(0-p_1)\| \cdot \Delta_{d_1}$, $\|p_1(0-p_1)\| \cdot \Delta_{d_2}$, p., compared of state x_1 , $\|p_1(0-p_1)\| \cdot \Delta_{d_2}$, p., compared of state x_1 , $\|p_1(0-p_1)\| \cdot \Delta_{d_2}$, and if since $x_1 \in V$ and it is successfully at all times $t \geq 0$, $\|p_1(0-p_1)\| \cdot \|p_1(0-p_2)\| \cdot \|p_1(0-p_2)\|$

 $\|\mathbf{x}_i(\infty) - \mathbf{x}_{i,dm}\| = 0$

Now the last thing is the actual stabilization.

How is stabilization achieved?

 $\|\mathbf{x}_i(\infty) - \mathbf{x}_{i,\text{des}}\| \times 0$ $\mathbf{e}_i^{\top} P \mathbf{e}_i \leq \varepsilon_{\Omega_i}$

it turns out that we cannot drive the system to complete equilibrium, but we can only trap it in a neighbourhood of it. this neighbourhood is smaller than that which it would have been if the controller didn't manage to attenuate the disturbances.

We can show that the cost that results from the application of the suggested control input, the optimal cost, rather than monotonically deacreasing, is bounded. In this case we can prove that the optimal cost is an input-to-state lyapunov function, which makes the closed-loop system input-to-state stable within its constraints set

Example system: the unicycle

simple system: the uniquie $\begin{aligned} & \text{State} \quad x_{-k} \text{ is, } \delta, \\ & \lambda(t) &= u(t) \cos \theta_{\lambda}(t) + \lambda(t) & \text{Spate} \quad x_{-k} \text{ is, } \delta, \\ & \lambda(t) &= u(t) \sin \theta_{\lambda}(t) + \lambda(t) & \text{Denotence, } \delta_{\lambda} = 0.1 \\ & \lambda(t) &= u(t) \sin \theta_{\lambda}(t) + \lambda(t) & \text{detections, } \delta_{\lambda} = 0.1 \\ & \lambda(t) &= \lambda(t) \sin 2t \end{aligned}$

From this point on I will only show you how the control regime that I have described to you performs in simulations. We will use the standard unicycle model moving in x-y and rotating around the z axis. Its states are $xy\theta$ and its inputs are the longitudinal velocity and the angular velocity

—Simulation example I/II: 2D Trajectories

2D trajectories of agent 1 (blue, middle), agent 2 (yellow, below) and agent 3 (red, above). Black circles are obstacles. Agents 2 and 3 must keep connectivity with agent 1. Disturbances are absent. Trajectories with and without disturbances are more or less similar. Only steady-atate charges.

Here we see three agents trying to get to their desired positions marked with x, that have to bypass these two obstacles. The blue one needs to be within certain bounds from the other two, and they all must not hit the obstacles or each other.

Simulation example II/II: Equilibrium close-up

The effect of unattenuated disturbances on the steady-state configurations. Left: no disturbance. Right: unattenuated additive bounded disturbances with $\overline{\delta}_i=0.1$.

If we look at what happens around the desired equilibrium, we see on the left that we can totally stabilize the system when there are no disturbances. On the righthand side we see the effect of disturbances which have a supremum of 0.1, that have not been attenuated to the trajectories of the systems around the equilibrium.

—Simulation example II/II: Equilibrium close-up

Steady-state configurations. Left: no disturbance. Middle: unattenuated additive bounded disturbances with $\overline{\delta}_i=0.1$. Right: attenuated additive bounded disturbances with $\overline{\delta}_i=0.1$ as a result of the proposed control regime.

if we design the control system the way i described you here today, then we can make the system get closer to its desired state.

—Conclusion

It is possible to design a feasible control regime based on the principle of Recoding Morizon Control such that, a multi-agent system constrained with connectivity and collision constraints on the states and input constraints, is stable under the influence of disturbances of disturbances.

It is possible to design a feasible control regime based on the principle of Receding Horizon Control such that, a multi-agent system constrained with connectivity and collision constraints on the states and input constraints, is stable under the influence of disturbances.