Robust Decentralized Control of Cooperative Multi-robot Systems

an inter-constraint Receding Horizon approach

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Introduction Contents

- What / how / why

What?

Robust Decentralized Control of Cooperative Multi-robot Systems

- *Simultaneous* control of *multiple* systems
- Decentralized: *no* central control authority
- Robust: uncertainty *is* present
- Cooperative: an agent-binding, resource-sharing task

Cooperation realizations (1/2)

DARPA making progress to autonomous attacking drone swarms

brian wang | June 19, 2016 | 5 comments

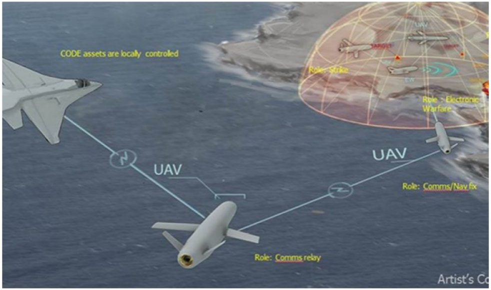

Figure : Controller—Relay—End effector (Goals-setter – autonomous agents)

Sources: `http://bit.ly/2pTljEf`, `http://bit.ly/2qt1kOW`

Cooperation realizations (2/2)

DARPA making progress to autonomous attacking drone swarms

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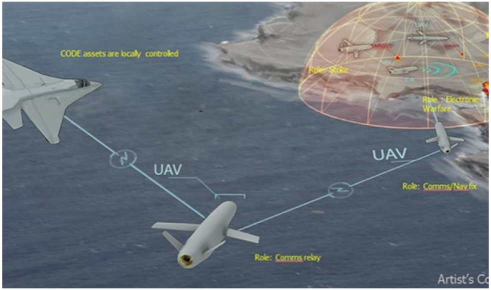

Figure : Controller—Relay—End effector (Goals-setter – autonomous agents)

Prerequisites-tier:

- Connectivity maintenance
- Collision avoidance (agents plus obstacles)

Control in what sense? (1/2)

Reference tracking

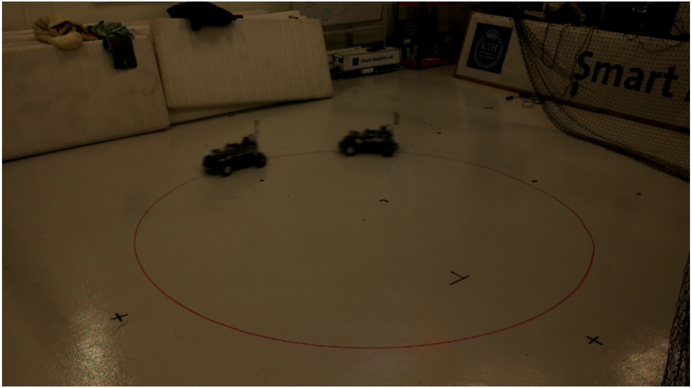

Figure : Vehicles tracking a desired trajectory in tandem.

Control in what sense? (2/2)

Stabilization

source: http://www.popularmechanics.com/military/weapons/a18362/tank-carry-beer/

In particular: stabilization

How?

an inter-constraint Receding Horizon approach

- ordinary Receding Horizon / Model Predictive Control strategy
- cooperating agents are *inter-constrained*

Why MPC?

- direct incorporation of connectivity / collision constraints
- direct incorporation of input / state constraints
- theoretic guarantees of stability
- effective in aspects other methods are not

Prior approaches

- Navigation Functions
- Cost-coupled Decentralized MPC

The specifics: Contents

- Terminology - Notation
- Problem
- Solutions
- Simulations

Model in Space

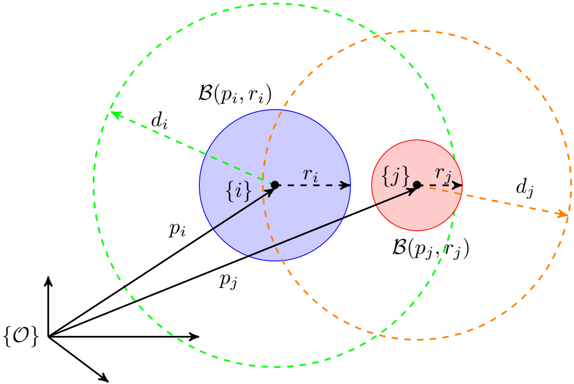

Figure : Agents i, j: spherical rigid bodies with radii $r_i > r_j$. Vectors p_i, p_j show center mass positions. $d_i > d_j$ are their sensing ranges.

Dynamics

Agents $i \in \mathcal{V} = \{1, 2, \ldots, N\}$, with radii r_i and ranges d_i

Each agent $i \in \mathcal{V}$ is described by a continuous-time non-linear model

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i, \mathbf{u}_i) + \boldsymbol{\delta}_i$$

$\mathbf{x}_i \in X_i$, $\mathbf{u}_i \in U_i$

with *unknown* disturbance $\boldsymbol{\delta}_i \in \Delta_i : \|\boldsymbol{\delta}_i\|_\infty = \overline{\delta}_i$

Solution requirements (0/III)

Design decentralized control policies $\mathbf{u}_i \in \mathcal{U}_i$ such that

- Collisions are avoided for all $i, j \in \mathcal{V},\ \ell \in \mathcal{L},\ t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}$$
$$\|\mathbf{p}_i(t) - \mathbf{p}_\ell\| > \underline{d}_{i\ell,o}$$

  $\mathbf{p}_i$ component of state $\mathbf{x}_i$

- Neighbours $i \in \mathcal{N}_j$, $j \in \mathcal{N}_i$ maintain connectivity at all times $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| < \min(\overline{d}_i, \overline{d}_j)$$

- (Closed-loop) System is stable at desired state

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Design decentralized control policies $\mathbf{u}_i \in \mathcal{U}_i$ such that

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- (Closed-loop) System is stable at desired state

Solution requirements (I/III): constraint design

Design decentralized control policies $\mathbf{u}_i \in \mathcal{U}_i$ such that

- Collisions are avoided for all $i, j \in \mathcal{V}$, $\ell \in \mathcal{L}$, $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}$$
$$\|\mathbf{p}_i(t) - \mathbf{p}_\ell\| > \underline{d}_{i\ell,o}$$

  $\mathbf{p}_i$ component of state $\mathbf{x}_i$

- Neighbours $i \in \mathcal{N}_j$, $j \in \mathcal{N}_i$ maintain connectivity at all times $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| < \text{min}(\overline{d}_i, \overline{d}_j)$$

- (Closed-loop) System is stable at desired state

$$\lim_{t \to \infty} \|\mathbf{x}_i(t) - \mathbf{x}_{i,\text{des}}\| \to 0$$

State constraint set

Encode all state requirements on agent i through time-varying set $\mathcal{Z}_i$.

$$
\begin{aligned}
\mathcal{Z}_i = \{\mathbf{x}_i(t) \in X_i : & \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}, \quad \forall j \in \mathcal{R}_i(t), \\
& \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| < \overline{d}_i, \qquad \forall j \in \mathcal{N}_i, \\
& \|\mathbf{p}_i(t) - \mathbf{p}_\ell\| > \underline{d}_{i\ell,o}, \qquad \forall \ell \in \mathcal{L}\}
\end{aligned}
$$

$\mathcal{R}_i(t)$: agents within range of agent i at time t
$\mathcal{N}_i$: assigned neighbours of agent i

The problem requires $\mathbf{x}_i(t) \in \mathcal{Z}_i$ for $t \geq 0$

error-wise

The *error* of agent $i \in \mathcal{V}$ with respect to $\mathbf{x}_{i,\text{des}}$ is

$$\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_{i,\text{des}}$$

Described by a continuous-time non-linear model

$$\dot{\mathbf{e}}_i = g_i(\mathbf{e}_i, \mathbf{u}_i) + \boldsymbol{\delta}_i$$

$$\begin{aligned} \mathbf{e}_i &\in \mathcal{E}_i \equiv \mathcal{Z}_i \ominus \mathbf{x}_{i,\text{des}} \\ \mathbf{u}_i &\in U_i \\ \boldsymbol{\delta}_i &\in \Delta_i \end{aligned}$$

where $\boldsymbol{A} \ominus \mathbf{b} = \{\mathbf{a} - \mathbf{b}, \mathbf{a} \in \boldsymbol{A}\}$ is the Minkowski (Pontryagin) difference operation

Solution requirements (II/III): control input

Design decentralized $\mathbf{u}_i \in \mathcal{U}_i$ such that

- Collisions are avoided for all $i, j \in \mathcal{V}$, $\ell \in \mathcal{L}$, $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}$$
$$\|\mathbf{p}_i(t) - \mathbf{p}_\ell\| > \underline{d}_{i\ell,o}$$

  $\mathbf{p}_i$ component of state $\mathbf{x}_i$

- Neighbours $i \in \mathcal{N}_j$, $j \in \mathcal{N}_i$ maintain connectivity at all times $t \geq 0$

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- (Closed-loop) System is stable at desired state

$$\lim_{t\to\infty} \|\mathbf{x}_i(t) - \mathbf{x}_{i,\mathsf{des}}\| \to 0$$

Solution requirements (II/III): control input

Designed control input $\mathbf{u}_i$ must

- result in states that satisfy the state constraints
- drive trajectory to / stabilize at desired state (how close?)
- exist and be feasible $\forall t \geq 0$ $(\mathbf{u}_i \in \mathcal{U}_i)$

The optimization problem

Given measurement $\mathbf{e}_i(t_k)$, find

$$\overline{\mathbf{u}}_i^{\star}(\cdot;\ \mathbf{e}_i(t_k)) \triangleq \underset{\overline{\mathbf{u}}_i(\cdot)}{\text{argmin}} \quad J_i\big(\mathbf{e}_i(t_k), \overline{\mathbf{u}}_i(\cdot)\big)$$

where

$$J_i\big(\mathbf{e}_i(t_k), \overline{\mathbf{u}}_i(\cdot)\big) \triangleq \int_{t_k}^{t_k+T_p} F_i\big(\overline{\mathbf{e}}_i(s), \overline{\mathbf{u}}_i(s)\big) ds \ + \ V_i\big(\overline{\mathbf{e}}_i(t_k + T_p)\big)$$

subject to:

$$\dot{\overline{\mathbf{e}}}_i(s) = g_i\big(\overline{\mathbf{e}}_i(s), \overline{\mathbf{u}}_i(s)\big), \quad \overline{\mathbf{e}}_i(t_k) = \mathbf{e}_i(t_k)$$

$$\overline{\mathbf{u}}_i(s) \in \mathcal{U}_i, \quad \overline{\mathbf{e}}_i(s) \in \mathcal{E}_{i,s-t_k}, \quad s \in [t_k, t_k + T_p]$$

$$\overline{\mathbf{e}}_i(t_k + T_p) \in \Omega_i$$

Issue #1

The disturbance is unknown to the controller.

How can we guarantee that $\mathbf{e}_i \in \mathcal{E}_i$?

Somehow the disturbance needs to be accounted for within the optimization problem.

The optimization problem

Given measurement $\mathbf{e}_i(t_k)$, find

$$\overline{\mathbf{u}}_i^{\star}(\cdot;\ \mathbf{e}_i(t_k)) \triangleq \underset{\overline{\mathbf{u}}_i(\cdot)}{\text{argmin}} \quad J_i\big(\mathbf{e}_i(t_k), \overline{\mathbf{u}}_i(\cdot)\big)$$

where

$$J_i\big(\mathbf{e}_i(t_k), \overline{\mathbf{u}}_i(\cdot)\big) \triangleq \int_{t_k}^{t_k+T_p} F_i\big(\overline{\mathbf{e}}_i(s), \overline{\mathbf{u}}_i(s)\big) ds \ + \ V_i\big(\overline{\mathbf{e}}_i(t_k+T_p)\big)$$

subject to:

$$\dot{\overline{\mathbf{e}}}_i(s) = g_i\big(\overline{\mathbf{e}}_i(s), \overline{\mathbf{u}}_i(s)\big), \quad \overline{\mathbf{e}}_i(t_k) = \mathbf{e}_i(t_k)$$

$$\overline{\mathbf{u}}_i(s) \in \mathcal{U}_i, \quad \overline{\mathbf{e}}_i(s) \in \mathcal{E}_{i,s-t_k}, \quad s \in [t_k, t_k+T_p]$$

$$\overline{\mathbf{e}}_i(t_k+T_p) \in \Omega_i$$

Restricted constraint set (1/5)

$\mathcal{E}_{i,s-t_k} \equiv \mathcal{E}_i \ominus \mathcal{B}_{i,s-t_k}$

$\mathcal{B}_{i,s-t_k} \equiv \{\overline{\mathbf{e}}_i : \|\overline{\mathbf{e}}_i\| \leq \zeta(\overline{\delta}_i, s)\}, \;\; \zeta(\nearrow, \nearrow)$

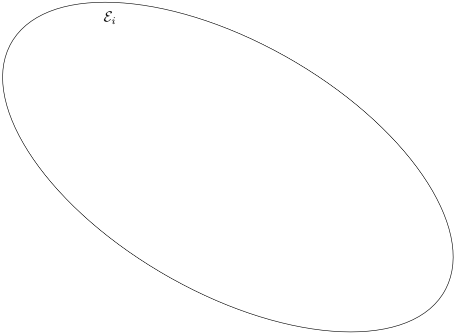

Figure : The set where all real trajectories must lie in.

Restricted constraint set (2/5)

$$\mathcal{E}_{i,s-t_k} \equiv \mathcal{E}_i \ominus \mathcal{B}_{i,s-t_k}$$

$$\mathcal{B}_{i,s-t_k} \equiv \{\overline{\mathbf{e}}_i : \|\overline{\mathbf{e}}_i\| \leq \zeta(\overline{\delta}_i, s)\}, \;\; \zeta(\nearrow, \nearrow)$$

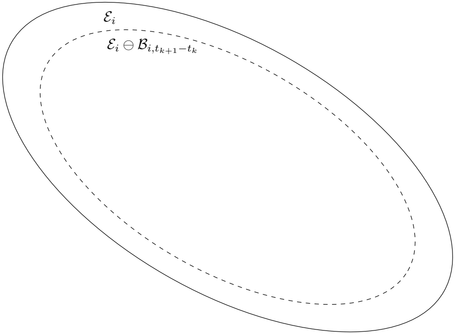

Figure : The set where all *predicted* trajectories are restricted in, when $t = t_k + h$, dashed.

Restricted constraint set (3/5)

$\mathcal{E}_{i,s-t_k} \equiv \mathcal{E}_i \ominus \mathcal{B}_{i,s-t_k}$

$\mathcal{B}_{i,s-t_k} \equiv \{\overline{\mathbf{e}}_i : \|\overline{\mathbf{e}}_i\| \leq \zeta(\overline{\delta}_i, s)\}, \;\; \zeta(\nearrow, \nearrow)$

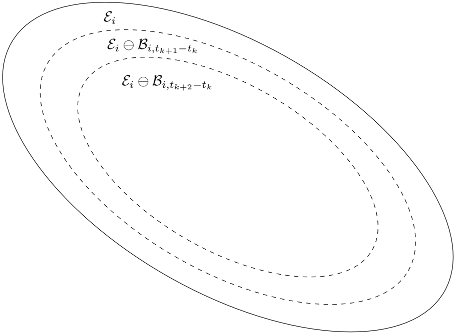

Figure : The set where all *predicted* trajectories are restricted in, when $t = t_k + 2h$, dashed.

Restricted constraint set (4/5)

$$\mathcal{E}_{i,s-t_k} \equiv \mathcal{E}_i \ominus \mathcal{B}_{i,s-t_k}$$

$$\mathcal{B}_{i,s-t_k} \equiv \{\overline{\mathbf{e}}_i : \|\overline{\mathbf{e}}_i\| \leq \zeta(\overline{\delta}_i, s)\}, \;\; \zeta(\nearrow, \nearrow)$$

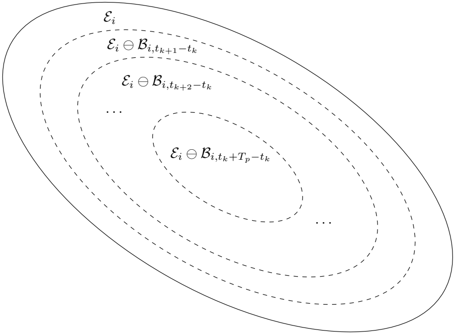

Figure : The set where all *predicted* trajectories are restricted in, when $t = t_k + T_p$, dashed.

Restricted constraint set (5/5)

Can be proved that this tightening of constraints results in (real) $\mathbf{e}_i \in \mathcal{E}_i$

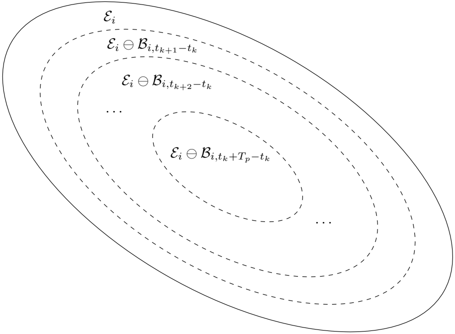

Figure : Progressive tightening of the original constraint set $\mathcal{E}_i$.

Issue #2

In principle:

1. agents > 1
2. agents need to cooperate

How can agents plan...

...when the values of their constraints are fundamentally unknown?

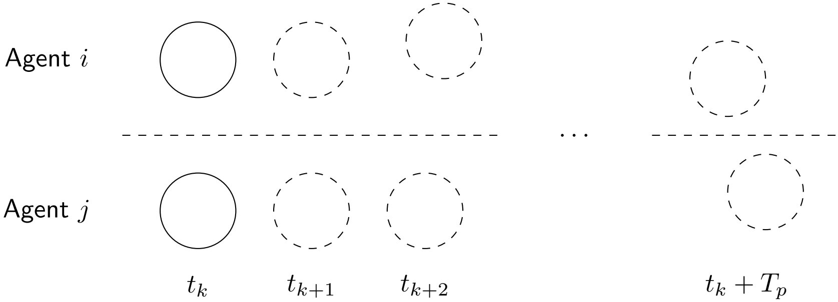

Figure : Suppose agent i solves its optimization problem before agent j. Within $[t_k, t_k + T_p]$, agent j will not be still. Furthermore, its trajectory is fundamentally unknown.

They access the open-loop trajectories of agents in $\mathcal{R}_i$

When at time t_k agent i solves a finite horizon optimization problem, he has access to

1. *measurements* of the states $\mathbf{x}_j(t_k)$

   of all agents $j \in \mathcal{R}_i(t_k)$ within its sensing range at time t_k

2. the *predicted states* $\overline{\mathbf{x}}_j(\tau)$, $\tau \in (t_k, t_k + T_p]$

   of all agents $j \in \mathcal{R}_i(t_k)$ within its sensing range

Inter-agent intra-horizon constraint design

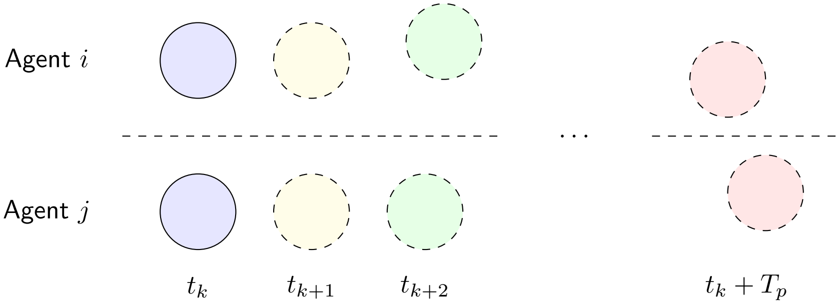

Figure : The state of agent i at each timestep is constrained by the state of j at the same step.

Steering to the desired state

Remember... the designed control input $\mathbf{u}_i$ must

- result in states that satisfy the state constraints
- drive trajectory to / stabilize at the desired state (how close?)
- exist and be feasible $\forall t \geq 0$ $(\mathbf{u}_i \in \mathcal{U}_i)$

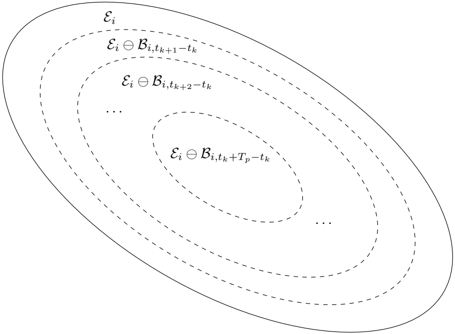

Figure : Progressive tightening of the original constraint set $\mathcal{E}_i$.

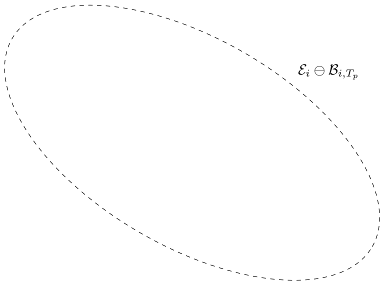

Figure : The furthest the predicted trajectories must be restricted to: $\mathcal{E}_{i,t_k+T_p-t_k}$

Road-trip to equilibrium (3/7)

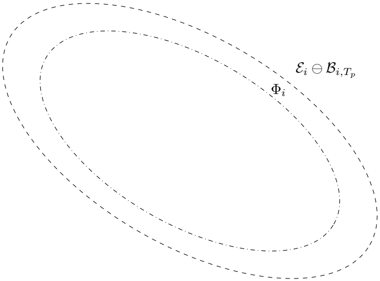

Figure : If linearized g_i is stabilizable, there exists $\overline{\mathbf{e}}_i \in \Phi_i$: $\mathbf{u}_i = h(\overline{\mathbf{e}}_i) \in \mathcal{U}_i$.

Road-trip to equilibrium (4/7)

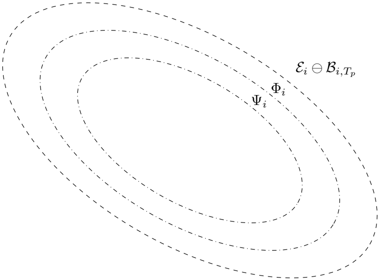

Figure : If this $\mathbf{u}_i$ is applied to a state in Ψ_i...

$$\Psi_i = \{\mathbf{e}_i \in \mathcal{E}_i : V_i(\mathbf{e}_i) = \mathbf{e}_i^\top \boldsymbol{P} \mathbf{e}_i \leq \varepsilon_{\Psi_i}, \;\; \varepsilon_{\Psi_i} > 0\}$$

Road-trip to equilibrium (5/7)

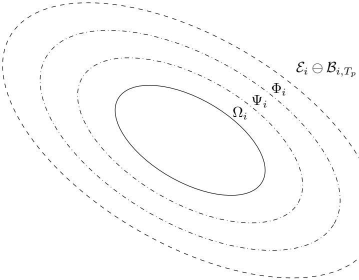

Figure : If this $\mathbf{u}_i$ is applied to a state in Ψ_i, the resulting state will end up in Ω_i.

$$\Omega_i = \{\mathbf{e}_i \in \mathcal{E}_i : V_i(\mathbf{e}_i) = \mathbf{e}_i^\top \boldsymbol{P} \mathbf{e}_i \leq \varepsilon_{\Omega_i}, \quad \varepsilon_{\Omega_i} \in (0, \varepsilon_{\Psi_i})\}$$

Road-trip to equilibrium (6/7)

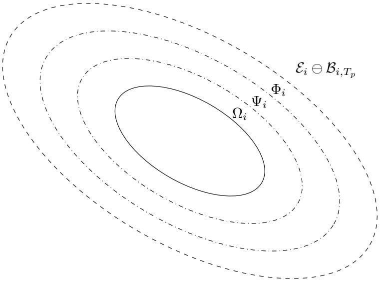

Figure : Once inside Ω_i, the trajectory is trapped there (Ω_i is invariant).

Road-trip to equilibrium (7/7)

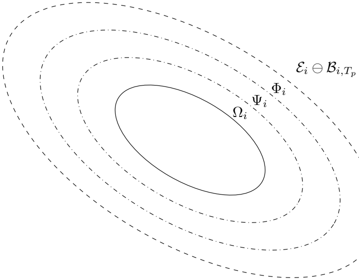

Figure : Once inside Ω_i, the trajectory is trapped there (Ω_i is invariant).

Provided that $\overline{\delta}_i \leq \xi(\varepsilon_{\Psi_i} - \varepsilon_{\Omega_i}, T_p)$, $\xi(\nearrow, \searrow)$, the trajectory never escapes Ω_i.

Recursive feasibility

Remember… the designed control input $\mathbf{u}_i$ must

- result in states that satisfy the state constraints
- drive the trajectory / stabilize at the desired state (how close?)
- exist and be feasible $\forall t \geq 0$ $(\mathbf{u}_i \in \mathcal{U}_i)$

Can show that
if the solution is feasible at $t = 0$, then there are past controls and an auxiliary feedback controller $\forall t > 0$, both in $\mathcal{U}_i$.

Solution requirements (III/III): stabilization

Design decentralized control policies $\mathbf{u}_i \in \mathcal{U}_i$ such that

- Collisions are avoided for all $i, j \in \mathcal{V}$, $\ell \in \mathcal{L}$, $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}$$
$$\|\mathbf{p}_i(t) - \mathbf{p}_\ell\| > \underline{d}_{i\ell,o}$$

  $\mathbf{p}_i$ component of state $\mathbf{x}_i$

- Neighbours $i \in \mathcal{N}_j$, $j \in \mathcal{N}_i$ maintain connectivity at all times $t \geq 0$

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| < \min(\overline{d}_i, \overline{d}_j)$$

- (Closed-loop) System is stable at desired state

$$\lim_{t \to \infty} \|\mathbf{x}_i(t) - \mathbf{x}_{i,\text{des}}\| \to 0$$

How is stabilization achieved?

$$\lim_{t \to \infty} \|\mathbf{x}_i(t) - \mathbf{x}_{i,\mathsf{des}}\| \not\to 0$$

$$\mathbf{e}_i^\top \boldsymbol{P} \mathbf{e}_i \leq \varepsilon_{\Omega_i}$$

How is stabilization achieved?

Can be shown that the optimal cost $J_i^\star = J_i\big(\mathbf{e}_i, \overline{\mathbf{u}}_i^\star\big)$

is an *Input-to-state* Lyapunov function in $\mathcal{E}_i$ for all $\boldsymbol{\delta}_i : \|\boldsymbol{\delta}_i\|_\infty \leq \overline{\delta}_i$

In essence:

- the *trajectory* is bounded in Ω_i: it does not escape Ω_i
- the *desired state* is not asymptotically stable unless the disturbance is decaying

Simulations

Example system: the unicycle

Unicycle dynamics

$$\dot{x}_i(t) = v_i(t)\cos\theta_i(t) + \delta_i(t)$$

$$\dot{y}_i(t) = v_i(t)\sin\theta_i(t) + \delta_i(t)$$

$$\dot{\theta}_i(t) = \omega_i(t) + \delta_i(t)$$

$$\delta_i(t) = \overline{\delta}_i \sin 2t$$

States: x_i, y_i, θ_i

Inputs: v_i, ω_i

Disturbance: $\overline{\delta}_i = 0.1$

$i \in \{1, 2, 3\}$

Simulation example I/II: 2D Trajectories

2D trajectories of agent 1 (blue, middle), agent 2 (yellow, below) and agent 3 (red, above). Black circles are obstacles. Agents 2 and 3 must keep connectivity with agent 1. Disturbances are absent. Trajectories with and without disturbances are more or less similar. Only steady-state changes.

Simulation example II/II: Equilibrium close-up

The effect of unattenuated disturbances on the steady-state configurations. Left: no disturbance. Right: unattenuated additive bounded disturbances with $\overline{\delta}_i = 0.1$.

Simulation example II/II: Equilibrium close-up

Steady-state configurations. Left: no disturbance. Middle: unattenuated additive bounded disturbances with $\overline{\delta}_i = 0.1$. Right: attenuated additive bounded disturbances with $\overline{\delta}_i = 0.1$ as a result of the proposed control regime.

Conclusion

It is possible to design a feasible control regime based on the principle of Receding Horizon Control such that, a multi-agent system constrained with connectivity and collision constraints on the states and input constraints, is stable under the influence of disturbances.

Thank you for your attention.