Multi-Robot Control by Utilizing Distributed Model Predictive Controllers

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Abstract

This paper addresses the problem of position- and orientation-based formation control of a class of second-order nonlinear multi-agent systems in 3D space, under static and undirected communication topologies. More specifically, we design a decentralized control protocol for each agent in the sense that each agent uses only local information from its neighbors to calculate its own control signal. Additionally, by introducing certain inter-agent distance constraints, we guarantee collision avoidance both between among the agents and between the agents and possible obstacles of the workspace. Connectivity maintenance between agents that are initially connected is also achieved by the proposed controller scheme. Finally, simulation results verify the performance of the proposed controllers.

Contents

1 Introduction

formation of multi-agent systems, mpc intro etc.

motivation why we need mpc controllers...

In many control problems it is desired to design a stabilizing feedback such that a performance criterion is minimized while satisfying constraints on the controls and the states. Ideally one would look for a closed solution for the feedback law satisfying the constraints while optimizing the performance. However, typically the optimal feedback law cannot be found analytically, even in the unconstrained case, since it involves the solution of the corresponding Hamilton-Jacobi-Bellman partial differential equations. One approach to circumvent this problem is the repeated solution of an open-loop optimal control problem for a given state. The first part of the resulting open-loop input signal is implemented and the whole process is repeated. Control approaches using this strategy are referred to as Model Predictive Control (MPC).

2 Notation and Preliminaries

2.1 Notation

The set of positive integers is denoted by$\mathbb{N}$. The real n-coordinate space, with $n \in \mathbb{N}$, is denoted by $\mathbb{R}^n$; $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively. Given a set S, we denote as $|S|$ its cardinality. The notation $\|\mathbf{x}\|$ is used for the Euclidean norm of a vector $\mathbf{x} \in \mathbb{R}^n$. Given a symmetric matrix $\boldsymbol{A} = \boldsymbol{A}^T$, $\lambda_{\min}(\boldsymbol{A}) = \min\{|\lambda| : \lambda \in \sigma(\boldsymbol{A})\}$ denotes the minimum eigenvalue of $\boldsymbol{A}$, respectively, where $\sigma(\boldsymbol{A})$ is the set of all the eigenvalues of $\boldsymbol{A}$ and $\text{rank}(\boldsymbol{A})$ is its rank; $\boldsymbol{A} \otimes \boldsymbol{B}$ denotes the Kronecker product of matrices $\boldsymbol{A}, \boldsymbol{B} \in \mathbb{R}^{m \times n}$, as was introduced in [**?**]. Define by $\mathbf{1}_n \in \mathbb{R}^n, \boldsymbol{I_n} \in \mathbb{R}^{n \times n}, \boldsymbol{0}_{m \times n} \in \mathbb{R}^{m \times n}$ the column vector with all entries 1, the unit matrix and the $m \times n$ matrix with all entries zeros, respectively. A matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is called skew-symmetric if and only if $\boldsymbol{A}^\top = -\boldsymbol{A}$. $\mathcal{B}(\mathbf{c}, r) \triangleq \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x} - \mathbf{c}\| \leq r\}$ is the 3D sphere of radius $r \in \mathbb{R}_{\geq 0}$ and center $\mathbf{c} \in \mathbb{R}^3$.

The vector expressing the coordinates of the origin of frame $\{j\}$ in frame $\{i\}$ is denoted by $\mathbf{p}_{j \triangleright i}$. When this vector is expressed in 3D space in a third frame, frame $\{k\}$, it is denoted by $\mathbf{p}^k_{j \triangleright i}$. The angular velocity of frame $\{j\}$ with respect to frame $\{i\}$, expressed in frame $\{k\}$ coordinates, is denoted by $\boldsymbol{\omega}^k_{j \triangleright i} \in \mathbb{R}^3$. We also use the notation $\mathbb{M} = \mathbb{R}^3 \times \mathbb{T}^3$. We further denote as $\mathbf{q}_{j \triangleright i} \in \mathbb{T}^3$ the Euler angles representing the orientation of frame $\{j\}$ with respect to frame $\{i\}$, where $\mathbb{T}^3$ is the 3D torus. For notational brevity, when a coordinate frame corresponds to the inertial frame of reference $\{\mathcal{O}\}$, we will omit its explicit notation (e.g., $\mathbf{p}_i = \mathbf{p}_{i \triangleright \mathcal{O}} = \mathbf{p}^{\mathcal{O}}_{i \triangleright \mathcal{O}}$, $\boldsymbol{\omega}_i = \boldsymbol{\omega}_{i \triangleright \mathcal{O}} = \boldsymbol{\omega}^{\mathcal{O}}_{i \triangleright \mathcal{O}}$). All vector and matrix differentiations are derived with respect to the inertial frame $\{\mathcal{O}\}$ unless stated otherwise.

2.2 Graph Theory

?? ISTORISOU // EXPAND An *undirected graph* $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a finite set of nodes, representing a team of agents, and $\mathcal{E} \subseteq \big\{\{i,j\} : i, j \in \mathcal{V}, i \neq j\big\}$, with $M = |\mathcal{E}|$, is the set of edges that model the communication capability between neighboring agents. For each agent, its neighbors' set $\mathcal{N}_i$ is defined as $\mathcal{N}_i = \{i_1, \ldots, i_{N_i}\} = \big\{j \in \mathcal{V} : \{i,j\} \in \mathcal{E}\big\}$, where $i_1, \ldots, i_{N_i}$ is an enumeration of the neighbors of agent i and $N_i = |\mathcal{N}_i|$.

If there is an edge $\{i,j\} \in \mathcal{E}$, then i, j are called *adjacent*. A *path* of length r from vertex i to vertex j is a sequence of $r+1$ distinct vertices, starting with i and ending with j, such that consecutive vertices are adjacent. For $i = j$, the path is called a *cycle*. If there is a path between any two vertices of the graph $\mathcal{G}$, then $\mathcal{G}$ is called *connected*. A connected graph is called a *tree* if it contains no cycles.

2.3 Non-linear Model Predictive Control

3 Problem Formulation

3.1 System Model

Consider a set of N rigid bodies, with $\mathcal{V} = \{1, 2, \ldots, N\}$, $N \geq 2$, operating in a workspace $W \subseteq \mathbb{R}^3$. A coordinate frame $\{i\}, i \in \mathcal{V}$ is attached to each body's center of mass. The workspace is assumed to be modeled as a bounded sphere $\mathcal{B}(\mathbf{0}, r_W)$ expressed in an inertial frame $\{\mathcal{O}\}$.

We consider that over time t each agent i occupies the space of a sphere $\mathcal{B}(\mathbf{p}_i(t), r_i)$, where $\mathbf{p}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$ is the position of the agent's center of mass, and $r_i < r_W$ is the radius of the agent's body. We denote $\mathbf{q}_i(t) : \mathbb{R}_{\geq 0} \to \mathbb{T}^3, i \in \mathcal{V}$, the Euler angles representing the agents' orientation with respect to the inertial frame $\{\mathcal{O}\}$, with $\mathbf{q}_i \triangleq [\phi_i, \theta_i, \psi_i]^\top$. We define

$$\mathbf{x}_i \triangleq [\mathbf{p}_i^\top, \mathbf{q}_i^\top]^\top, \mathbf{x}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^3 \times \mathbb{T}^3 \equiv \mathbb{M}$$
$$\mathbf{v}_i \triangleq [\dot{\mathbf{p}}_i^\top, \boldsymbol{\omega}_i^\top]^\top, \mathbf{v}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^3 \times \mathbb{R}^3 \equiv \mathbb{R}^6$$

and model the motion of agent i under second order dynamics:

$$\dot{\mathbf{x}}_i(t) = \boldsymbol{J}_i^{-1}(\mathbf{x}_i)\mathbf{v}_i(t), \tag{1a}$$
$$\mathbf{u}_i = \boldsymbol{M}_i(\mathbf{x}_i)\dot{\mathbf{v}}_i(t) + \boldsymbol{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)\mathbf{v}_i(t) + \mathbf{g}_i(\mathbf{x}_i), \tag{1b}$$

In equation (1a), $\boldsymbol{J}_i : \mathbb{T}^3 \to \mathbb{R}^{6\times 6}$ is a Jacobian matrix that maps the non-orthogonal Euler angle rates to the orthogonal angular velocities $\mathbf{v}_i$:

$$\boldsymbol{J}_i(\mathbf{x}_i) = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{J}_q(\mathbf{x}_i) \end{bmatrix}, \text{ with } \boldsymbol{J}_q(\mathbf{x}_i) = \begin{bmatrix} 1 & 0 & -\sin\theta_i \\ 0 & \cos\phi_i & \cos\theta_i \sin\phi_i \\ 0 & -\sin\phi_i & \cos\phi_i\cos\theta_i \end{bmatrix}$$

The matrix $\boldsymbol{J}_i$ is singular when $det(\boldsymbol{J}_i) = \cos\theta_i = 0 \Leftrightarrow \theta_i = \pm\frac{\pi}{2}$. The controller proposed in this thesis guarantees that this is always avoided, and hence equation (1a) is well defined. This gives rise to the following remark:

Remark 3.1. $det(\boldsymbol{J}_i) = \cos\theta_i \leq 1, \forall i \in \mathcal{V}$

In equation (1b), $\boldsymbol{M}_i : \mathbb{M} \to \mathbb{R}^{6\times 6}$ is the symmetric and positive definite *inertia matrix*, $\boldsymbol{C}_i : \mathbb{M} \times \mathbb{R}^6 \to \mathbb{R}^{6\times 6}$ is the *Coriolis matrix* and $\mathbf{g}_i : \mathbb{M} \to \mathbb{R}^6$ is the *gravity vector*. Finally, $\mathbf{u}_i \in \mathbb{R}^6$ is the control input vector representing the 6D generalized *actuation force* acting on the agent.

Remark 3.2. According to [?], the matrices $\dot{\boldsymbol{M}}_i - 2\boldsymbol{C}_i, i \in \mathcal{V}$ are skew-symmetric. The quadratic form of a skew-symmetric matrix is always equal to 0 [?], hence:

$$\mathbf{y}^\top \left[\dot{\boldsymbol{M}}_i - 2\boldsymbol{C}_i\right] \mathbf{y} = 0, \forall \mathbf{y} \in \mathbb{R}^n, i \in \mathcal{V}. \tag{2}$$

However, access to measurements of, or knowledge about these matrices and vectors was not considered up until now. At this point we make the following assumption:

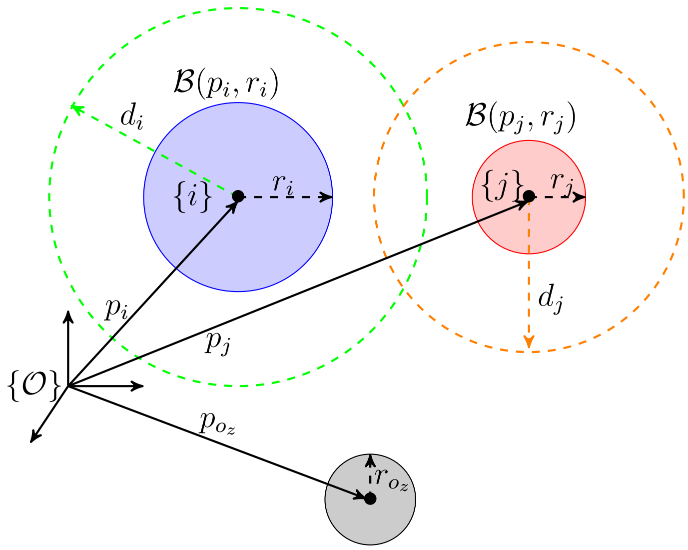

Figure 1: Illustration of two agents $i, j \in \mathcal{V}$ and an static obstacle o_z in the workspace; $\{\mathcal{O}\}$ is the inertial frame, $\{i\}, \{j\}$ are the frames attached to the agents' center of mass, $\mathbf{p}_i, \mathbf{p}_j, \mathbf{p}_{o_z} \in \mathbb{R}^3$ are the positions of the center of mass of the agents i, j and the obstacle o_z respectively, expressed in frame $\{\mathcal{O}\}$. r_i, r_j, r_{o_z} are the radii of the agents i, j and the obstacle o_z respectively. d_i, d_j with $d_i > d_j$ are the agents' sensing ranges. In this figure, agents i and j are not neighbours, since the center of mass of agent j is not within the sensing range of agent i and vice versa: $\mathbf{p}_j \notin \mathcal{B}(\mathbf{p}_i(t), d_i)$ and $\mathbf{p}_i \notin \mathcal{B}(\mathbf{p}_j(t), d_j)$.

Assumption 3.1. (Measurements and Access to Information Assumption From an Inter-agent Perspective)

1. Agent i has access to measurements $\mathbf{p}_i, \mathbf{q}_i, \dot{\mathbf{p}}_i, \boldsymbol{\omega}_i, i \in \mathcal{V}$, that is, vectors $\mathbf{x}_i, \mathbf{v}_i$ pertaining to himself,

2. Agent i has a (upper-bounded) sensing range d_i such that

$$d_i > \max\{r_i + r_j : \forall i, j \in \mathcal{V}, i \neq j\}$$

3. the inertia $\boldsymbol{M}$ and Coriolis $\boldsymbol{C}$ vector fields are bounded and unknown

4. the gravity vectors $\mathbf{g}$ are bounded and known

The consequence of points 1 and 2 is that, by defining the set of agents j that are within the sensing range of agent i at time t as

$$\mathcal{R}_i(t) \triangleq \{j \in \mathcal{V} : \mathbf{p}_j(t) \in \mathcal{B}(\mathbf{p}_i(t), d_i)\}$$

or equivalently

$$\mathcal{R}_i(t) \triangleq \{j \in \mathcal{V} : \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| \leq d_i\}$$

agent i also knows at each time instant t all

$$\mathbf{p}_{j \triangleright i}(t), \mathbf{q}_{j \triangleright i}(t), \dot{\mathbf{p}}_{j \triangleright i}(t), \boldsymbol{\omega}_{j \triangleright i}(t)$$

Therefore, agent i assumes access to all measurements

$$\mathbf{p}_j(t), \mathbf{q}_j(t), \dot{\mathbf{p}}_j(t), \boldsymbol{\omega}_j(t), \forall j \in \mathcal{R}_i(t), t \in \mathbb{R}_{\geq 0}$$

of agent j by virtue of being able to calculate them using knowledge of its own $\mathbf{p}_i(t)$, $\mathbf{q}_i(t)$, $\dot{\mathbf{p}}_i(t)$, $\boldsymbol{\omega}_i(t)$.

In the workspace there are $|Z|$ *static obstacles*, modeled as spheres with centers at positions $\mathbf{p}_{o_z}$ with radii $r_{o_z} \in \mathbb{R}^3, z \in Z = \{1, \ldots, |Z|\}$. Thus, the obstacles are modeled by the spheres $\mathcal{B}(\mathbf{p}_{o_z}, r_{o_z}), z \in \{1, \ldots, |Z|\}$. The geometry in workspace W of two agents i and j as well as an obstacle z is depicted in Fig. 1.

Let us also define the distance between any two agents i, j as $d_{ij,a} : \mathbb{R}^6 \to \mathbb{R}_{\geq 0}$, and that between agent i and obstacle z as $d_{iz,o} : \mathbb{R}^3 \to \mathbb{R}_{\geq 0}$:

$$d_{ij,a}(t) \triangleq \|\mathbf{p}_i(t) - \mathbf{p}_j(t)\|, \tag{3a}$$

$$d_{iz,o}(t) \triangleq \|\mathbf{p}_i(t) - \mathbf{p}_z(t)\| \tag{3b}$$

$\forall i, j \in \mathcal{V}, i \neq j, z \in Z$, as well as constants $\underline{d}_{ij,a} \triangleq r_i + r_j, \underline{d}_{iz,o} \triangleq r_i + r_{o_z}$. The latter stand for the minimum distance between two agents, and the minimum distance between an agent and an obstacle. These arise spatially as physical limitations and will be utilized as collision-avoidance constraints.

The *topology* of the multi-agent network is modeled through the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} = \{1, \ldots, N\}$ and

$$\mathcal{E} \triangleq \big\{\{i, j\} \in \mathcal{V} \times \mathcal{V} : i \neq j, j \in \mathcal{N}_i(0) \subseteq \mathcal{R}_i(0) \text{ and } i \in \mathcal{N}_j(0) \subseteq \mathcal{R}_j(0)\big\} \tag{4}$$

Agents $j \in \mathcal{N}_i(0)$ are defined as the *neighbours* of agent i. They are the set of indices of agents $j \in \mathcal{V}$ which

1. are within the sensing range of agent i at time $t = 0$, i.e. $j \in \mathcal{R}_i(0)$, and
2. are intended to be kept within the sensing range of agent i at all times $t \in \mathbb{R}_{>0}$

Therefore, while the composition of the set $\mathcal{R}_i(t)$ evolves and varies through time in general, the set $\mathcal{N}_i(t)$ should remain constant through time. That means that we consider that the connected graph $\mathcal{G}$ is static, in the sense that no edges are added to it through time. We do not exclude, however, edge removal through connectivity loss between initially neighboring agents, which we guarantee to avoid, as presented in the sequel.

Definition (4) implies that the graph is undirected, i.e.,

$$d_{ij,a}(0) < d_i, \forall i \in \mathcal{V}, j \in \mathcal{N}_i(0) \tag{5}$$

$$d_{ji,a}(0) < d_j, \forall j \in \mathcal{V}, i \in \mathcal{N}_j(0) \tag{6}$$

It is also assumed that at $t = 0$ the neighboring agents are in a *collision-free configuration*, i.e.

$$\underline{d}_{ij,a} < d_{ij,a}(0), \forall i, j \in \mathcal{V}, i \neq j \tag{7}$$

Furthermore, it is assumed that, initially, the Jacobians $\boldsymbol{J}_i$ are well-defined $\forall i \in \mathcal{V}$. These four assumptions, which concern the initial conditions of the problem, are summarized in assumption 3.2:

Assumption 3.2. (Initial Conditions Assumption)

At time $t = 0$

1. the communication graph $\mathcal{G}$ is connected,
2. all agents are in a collision-free configuration:

$$\underline{d}_{ij,a} < d_{ij,a}(0) < d_i$$

3. all agents are in a singularity-free configuration:

$$\theta_i(0) \neq \pm\frac{\pi}{2}$$

$\forall i \in \mathcal{V}$ and all agents j within the sensing range of i.

As for $t \geq 0$, agent i is in a collision-and-singularity-free configuration, and is connected to the neighbours it had at time $t = 0$, when

$$\sum_{j \in \mathcal{N}_i(0)} \|\mathbf{x}_i(t) - \mathbf{x}_j(t) - \mathbf{x}_{ji,des}\| < \mu_i, \forall i \in \mathcal{V}$$

where μ_i is an arbitrarily small positive constant. Intuitively, this means that when the agents reach very close to their desired formation, they will be sufficiently away from collisions, connectivity losses and singular configurations. In particular, this concerns collisions between an agent and an obstacle or the workspace boundary (inter-agent collisions and connectivity losses are avoided by choosing feasible formation displacements $p_{ij,des}$), and singular configurations. In other words, the multi-agent formation should be performed sufficiently away from obstacles and the workspace boundary.

The desired steady-state displacements $\mathbf{x}_{ij,des} = [\mathbf{p}_{ij,des}^\top, \mathbf{q}_{ij,des}^\top]^\top$ are themselves *feasible* if $\forall i \in \mathcal{V}$, $j \in \mathcal{N}_i(0)$,

$$\bigcap \{(\mathbf{x}_i, \mathbf{x}_j) \in W^{N_i+1} : \|\mathbf{x}_i - \mathbf{x}_j - \mathbf{x}_{ij,des}\| = 0\} \neq \varnothing$$

At this point, let us define d_o

$$d_o \triangleq \min\{\|\mathbf{p}_{o_z} - \mathbf{p}_{o_{z'}}\| : z, z' \in Z, z \neq z'\},$$

as the distance between the two least distant obstacles in the workspace, $d_{o,W}$

$$d_{o,W} \triangleq \min\{r_W - \left(r_{o_z} + \|\mathbf{p}_{o_z}\|\right) : z \in Z\},$$

as the distance between the least distant obstacle from the boundary of the workspace and the boundary itself, D

$$D \triangleq \min\{d_o, d_{o,W}\}$$

as the least of these two distances, and Δ

$$\Delta \triangleq \max\Big\{d_{ij,a} + r_i + r_j : i, j \in \mathcal{V}, i \neq j, \\ \|\mathbf{p}_k - \mathbf{p}_\ell\| = \mathbf{p}_{k\ell,\text{des}}, \\ \|\mathbf{q}_k - \mathbf{q}_\ell\| = \mathbf{q}_{k\ell,\text{des}}, \\ k \in \mathcal{V}, \ell \in \mathcal{N}_k(0)\Big\}$$

as the *diameter of formation.* Δ is the distance between the two most distant agents when formation is achieved. Given these notions, we can state an assumption on the feasibility of a solution to the problem that this thesis addresses:

Assumption 3.3. (After-formation Geometric Assumption)

- When the multi-agent system reaches the desired formation, it should be able to pass between two of the obstacles and between an obstacle and the boundary of the workspace. Thus, it is required that $D > \Delta$.
- As a consequence, at the very least, all agents should be able to pass between any two obstacles and between all obstacles and the boundary of the workspace, without, simultaneously, any of them colliding with each other or with the obstacles or the boundary of the workspace. Thus, it is required that $D > \sum_{i \in \mathcal{V}} 2r_i$.

These geometric assumptions can be summarized in the following inequality:

$$D > \max\Big\{\Delta, \sum_{i \in \mathcal{V}} 2r_i\Big\} \tag{8}$$

3.2 Problem Statement

Due to the fact that the agents are not dimensionless and their communication capabilities are limited, the control protocol should for all neighboring agents $i \in \mathcal{V}, j \in \mathcal{N}_i(0)$ make sure that:

1. the desired position formation $\mathbf{p}_{ij,\text{des}}$ is achieved in finite time
2. the desired formation angles $\mathbf{q}_{ij,\text{des}}$ are achieved in finite time
3. connectivity between initially connected agents (neighbours) is maintained at all times, i.e. all edges of $\mathcal{G}(t = 0)$ are maintained

Furthermore, for all agents $i \in \mathcal{V}$, obstacles $z \in \mathcal{Z}$ and the workspace boundary W, it should guarantee for all $t \in \mathbb{R}_{\geq 0}$ that:

1. all agents avoid collision with each other
2. all agents avoid collision with all obstacles

3. all agents avoid collision with the workspace boundary

4. singularity of the Jacobian matrices $\boldsymbol{J}_i$ is avoided

Therefore, all neighboring agents of agent i must remain within a distance less than d_i from him, for all $i \in \mathcal{V}$, and all agents $i, j \in \mathcal{V}, i \neq j$ must remain within distance greater than $\underline{d}_{ij,a}$ with one another.

Formally, the control problem under the aforementioned constraints is formulated as follows:

Problem 3.1. Consider N agents modeled as bounded spheres $\mathcal{B}(\mathbf{p}_i, r_i)$, $i \in \mathcal{V}, |V| = N$ that operate in workspace W that is also modeled as a bounded sphere $\mathcal{B}(0, r_W)$, featuring $|Z|$ spherical obstacles, also modeled as bounded spheres $\mathcal{B}(p_{o_z}, r_{o_z}), z \in \mathcal{Z}$. Each agent i is governed by the dynamics (1), under assumptions 3.1, 3.2, 3.3. Given desired *feasible* inter-agent displacements $\mathbf{p}_{ij,des}$, $\mathbf{q}_{ij,des}$, $\forall i \in \mathcal{V}, j \in \mathcal{N}_i(0)$ such that

$$\underline{d}_{ij,a} < d_{ij,a} < d_i, \forall (i,j) \in \{(i,j) \in \mathcal{V} \times \mathcal{V} : \|\mathbf{p}_i - \mathbf{p}_j - \mathbf{p}_{ij,des}\| = 0\}$$

design decentralized control laws $\mathbf{u}_i \in \mathbb{R}^6$ such that $\forall i \in \mathcal{V}$ and $t \in \mathbb{R}_{\geq 0}$ the following hold:

1. Position and orientation formation is achieved in steady-state

$$\lim_{t \to \infty} \|\mathbf{x}_i(t) - \mathbf{x}_j(t) - \mathbf{x}_{ij,des}(t)\| < \mu_i, \forall j \in \mathcal{N}_i(0)$$

2. Inter-agent collision is avoided

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| > \underline{d}_{ij,a}, \forall j \in \mathcal{V} \backslash \{i\}$$

3. Agent-with-obstacle collision is avoided

$$\|\mathbf{p}_i(t) - \mathbf{p}_{o_z}(t)\| > \underline{d}_{iz,o}, \forall z \in \mathcal{Z}$$

4. Agent-with-workspace-boundary collision is avoided

$$\|\mathbf{p}_i(t)\| + r_i < r_W$$

5. Inter-agent connectivity loss is avoided

$$\|\mathbf{p}_i(t) - \mathbf{p}_j(t)\| < d_i, \forall j \in \mathcal{N}_i(0)$$

6. All maps $\boldsymbol{J}_i$ are well defined

$$\theta_i(t) \neq \pm \frac{\pi}{2}$$

4 Problem Solution

In this section, a systematic solution to Problem 3.1 is introduced. Our overall approach builds on formulating a model predictive control optimization problem for each agent such that solution this captures all the desired control specifications. The following analysis is performed:

1. The form of the proposed optimizaton problem is described in Section 4.1.
2. The feasibility of the problem is given in 4.2.
3. The stability analysis is given in 4.3.

4.1 Distributed MPC

4.1.1 Model

The state evolution of agent i is modeled by a non-linear continuous-time differential equation of the form

$$\dot{\mathbf{x}}_i(t) = f_i\big(\mathbf{x}_i(t), \mathbf{u}_i(t)\big) \tag{9}$$
$$\mathbf{x}_i(0) = \mathbf{x}_0 \tag{10}$$
$$\mathbf{x}_i(t) \in \mathcal{X}_i \subset \mathbb{R}^n \tag{11}$$
$$\mathbf{u}_i(t) \in \mathcal{U}_i \subset \mathbb{R}^m \tag{12}$$

where state $\mathbf{x}_i$ is directly measurable. Equation 9 does not consider model-plant mismatches or external disturbances. The applied input $\mathbf{u}_i$ is a portion of the optimal solution to an optimization problem where the *predicted* states of the neighbouring agents of agent i are taken into account in the cost function, as opposed to their *true* states.

Assumption 4.1. (Functions f_i are Lipschitz continuous)
The functions $f_i(\mathbf{x}, \mathbf{u}), \forall i \in \mathcal{V}, x \in \mathcal{X}_i$ are Lipschitz continuous with Lipschitz constants L_{f_i}:

$$\|f_i(\mathbf{x}_1, \mathbf{u}) - f_i(\mathbf{x}_2, \mathbf{u})\| \leq L_{f_i}\|\mathbf{x}_1 - \mathbf{x}_2\| \tag{13}$$

4.1.2 Cost function

$$J_i\big(\overline{\mathbf{u}}_i(\cdot);\ \mathbf{x}_i(t_k), \mathbf{x}_{\mathcal{N}_i}(t_k)\big) = J_i^U\big(\overline{\mathbf{u}}_i(\cdot)\big) + J_i^X\big(\mathbf{x}_i(t_k), \mathbf{x}_{\mathcal{N}_i}(t_k)\big) \tag{14}$$

where

$$J_i^U\big(\overline{\mathbf{u}}_i(\cdot)\big) = \int_{t_k}^{t_k+T_p} h_i\big(\overline{\mathbf{u}}_i(\tau)\big)d\tau \tag{15}$$

$$J_i^X\big(\mathbf{x}_i(t_k), \mathbf{x}_{\mathcal{N}_i}(t_k)\big) = \sum_{j\in\mathcal{N}_i}\Bigg(\int_{t_k}^{t_k+T_p} g_{ij}(\overline{\mathbf{x}}_i(\tau), \overline{\mathbf{x}}_j(\tau)\big)d\tau + V_{ij}\big(\overline{\mathbf{x}}_i(t_k+T_p), \overline{\mathbf{x}}_j(t_k+T_p)\big)\Bigg) \tag{16}$$

and

$$h_i(\mathbf{u}_i) = \|\mathbf{u}_i\|^2_{R_i} \tag{17}$$

$$g_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \|(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{x}_{ji,des}\|^2_{G_{ij}} \tag{18}$$

Assumption 4.2. (Functions h_i are Lipschitz continuous)
The functions $h_i(\mathbf{u}), \forall i \in \mathcal{V}, \mathbf{u} \in \mathcal{U}_i$ are Lipschitz continuous with Lipschitz constants L_{h_i}:

$$\|h_i(\mathbf{u}_1) - h_i(\mathbf{u}_2)\| \leq L_{h_i}\|\mathbf{u}_1 - \mathbf{u}_2\| \tag{19}$$

Assumption 4.3. (Functions g_{ij} are Lipschitz continuous)
The functions $g_{ij}(\mathbf{x}_i, \mathbf{x}_j), \forall i \in \mathcal{V}, j \in \mathcal{N}_i$ are Lipschitz continuous with Lipschitz constants L_{g_i}:

$$\|g_{ij}(\mathbf{x}_{i,1}, \mathbf{x}_j) - g_{ij}(\mathbf{x}_{i,2}, \mathbf{x}_j)\| \leq L_{g_i}\|\mathbf{x}_{i,1} - \mathbf{x}_{i,2}\| \tag{20}$$

4.1.3 Optimization problem

$$\min_{\overline{\mathbf{u}}_i(\cdot)} J_i\big(\overline{\mathbf{u}}_i(\cdot);\ \mathbf{x}_i(t_k), \mathbf{x}_{\mathcal{N}_i}(t_k), \mathbf{x}_{\mathcal{Z}}\big) \tag{21}$$

$$\text{subject to:} \tag{22}$$

$$\dot{\overline{\mathbf{x}}}_i(t) = f_i(\overline{\mathbf{x}}_i(t), \overline{\mathbf{u}}_i(t)) \tag{23}$$

$$\overline{\mathbf{x}}_i(t_k) = \mathbf{x}_i(t_k) \tag{24}$$

$$\overline{\mathbf{u}}_i \in \mathcal{U}_i \tag{25}$$

$$\overline{\mathbf{x}}_i(t) \in \mathcal{X}_i^{t-t_k}, t \in [t_k, t_k+T_p] \tag{26}$$

$$\overline{\mathbf{x}}_i(t_k+T_p) \in \mathcal{X}_{f_i} \tag{27}$$

$$J_i\big(\overline{\mathbf{u}}_i(\cdot);\ \mathbf{x}_i(t_k)\big) \leq J_i^{\sup}(t) \tag{28}$$

where the constraint on $\overline{\mathbf{x}}_i(t)$ is narrowed to $\overline{\mathbf{x}}_i(t) \in \mathcal{X}_i^{t-t_k} \subseteq \mathcal{X}_i$ so as to ensure that there is a robust positively invariant set for the closed-loop system where a solution to the FHOCP exists **??**.

Specifically, $\mathcal{X}_i^{t-t_k} = \mathcal{X}_i \sim \mathcal{B}_i^{t-t_k}$, where

$$\mathcal{B}_i^{t-t_k} = \Big\{ \mathbf{x}_i \in \mathbb{R}^n : \|\mathbf{x}_i\| \leq \gamma_i(t-t_k) = \frac{\overline{\mathbf{w}}_i}{L_{f_i}}(e^{L_{f_i}(t-t_k)} - 1) \Big\} \tag{29}$$

and the operator $\sim$ denotes the Pontryagin difference.

The constraint $J_i\big(\overline{\mathbf{u}}_i(\cdot);\ \mathbf{x}_i(t_k)\big) \leq J_i^{\text{sup}}(t_k)$ is imposed for ensuring the stability for each agent; it will be defined later on in the text.

Assumption 4.4. There exists a local stabilizing controller $\boldsymbol{\kappa}_i(\mathbf{x}) \in \mathcal{U}_i$ such that

$$\frac{\partial V_i}{\partial \mathbf{x}} f_i\Big(\mathbf{x}(\tau), \boldsymbol{\kappa}_i\big(\mathbf{x}(\tau)\big)\Big) + h_i\Big(\mathbf{x}(\tau), \boldsymbol{\kappa}_i\big(\mathbf{x}(\tau)\big)\Big) \leq 0, \forall \mathbf{x} \in \Phi_i \tag{30}$$

where $\Phi_i = \{\mathbf{x} \in \mathbb{R}^n : V_i(\mathbf{x}) \leq \alpha_i\}$, and $\Phi_i \subseteq \mathcal{X}_i^{T_p} = \{x \in \mathbb{R}^n : \boldsymbol{\kappa}_i(\mathbf{x}) \in \mathcal{U}_i\}$

Assumption 4.5. The terminal costs $V_i(\mathbf{x})$ are Lipschitz continuous in $\mathbf{x} \in \Phi_i$ with Lipschitz constants L_{V_i}:

$$\|V_i(\mathbf{x}_1) - V_i(\mathbf{x}_2)\| \leq L_{V_i}\|\mathbf{x}_1 - \mathbf{x}_2\| \tag{31}$$

Assumption 4.6. The terminal set $\mathcal{X}_{f_i} = \{\mathbf{x} \in \mathbb{R}^n : V_i(\mathbf{x}) \in \alpha_{V_i}\}$ is such that for all $\mathbf{x} \in \Phi_i,\ f_i\big(\mathbf{x}, \boldsymbol{\kappa}_i(\mathbf{x})\big) \in \mathcal{X}_{f_i} \subseteq \Phi_i$.

4.2 Feasibility Analysis

4.3 Stability Analysis

5 Simulation Results

6 Conclusions and Future Work