DD2423 - Lab I

1 Question 1

What we see here is that:

- The further the non-zero point (p, q) from the origin $O(0, 0)$, the smaller the wavelength of the spatial image (more dense lines in the real and imaginary part of the spatial image),
- The amplitude of all the spatial images is the same,
- The direction of the waveforms in the spatial images is dictated by the position of the non-zero point (p, q) relative to the origin $O(0, 0)$

2 Question 2

We exploit equation $4.2 - 33$ from [?]

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} s(x, y) \cdot A\delta(x - x_0, y - y_0) = A \cdot s(x_0, y_0) \tag{1}$$

knowing that the output Fourier transform is a delta function at (p, q). Hence, for a quadratic image $M = N$ and in the spatial domain:

$$f(x, y) = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \delta(u-p, v-q) \cdot e^{\frac{2\pi i \cdot (xu + yv)}{N}} = \frac{1}{N^2} \cdot e^{\frac{2\pi i \cdot (px + qy)}{N}} \tag{2}$$

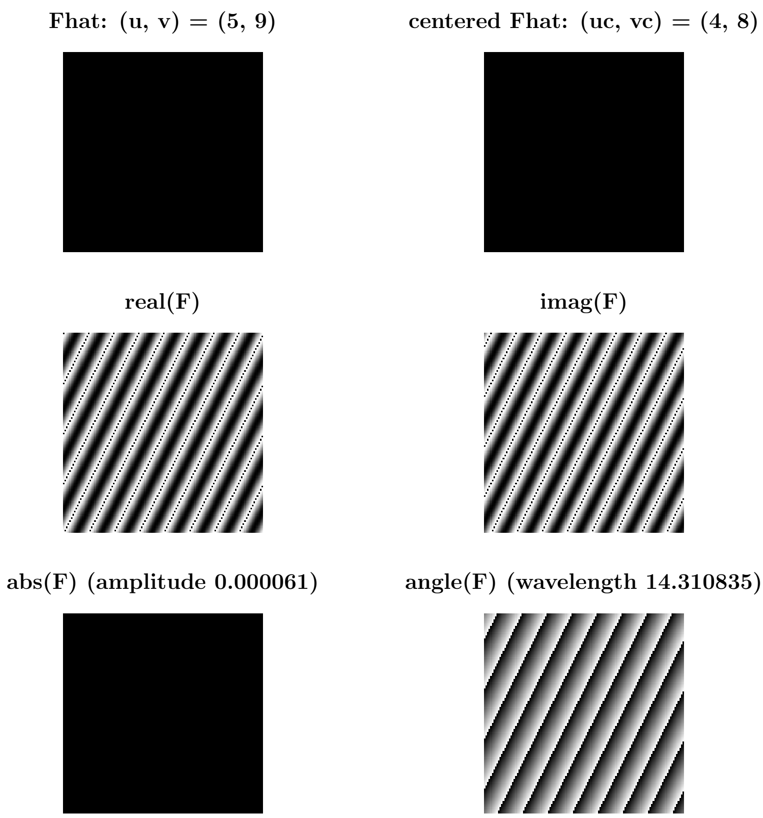

Figure 1: $(p, q) = (5, 9)$

Hence,

$$f(x, y) = \frac{1}{N^2} \cdot (\cos(\frac{2\pi \cdot (px + qy)}{N}) + i \sin(\frac{2\pi \cdot (px + qy)}{N})) \tag{3}$$

TODO: add figures

3 Question 3

As can be seen in equation 3, the amplitude of the waveform is

$$A = \frac{1}{N^2} \tag{4}$$

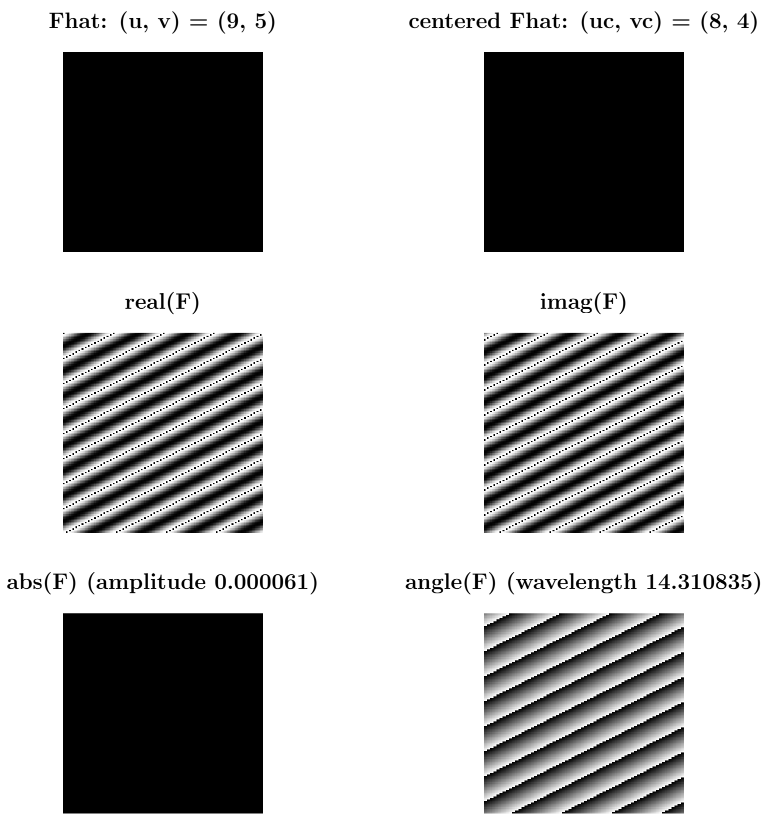

Figure 2: $(p, q) = (9, 5)$

4 Question 4

As seen in the lecture notes,

$$\lambda = \frac{2\pi}{|\omega|} \tag{5}$$

and

$$\omega = [\frac{2\pi u}{N} \ \frac{2\pi v}{N}]^T \tag{6}$$

Hence, equation 5 for $(u, v) = (p, q)$ becomes

$$\lambda = \frac{N}{\sqrt{p^2 + q^2}} \tag{7}$$

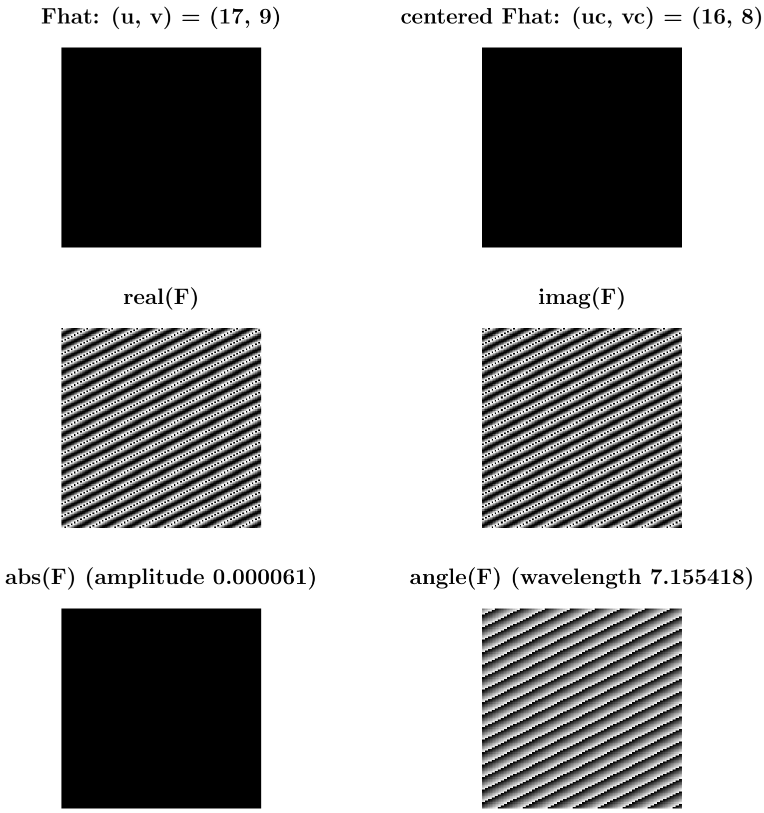

Figure 3: $(p, q) = (17, 9)$

TODO: add figures

5 Question 5

For an quadratic image of size N, the highest number of cycles that can fit in it is $N/2$. Hence, when either p or q exceed the value of $N/2$, which in our case is $N/2 = 64$, the Nyquist frequency is exceeded and the corresponding waveform in the spatial domain is no longer a sinusoid.

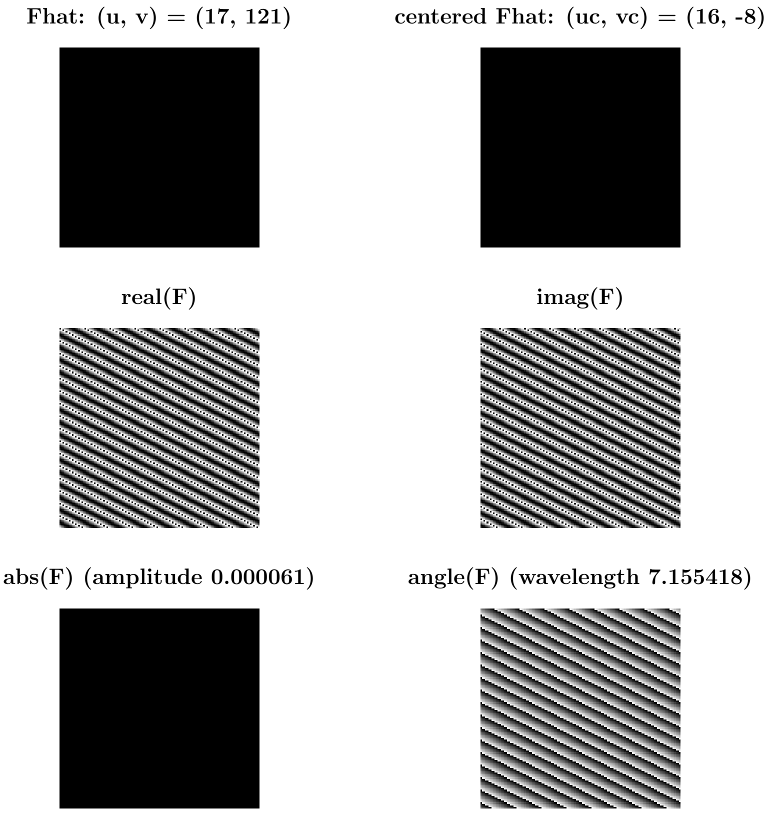

Figure 4: $(p, q) = (17, 121)$

6 Question 6

The purpose of these lines is to correctly map the angular frequency values ω_x and ω_y inside the interval

$$-\frac{N}{2} \leq \omega_x, \omega_y \leq \frac{N}{2} \tag{8}$$

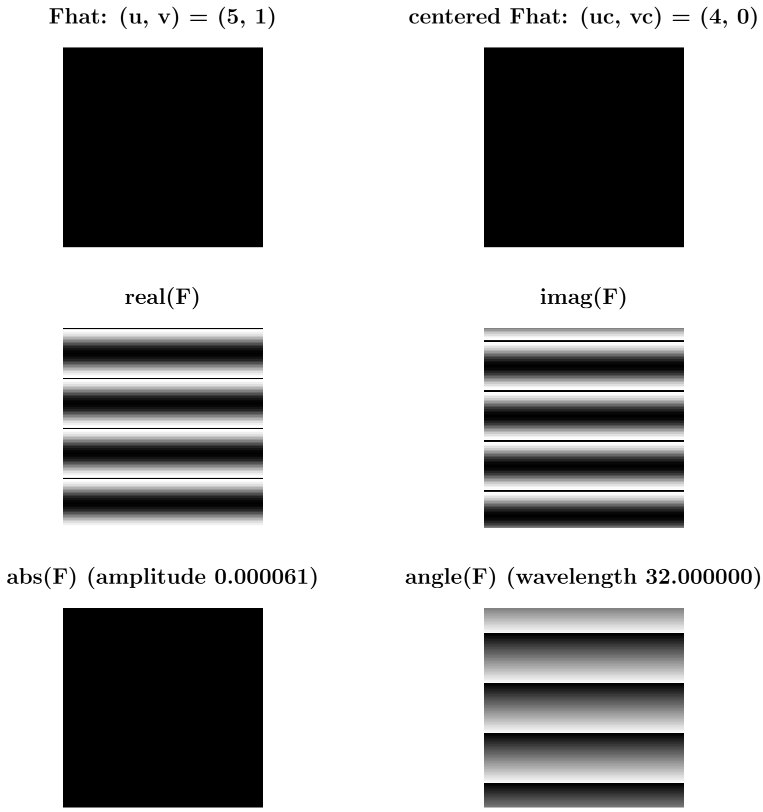

Figure 5: $(p, q) = (5, 1)$

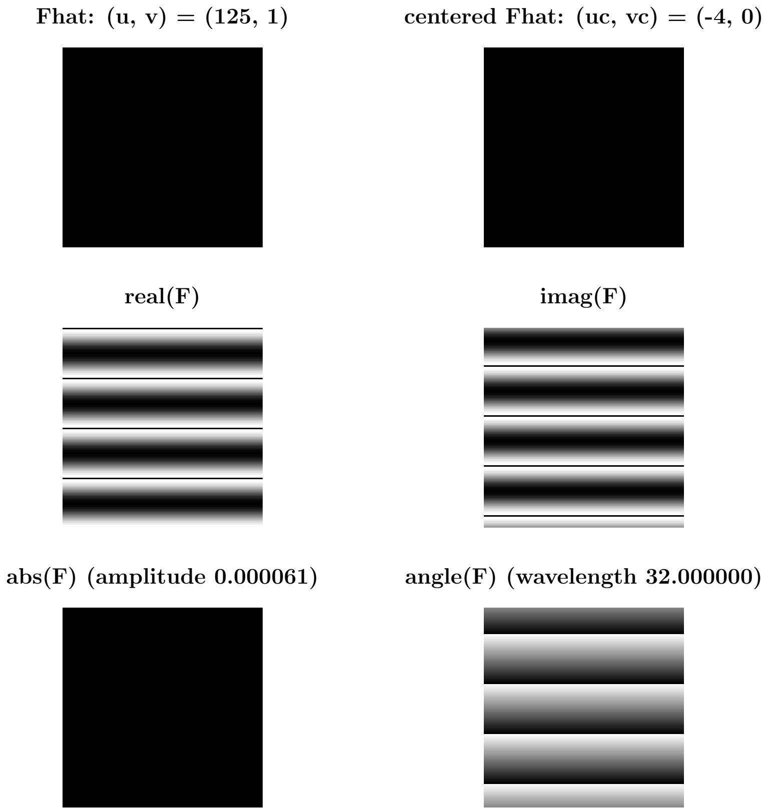

Figure 6: $(p, q) = (125, 1)$

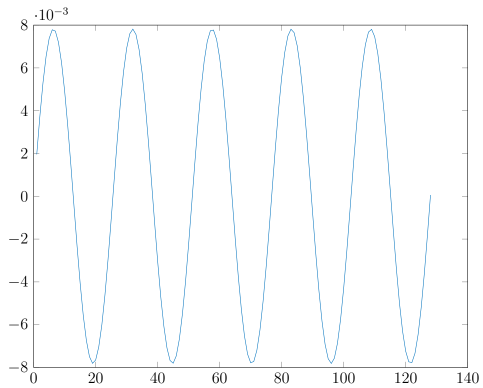

Figure 7:

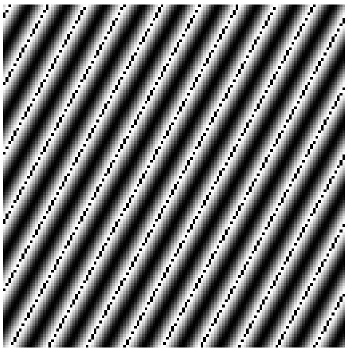

Figure 8:

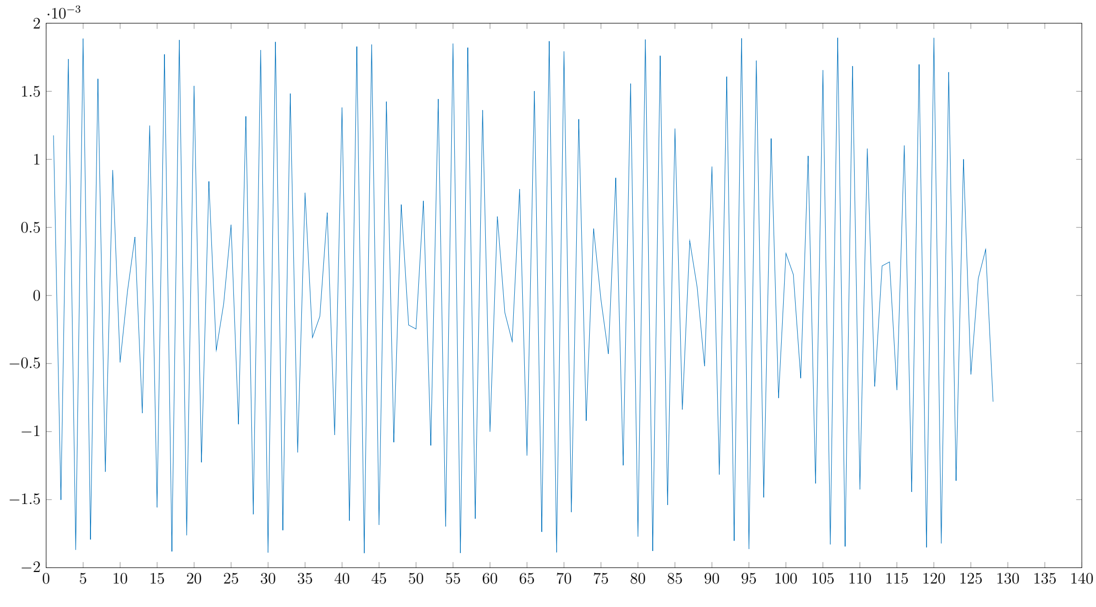

Figure 9: Example waveform in the spatial domain for $(p, q) = (69, 120)$