EL2320 - Applied Estimation
Lab I
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1 Part I

1.1 Question 1

Since

$$\hat{x}_t = g(u_t, x_{t-1}) + \epsilon_t \tag{1}$$

and

$$z_t = h(x_t) + \delta_t \tag{2}$$

u_t models a part of the dynamics between x_{t-1} and x_t. So, it is an action that carries information about the change of state in the environment. For example, u_t can be the setting of the desired temperature in a room.

On the other hand, x_t is the environmental state at time t and depends only on the state at the previous timestep x_{t-1} and the control u_t at time t. As an example, x_t can be the actual temperature of a room.

z_t is the measurement on x_t and depends only on the environmental state x_t at time t. For example z_t can be the reading on a thermometer, when trying to estimate the true temperature of a room.

1.2 Question 2

The uncertainty in the belief during a Kalman filter update step is given by

$$\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \tag{3}$$

where

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} \tag{4}$$

If we plug equation 4 into equation 3 and use equation the Sherman/Morrison formula, then

$$\Sigma_t = \bar{\Sigma}_t - \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} C_t \bar{\Sigma}_t = (\bar{\Sigma}_t^{-1} + C_t^T Q_t^{-1} C_t)^{-1} \tag{5}$$

where we know that $C_t^T Q_t^{-1} C_t$ is positive semi-definite. Hence, the uncertainty in the belief during an update cannot increase.

1.3 Question 3

From lecture's 5 notes:

$$\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t) = (I - K_tC_t)\bar{\mu}_t + K_tz_t = (I - W)\bar{\mu}_t + W\mu_z \tag{6}$$

where $W = \Sigma_t C_t^T Q_t^{-1} C_t$ and $C_t\mu_z = (z_t - \bar{z}_t + C_t\bar{\mu}_t)$. Hence, essentially the relation between Σ_t and Q_t decides the weighting between the measurements and the belief.

1.4 Question 4

If Q is large, then the Kalman Gain will be small and the filter would take more time to converge.

1.5 Question 5

For the measurements to have an increased effect, the Kalman Gain should be large, since $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$. For the Kalman Gain to be large, Q_t, that is, the uncertainty regarding the measurements should be small.

1.6 Question 6

The belief uncertainty during prediction is given by equation 7:

$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \tag{7}$$

If $A_t \geq I$ then $A_t \Sigma_{t-1} A_t^T$ increases, and noise (R_t) is added. Hence, in general, $\bar{\Sigma}_t$ increases but it depends on A_t and R_t.

1.7 Question 7

1.8 Question 8

If there is no a priori knowledge about the distribution, then the mean of the Kalman Filter can be the MLE of the Gaussian belief. If there is a priori knowledge about the distribution, then a Kalman Filter can be a MAP if the distribution is Gaussian.

1.9 Question 9

The EKF revokes the assumption that a state is a linear function of its previous state, and applies the KF by approximating the non-linear function that is tangent to g at the mean of the posterior. Furthermore, $A_t x_{t-1} + B_t u_t$ is replaced by $g(u_t, x_{t-1})$, $C_t x_t$ by $h(x_t)$, A_t by G_t and C_t by H_t.

1.10 Question 10

No. It depends on the the local nonlinearity of the g function.

1.11 Question 11

We could increase the model uncertainties by increasing the covariances Q_t and R_t.

1.12 Question 12

2 Part II

2.1 Question 1

In this case $\epsilon_k : 2 \times 1$, δ_k : scalar. In the general case, ϵ_k will be $N \times 1$, where N is the number of state variables, and δ_k will be $M \times 1$, where M is the number of variables the EKF tries to estimate.
A scalar Gaussian is characterized by a mean value μ and a variance σ^2. A

white Gaussian has $\mu = 0$. In the general case, $\boldsymbol{\mu}$ is a single-column matrix and the scalar variance is replaced by a covariance matrix $\mathbf{\Sigma}$. In this case, a white Gaussian has $\boldsymbol{\mu} = \mathbf{0}$ and $\mathbf{\Sigma}$ is a diagonal matrix because the noise in each state variable is independent of one another.

2.2 Question 2

Variable	Usage
x	The true state of the system.
$\hat{x}$	The estimate of the true state of the system by the EKF.
P	Estimate error covariance matrix.
G	Identity matrix for mathematical consistency.
D	Identity matrix for mathematical consistency. Scalar here.
Q	Process noise variance.
R	Measurement noise covariance matrix.
$wStdP$	The actual (simulated) standard deviation of the error in position.
$wStdV$	The actual (simulated) standard deviation of the error in velocity.
$vStd$	The actual (simulated) standard deviation of the error in position estimation.
u	Control signal, the acceleration.
PP	Estimate error covariances over time.