EL2425 - Slip Control
Meeting agenda 2016-11-16

December 14, 2016

1 Done

1.1 Tracking the centerline of a lane using a PID controller

The problem can be decomposed into two separate and independent components involving a translation and a rotation of the vehicle.

Translational component

We assume that the pose of the vehicle at time t is (x_c, y_c, v_c, ψ_v) and that two range scans at $+90°$ and $-90°$ with respect to the longitudinal axis of the vehicle are available, which are denoted as CL and CR respectively. Also, in figure 1, point O' is a future reference and the angle λ is the angle that the vehicle should make in order to reach O' in a future moment.

Then, since $CL + CR = L = OL + OR$ and $CL = OC + OL$, this means that

$$OC = OR - CR = \frac{L}{2} - CR = \frac{CL + CR}{2} - CR = \frac{CL - CR}{2} \tag{1}$$

where L is the width of the lane whose centerline the vehicle is to track.

Furthermore, in the $CC'O'$ triangle

$$\tan\lambda = \frac{O'C'}{CC'} = \frac{OC}{CC'} = \frac{CL - CR}{2CC'} \tag{2}$$

$$\lambda = \tan^{-1}\frac{CL - CR}{2CC'} \tag{3}$$

The length CC' is unknown and can be set beforehand. Its magnitude will determine the vehicle's rate of convergence to the centerline of the lane.

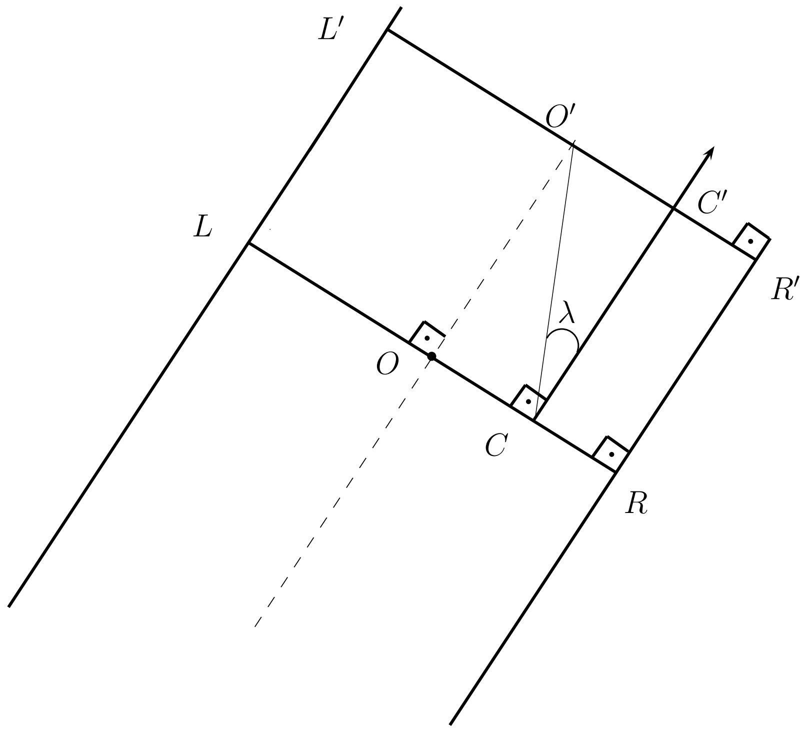

Figure 1: The vehicle's heading angle is equal to that of the lane's. However, its position is off track.

The angular error of the translational component is then

$$e_t = -\tan^{-1}\frac{CL - CR}{2CC'} \tag{4}$$

where the minus sign is introduced due to the convention that a left turn is assigned a negative value.

Rotational component

The rotational component problem can be solved in two ways, of which the second is more robust than the first.

In the first solution, we assume that the the pose of the vehicle at time t as (x_c, y_c, v_c, ψ_v) and that three range scans at $+90°$, $0°$ and $-90°$ with respect to the longitudinal axis of the vehicle are available, which are denoted as CL, CF and CR respectively.

Here we can distinguish two cases: one where the vehicle is facing the right lane boundary and one where it is facing the left one. It is not obvious in this configuration where the vehicle is heading: the only available information so far is only the three range scans.

In the first case, the heading angle error is

$$\phi = \tan^{-1}\frac{CR}{CF} \tag{5}$$

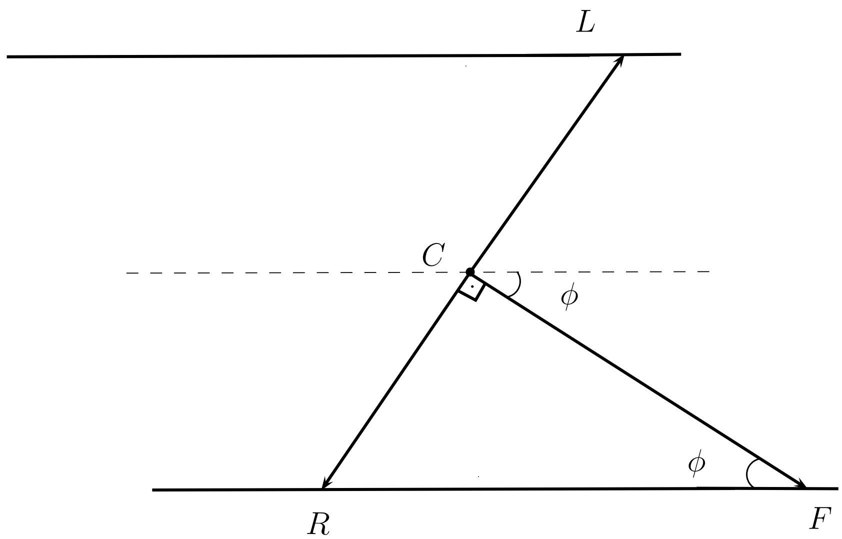

Figure 2: The vehicle's position and its reference are equal. However, the vehicle's heading is off track. Its heading is towards the right lane boundary.

The rotational error of the vehicle in this case is

$$e_r = -\tan^{-1}\frac{CR}{CF} \tag{6}$$

therefore the overall angular error of the vehicle in this case is

$$e = -\tan^{-1}\frac{CL - CR}{2CC'} - \tan^{-1}\frac{CR}{CF} \tag{7}$$

where the minus signs are introduced due to the convention that a left turn is assigned a negative value.

In the second case, the heading angle error is

$$\phi = \tan^{-1}\frac{CL}{CF} \tag{8}$$

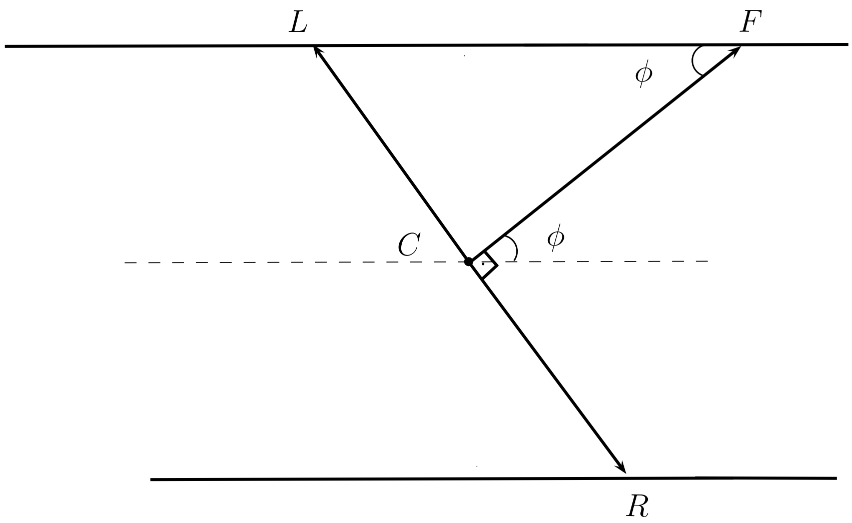

Figure 3: The vehicle's position and its reference are equal. However, the vehicle's heading is off track. Its heading is towards the left lane boundary.

The rotational error of the vehicle in this case is

$$e_r = \tan^{-1}\frac{CL}{CF} \tag{9}$$

therefore the overall angular error of the vehicle in this case is

$$e = -\tan^{-1}\frac{CL - CR}{2CC'} + \tan^{-1}\frac{CL}{CF} \tag{10}$$

where the minus sign of the translational error is introduced due to the convention that a left turn is assigned a negative value.

In order to deduce the correct value of ϕ ($-\tan^{-1}\frac{CR}{CF}$ or $\tan^{-1}\frac{CL}{CF}$) further ranges scans are needed. To this end, a difference between ranges around point F is taken: starting at the right of F and moving anti-clockwise, we calculate the difference between two range scans for a given angle between them. If its sign is negative then the vehicle is facing the right lane boundary; if not, it is facing the left lane boundary. This concept is depicted in figures 4 and 5.

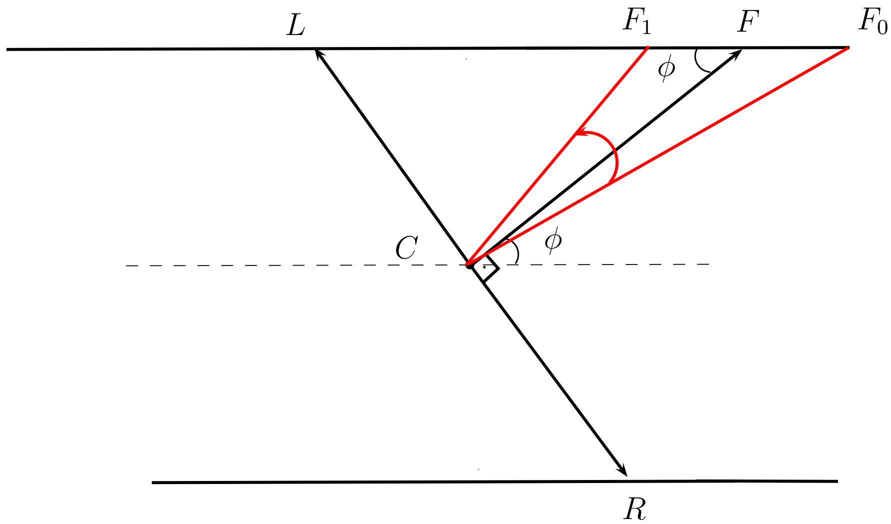

Figure 4: $CF_0 > CF > CF_1$, hence $CF_0 - CF_1 > 0$ and $\phi = tan^{-1}\dfrac{CL}{CF}$

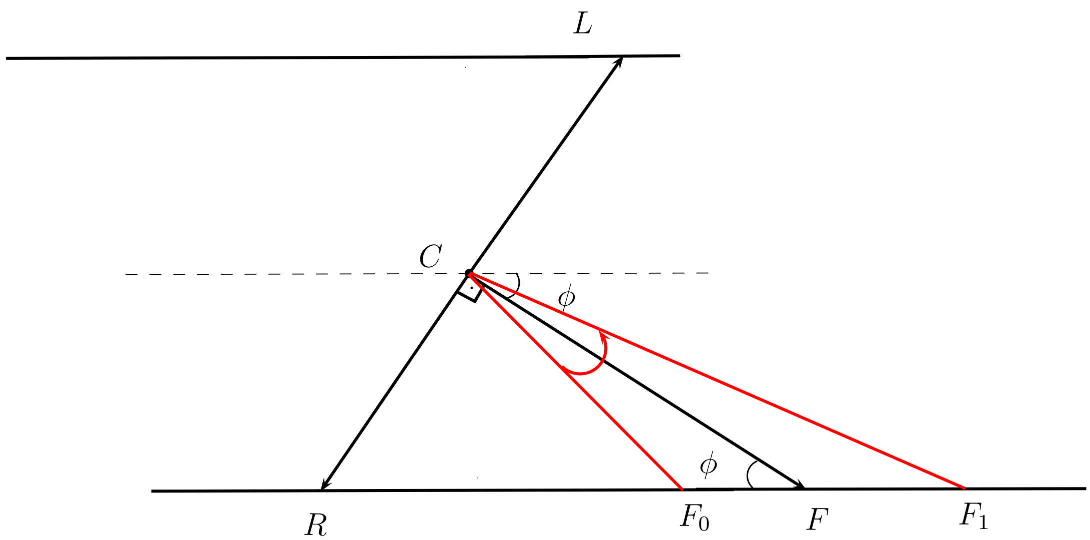

Figure 5: $CF_0 < CF < CF_1$, hence $CF_0 - CF_1 < 0$ and $\phi = -tan^{-1}\dfrac{CR}{CF}$

In the second solution we assume that the the pose of the vehicle at time t as (x_c, y_c, v_c, ψ_v) and that *two* range scans at $+90°$ and $-90°$ with respect to the longitudinal axis of the vehicle are available, which are denoted as CL and CR respectively. Here, again we distinguish two cases, one where the vehicle is facing the right lane boundary and one where it is facing the left one. It is not obvious in this configuration where the vehicle is heading: the only available information so far is only the two ranges CR and CL.

We retrieve the first minimum distance from the range scan available at time t, and denote the point which corresponds to this distance with M_0. The angle between point M_0 and the longitudinal axis of the vehicle is denoted with α. In order to find this angle, we can exploit the fact that each ray of the scan is separated from the next by *res* degrees, while all of them are stored in an array sequentially, in an anti-clockwise manner. In the case of the `HOKUYO-UST-10LX-LASER`, the angular resolution is $res = 0.45°$, and the starting angle is $-135°$ with respect to the longitudinal axis of the vehicle. This means that we can retrieve the angle α by first calculating the number of indices between the one that corresponds to the ray with the minimum range and the one that corresponds to $+135°$ (which in our case is $135/0.25 = 540$) with regard to the laser's system of reference, and then by multiplying this number Δi by the angular resolution res. Hence, $\alpha = 0.25 \times \Delta i$.

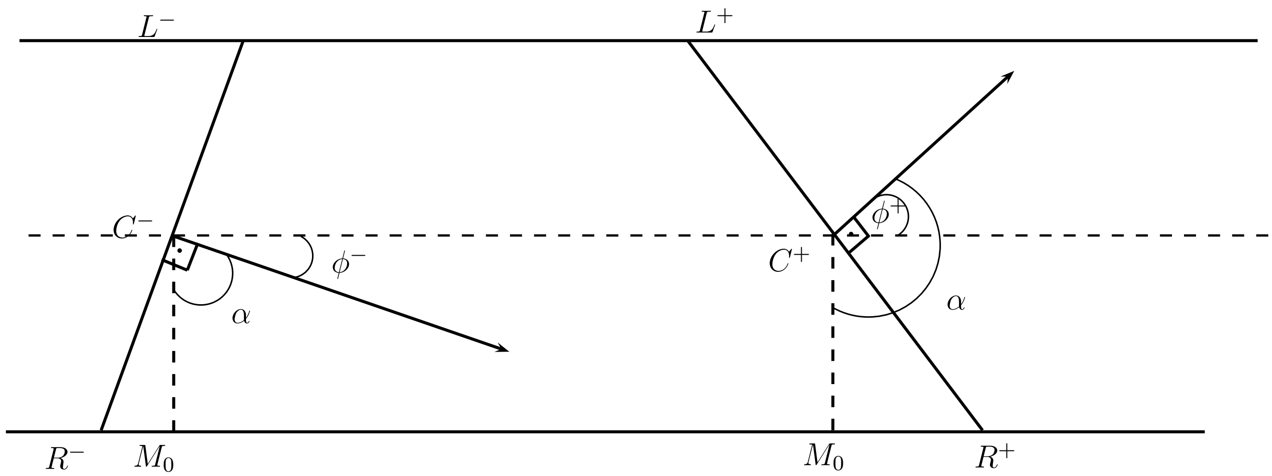

Figure 6

At this point we do not know whether the vehicle is pointing to the left or to the right lane boundary yet. However, we can determine the sign and the magnitude of the orientation of the vehicle with respect to the orientation of the lane by examining the sign of the difference $\alpha - 90°$: if it is negative, the vehicle is pointing towards the right boundary lane, if it is positive, towards the left. Furthermore, we can now calculate the magnitude of the orientation of the vehicle as the difference $|\alpha - 90°|$, as shown in figure 8. In conclusion, when the car is located at the centerline, its orientation with respect to the orientation of the lane is

$$\phi = \alpha - 90°$$

therefore the overall angular error of the vehicle in this case is

$$e = -\tan^{-1}\frac{CL - CR}{2CC'} + \alpha - 90 \tag{11}$$

Since the input to the plant is in terms of angular displacement, this is in fact the error that the PID controller can include and utilize in order to determine the plant's input.

The angular input to the vehicle will then be

$$\delta = K_p \cdot e + K_d \cdot \frac{de}{dt} + K_i \cdot \int edt \tag{12}$$

1.2 Tracking the centerline of a lane using a MPC controller

In figure 7, the x axis is fixed on the lane's centerline, while axis y is perpendicular to it, facing the left boundary of the lane. The origin is at point O. The vehicle is represented by point C. The orientation of the vehicle with respect to the lane (the x-axis) is ϕ. Given this configuration and two ranges from the range scan, at $-90°$ and $90°$ with respect to the longitudinal axis of the vehicle (denoted by CR and CL respectively), the objective is to find the distance OC and the angle ϕ so that a MPC optimization problem can be solved with OC and ϕ acting as initial conditions. The only source of information is the lidar itself.

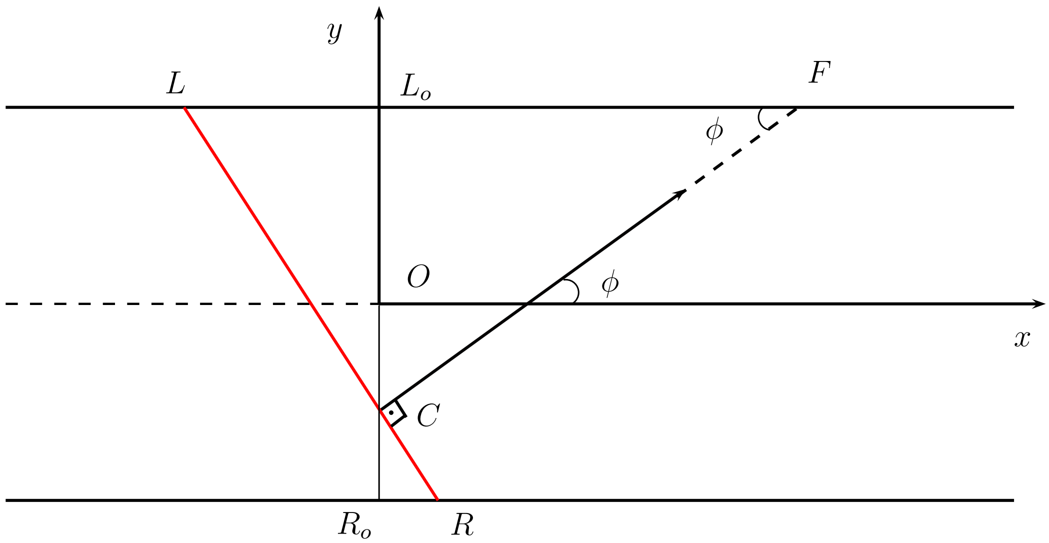

Figure 7

Initial conditions

Finding ϕ

Here we can distinguish two cases, one where the vehicle is facing the right lane boundary and one where it is facing the left one. It is not obvious in this configuration where the vehicle is heading: the only available information so far is only the two ranges CR and CL.

In the first case, when the vehicle is on the right semilane, $CL - CR > 0$. We retrieve the minimum distance from the range scan available at time t, and denote the point which corresponds to this distance with M. The angle between point M and the longitudinal axis of the vehicle is denoted with α. In order to find this angle, we can exploit the fact that each ray of the scan is separated from the next by res degrees, while all of them are stored in an array sequentially, in an anti-clockwise manner. In the case of the `HOKUYO-UST-10LX-LASER`, the angular resolution is $res = 0.45°$, and the starting angle is $-135°$ with respect to the longitudinal axis of the vehicle. This means that we can retrieve the angle α by first calculating the number of indices between the one that corresponds to the ray with the minimum range and the one that corresponds to $+135°$ (which in our case is $135/0.25 = 540$) with regard to the laser's system of reference, and then by multiplying this number Δi by the angular resolution res. Hence, $\alpha = 0.25 \times \Delta i$.

At this point we do not know whether the vehicle is pointing to the left or to the right lane boundary. However, we can determine the sign and the magnitude of the orientation of the vehicle with respect to the orientation of the lane by examining the sign of the difference $\alpha - 90°$: if it is negative, the vehicle is pointing towards the right boundary lane, if it is positive, towards the left. Furthermore, we can now calculate the magnitude of the orientation of the vehicle as the difference $|\alpha - 90°|$, as shown in figure 8. In conclusion, if the car is located at the right semilane its orientation with respect to the orientation of the lane is

$$\phi = sign(CL - CR)(\alpha - 90)$$

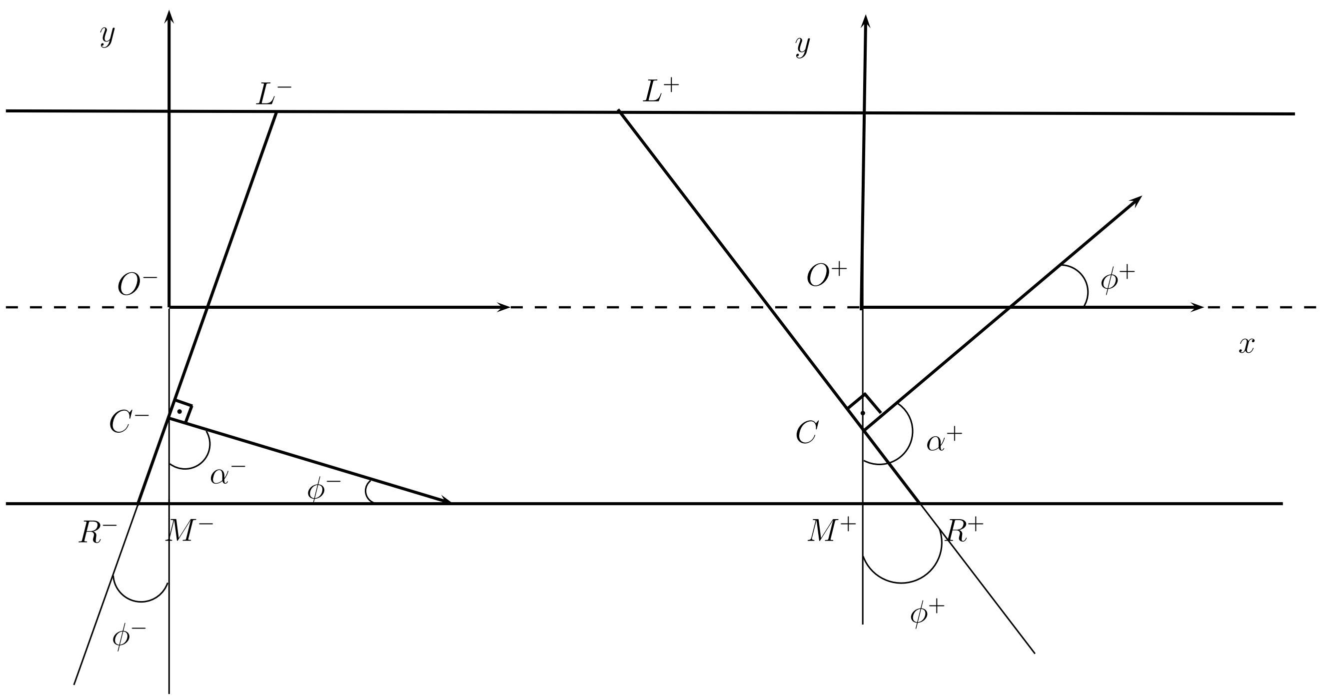

Figure 8: The vehicle is at the right side of the lane.

In the second case, when the vehicle is on the left semilane, $CL - CR < 0$. We retrieve the minimum distance from the range scan available at time t, and denote the point which corresponds to this distance with M. The angle between point M and the longitudinal axis of the vehicle is denoted with α. In order to find this angle, we can exploit the fact that each ray of the scan is separated from the next by res degrees, while all of them are stored in an array sequentially, in an anti-clockwise manner. In the case of the `HOKUYO-UST-10LX-LASER`, the angular resolution is $res = 0.45°$, and the starting angle is $-135°$ with respect to the longitudinal axis of the vehicle. This means that we can retrieve the angle α by first calculating the number of indices between the one that corresponds to the ray with the minimum range and the one that corresponds to $+225°$ (which in our case is $225/0.25 = 900$) with regard to the laser's system of reference, and then by multiplying this number Δi by the angular resolution res. Hence, $\alpha = 0.25 \times \Delta i$.

At this point we do not know whether the vehicle is pointing to the left or to the right lane boundary. However, we can determine the sign and the magnitude of the orientation of the vehicle with respect to the orientation of the lane by examining the sign of the difference $\alpha - 90°$: if it is positive, the vehicle is pointing towards the right boundary lane, if it is negative, towards the left. Furthermore, we can now calculate the magnitude of the orientation of the vehicle as the difference $|\alpha - 90°|$, as shown in figure 9. In conclusion, if the car is located at the left semilane its orientation with respect to the orientation of the lane is

$$\phi = sign(CL - CR)(\alpha - 90)$$

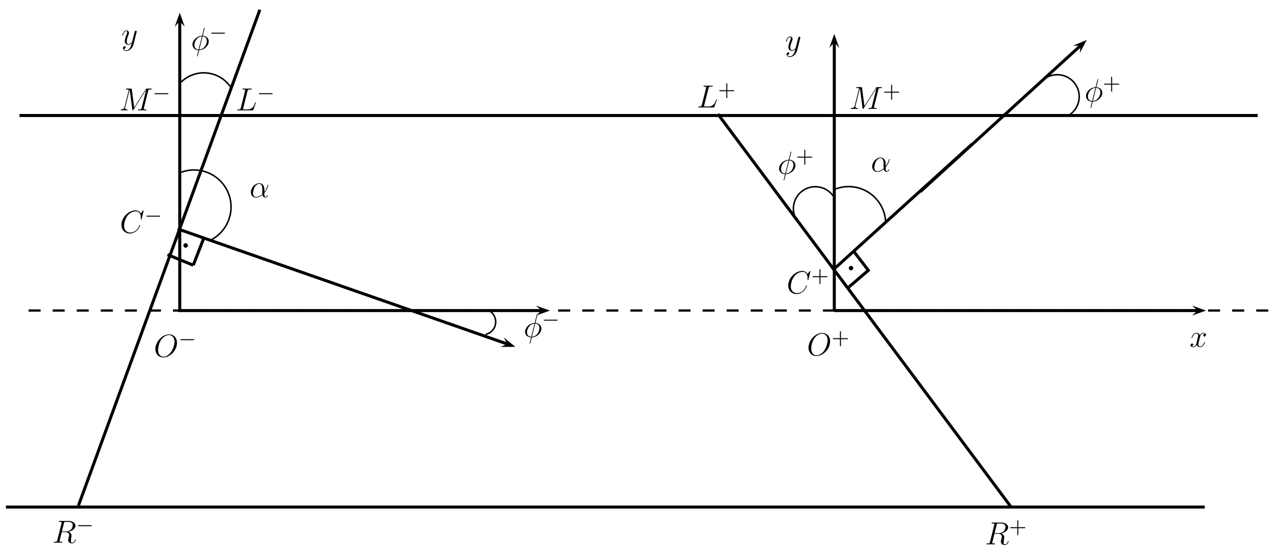

Figure 9: The vehicle is at the left side of the lane.

In conclusion, angle $\phi = sign(CL - CR)(\alpha - 90)$, where angle α is calculated as a (weighted by angular resolution) difference between the indices of the minimum range provided by the range scan at time t and that of the minimum range between CL and CR.

Finding OC

Looking at figure 7 we can see that

$$L_oO + OC + CR_o = W = CR_o + CL_o \tag{13}$$

where W is the width of the lane. But $L_oO = \dfrac{W}{2}$ hence

$$OC + CR_o = \frac{1}{2}(CR_o + CL_o) \Leftrightarrow \tag{14}$$

$$OC = \frac{1}{2}(CL_o - CR_o) \tag{15}$$

But $CL_o = CLsin\lambda$ and $CR_o = CRcos\phi$, hence

$$OC = \frac{1}{2}(CLsin\lambda - CRcos\phi) \tag{16}$$

From triangle LCF in the case where the vehicle is facing the left lane boundary, we note that $\phi + \lambda + \frac{\pi}{2} = \pi$, hence $\lambda = \frac{\pi}{2} - \phi$. Then, we conclude that

$$OC = \frac{1}{2}(CL - CR)cos\phi \tag{17}$$

Obtaining the linearized kinematic model

The model constitutes the equations of motion of the vehicle, and has four states (x, y, v and ψ) and two inputs (v_i and δ). The equations of the vehicle's motion that are relevant here are

$$\dot{x} = vcos(\psi + \beta) \tag{18}$$

$$\dot{y} = vsin(\psi + \beta) \tag{19}$$

$$\dot{v} = \frac{v_i - v}{\tau} \tag{20}$$

$$\dot{\psi} = \frac{v}{l_r}sin\beta \tag{21}$$

Sampling with a sampling time of T_s gives

$$x_{k+1} = x_k + T_s v_k cos(\psi_k + \beta_k) \tag{22}$$

$$y_{k+1} = y_k + T_s v_k sin(\psi_k + \beta_k) \tag{23}$$

$$v_{k+1} = v_k + \frac{T_s}{\tau}(v_{i,k} - v_k) \tag{24}$$

$$\psi_{k+1} = \psi_k + T_s \frac{v}{l_r} sin\beta_k \tag{25}$$

where

$$\beta_k = tan^{-1}\Big(\frac{l_r}{l_r + l_f}tan\delta_{k-1}\Big) \tag{26}$$

Forming the Jacobians for matrices A, B and evaluating them at time $t = k$ around the state $\psi = \phi$, $v = v_k$ and $\delta = \delta_{k-1}$ (δ_k is to be determined at time k):

$$A = \begin{bmatrix} 1 & 0 & T_s cos(\phi + \beta_k) & -T_s v_k sin(\phi + \beta_k) \\ 0 & 1 & T_s sin(\phi + \beta_k) & T_s v_k cos(\phi + \beta_k) \\ 0 & 0 & 1 - \dfrac{T_s}{\tau} & 0 \\ 0 & 0 & \dfrac{T_s}{l_r} sin(\beta_k) & 1 \end{bmatrix} \tag{27}$$

$$A = \begin{bmatrix} 1 & 0 & T_s cos\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) & -T_s v_k sin\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ 0 & 1 & T_s sin\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) & T_s v_k cos\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ 0 & 0 & 1 - \dfrac{T_s}{\tau} & 0 \\ 0 & 0 & \dfrac{T_s}{l_r} sin(tan^{-1}(l_q tan\delta_{k-1})) & 1 \end{bmatrix} \tag{28}$$

$$B = \begin{bmatrix} 0 & -T_s v_k sin(\phi + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ 0 & T_s v_k cos(\phi + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ \dfrac{T_s}{\tau} v_k & 0 \\ 0 & \dfrac{T_s v_k}{l_r} cos(\beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \end{bmatrix} \tag{29}$$

$$B = \begin{bmatrix} 0 & -T_s v_k sin\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ 0 & T_s v_k cos\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ \dfrac{T_s}{\tau} v_k & 0 \\ 0 & \dfrac{T_s v_k}{l_r} cos\Bigg(tan^{-1}\Big(l_q tan\delta_{k-1}\Big)\Bigg) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \end{bmatrix} \tag{30}$$

where $l_q = \dfrac{l_r}{l_r + l_f}$
Now we can express the linear model as

$$s_{k+1} = A s_k + B u_k \tag{31}$$

where

$$s_k = \begin{bmatrix} x_k \\ y_k \\ v_k \\ \psi_k \end{bmatrix}, u_k = \begin{bmatrix} v_{i,k} \\ \delta_k \end{bmatrix} \tag{32}$$

However, states x and v cannot be measured under our configuration, since no SLAM module is employed and no encoders are mounted on the wheels of the vehicle. Hence, the model has to be reduced, while now the velocity will be constant and set to v_0. (Hence the input throttle also needs to be excluded as an input). Nevertheless, state x needs to be included in the model even if it is not possible to measure it due to the formulation of the problem in terms of the MPC framework: the reference of the vehicle in terms of x needs to be (practical) infinity in order for it to keep moving along the lane. The new system matrices and the new states are modified as

$$A = \begin{bmatrix} 1 & 0 & -T_s v_k sin\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ \\ 0 & 1 & T_s v_k cos\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

$$B = \begin{bmatrix} -T_s v_k sin\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ T_s v_k cos\Big(\phi + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ \dfrac{T_s v_k}{l_r} cos\Bigg(tan^{-1}\Big(l_q tan\delta_{k-1}\Big)\Bigg) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \end{bmatrix} \tag{34}$$

the model being

$$s_{k+1} = As_k + Bu_k, s_k = \begin{bmatrix} x_k \\ y_k \\ \psi_k \end{bmatrix}, u_k = \begin{bmatrix} \delta_k \end{bmatrix} \tag{35}$$

Stating the optimization problem

We can now form the optimization problem to be solved at time step t as

$$\text{minimize} \sum_{k=0}^{N-1} (s_k - s_{ref})^T Q (s_k - s_{ref}) + u_k^T R u_k + (s_N - s_{ref})^T Q_f x_(s_N - s_{ref}) \tag{36}$$

$$\text{subject to } s_{k+1} = As_k + Bu_k \tag{37}$$

$$\delta^{min} \leq \delta_k \leq \delta^{max} \tag{38}$$

$$s_0 = (x_t, y_t, \psi_t) \equiv (0, OC, \phi) \tag{39}$$

$$s_{ref} = (x^{max}, 0, 0) \tag{40}$$

$$Q > 0, R > 0, Q_f > 0 \tag{41}$$

1.3 Tracking the circumference of a circle using a MPC controller

In figure 10, the vehicle C, whose velocity is constant and denoted by v, is to track a circle whose center is O' and whose radius is R. Its orientation relative to the global coordinate system is ψ. The vehicle's coordinates are (x_c, y_c). The circular trajectory is known a priori, and is comprised of a set of ordered points who encode the x-wise, y-wise and yaw-wise states that the vehicle ought to have, that is, each point on the circle serves as a reference to the vehicle. In terms of yaw angle, the longitudinal axis of the vehicle ought to always be tangent to the circle.

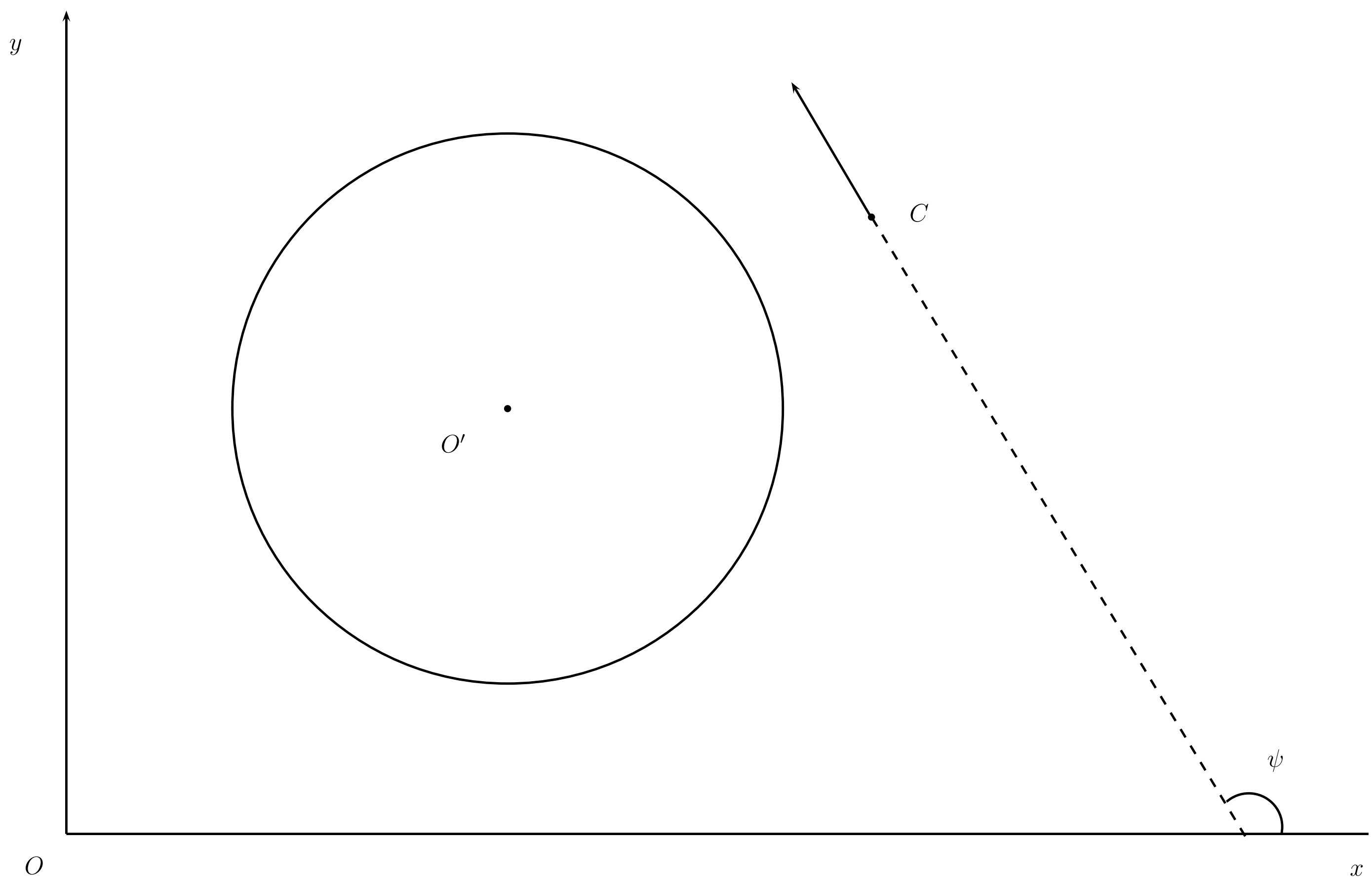

Figure 10

In this setting, since an optimization problem will have to be solved at each time instant t, the first thing that needs to be identified is the reference pose of the vehicle at each time instant t. More precisely, since the objective is the minimization of the deviation of the vehicle's trajectory from the circular one, and the framework is that of MPC, we need to identify the sequence of points on the circle that shall act as references while iterating through the prediction equations of the linearized model of the vehicle's behaviour. This task is divided into two sub-tasks: first, we identify the point T of the circle that lies on the line that is tangent to the circle and that passes through the position of the vehicle. When this point is known, the sequence of $N-1$ reference poses can be calculated, given knowledge of the circle's radius and the vehicle's velocity. N is the horizon length. The following two sections illustrate the procedures by which the N reference poses are identified.

1.3.1 Finding T

Here, we are not concerned with the orientation of the vehicle; its coordinates will only be used as a point to identify the tangent that passes through the position of the vehicle.

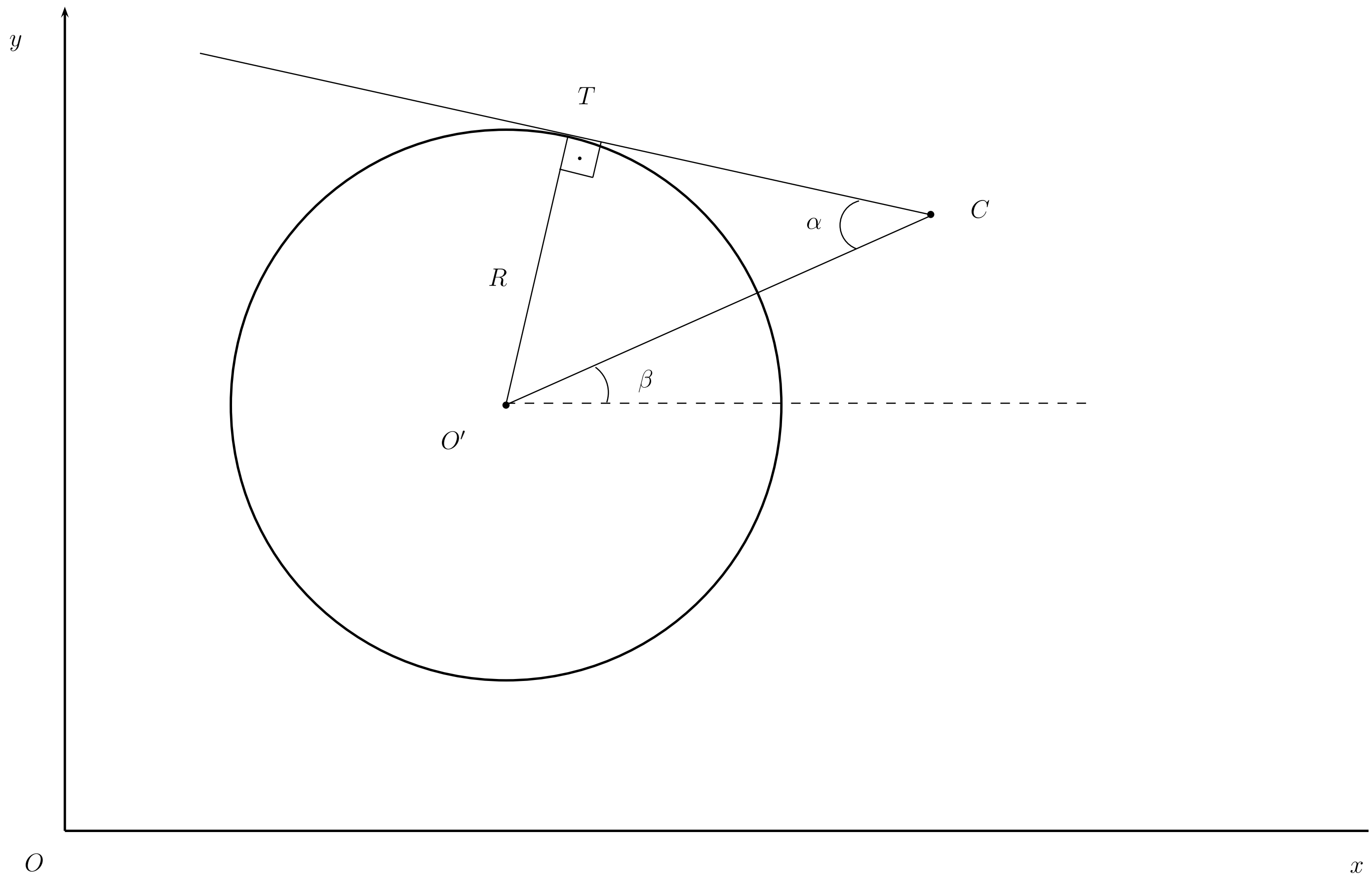

Figure 11

In triangle $TO'C$, $\angle O'TC$ is right, and

$$\alpha = sin^{-1}\frac{R}{O'C} \tag{42}$$

while

$$\beta = tan^{-1}\frac{y_c - y_{O'}}{x_c - x_{O'}} \tag{43}$$

Point T is then rudimentary to find: it is the point that lies on the circle and is found while rotating by $\beta + \dfrac{\pi}{2} - \alpha$ from the x axis. The coordinates of point T are

$$x_T = x_{O'} + Rcos\theta_T \tag{44}$$
$$y_T = y_{O'} + Rsin\theta_T \tag{45}$$
$$\theta_T = \beta + \frac{\pi}{2} - \alpha \tag{46}$$
$$\tag{47}$$

The same reasoning applies to when the vehicle is located at every other quadrant. The pose that the vehicle ought to have at point T of the circle can now be retrieved from the given set of poses. Point T shall serve as the first reference for the vehicle in the statement of the optimization problem, that is, $s_T = [x_T, y_T, \psi_T]^T = s_0^{ref}$. The next task is now to find the next $N-1$ reference poses in the circle that the vehicle ought to refer to.

1.3.2 Beyond point T

The next reference points depend on the velocity of the vehicle. Since its velocity is constant, we know from elementary physics that $v = \omega R$, which means that $\omega = v/R$, which is known, since the radius of the circle is set by us in advance, and the velocity of the vehicle is measurable. If we denote with θ_T the angle that the line passing through O' and T makes with the x axis, with θ_1 the angle that the line passing through O' and the first reference point after point T makes with the x axis, and T_s the sampling time of the pose of the vehicle, then

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_1 - \theta_T}{T_s} \Leftrightarrow \tag{48}$$
$$\theta_1 = \theta_T + T_s \frac{v}{R} \tag{49}$$
$$\tag{50}$$

In general, the k-th point in the sequence of the $N-1$ reference points will lie at an angle of

$$\theta_k = \theta_T + kT_s \frac{v}{R} \tag{51}$$
$$\tag{52}$$

The coordinates of the k-th reference point ($1 \leq k \leq N-1$) are

$$x_k^{ref} = x_{O'} + Rcos\theta_k \tag{53}$$
$$y_k^{ref} = y_{O'} + Rsin\theta_k \tag{54}$$
$$\tag{55}$$

The pose that the vehicle ought to have at each point can be retrieved again from the set of given poses.

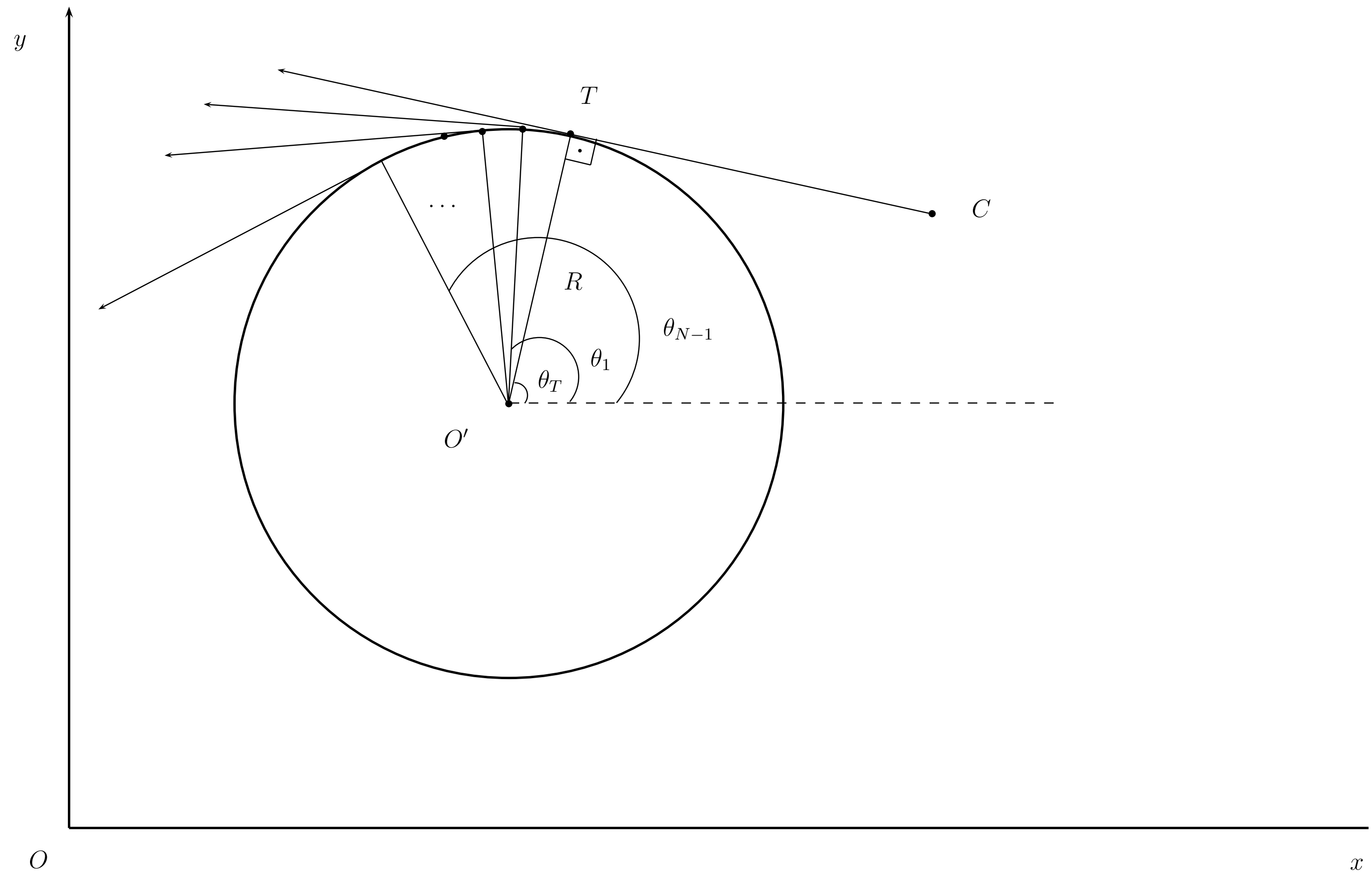

Figure 12

1.3.3 Obtaining the linearized kinematic model

The model constitutes the equations of motion of the vehicle, and has three states (x, y, and ψ) and one input (δ), since we assume a constant velocity. The equations of the vehicle's motion that are relevant here are

$$\dot{x} = vcos(\psi + \beta) \tag{56}$$

$$\dot{y} = vsin(\psi + \beta) \tag{57}$$

$$\dot{\psi} = \frac{v}{l_r} sin\beta \tag{58}$$

Sampling with a sampling time of T_s gives

$$x_{k+1} = x_k + T_s v_k cos(\psi_k + \beta_k) \tag{59}$$

$$y_{k+1} = y_k + T_s v_k sin(\psi_k + \beta_k) \tag{60}$$

$$\psi_{k+1} = \psi_k + T_s \frac{v}{l_r} sin\beta_k \tag{61}$$

where

$$\beta_k = tan^{-1}\Big(\frac{l_r}{l_r + l_f} tan\delta_{k-1}\Big) \tag{62}$$

Forming the Jacobians for matrices A, B and evaluating them at time $t = k$ around the current state $\psi = \psi_k$, and $\delta = \delta_{k-1}$ (δ_k is to be determined at time k):

$$A = \begin{bmatrix} 1 & 0 & -T_s v_k sin(\psi_k + \beta_k) \\ \\ 0 & 1 & T_s v_k cos(\psi_k + \beta_k) \\ \\ 0 & 0 & 1 \end{bmatrix}, \text{or} \tag{63}$$

$$A = \begin{bmatrix} 1 & 0 & -T_s v_k sin\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ \\ 0 & 1 & T_s v_k cos\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \\ \\ 0 & 0 & 1 \end{bmatrix} \tag{64}$$

$$B = \begin{bmatrix} -T_s v_k sin(\psi_k + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ T_s v_k cos(\psi_k + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ \dfrac{T_s v_k}{l_r} cos(\beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \end{bmatrix}, \text{or} \tag{65}$$

$$B = \begin{bmatrix} -T_s v_k sin\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ T_s v_k cos\Big(\psi_k + tan^{-1}(l_q tan\delta_{k-1})\Big) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \\ \dfrac{T_s v_k}{l_r} cos\Bigg(tan^{-1}\Big(l_q tan\delta_{k-1}\Big)\Bigg) \dfrac{l_q}{l_q^2 sin^2\delta_{k-1} + cos^2\delta_{k-1}} \end{bmatrix} \tag{66}$$

where $l_q = \dfrac{l_r}{l_r + l_f}$

Now we can express the linear model as

$$s_{k+1} = As_k + Bu_k \tag{67}$$

where

$$s_k = \begin{bmatrix} x_k \\ y_k \\ \psi_k \end{bmatrix} \tag{68}$$

and

$$u_k = \delta_k \tag{69}$$

However, here there is another option. We can keep the A and B matrices constant during the solution of the optimization problem, which means that only one linearization is made (around the current state of the vehicle), or we can solve the problem once with these matrices, extract the sequence of predicted states, and linearize around them. This will result in $N-1$ new linearizations around each predicted state of the vehicle, and hence $N-1$ additional A_t matrices. In the same spirit, we linearize around the corresponding optimal input $N-1$ times in total, and we obtain $N-1$ additional B_t matrices.

1.3.4 Stating the optimization problem

We can now form the optimization problem to be solved at time t as

$$\text{minimize} \sum_{k=0}^{N-1} (s_k - s_k^{ref})^T Q (s_k - s_k^{ref}) + u_k^T R u_k \tag{70}$$

$$\text{subject to } s_{k+1} = A_k s_k + B_k u_k, \text{ where } s_k = [x_k, y_k, \psi_k]^T, u_k = \delta_k \tag{71}$$

$$\delta^{min} \leq \delta_k \leq \delta^{max} \tag{72}$$

$$s_k^{ref} = (x_k^{ref}, y_k^{ref}, \psi_k^{ref}) \tag{73}$$

$$s_0 = (x_t, y_t, \psi_t) \tag{74}$$

where A_k or B_k are the Jacobians obtained via linearization around the state of the vehicle at time t, in which case they are constant across all k's, ($A_k = A$, $B_k = B$, $\forall k$) and the problem is solved only once per sampling time; or they are obtained by linearizing around the predicted state of the vehicle, in which case the problem is solved once with $A_k = A$, $B_k = B$, and one final time with the variant-time A and B matrices.

2 Ongoing

3 Issues

4 To do

5 Misc.

The progress of the project can be observed in `trello` and `github`:

- `https://trello.com/b/uEP0jl0B/slip-control`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425_resources`