EL2425 - Slip Control
Meeting agenda 2016-11-16

November 18, 2016

1 Done

2 Ongoing

2.1 Theoretical solution to tracking the centerline of a lane using a PID controller

The problem can be decomposed into two separate and independent components involving a translation and a rotation of the vehicle.

Translational component We assume that the pose of the vehicle at time t is (x_c, y_c, v_c, ψ_v) and that two range scans at $+90°$ and $-90°$ with respect to the longitudinal axis of the vehicle are available, which are denoted as CL and CR respectively. Also, in figure 1, point O' is a future reference and the angle λ is the angle that the vehicle should make in order to reach O' in a future moment.

Then, since $CL + CR = L = OL + OR$ and $CL = OC + OL$, this means that

$$OC = OR - CR = \frac{L}{2} - CR = \frac{CL + CR}{2} - CR = \frac{CL - CR}{2} \tag{1}$$

where L is the width of the lane whose centerline the vehicle is to track.

Furthermore, in the $CC'O'$ triangle

$$\tan\lambda = \frac{O'C'}{CC'} = \frac{OC}{CC'} = \frac{CL - CR}{2CC'} \tag{2}$$

$$\lambda = \tan^{-1}\frac{CL - CR}{2CC'} \tag{3}$$

The length CC' is unknown and can be set beforehand. Its magnitude will determine the vehicle's rate of convergence to the centerline of the lane.

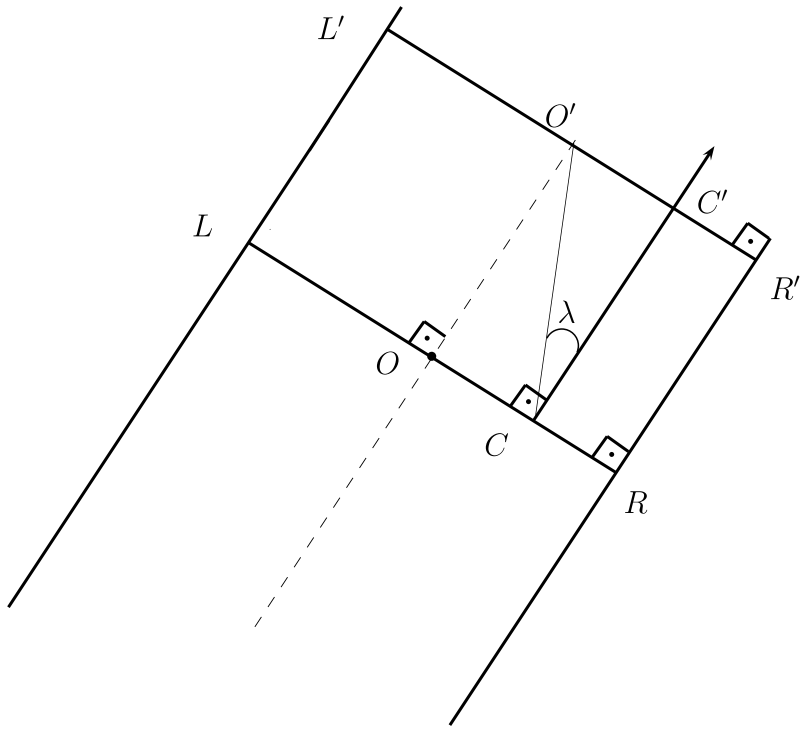

Figure 1: The vehicle's heading angle is equal to that of the lane's. However, its position is off track.

The angular error of the translational component is then

$$e_t = -\tan^{-1}\frac{CL - CR}{2CC'} \tag{4}$$

where the minus sign is introduced due to the convention that a left turn is assigned a negative value.

Rotational component

We assume that the the pose of the vehicle at time t as (x_c, y_c, v_c, ψ_v) and that three range scans at $+90°$, $0°$ and $-90°$ with respect to the longitudinal axis of the vehicle are available, which are denoted as CL, CF and CR respectively.

Here we can distinguish two cases: one where the vehicle is facing the right lane boundary and one where it is facing the left one. It is not obvious in this configuration where the vehicle is heading: the only available information so far is only the three range scans.

In the first case, the heading angle error is

$$\phi = \tan^{-1}\frac{CR}{CF} \tag{5}$$

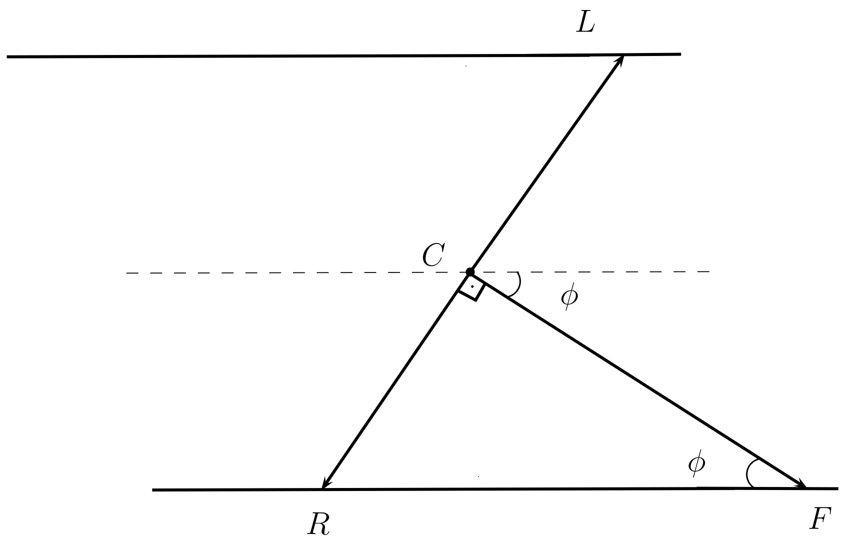

Figure 2: The vehicle's position and its reference are equal. However, the vehicle's heading is off track. Its heading is towards the right lane boundary.

The rotational error of the vehicle in this case is

$$e_r = -\tan^{-1}\frac{CR}{CF} \tag{6}$$

therefore the overall angular error of the vehicle in this case is

$$e = -\tan^{-1}\frac{CL - CR}{2CC'} - \tan^{-1}\frac{CR}{CF} \tag{7}$$

where the minus signs are introduced due to the convention that a left turn is assigned a negative value.

In the second case, the heading angle error is

$$\phi = \tan^{-1}\frac{CL}{CF} \tag{8}$$

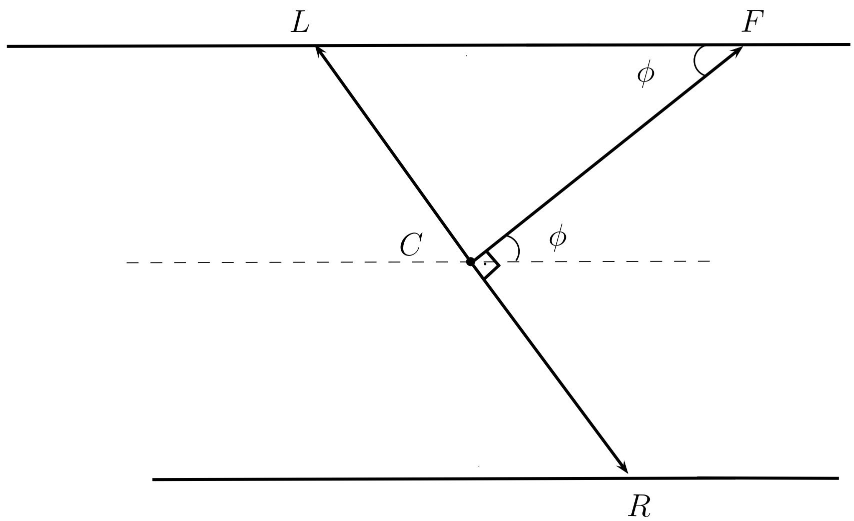

Figure 3: The vehicle's position and its reference are equal. However, the vehicle's heading is off track. Its heading is towards the left lane boundary.

The rotational error of the vehicle in this case is

$$e_r = \tan^{-1}\frac{CL}{CF} \tag{9}$$

therefore the overall angular error of the vehicle in this case is

$$e = -\tan^{-1}\frac{CL - CR}{2CC'} + \tan^{-1}\frac{CL}{CF} \tag{10}$$

where the minus sign of the translational error is introduced due to the convention that a left turn is assigned a negative value.

In order to deduce the correct value of ϕ ($-\tan^{-1}\dfrac{CR}{CF}$ or $\tan^{-1}\dfrac{CL}{CF}$) further ranges scans are needed. To this end, a difference between ranges around point F is taken: starting at the right of F and moving anti-clockwise, we calculate the difference between two range scans for a given angle between them. If its sign is negative then the vehicle is facing the right lane boundary; if not, it is facing the left lane boundary. This concept is depicted in figures 4 and 5.

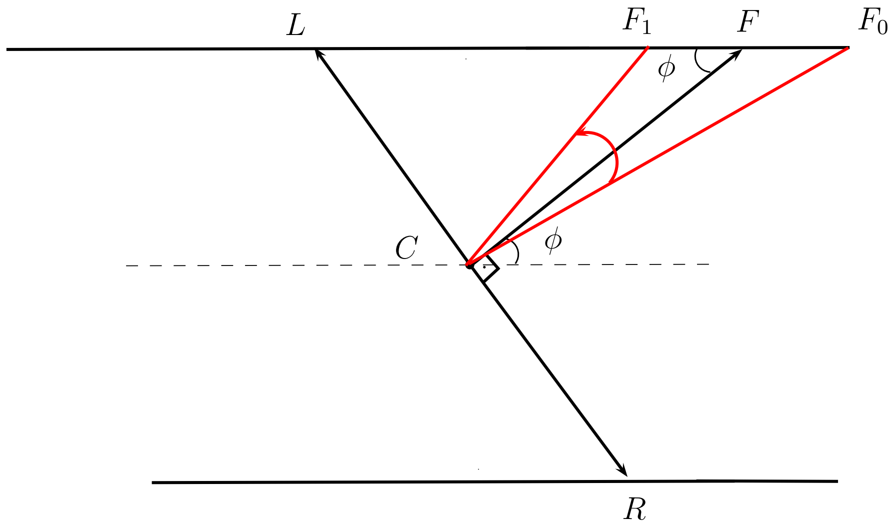

Figure 4: $CF_0 > CF > CF_1$, hence $CF_0 - CF_1 > 0$ and $\phi = tan^{-1}\dfrac{CL}{CF}$

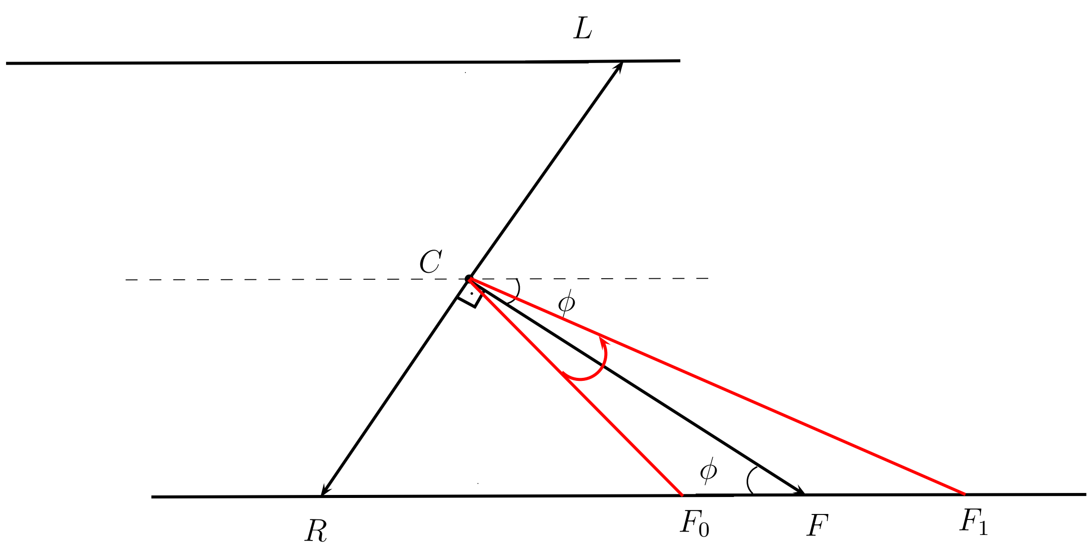

Figure 5: $CF_0 < CF < CF_1$, hence $CF_0 - CF_1 < 0$ and $\phi = -tan^{-1}\dfrac{CR}{CF}$

Since the input to the plant is in terms of angular displacement, this is in fact the error that the PID controller can include and utilize in order to determine the plant's input.

The angular input to the vehicle will then be

$$\delta = K_p \cdot e + K_d \cdot \frac{de}{dt} + K_i \cdot \int edt \qquad (11)$$

2.2 Theoretical solution to tracking the centerline of a lane using a MPC controller

In figure 6, the x axis is fixed on the lane's centerline, while axis y is perpendicular to it. The origin is at point O. The vehicle is represented by point C. The orientation of the vehicle with respect to the lane (the x-axis) is ϕ. Given this configuration and three scans, at $-90°$, $0°$ and $90°$ with respect to the longitudinal axis of the vehicle, denoted by CR, CF and CL respectively, the objective is to find the distance OC and the angle ϕ so that a MPC optimization problem can be solved with OC and ϕ acting as initial conditions. The velocity of the vehicle, which is constant, and its displacement along the x-axis are at this point irrelevant to the optimization problem given this configuration. The only source of information is the lidar itself.

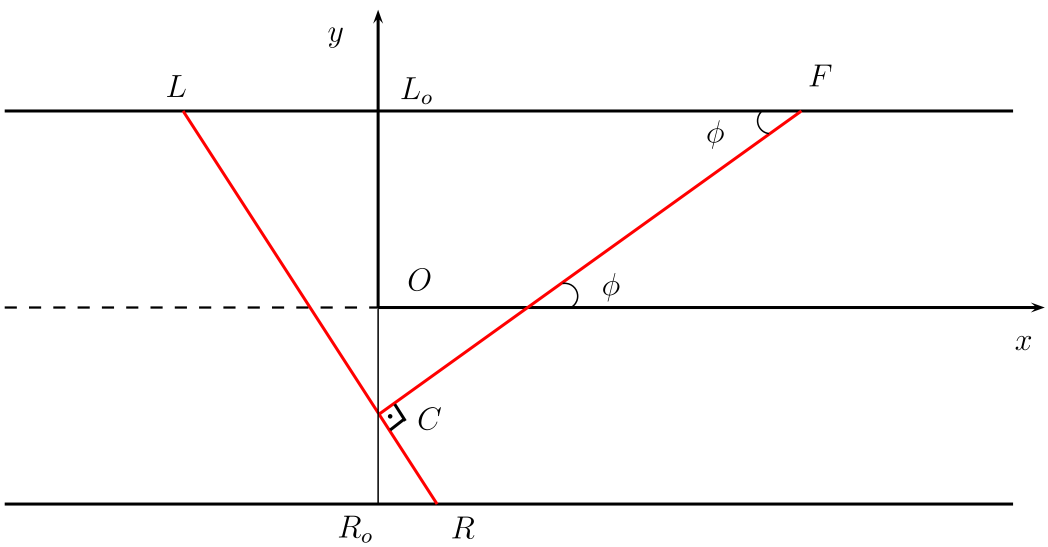

Figure 6

2.2.1 Initial conditions

Finding ϕ

Here we can distinguish two cases: one where the vehicle is facing the right lane boundary and one where it is facing the left one. It is not obvious in this configuration where the vehicle is heading: the only available information so far is only the three range scans.

In the first case, the heading angle error is

$$\phi = \tan^{-1}\frac{CR}{CF} \tag{12}$$

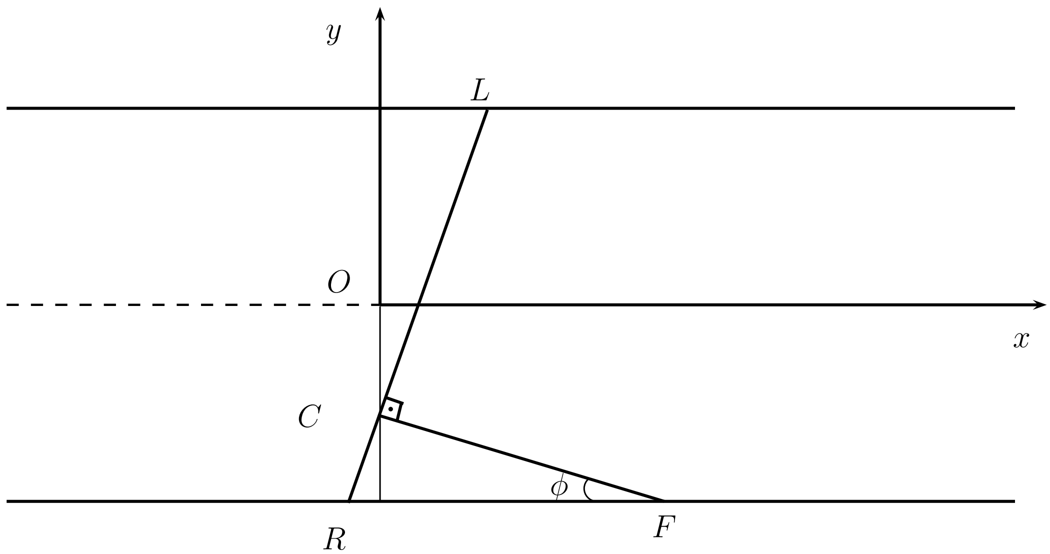

Figure 7: The vehicle's heading is towards the right lane boundary.

In the second case, the heading angle error is

$$\phi = \tan^{-1}\frac{CL}{CF} \tag{13}$$

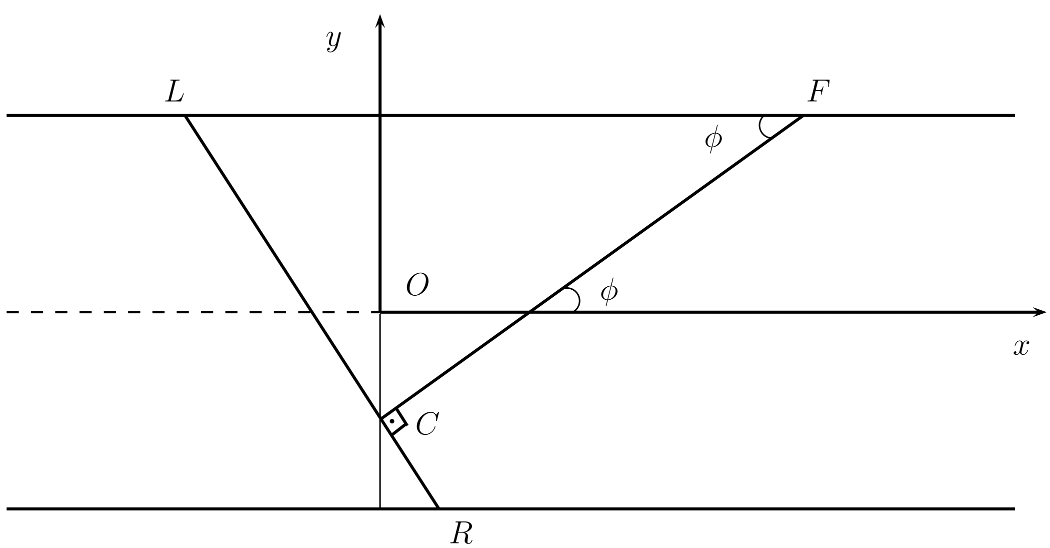

Figure 8: The vehicle's heading is towards the left lane boundary.

In order to deduce the correct value of ϕ ($-\tan^{-1}\dfrac{CR}{CF}$ or $\tan^{-1}\dfrac{CL}{CF}$) further ranges scans are needed. To this end, a difference between ranges around point F is taken: starting at the right of F and moving anti-clockwise, we calculate the difference between two range scans for a given angle between them. If its sign is negative then the vehicle is facing the right lane boundary; if not, it is facing the left lane boundary. This concept is depicted in figures 9 and 10.

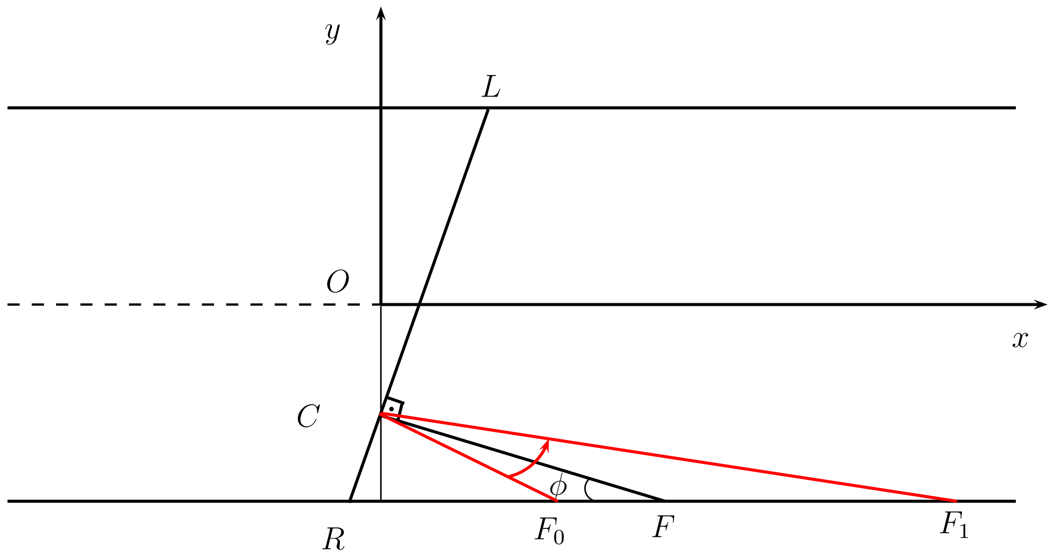

Figure 9: $CF_0 < CF < CF_1$, hence $CF_0 - CF_1 < 0$ and $\phi = -tan^{-1}\dfrac{CR}{CF}$

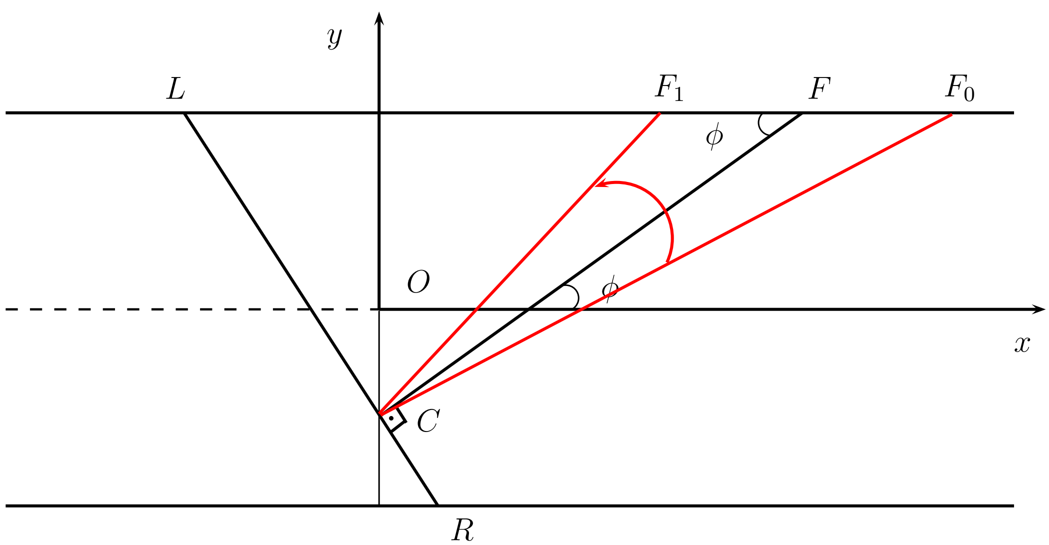

Figure 10: $CF_0 > CF > CF_1$, hence $CF_0 - CF_1 > 0$ and $\phi = tan^{-1}\dfrac{CL}{CF}$

Finding OC

First we note that OC does not depend on the orientation of the vehicle. Then

$$L_oO + OC + CR_o = W = CR_o + CL_o \tag{14}$$

where W is the width of the lane. But $L_oO = \dfrac{W}{2}$ hence

$$OC + CR_o = \frac{1}{2}(CR_o + CL_o) \Leftrightarrow \tag{15}$$
$$OC = \frac{1}{2}(CL_o - CR_o) \tag{16}$$

But $CL_o = CLsin\lambda$ and $CR_o = CRcos\phi$, hence

$$OC = \frac{1}{2}(CLsin\lambda - CRcos\phi) \tag{17}$$

From triangle LCF in the case where the vehicle is facing the left lane boundary, we note that $\phi + \lambda + \frac{\pi}{2} = \pi$, hence $\lambda = \frac{\pi}{2} - \phi$. Then, we conclude that

$$OC = \frac{1}{2}(CL - CR)cos\phi \tag{18}$$

2.2.2 Obtaining the relevant linearized kinematic model

The model constitutes the equations of motion of the vehicle, and has two states (y and ϕ) and one input (δ). The equations of the vehicle's motion that are relevant here are

$$\dot{y} = vsin(\phi + \beta) \tag{19}$$
$$\dot{\phi} = \frac{v}{l_r}sin\beta \tag{20}$$

Sampling with a sampling time of T_s gives

$$y_{k+1} = y_k + T_s vsin(\phi_k + \beta_k) \tag{21}$$
$$\phi_{k+1} = \phi_k + T_s\frac{v}{l_r}sin\beta_k \tag{22}$$

where

$$\beta_k = tan^{-1}\left(\frac{l_r}{l_r + l_f}tan\delta_k\right) \tag{23}$$

Forming the Jacobians for matrices A, B and evaluating them at time $t = k$ around $\delta = 0$ (which makes $\beta = 0$) gives

$$A = \begin{bmatrix} 1 & T_s vcos(\phi_k + \beta_k) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & T_s vcos\Big(\phi_k + tan^{-1}(l_q tan\delta_k)\Big) \\ 0 & 1 \end{bmatrix} \tag{24}$$

$$A = \begin{bmatrix} 1 & T_s v cos\phi_k \\ \\ 0 & 1 \end{bmatrix} \tag{25}$$

$$B = \begin{bmatrix} T_s v cos(\phi_k + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \dfrac{T_s v}{l_r} cos(\beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} = \begin{bmatrix} T_s v cos\Big(\phi_k + tan^{-1}(l_q tan\delta_k)\Big) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \dfrac{T_s v}{l_r} cos\left(tan^{-1}\Big(l_q tan\delta_k\Big)\right) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} \tag{26}$$

$$B = \begin{bmatrix} \dfrac{T_s l_r v}{l_r + l_f} cos\phi_k \\ \\ \dfrac{T_s v}{l_r + l_f} \end{bmatrix} \tag{27}$$

where $l_q = \dfrac{l_r}{l_r + l_f}$

Now we can express the linear model as

$$s_{k+1} = As_k + B\delta_k \tag{28}$$

where

$$s = \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} \tag{29}$$

or

$$\begin{bmatrix} y_{k+1} \\ \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s v cos\phi_k \\ \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} \dfrac{T_s l_r v}{l_r + l_f} cos\phi_k \\ \\ \dfrac{T_s v}{l_r + l_f} \end{bmatrix} \delta_k \tag{30}$$

2.2.3 Stating the optimization problem

We can now form the optimization problem as

$$\text{minimize} \sum_{k=0}^{N} y_k^2 q_y + \phi_k^2 q_\phi + \delta_k^2 r \tag{31}$$

$$\text{subject to } s_{k+1} = As_k + B\delta_k, \text{ where } s_k = [y_k, \phi_k]^T \tag{32}$$

$$\delta_{min} \le \delta_k \le \delta_{max} \tag{33}$$

$$y_0 = -\frac{1}{2}(CL - CR)cos\phi_0 \tag{34}$$

$$\phi_0 = tan^{-1}\frac{CL}{CF} \text{ or } \phi_0 = -tan^{-1}\frac{CR}{CF} \tag{35}$$

2.3 Theoretical solution to tracking the circumference of a circle using a MPC controller

In figure 11, the vehicle C, whose velocity is constant and denoted by v, is to track a circle whose center is O' and whose radius is $O'R$. Its orientation relative to the global coordinate system is ψ. R is the point C is to track. The orientation of R relative to the global coordinate system is θ. The vehicle's coordinates are (x_c, y_c). The aim is to find the vehicle's deviation from R in terms of translation along the x and y axes and rotation around the x-axis.

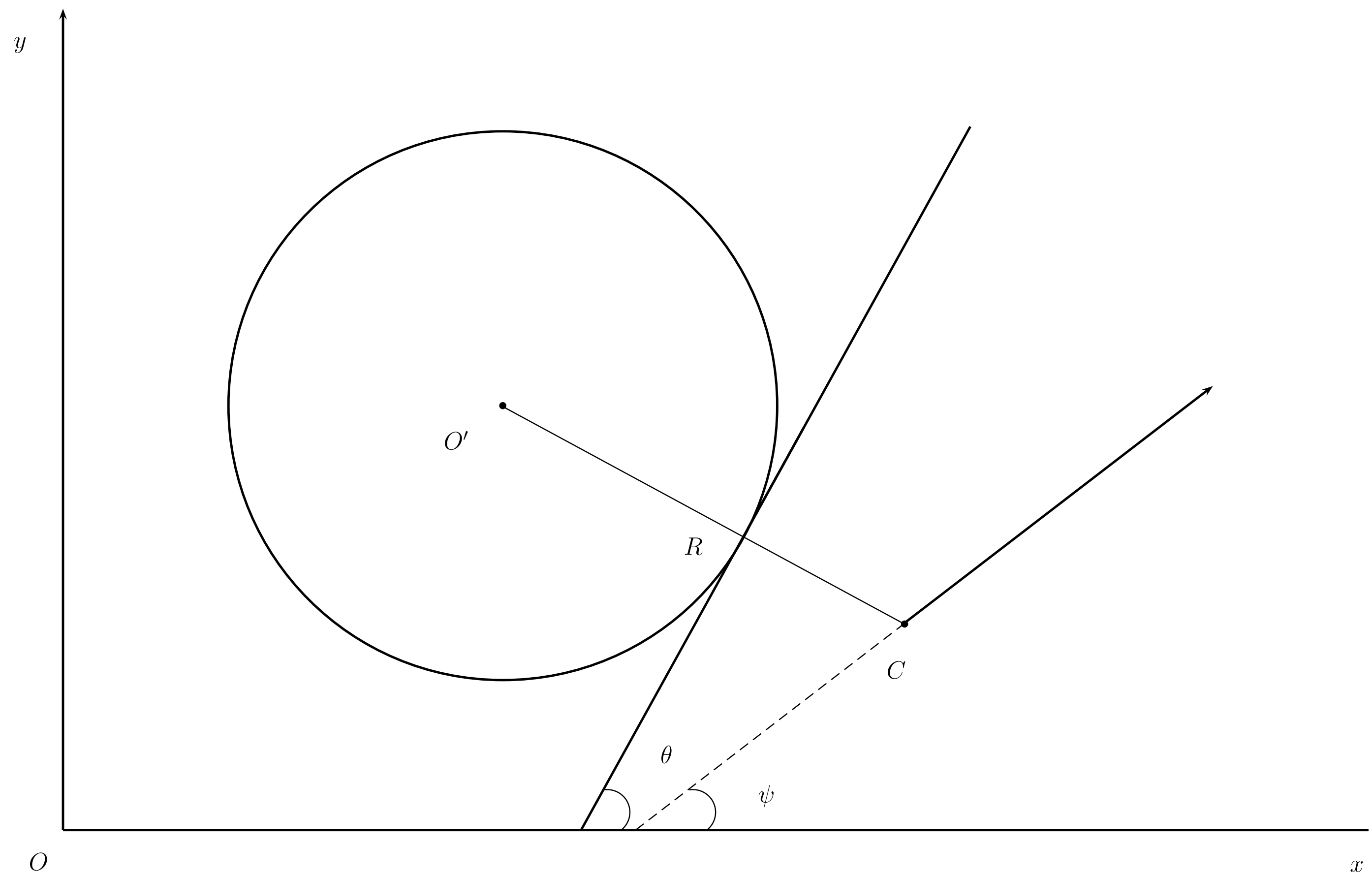

Figure 11

The solution to the problem can be decomposed into two separate components: a translational and a rotational one.

2.3.1 Translational component

Here, we assume that the orientation of the tangent at R is the same of that of the vehicle and equal to ψ. Given the distance RC (the coordinates of both R and C are known) we are interested in finding the displacements RC_x and RC_y. From triangle C_yRC we can immediately deduce that

$$RC_x = RC sin\psi \tag{36}$$
$$RC_y = -RC cos\psi \tag{37}$$
$$\tag{38}$$

where $RC = \sqrt{(x_C - x_R)^2 + (y_C - y_R)^2}$

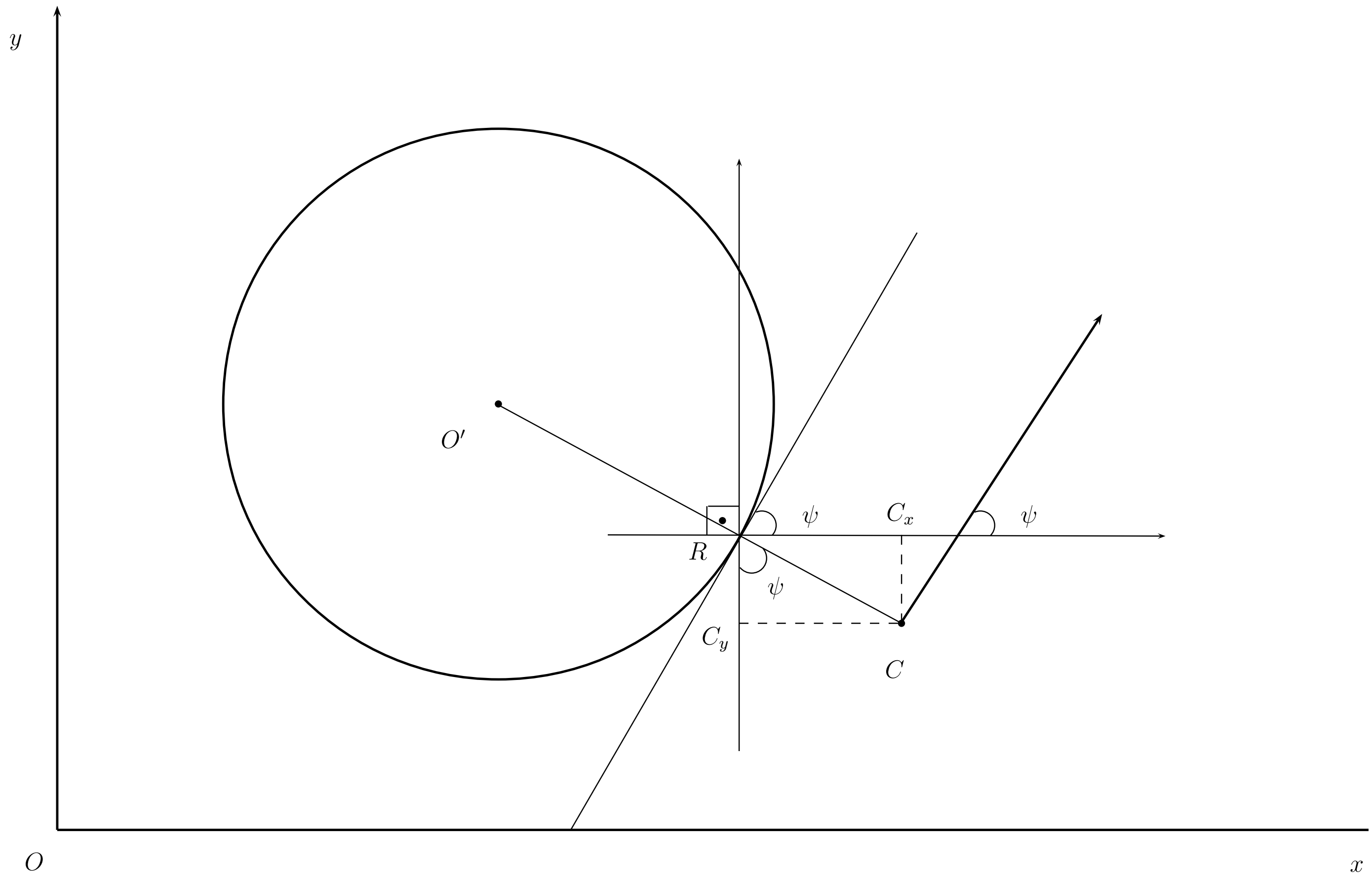

Figure 12

2.3.2 Rotational component

Here, we assume that the only deviation of C from R is only in terms of orientation. From figure 13 we can easily discern that the angular error is $\theta - \psi$.

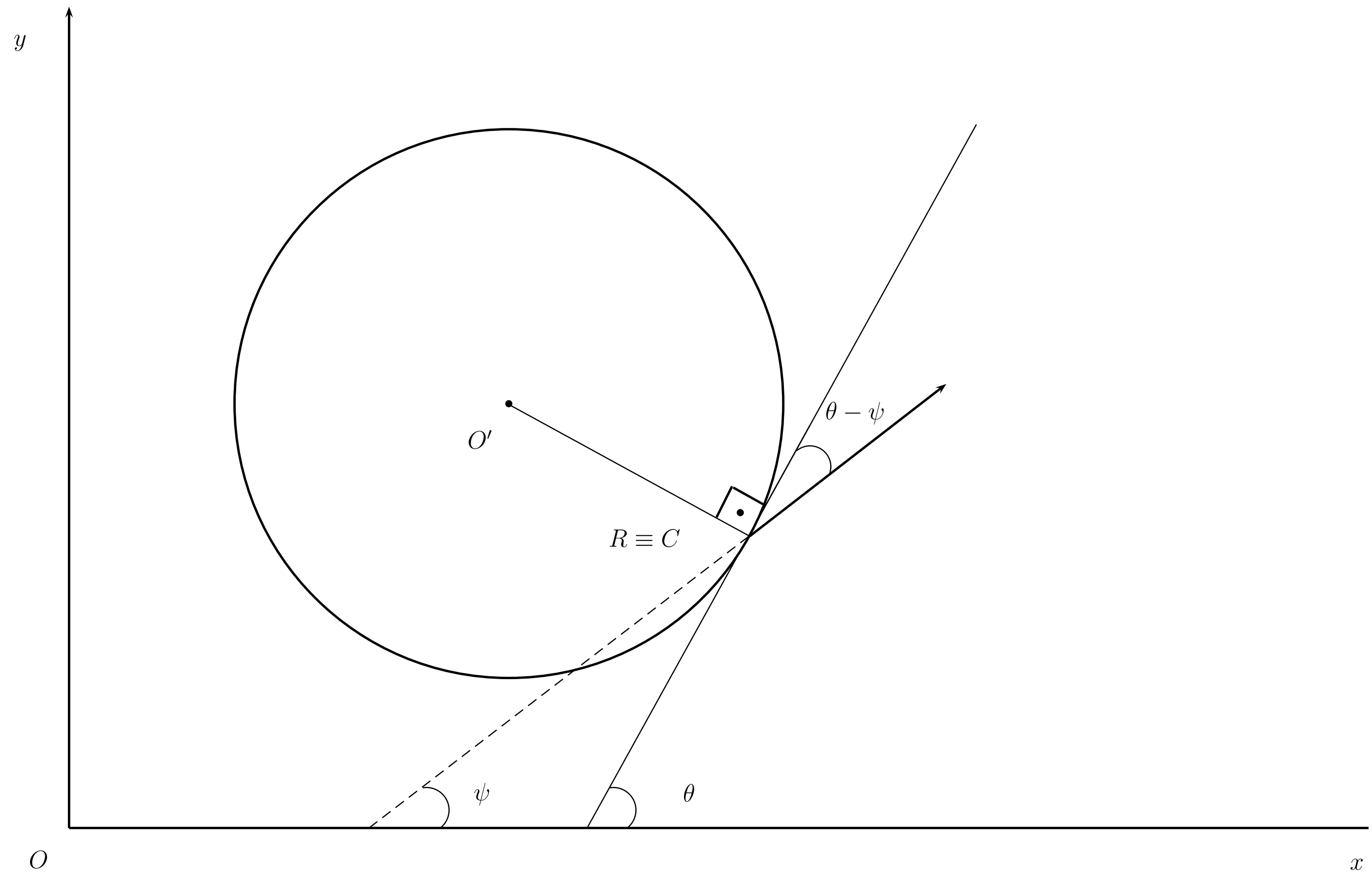

Figure 13

2.3.3 Obtaining the relevant linearized kinematic model

The model constitutes the equations of motion of the vehicle, and has three states (x, y and ψ) and one input (δ). The equations of the vehicle's motion that are relevant here are

$$\dot{x} = vcos(\psi + \beta) \tag{39}$$
$$\dot{y} = vsin(\psi + \beta) \tag{40}$$
$$\dot{\psi} = \frac{v}{l_r} sin\beta \tag{41}$$

Sampling with a sampling time of T_s gives

$$x_{k+1} = x_k + T_s vcos(\psi_k + \beta_k) \tag{42}$$
$$y_{k+1} = y_k + T_s vsin(\psi_k + \beta_k) \tag{43}$$
$$\psi_{k+1} = \psi_k + T_s \frac{v}{l_r} sin\beta_k \tag{44}$$

where

$$\beta_k = tan^{-1}\Big(\frac{l_r}{l_r + l_f} tan\delta_k\Big) \tag{45}$$

Forming the Jacobians for matrices A, B and evaluating them at time $t = k$ around $\delta = 0$ (which makes $\beta = 0$) gives

$$A = \begin{bmatrix} 1 & 0 & -T_s v sin(\psi_k + \beta_k) \\ \\ 0 & 1 & T_s v cos(\psi_k + \beta_k) \\ \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -T_s v sin\Big(\psi_k + tan^{-1}(l_q tan\delta_k)\Big) \\ \\ 0 & 1 & T_s v cos\Big(\psi_k + tan^{-1}(l_q tan\delta_k)\Big) \\ \\ 0 & 0 & 1 \end{bmatrix} \tag{46}$$

$$A = \begin{bmatrix} 1 & 0 & -T_s v sin(\psi_k) \\ \\ 0 & 1 & T_s v cos(\psi_k) \\ \\ 0 & 0 & 1 \end{bmatrix} \tag{47}$$

$$B = \begin{bmatrix} -T_s v sin(\psi_k + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ T_s v cos(\psi_k + \beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \dfrac{T_s v}{l_r} cos(\beta_k) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} = \begin{bmatrix} -T_s v sin\Big(\psi_k + tan^{-1}(l_q tan\delta_k)\Big) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ T_s v cos\Big(\psi_k + tan^{-1}(l_q tan\delta_k)\Big) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \dfrac{T_s v}{l_r} cos\Bigg(tan^{-1}\Big(l_q tan\delta_k\Big)\Bigg) \dfrac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} \tag{48}$$

$$B = \begin{bmatrix} -\dfrac{T_s l_r v}{l_r + l_f} sin\psi_k \\ \\ \dfrac{T_s l_r v}{l_r + l_f} cos\psi_k \\ \\ \dfrac{T_s v}{l_r + l_f} \end{bmatrix} \tag{49}$$

where $l_q = \dfrac{l_r}{l_r + l_f}$
Now we can express the linear model as

$$s_{k+1} = A s_k + B \delta_k \tag{50}$$

where

$$s = \begin{bmatrix} x_k \\ y_k \\ \psi_k \end{bmatrix} \tag{51}$$

or

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \psi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -T_s v sin(\psi_k) \\ 0 & 1 & T_s v cos(\psi_k) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \psi_k \end{bmatrix} + \begin{bmatrix} -\frac{T_s l_r v}{l_r + l_f} sin\psi_k \\ \frac{T_s l_r v}{l_r + l_f} cos\psi_k \\ \frac{T_s v}{l_r + l_f} \end{bmatrix} \delta_k \tag{52}$$

2.3.4 Stating the optimization problem

We can now form the optimization problem as

$$\text{minimize } \sum_{k=0}^{N} x_k^2 q_x + y_k^2 q_y + \phi_k^2 q_\phi + \delta_k^2 r \tag{53}$$

$$\text{subject to } s_{k+1} = A s_k + B \delta_k, \text{ where } s_k = [x_k, y_k, \phi_k]^T \tag{54}$$

$$\delta_{min} \leq \delta_k \leq \delta_{max} \tag{55}$$

$$x_0 = RC sin\phi_0 \tag{56}$$

$$y_0 = -RC cos\phi_0 \tag{57}$$

$$\phi_0 = \theta_R - \psi \tag{58}$$

3 Issues

4 To do

5 Misc.

The progress of the project can be observed in `trello` and `github`:

- `https://trello.com/b/uEP0jl0B/slip-control`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425_resources`