EL2425 - Slip Control
Meeting agenda 2016-11-16

November 17, 2016

1 Done

2 Ongoing

2.1 Theoretical solution involving MPC and tracking the centerline of a lane

In figure 1, the x axis is fixed on the lane's centerline, while axis y is perpendicular to it. The origin is at point O. The vehicle is represented by point C. The orientation of the vehicle with respect to the lane (the x-axis) is ϕ. Given this configuration and three scans, at $-90°$, $0°$ and $90°$ with respect to the longitudinal axis of the vehicle, denoted by CR, CF and CL respectively, the objective is to find the distance OC and the angle ϕ so that a MPC optimization problem can be solved with OC and ϕ acting as initial conditions. The velocity of the vehicle, which is constant, and its displacement along the x-axis are at this point irrelevant to the optimization problem given this configuration. The only source of information is the lidar itself and nothing else.

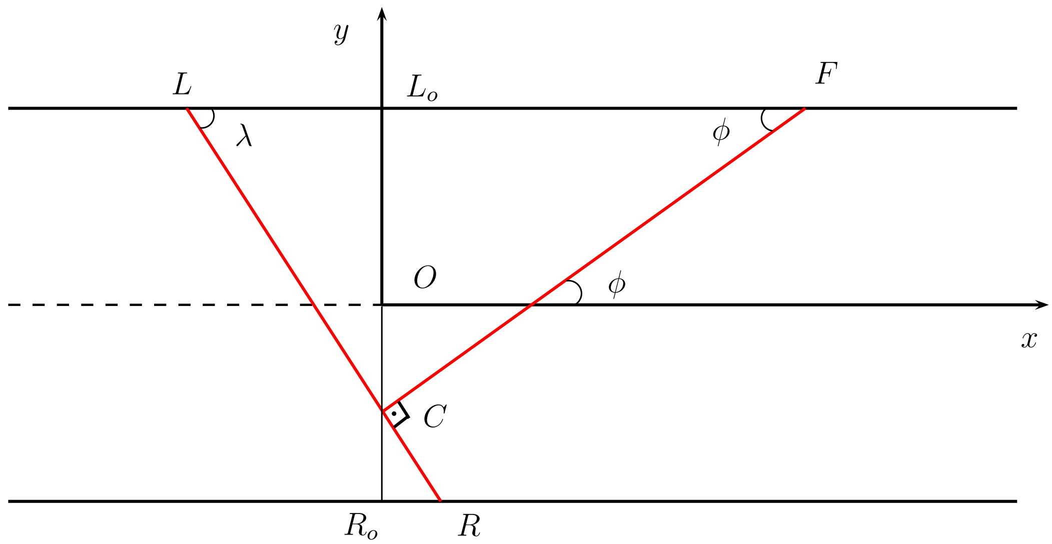

Figure 1

2.1.1 Initial conditions

Finding ϕ

We turn our attention to triangle LCF, where

$$tan\phi = \frac{CL}{CF} \Leftrightarrow \phi = tan^{-1}\frac{CL}{CF} \tag{1}$$

$$\tag{2}$$

Finding OC

First we note that

$$L_oO + OC + CR_o = W = CR_o + CL_o \tag{3}$$

where W is the width of the lane. But $L_oO = \frac{W}{2}$ hence

$$OC + CR_o = \frac{1}{2}(CR_o + CL_o) \Leftrightarrow \tag{4}$$

$$OC = \frac{1}{2}(CL_o - CR_o) \tag{5}$$

But $CL_o = CLsin\lambda$ and $CR_o = CRcos\phi$, hence

$$OC = \frac{1}{2}(CLsin\lambda - CRcos\phi) \tag{6}$$

From triangle LCF we note that $\phi + \lambda + \frac{\pi}{2} = \pi$, hence $\lambda = \frac{\pi}{2} - \phi$. Then, we conclude that

$$OC = \frac{1}{2}(CL - CR)cos\phi \tag{7}$$

2.1.2 Obtaining the relevant linearized kinematic model

The model constitutes the equations of motion of the vehicle, and has two states (y and ϕ) and one input (δ). The equations of the vehicle's motion that are relevant here are

$$\dot{y} = vsin(\phi + \beta) \tag{8}$$

$$\dot{\phi} = \frac{v}{l_r}sin\beta \tag{9}$$

Sampling with a sampling time of T_s gives

$$y_{k+1} = y_k + T_s vsin(\phi_k + \beta_k) \tag{10}$$

$$\phi_{k+1} = \phi_k + T_s\frac{v}{l_r}sin\beta_k \tag{11}$$

where

$$\beta_k = tan^{-1}\left(\frac{l_r}{l_r + l_f}tan\delta_k\right) \tag{12}$$

Forming the Jacobians for matrices A, B and evaluating them at time $t = k$ around $\delta = 0$ (which makes $\beta = 0$) gives

$$A = \begin{bmatrix} 1 & T_s vcos(\phi + \beta) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & T_s vcos\left(tan^{-1}\frac{CL_k}{CF_k} + tan^{-1}(l_q tan\delta_k)\right) \\ 0 & 1 \end{bmatrix} \tag{13}$$

$$A = \begin{bmatrix} 1 & T_s vcos\left(tan^{-1}\frac{CL_k}{CF_k}\right) \\ 0 & 1 \end{bmatrix} \tag{14}$$

$$B = \begin{bmatrix} T_s vcos(\phi + \beta)\frac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \frac{T_s v}{l_r}cos(\beta)\frac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} = \begin{bmatrix} T_s vcos\left(tan^{-1}\frac{CL_k}{CF_k} + tan^{-1}(l_q tan\delta_k)\right)\frac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \\ \frac{T_s v}{l_r}cos\left(tan^{-1}\left(l_q tan\delta_k\right)\right)\frac{l_q}{l_q^2 sin^2\delta_k + cos^2\delta_k} \end{bmatrix} \tag{15}$$

$$B = \begin{bmatrix} \frac{T_s l_r v}{l_r + l_f}cos\left(tan^{-1}\frac{CL_k}{CF_k}\right) \\ \frac{T_s v}{l_r + l_f} \end{bmatrix} \tag{16}$$

where $l_q = \dfrac{l_r}{l_r + l_f}$

Now we can express the linear model as

$$s_{k+1} = As_k + B\delta_k \tag{17}$$

where

$$s = \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} \tag{18}$$

or

$$\begin{bmatrix} y_{k+1} \\ \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s v cos\Big(tan^{-1}\dfrac{CL_k}{CF_k}\Big) \\ \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} \dfrac{T_s l_r v}{l_r + l_f} cos\Big(tan^{-1}\dfrac{CL_k}{CF_k}\Big) \\ \\ \dfrac{T_s v}{l_r + l_f} \end{bmatrix} \delta_k \tag{19}$$

2.1.3 Stating the optimization problem

We can now form the optimization problem as

$$\text{minimize} \sum_{k=0}^{N} y_k^2 q_y + \phi_k^2 q_\phi + \delta_k^2 r \tag{20}$$

$$\text{subject to } s_{k+1} = As_k + B\delta_k, \text{ where } s_k = [y_k, \phi_k]^T \tag{21}$$

$$\delta_{min} \leq \delta_k \leq \delta_{max} \tag{22}$$

$$y_0 = -\frac{1}{2}(CL - CR)cos\phi_0 \tag{23}$$

$$\phi_0 = tan^{-1}\frac{CL}{CF} \tag{24}$$

3 Issues

4 To do

5 Misc.

The progress of the project can be observed in `trello` and `github`:

- `https://trello.com/b/uEP0jl0B/slip-control`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425`
- `https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425_resources`