

# EL2425 - Slip Control

## Meeting agenda 2016-11-16

November 16, 2016

### 1 Done

### 2 Ongoing

#### 2.1 Theoretical solution involving MPC and tracking the center-line of a lane

In figure 1, the  $x$  axis is fixed on the lane's centerline, while axis  $y$  is perpendicular to it. The origin is at point  $O$ . The vehicle is represented by point  $C$ . The orientation of the vehicle with respect to the lane (the  $x$ -axis) is  $\phi$ . Given this configuration and three scans, at  $-90^\circ$ ,  $0^\circ$  and  $90^\circ$  with respect to the longitudinal axis of the vehicle, denoted by  $CR$ ,  $CF$  and  $CL$  respectively, the objective is to find the distance  $OC$  and the angle  $\phi$  so that a MPC optimization problem can be solved with  $OC$  and  $\phi$  acting as initial conditions. The velocity of the vehicle, which is constant, and its displacement along the  $x$ -axis are at this point irrelevant to the optimization problem given this configuration. The only source of information is the lidar itself and nothing else.

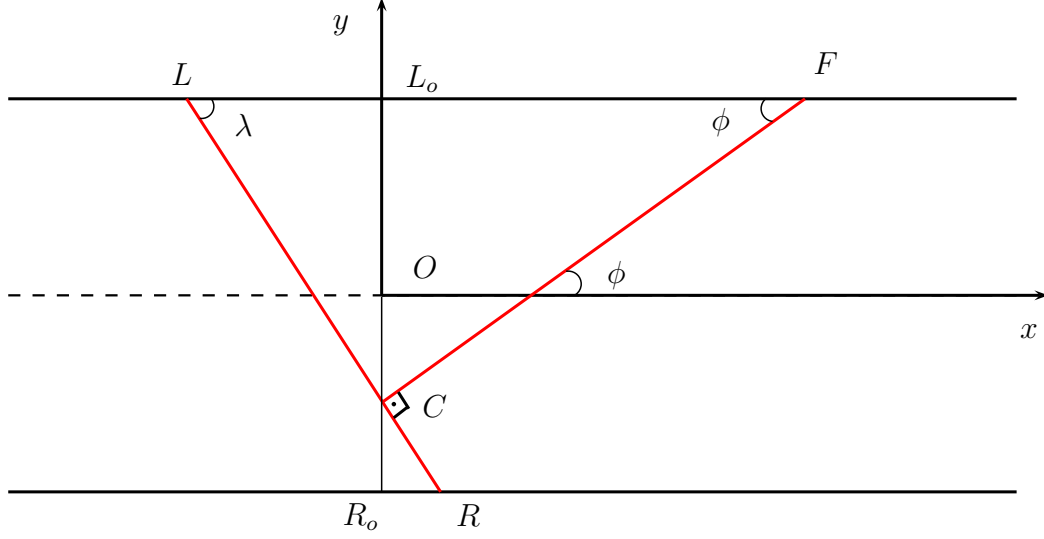


Figure 1

### 2.1.1 Initial conditions

#### Finding $\phi$

We turn our attention to triangle LCF, where

$$\tan\phi = \frac{CL}{CF} \Leftrightarrow \phi = \tan^{-1} \frac{CL}{CF} \quad (1)$$

(2)

#### Finding $OC$

First we note that

$$L_oO + OC + CR_o = W = CR_o + CL_o \quad (3)$$

where  $W$  is the width of the lane. But  $L_oO = \frac{W}{2}$  hence

$$OC + CR_o = \frac{1}{2}(CR_o + CL_o) \Leftrightarrow \quad (4)$$

$$OC = \frac{1}{2}(CL_o - CR_o) \quad (5)$$

But  $CL_o = CL\sin\lambda$  and  $CR_o = CR\cos\phi$ , hence

$$OC = \frac{1}{2}(CL\sin\lambda - CR\cos\phi) \quad (6)$$

From triangle LCF we note that  $\phi + \lambda + \frac{\pi}{2} = \pi$ , hence  $\lambda = \frac{\pi}{2} - \phi$ . Then, we conclude that

$$OC = \frac{1}{2}(CL - CR)\cos\phi \quad (7)$$

### 2.1.2 Obtaining the relevant linearized kinematic model

The equations of the vehicle's motion that are relevant are

$$\dot{y} = v\sin(\phi + \beta) \quad (8)$$

$$\dot{\phi} = \frac{v}{l_r}\sin\beta \quad (9)$$

Sampling with a sampling time of  $T_s$  gives

$$y[k+1] = y[k] + T_s v \sin(\phi + \beta) \quad (10)$$

$$\phi[k+1] = \phi[k] + T_s \frac{v}{l_r} \sin\beta \quad (11)$$

Forming the Jacobians for matrices  $A$ ,  $B$  and evaluating them at time  $t = k$  around  $\delta = 0$  (which makes  $\beta = 0$ ) gives

$$A = \begin{bmatrix} 1 & T_s v \cos(\phi + \beta) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & T_s v \cos\left(\tan^{-1}\frac{CL_k}{CF_k} + \tan^{-1}(l_q \tan\delta_k)\right) \\ 0 & 1 \end{bmatrix} \quad (12)$$

$$A = \begin{bmatrix} 1 & T_s v \cos\left(\tan^{-1}\frac{CL_k}{CF_k}\right) \\ 0 & 1 \end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix} T_s v \cos(\phi + \beta) \frac{l_q}{l_q^2 \sin^2 \delta_k + \cos^2 \delta_k} \\ \frac{T_s v}{l_r} \cos(\beta) \frac{l_q}{l_q^2 \sin^2 \delta_k + \cos^2 \delta_k} \end{bmatrix} = \begin{bmatrix} T_s v \cos\left(\tan^{-1}\frac{CL_k}{CF_k} + \tan^{-1}(l_q \tan\delta_k)\right) \frac{l_q}{l_q^2 \sin^2 \delta_k + \cos^2 \delta_k} \\ \frac{T_s v}{l_r} \cos\left(\tan^{-1}(l_q \tan\delta_k)\right) \frac{l_q}{l_q^2 \sin^2 \delta_k + \cos^2 \delta_k} \end{bmatrix} \quad (14)$$

$$B = \begin{bmatrix} \frac{T_s l_r v}{l_r + l_f} \cos\left(\tan^{-1}\frac{CL_k}{CF_k}\right) \\ \frac{T_s v}{l_r + l_f} \end{bmatrix} \quad (15)$$

where  $l_q = \frac{l_r}{l_r + l_f}$

Now we can express the linear model as

$$\begin{bmatrix} y_{k+1} \\ \phi_{k+1} \end{bmatrix} = A \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} + B\delta_k, \text{ or} \quad (16)$$

$$\begin{bmatrix} y_{k+1} \\ \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s v \cos\left(\tan^{-1} \frac{CL_k}{CF_k}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} \frac{T_s l_r v}{l_r + l_f} \cos\left(\tan^{-1} \frac{CL_k}{CF_k}\right) \\ \frac{T_s v}{l_r + l_f} \end{bmatrix} \delta_k \quad (17)$$

### 3 Issues

### 4 To do

### 5 Misc.

The progress of the project can be observed in `trello` and `github`:

- <https://trello.com/b/uEP0jl0B/slip-control>
- [https://gits-15.sys.kth.se/alefil/HT16\\_P2\\_EL2425](https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425)
- [https://gits-15.sys.kth.se/alefil/HT16\\_P2\\_EL2425\\_resources](https://gits-15.sys.kth.se/alefil/HT16_P2_EL2425_resources)